ORTA DOĞU TEKNİK ÜNİVERSİTESİ
MIDDLE EAST TECHNICAL UNIVERSITY

Aerospace Engineering Department

AEE486: Spacecraft Dynamics

Homework 4

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Contents

1 Question 1

We will use Magnetic B-dot control law to de-tumble the s/c given to us in this question. We have the given MOI tensor, the initial quaternion value, and the initial angular velocity value.

- $J = \begin{bmatrix} 6.9 & 0 & 0 \\ 0 & 7.5 & 0 \\ 0 & 0 & 8.4 \end{bmatrix}$
- $\mathbf{q_0} = \frac{\sqrt{2}}{2} \begin{bmatrix} 1 & 0 & 0 & 1 \end{bmatrix}^T$
- $\boldsymbol{\omega_{bi}^b} = \begin{bmatrix} 0.1 & 0.1 & 0.1 \end{bmatrix}^T$

Firstly, we have magnetic dipole moment formula,

$$\mathbf{M_b} = \frac{k}{||\mathbf{B_b}||}[\boldsymbol{\omega}\times]\mathbf{b} \tag{1.1}$$

where k is the control constant,

$$k = \frac{4\pi}{T_{orb}}(1 + sin\xi_m)J_{min} \tag{1.2}$$

where period and inclination is also given,

- $T = 5800sec$
- $\xi_m = 98deg$

We need magnetic field vector in body frame. We have given magnetic field in ECI frame. I converted this to in body frame by using attitude matrix. I calculated the attitude matrix from quaternions in every step by using quat2dcm function on MATLAB.

After, calculating magnetic moment, we will use it in magnetic torque equations. We used the same method previous homework.

$$\mathbf{N_{md}} = \mathbf{M_b} \times \mathbf{B_b} \tag{1.3}$$

After finding magnetic torque, we will do dynamics and kinematics together in every step to find quaternions and angular rates again same as previous homework. The Euler's rotational equation we will use for the dynamic part,

$$\frac{d\boldsymbol{\omega_{bi}^b}}{dt} = J^{-1}\left[\mathbf{N} - \boldsymbol{\omega_{bi}^b} \times (J\boldsymbol{\omega_{bi}^b})\right] \tag{1.4}$$

The quaternion kinematics equation we will use for the kinematics part,

$$\frac{dq}{dt} = \frac{1}{2}\Omega(\boldsymbol{\omega}\times)q \tag{1.5}$$

I used RK4 method to propogate the variables. Then I plotted the components of angular rates and dipole moments vs time graphs.

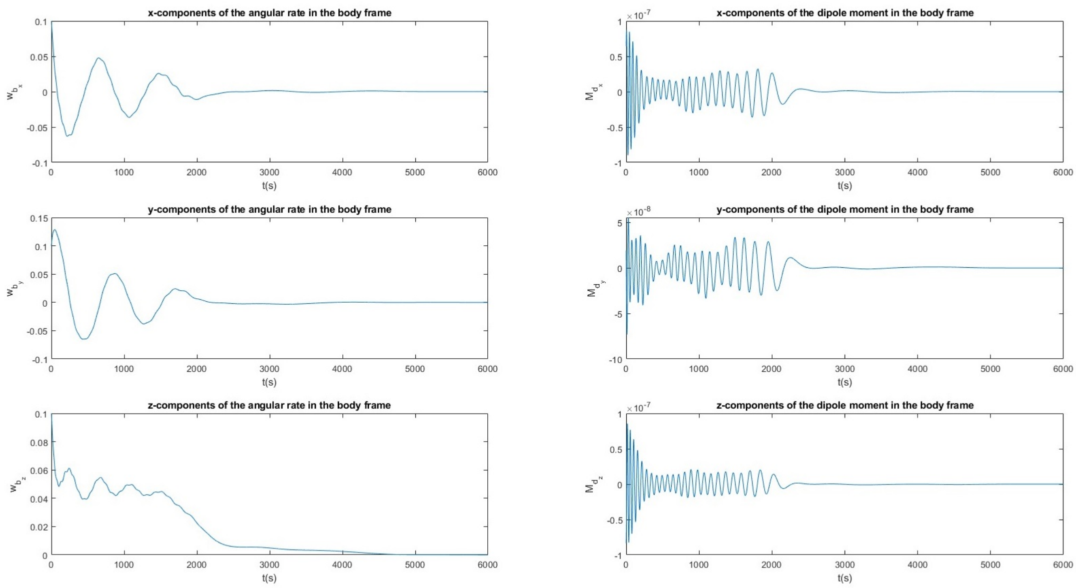

Figure 1.1: Components of the angular rate and dipole moment in body frame

As seen in Figure 1.1, components of angular rates shrink and remain constant in time due to effect of dipole moment. After about 5000 seconds they were fully controlled. Also magnitude of dipole moment component can be seen as decreasing with decreasing angular rates.

