ORTA DOĞU TEKNİK ÜNİVERSİTESİ
MIDDLE EAST TECHNICAL UNIVERSITY

Aerospace Engineering Department

AEE486: Spacecraft Dynamics

Homework 3

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1 Question 1

In this homework, we will do satellite kinematics and dynamics together. Firstly, we have given an initial attitude of s/c. We know the initial z axis of the s/c looking through the sun in ECI frame. We have to get this attitude matrix. So, I used Euler angles with 3-1-3 rotational sequence.

$$A_{bi} = R_3(\phi)R_1(\theta)R_3(\psi) \tag{1.1}$$

Third angle will be zero. Also third column of the transpose of this attitude matrix with other two angles have to give us initial sun vector. So, other two angles are,

$$\theta = -1.55069rad \tag{1.2a}$$

$$\psi = 1.52437rad \tag{1.2b}$$

Since we know the angles, the attitude matrix is,

$$A_{bi} = \begin{bmatrix} 0.0464 & 0.999 & 0 \\ -0.020 & 0.001 & -1 \\ -0.999 & -0.046 & 0.020 \end{bmatrix} \tag{1.3}$$

Then, I converted attitude matrix to quaternion by dcm2quat function on MATLAB and corrected the quaternion writing with ours.

Now, I have initial quaternion values. Now I can propogate the dynamics and kinematics equation together to get further values. The Euler's rotational equation we will use for the dynamic part,

$$\frac{d\boldsymbol{\omega}_{\boldsymbol{bi}}^{\boldsymbol{b}}}{dt} = J^{-1}\left[N - \boldsymbol{\omega}_{\boldsymbol{bi}}^{\boldsymbol{b}} \times (J\boldsymbol{\omega}_{\boldsymbol{bi}}^{\boldsymbol{b}})\right] \tag{1.4}$$

But, in this question, there is no external torque effected the s/c. So torque will be zero vector.

The quaternion kinematics equation we will use for the kinematics part,

$$\frac{dq}{dt} = \frac{1}{2}\Omega(\boldsymbol{\omega}\times)q \tag{1.5}$$

Also we have given MOI tensor matrix for our nanosatellite.

$$J = \begin{bmatrix} 0.018 & 0 & 0 \\ 0 & 0.018 & 0 \\ 0 & 0 & 0.065 \end{bmatrix} \tag{1.6}$$

And we know that, initially s/c has an angular velocity of 60rpm in Z axis.

$$\boldsymbol{\omega_{bi}^{b}} = \begin{bmatrix} 0 \\ 0 \\ 60rpm \end{bmatrix} \tag{1.7}$$

Before I start, I converted the rpm to rad/s. Then I started the propogate angular velocity and quaternions together by using RK4 method with time step of 0.001 and I normalized quaternions at the end of the loops.

After obtaining all the values, I converted the angular velocity vectors from body frame to inertial frame. Then I normalized and plotted them. Because we are asked to plot the elements of the axis of rotation.

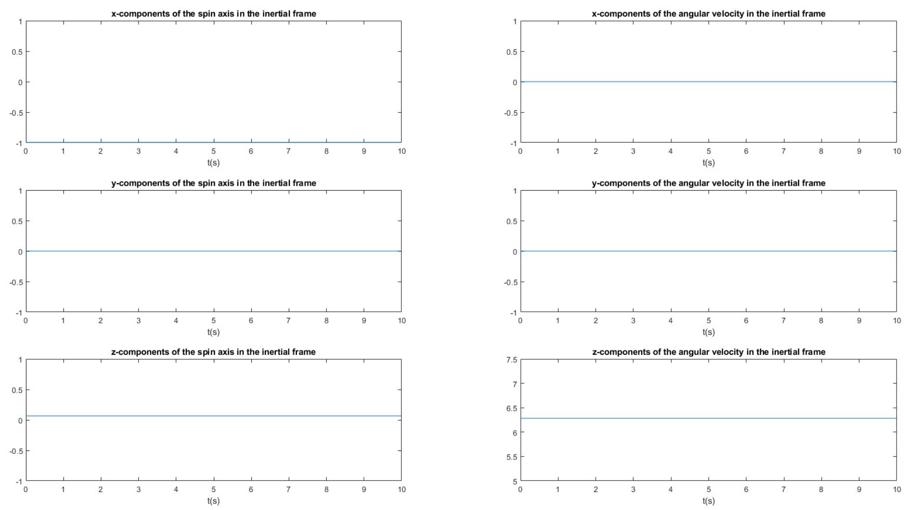

Figure 1.1: Components of the spin axis and angular velocity in the inertial frame

As we can see in Figure 1.1, all three components of spin axis do not changing in time. Because there is no external torques effecting on s/c. Also, angular velocity of the s/c remains same in time.

