ORTA DOĞU TEKNİK ÜNİVERSİTESİ
MIDDLE EAST TECHNICAL UNIVERSITY

AEROSPACE ENGINEERING DEPARTMENT

AEE486: SPACECRAFT DYNAMICS

Homework 5 - 6

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Contents

1 Question 1

We have given satellite's position and velocity vectors in ECI at time t_0 as:

$$\boldsymbol{r} = 2500\mathbf{i} + 16000\mathbf{j} + 4000\mathbf{k}\ (km) \tag{1.1}$$

$$\boldsymbol{v} = -3\mathbf{i} - 1\mathbf{j} + 5\mathbf{k}\ (km/s) \tag{1.2}$$

1.1 a - The Time Since or Till the Nearest Perigee Pass

To find the time since or till the nearest perigee pass for the spacecraft, we have to calculate,

$$h \Rightarrow \boldsymbol{e} \Rightarrow a \Rightarrow T$$

$$\boldsymbol{e} \Rightarrow \theta \Rightarrow E \Rightarrow M_e$$

$$T, Me \Rightarrow t$$

So, the specific angular momentum vector **h** is,

$$\boldsymbol{h} = \boldsymbol{r} \times \boldsymbol{v} = (2500\mathbf{i} + 16000\mathbf{j} + 4000\mathbf{k}) \times (-3\mathbf{i} - 1\mathbf{j} + 5\mathbf{k})\ (km^2/s) \tag{1.3}$$

$$\boldsymbol{h} = 84000\mathbf{i} - 24500\mathbf{j} + 45500\mathbf{k}\ (km^2/s) \tag{1.4}$$

Then, the eccentricity vector **e** is,

$$\boldsymbol{e} = \frac{\boldsymbol{v} \times \boldsymbol{h}}{\mu} - \frac{\boldsymbol{r}}{r} = 0.0433\mathbf{i} + 0.4370\mathbf{j} + 0.1553\mathbf{k} \tag{1.5}$$

We have to find semi major axis a also,

$$a = \frac{h^2}{\mu}\frac{1}{1-e^2} = 31162\ (km) \tag{1.6}$$

Finally, we can obtain period T with these parameters.

$$T = \frac{2\pi}{\sqrt{\mu}}a^{3/2} = 54744\ (s) \tag{1.7}$$

We have to obtain mean anomaly angle M_e. We know true anomaly angle θ is,

$$\theta = cos^{-1}\left[\frac{\boldsymbol{e}\cdot\boldsymbol{r}}{er}\right] = 0.1117\ (rad) \tag{1.8}$$

With using θ, the eccentric anomaly E is,

$$E = 2\ tan^{-1}\left[\sqrt{\frac{1-e}{1+e}}\ tan\frac{\theta}{2}\right] = 0.0675 \tag{1.9}$$

And, mean anomaly angle M_e is,

$$M_e = E - esinE = 0.0361 \tag{1.10}$$

Now, we have all sufficient parameters. The time since or till the nearest perigee pass t_0 is,

$$t_0 = \frac{M_e}{2\pi} T = 314.3\ (sec) \tag{1.11}$$

1.2 b - The Position and Velocity Vector After 1 Hour

In this part, we have,

$$t = t_0 + 3600 = 3914.3\ (sec) \tag{1.12}$$

To obtain the position and velocity vector at a time, we apply the previous steps in reverse. Since we know period T already,

$$\theta \Leftarrow E \Leftarrow M_e \Leftarrow t$$

So, firstly,

$$M_e = \frac{2\pi}{T} t = 0.4493 \tag{1.13}$$

Then, we use Newton-Raphson iteration method to find E,

Since we know,

$$M_e = E - esinE \tag{1.14}$$

$$F(E) = E - esinE - M_e \tag{1.15}$$

$$F'(E) = 1 - ecosE \tag{1.16}$$

The method is,

$$E_{i+1} = E_i - \frac{E_i - esinE_i - M_e}{1 - ecosE_i} \tag{1.17}$$

After the method is applied, E is found as,

$$E = 0.7752 \tag{1.18}$$

Then, true anomaly angle θ is,

$$\theta = 2\ tan^{-1} \sqrt{tan^2\left(\frac{E}{2}\right)\ \frac{1+e}{1-e}} = 1.0188\ (rad) \tag{1.19}$$