Now, we are asked to limit magnetic dipole moment with boundry of [-3 3]Am^2. I limited three components of the magnetic dipole moments separately in every step and plotted the results.

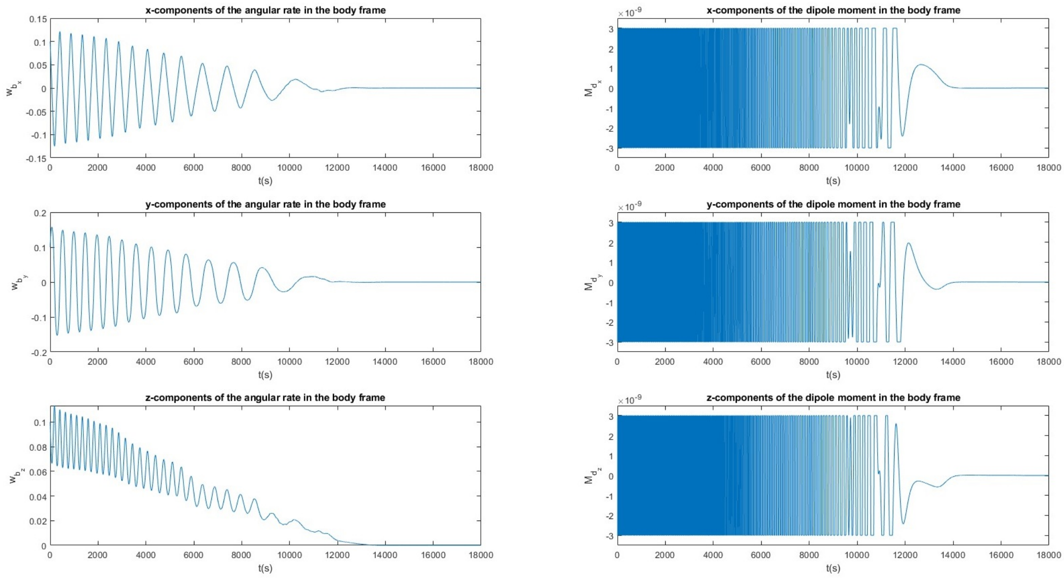

Figure 1.2: Components of the angular rate and dipole moment in body frame

In this figure, total time is 18000 second and angular rates complete their shrinking in about 15000 second. We can see that the shrinking happens much more slowly comparison to previous results due to limits in magnetic dipole moment.

2 Question 2

Now we are going to do attitude control with reaction wheels for the same satellite. We have given initial quaternions and initial angular rates again.

- $\mathbf{q_0} = \frac{\sqrt{2}}{2}\begin{bmatrix}0.6853 & 0.6953 & 0.1531 & 0.1531\end{bmatrix}^T$
- $\boldsymbol{\omega^b_{bi}} = \begin{bmatrix}0.53 & 0.53 & 0.053\end{bmatrix}^T$

Also, initial wheel angular momentum is set as $\mathbf{h}(0) = 0$ in all three axis. We will use control formula to find wheel torque for attitude control, we have two very similar formula,

$$\mathbf{N} = -k_p\delta_{\mathbf{q_{1:3}}} - k_d\boldsymbol{\omega} \tag{2.1}$$

$$\mathbf{N} = -k_p sign(\delta_{\mathbf{q_4}})\delta_{\mathbf{q_{1:3}}} - k_d\boldsymbol{\omega} \tag{2.2}$$

These are Eq.7.7 and Eq.7.12 from Fundamentals of S/C Attitude Determination and Control by Markley and Crassidis. Second formula is obtained by modification of first one, in order to solve the issue when the fourth component of the initial error quaternion is negative. I used both and there were no difference between the results for our case. Because our fourth component of the initial error quaternion has a positive value.