2 Question 2

This time we have magnetic disturbance and gravity gradient torques effected to our s/c. So, magnetic disturbance and gravity gradient torques are,

$$\mathbf{N_{md}} = \mathbf{M} \times \mathbf{B_b} \tag{2.1}$$

$$\mathbf{N_{gg}} = \frac{3\mu}{|\mathbf{r}|^3}\mathbf{n} \times (J\mathbf{n}) \tag{2.2}$$

where,

- $\mathbf{M} = \begin{bmatrix}-0.09 & 0.001 & 0.11\end{bmatrix}^T$
- $\mu = 3.986004418 \times 10^{14} m^3/s^2$

We still don't have the magnetic field vector in the body frame. We have that vector in ECI frame. I converted that in every loop by inertia-to-body attitude matrix which I get from quaternions.

Also I converted position vector from inertia to body frame similarly and normalized to get nadir vector.

To calculate distance, I calculated the magnitude of position vector by using norm() function on MATLAB.

After these calculations, We have now magnetic disturbance and gravity gradient torques.

So the total disturbance torque is,

$$\mathbf{N_t} = \mathbf{N_{md}} + \mathbf{N_{gg}} \tag{2.3}$$

Then, again using Euler's Rotational Equation with total torque, I propogated the angular velocity and using quaternion kinematics equation, I propogated the quaternions from these angular velocity values. After obtaining all the values, I converted the angular velocity vectors from body frame to inertial frame.

Then I normalized and plotted them like we did on first question.

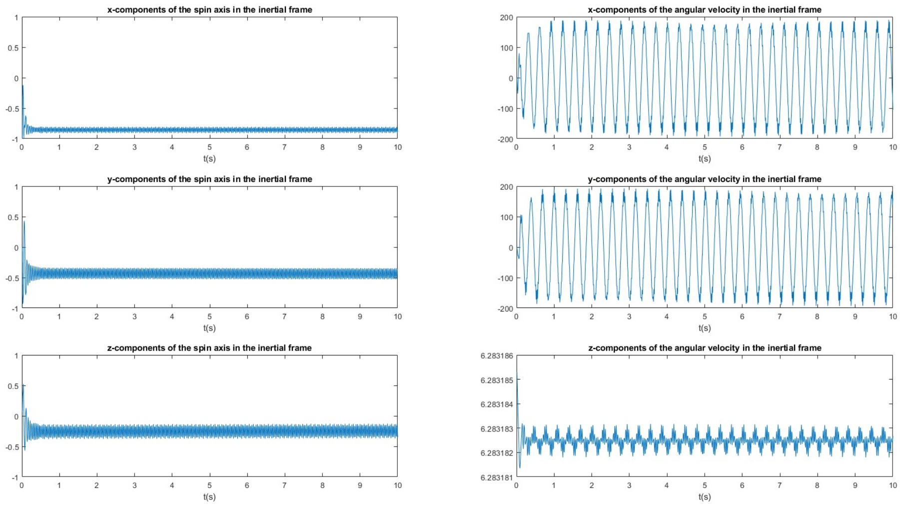

Figure 2.1: Components of the spin axis and angular velocity in the inertial frame

As seen in Figure 2.1, all three components of spin axis are starting with some values and settles down, then oscillating regularly. Also angular velocity components are oscillating with some value regularly.

3 Question 3

This time everything is the same but we have different MOI tensor value for bigger s/c.

$$J = \begin{bmatrix} 6.9 & 0 & 0 \\ 0 & 7.5 & 0 \\ 0 & 0 & 8.4 \end{bmatrix} \tag{3.1}$$

I used the same algorithm with that given MOI tensor matrix.

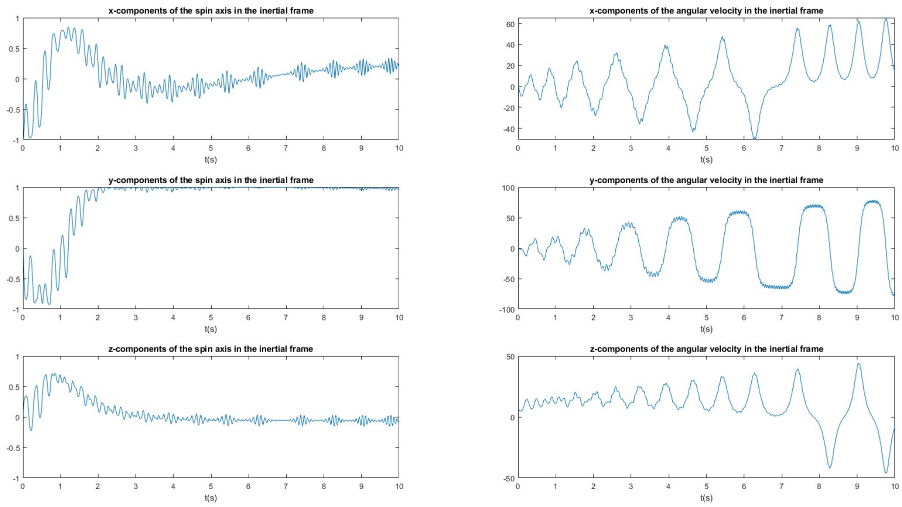

Figure 3.1: Components of the spin axis and angular velocity in the inertial frame

This time, we have bigger inertia values and due to this, we have bigger gravity gradient velues. So, as seen in Figure 3.1, we have bigger changes in spin axis initially. But, again they settle down and remains constant with some oscillation. Moreover, we have oscillating angular velocity values but this time they diverges in time due to the higher gravity gradient effects.