Now, we know where the satellite is in terms of θ. We have to find velocity and position vectors for that θ. First we will find the vectors of the satellite in the perifocal frame. Then we will convert these vectors to inertial frame using the C_{ip} rotation matrix.

$$\boldsymbol{r}_p = \frac{h^2}{\mu}\frac{1}{1+ecos\theta}(cos\theta\bar{p} + sin\theta\bar{q}) = \begin{bmatrix} 37611 \\ 18511 \\ 0 \end{bmatrix} (km) \tag{1.20}$$

$$\boldsymbol{v}_p = \frac{\mu}{h}[(-esin\theta)\bar{p} + (ecos\theta)\bar{q}] = \begin{bmatrix} -0.8312 \\ 1.689 \\ 0 \end{bmatrix} (km/s) \tag{1.21}$$

where,

- $\bar{p} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T$
- $\bar{q} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T$

Now, we have to convert these vectors into inertial frame by using C_{ip} rotation matrix.

$$\boldsymbol{r}_i = C_{ip}\boldsymbol{r}_p \tag{1.22}$$

$$\boldsymbol{v}_i = C_{ip}\boldsymbol{v}_p \tag{1.23}$$

To obtain C_{ip}, we need the find three orbital parameters which are inclination i, argument of periapsis ω, and longitude of ascending node Ω.

First, we will start with the inclination i,

$$i = cos^{-1}\left(\frac{\boldsymbol{h}\cdot z_i}{h}\right) = 1.0913\ (rad) \tag{1.24}$$

where,

- $z_i = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$

Then, we will continue with the argument of periapsis ω,

$$\omega = cos^{-1}\left(\frac{\mathbf{n}\cdot\mathbf{e}}{ne}\right) = 0.3854 \tag{1.25}$$

where,

- $\boldsymbol{n} = z_i \times \boldsymbol{h} = \begin{bmatrix} 24500 & 84000 & 0 \end{bmatrix}^T$

We have to control below to check the value of ω

$$\boldsymbol{e}\cdot z_i = 0.1553 \geq 0 \tag{1.26}$$

So, $0 \leq \omega \leq \pi$.

Lastly, we will obtain the longitude of ascending node Ω as,

$$\Omega = cos^{-1}\left(\frac{\boldsymbol{n} \cdot \boldsymbol{x}_i}{n}\right) = 1.2870 \tag{1.27}$$

where,

- $\boldsymbol{x}_i = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T$

Also, we have to control below to check the value of Ω,

$$\boldsymbol{n} \cdot \boldsymbol{y}_i = 84000 \geq 0 \tag{1.28}$$

where,

- $\boldsymbol{y}_i = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T$

Now, we know $0 \leq \Omega \leq \pi$.

We have all three angles to calculate rotation matrix from perifocal frame to inertial frame to find position and velocity vectors of the satellite at new position.

$$C_{ip} = \begin{bmatrix} c(\Omega)c(\omega) - s(\Omega)c(i)s(\omega) & -c(\Omega)s(\omega) - s(\Omega)c(i)c(\omega) & s(\Omega)s(i) \\ s(\Omega)c(\omega) + c(\Omega)c(i)s(\omega) & -s(\Omega)s(\omega) + c(\Omega)c(i)c(\omega) & -c(\Omega)s(i) \\ s(i)s(\omega) & s(i)c(\omega) & c(i) \end{bmatrix} \tag{1.29}$$

Now that we have the rotation matrix, we can convert the velocity and position vectors into the inertial frame.

$$\boldsymbol{r}_i = C_{ip}\boldsymbol{r}_p = \begin{bmatrix} 20863 \\ -23859 \\ 27436 \end{bmatrix} (km) \tag{1.30}$$

$$\boldsymbol{v}_i = C_{ip}\boldsymbol{v}_p = \begin{bmatrix} 1.5072 \\ 0.0213 \\ -1.1276 \end{bmatrix} (km/s) \tag{1.31}$$