For $k_p and k_d$, I made a guess based on the example in the book. I took,

- $k_p = \frac{50}{1000} = 0.05$
- $k_d = \frac{500}{1000} = 0.5$

So, to calculate wheel torque we have to calculate error quaternion first,

$$\delta q = q \otimes q_c^{-1} \tag{2.3}$$

In every loop, I calculated error quaternion and wheel torque respectively. Then, I propogated the quaternions and angular rates from wheel torque by using kinematics and dynamics equation which I used previously. Finally I plotted the components of error quaternions and wheel torques graphs.

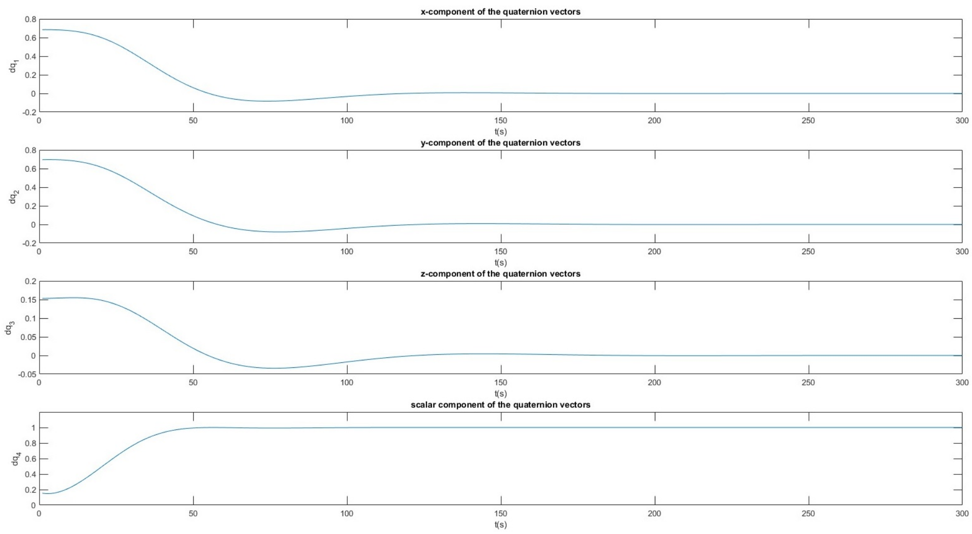

Figure 2.1: Components of the error quaternions

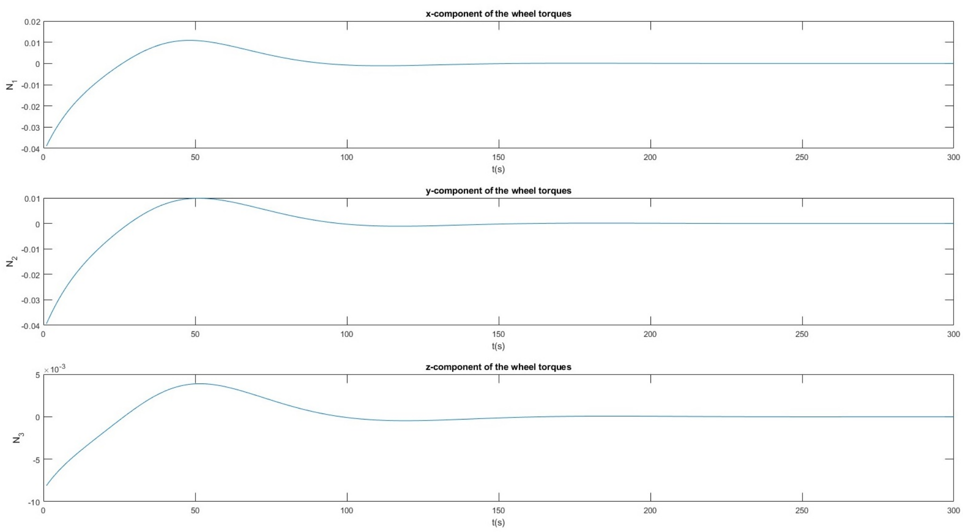

Figure 2.2: Components of the wheel torque

We can see from figures, quaternions approach the desired value over time and eventually reach there. In order to achieve this, the wheel torques are first loaded some, then they are loaded in the opposite direction a little, and become zero when the desired value is reached.