4 Bonus Question 1

This time, again everything is the same but we changed the MOI tensor values for Z and Y axis.

$$J = \begin{bmatrix} 6.9 & 0 & 0 \\ 0 & 8.4 & 0 \\ 0 & 0 & 7.5 \end{bmatrix} \tag{4.1}$$

I used the same algorithm with the new MOI tensor matrix.

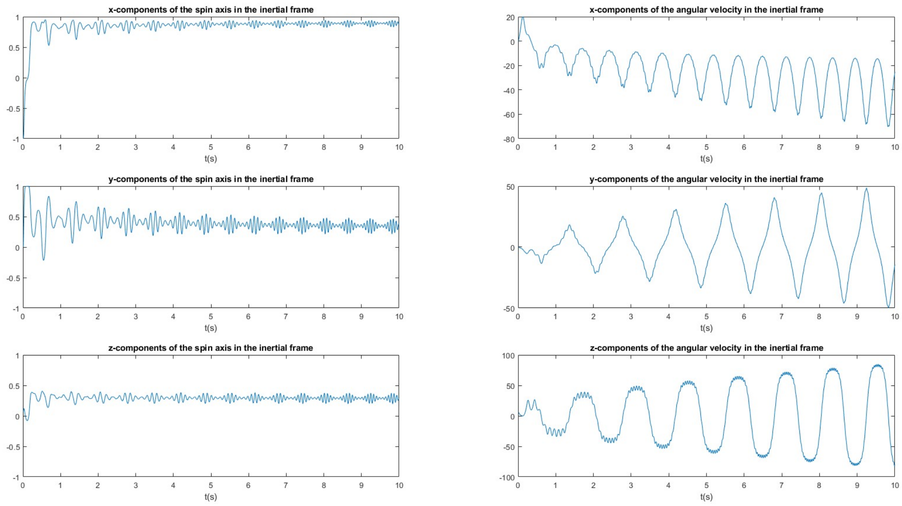

Figure 4.1: Components of the spin axis and angular velocity in the inertial frame

This time, there is less variation in the components of spin axis at the beginning and afterwards. But the oscillations getting smaller and settles with some magnitudes. Again, angular velocity of the s/c oscillating divergently. I think, this is due to the higher gravity gradient effects again.

5 Bonus Question 2

This time, again everything is the same but we have small non-diagonal terms on the MOI tensor. We have,

$$J = \begin{bmatrix} 6.9 & 0.05 & 0.1 \\ 0.05 & 8.4 & 0.15 \\ 0.15 & 0.1 & 7.5 \end{bmatrix} \tag{5.1}$$

I used the same algorithm with the new MOI tensor matrix.

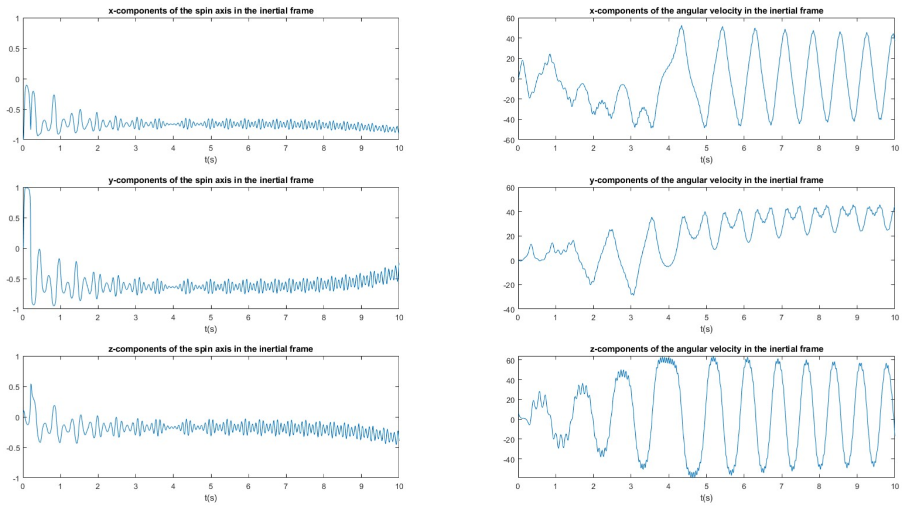

Figure 5.1: Components of the spin axis and angular velocity in the inertial frame

As seen in Figure 5.1, we have very different plots than the previous one and just small inertia terms cause that. We can say that the components of the spin axis are more irregular initially and they shift towards a direction as time progress. Also, angular velocity components of the s/c do not diverge this time. First two axis start with some value and settle in time with another value and oscillate about that value. As a result, angular velocity does not increase forever.