1.3 c - Using a Numerical Integrator

Here again we will find velocity and position vectors one hour later, but this time we will do a numerical integration. We have these,

$$\dot{\boldsymbol{r}}_i = \boldsymbol{v}_i \tag{1.32}$$

$$\dot{\boldsymbol{v}}_i = -\frac{\mu}{r^3}\boldsymbol{r}_i \tag{1.33}$$

We will use the Euler Method for numerical integration. We will use these equations to find the derivative of each vector at each time step. Then we will estimate the vector change over that time steps along with the time step size. Note that the vectors in these equations are in the inertial frame.

$$\boldsymbol{v}_{k+1} = \boldsymbol{v}_k + \left(-\frac{\mu}{r_k^3}\boldsymbol{r}_k\right) dt \tag{1.34}$$

$$\boldsymbol{r}_{k+1} = \boldsymbol{r}_k + \boldsymbol{v}_k dt \tag{1.35}$$

Vector results we found after integration,

$$\boldsymbol{r}_i = \boldsymbol{r}_{k=3915} = \begin{bmatrix} -8554 \\ 3969 \\ 17929 \end{bmatrix} (km) \tag{1.36}$$

$$\boldsymbol{v}_i = \boldsymbol{v}_{k=3915} = \begin{bmatrix} -2.2251 \\ -4.2882 \\ 1.7989 \end{bmatrix} (km/s) \tag{1.37}$$

In addition to the homework, visualization of the path traveled by the satellite in orbit,

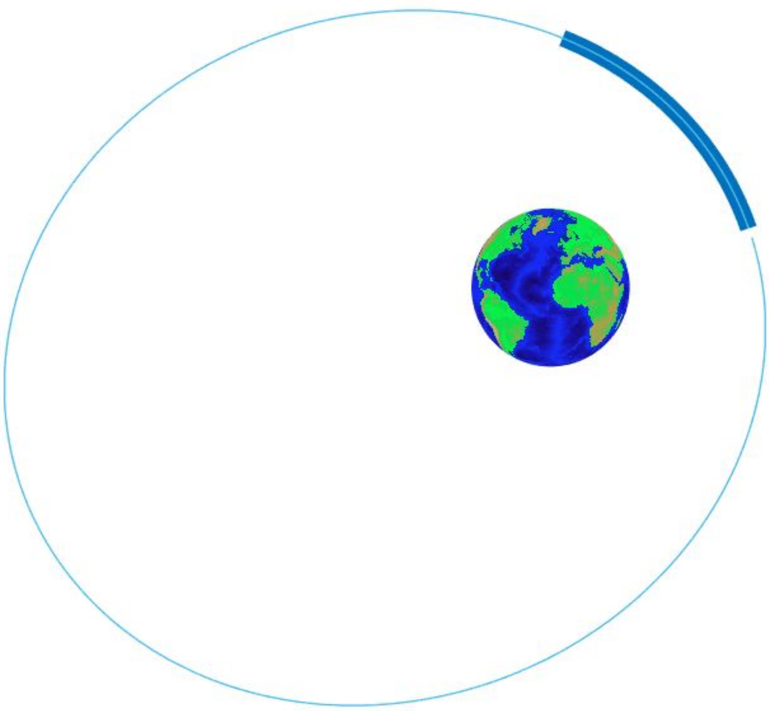

Figure 1.1: The total distance traveled by the satellite in its orbit

2 Question 2

Our communication spacecraft has a mass of 5000kg and is in a LEO circular parking orbit at an altitude of 200km. We are asked to find the amount of propellant we need to transfer it to geosynchronous mission orbit for given $I_{sp} = 300s$.

We will use the following path to transfer the satellite from the LEO orbit to the GEO orbit.