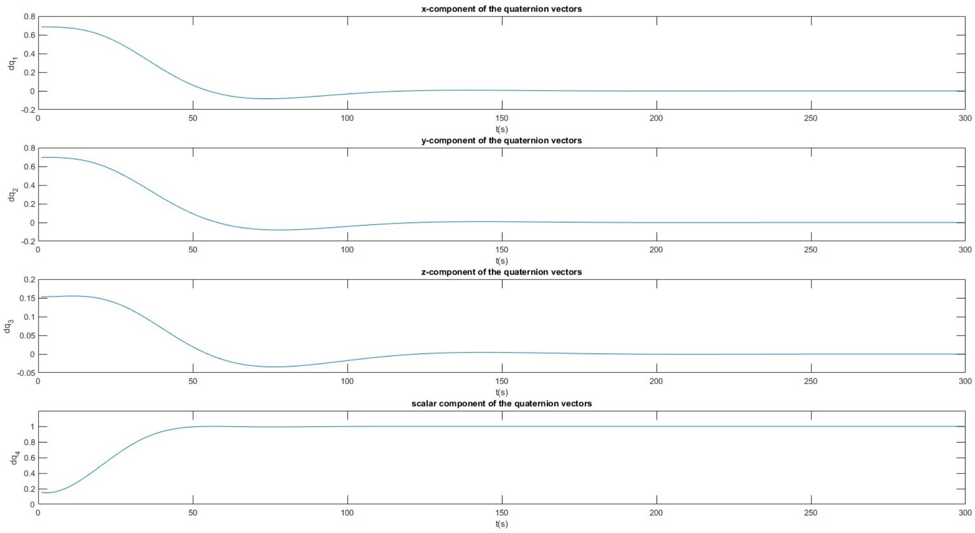

Figure 2.3: Components of the error quaternions

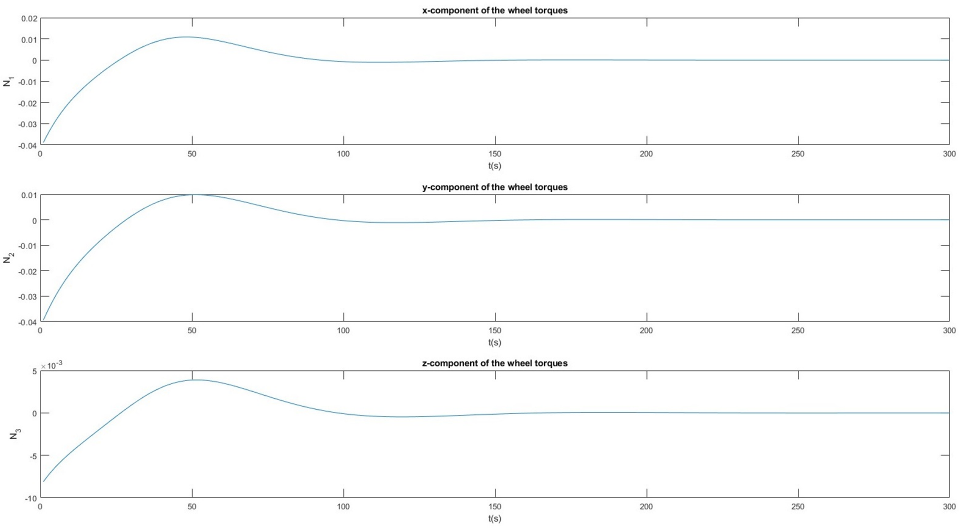

Figure 2.4: Components of the wheel torque

Results on Figure 2.3 and Figure 2.4 are calculated from Equation 2.2. As seen, there is no difference in the results.

Finally, we are asked to find angular momentums of four wheels. We have an equation for angular momentum rate,

$$\dot{h} = [\boldsymbol{\omega}\times]h - L \tag{2.4}$$

I used this formula in every loop and propagated the angular momentums with Euler method.