$$LEO \Rightarrow \text{Transfer Orbit} \Rightarrow GEO$$

To find velocity change Δv for each maneuver, we have to find radius and specific angular momentum of the orbits first. Our LEO and GEO Orbits are circular. So their radius,

$$r_L = 200 + 6378 = 6578 \ (km) \tag{2.1}$$

For GEO orbit, I used the value I found online,

$$r_G = 42164 \ (km) \tag{2.2}$$

For elliptic transfer orbit,

$$r_{t_p} = r_L = 6578 \ (km) \tag{2.3}$$

$$r_{t_a} = r_G = 42164 \ (km) \tag{2.4}$$

The angular momentum of the LEO orbit,

$$h_L = \sqrt{2\mu}\sqrt{\frac{r_G}{2}} = 51205 \ (km^2/s) \tag{2.5}$$

And, the angular momentum of the GEO orbit,

$$h_G = \sqrt{2\mu}\sqrt{\frac{r_L}{2}} = 129640 \ (km^2/s) \tag{2.6}$$

Lastly, the angular momentum of the transfer orbit,

$$h_t = \sqrt{2\mu}\sqrt{\frac{r_{t_p} r_{t_a}}{r_{t_p} + r_{t_a}}} = 67352 \ (km^2/s) \tag{2.7}$$

Now, we can find the velocities. LEO orbit velocity,

$$v_L = \frac{h_L}{r_L} = 7.7843\ (km/s) \tag{2.8}$$

Transfer orbit velocity at the perigee point,

$$v_{t_1} = \frac{h_t}{r_{t_p}} = 10.2390\ (km/s) \tag{2.9}$$

Transfer orbit velocity at the apogee point,

$$v_{t_2} = \frac{h_t}{r_{t_a}} = 1.5974\ (km/s) \tag{2.10}$$

GEO orbit velocity,

$$v_G = \frac{h_G}{r_G} = 3.0747\ (km/s) \tag{2.11}$$

Now, time to find the velocity changes Δv_1 , Δv_2,

$$\Delta v_1 = |v_{t_1} - v_L| = 2.4546\ (km/s) \tag{2.12}$$

$$\Delta v_2 = |v_G - v_{t_2}| = 1.4773\ (km/s) \tag{2.13}$$

Then, the amount of propellant needed for these maneuvers, we know,

$$\frac{\Delta m}{m} = (1 - e^{\frac{\Delta v}{I_{sp} g_0}}) \tag{2.14}$$

where,

- $g_0 = 6.6743 \times 10^{-11}\ \mathrm{m^3\ kg^{-1}\ s^{-2}}$
- $I_{sp} = 300\ s$

So, mass change for the first maneuver Δm_1 is,

$$\Delta m_1 = m_1(1 - e^{\frac{\Delta v_1}{I_{sp} g_0}}) = -4.1720\ (kg) \tag{2.15}$$

Now, the mass of the satellite has changed for the new maneuver,

$$m_2 = m_1 + \Delta m_1 = 4995.8\ (kg) \tag{2.16}$$

Then, Δm_2 is,

$$\Delta m_2 = m_2(1 - e^{\frac{\Delta v_2}{I_{sp} g_0}}) = -2.5084\ (kg) \tag{2.17}$$

Finally, the total amount of propellant is,

$$\Delta m = |\Delta m_1 + \Delta m_2| = 6.6804\ (kg) \tag{2.18}$$

3 Question 3

We have an asteroid in this part which is coming to the Earth with an altitude of 100000 km, a speed of 6 km/s, and a flight path angle γ -80 degree.

First, the magnitude of the position of the asteroid is,

$$r = 100000 + 6378 = 106378\ (km) \tag{3.1}$$

Flight path angle γ defined as,

$$\gamma = \frac{\boldsymbol{v}_r}{\boldsymbol{v}_T} \tag{3.2}$$

We have to define velocity and position of the asteroid. For simplifying the solution, we assume the position of the asteroid is in only y-axis, and velocity of the asteroid is in -y and z axis.

Note that we could have solved the question directly in the perifocal frame, but I chose to solve this way.