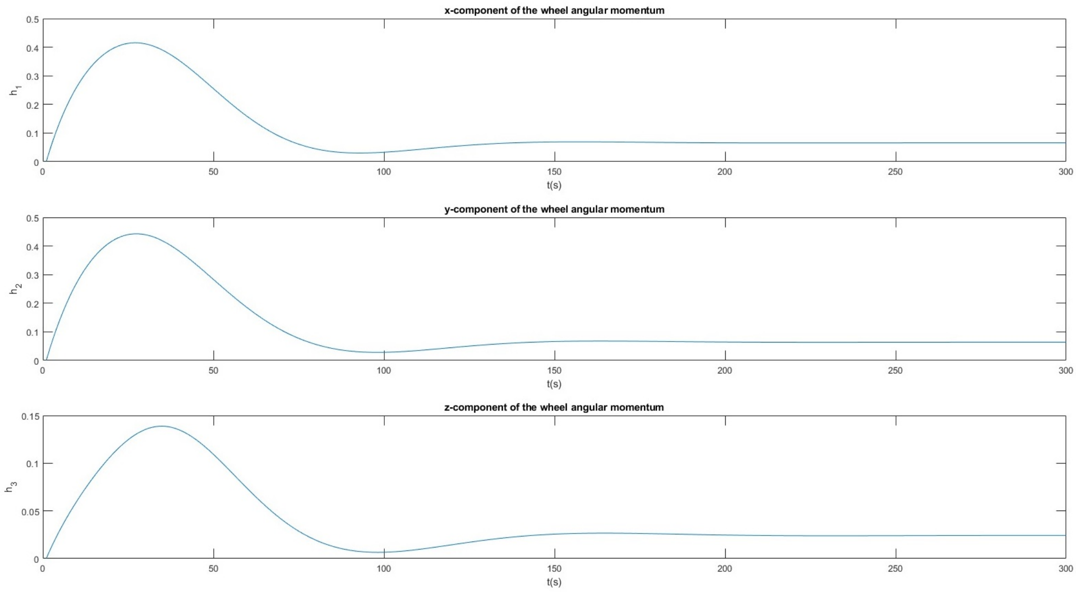

Figure 2.5: Components of the wheel angular momentum

However, we are asked to find angular momentum of four wheels. So, we need to use mapping formula,

$$[h]_{b,3\times 1} = [W]_{3\times 4}[h]_{w,4\times 1} \tag{2.5}$$

where the given mapping matrix W_4 is,

- $[W]_{3\times 4} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$

We cannot directly calculate inverse of W_4 here. We have to use pseudoinverse. I used pinv function on MATLAB and calculated $[h]_w$.

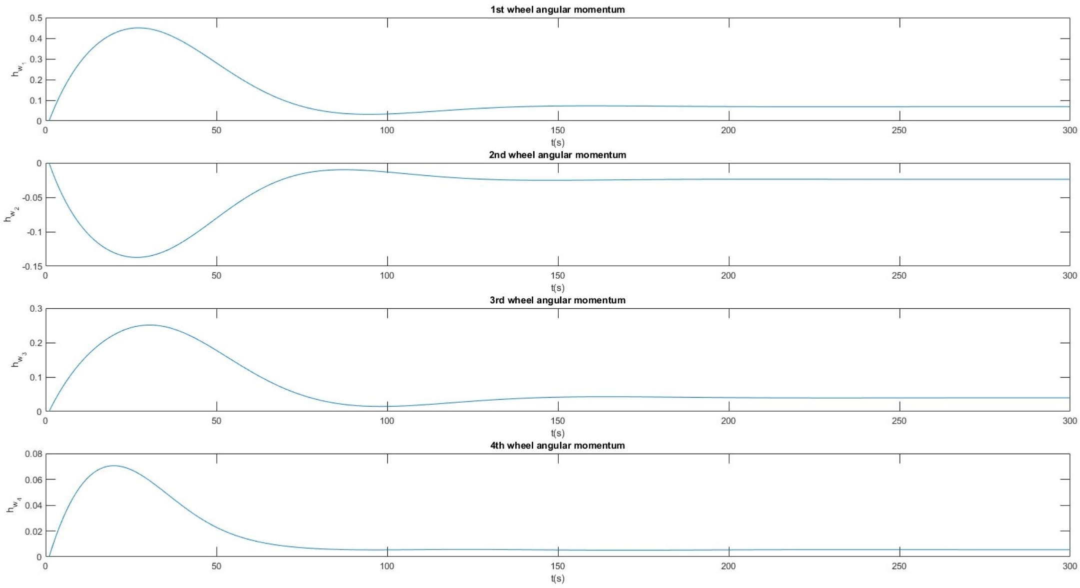

Figure 2.6: Components of the angular momentum for each four wheels

As seen in the figures, components of $[h]_b$ and $[h]_w$ are very different from each other. We converted the three angular momentum components into a quad layout. As expected, the angular momentum values initially vary slightly to control the attitude by producing torque. When the desired attitude is reached, they stay constant at their final value. And we know they don't have to stay at zero finally, because they do not produce torque unless their value changes.