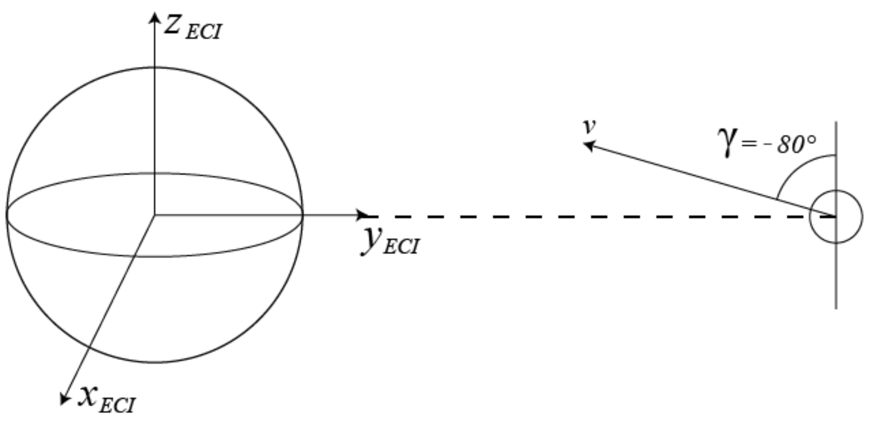

Figure 3.1: Representation of the asteroid relative to earth

Thus, we can say in ECI frame,

- $\boldsymbol{r} = 106378 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\ (km)$
- $\boldsymbol{v} = 6\ sin(80) \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} + 6\ cos(80) \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\ (km/s)$

So, the specific angular momentum vector of the asteroid **h** is,

$$\boldsymbol{h} = \boldsymbol{r} \times \boldsymbol{v} = (110830\mathbf{i}\ (km^2/s) \tag{3.3}$$

Then, the eccentricity vector **e** is,

$$\boldsymbol{e} = \frac{\boldsymbol{v} \times \boldsymbol{h}}{\mu} - \frac{\boldsymbol{r}}{r} = -0.7103\mathbf{j} + 1.6430\mathbf{k} \tag{3.4}$$

Since $||\boldsymbol{e}|| = 1.79$, as we expected from an asteroid, the asteroid has an hyperbolic orbit. So, if it doesn't crash to the earth, it will fly-by.

Now, we must control the perigee radius, whether greater or less than the earth's radius, to understand if it will crash into the planet.

$$r_p = \frac{h^2}{\mu}\frac{1}{1+e} = 11046\ (km) \tag{3.5}$$

$$h(altitude) = r_p - r_{earth} = 11046 - 6378 = 4668.1\ (km) \tag{3.6}$$

Since the perigee radius is greater than the earth's radius, the asteroid will not crash into the earth. It will fly-by at 4668.1 km altitude.

Now, we will find the time for closest approach of the asteroid. For this, we will follow the path below,

$$h \Rightarrow \boldsymbol{e} \Rightarrow \theta \Rightarrow F \Rightarrow M_h \Rightarrow t$$

Since we already calculate $\boldsymbol{h}$ and $\boldsymbol{e}$, we will continue with true anomaly angle θ,

$$\theta = cos^{-1}\left[\frac{\boldsymbol{e}\cdot\boldsymbol{r}}{er}\right] = 1.9788\ (rad) \tag{3.7}$$

With using θ, the hyperbolic eccentric anomaly F is,

$$F = 2\ tanh^{-1}\left[\sqrt{\frac{e-1}{e+1}}\ tan\frac{\theta}{2}\right] = 2.2526 \tag{3.8}$$

And, hyperbolic mean anomaly angle M_h is,

$$M_h = esinhF - F = 6.1669 \tag{3.9}$$

Now, we have all sufficient parameters. The time for closest approach is,

$$t = \frac{h^3}{\mu^2}\frac{1}{(e^2-1)^{3/2}}M_h = 16151\ (sec) \tag{3.10}$$

4 Question 4

In this question, we have the ISS and Soyuz spacecraft in circular orbits with altitudes of 450km and 200km respectively. Soyuz spacecraft will meet and dock with the ISS. Both spacecraft have coplanar orbits. Initially ISS is leading the Soyuz with a true anomaly difference of $\Delta\theta = 20$ deg.

Before starting the calculations, we will obtain the radius of orbits.

$$r_{pr} = 200 + 6378 = 6578 \ (km) \tag{4.1}$$

$$r_{iss} = 450 + 6378 = 6828 \ (km) \tag{4.2}$$

4.1 Waiting in the Parking Orbit then Using Hohmann Transfer

Firstly, we have to calculate the necessary time for Hohmann transfer and obtain where will the ISS be after the transfer.

We will calculate semi-major axis of the transfer orbit.

$$a_t = \frac{6578 + 6828}{2} = 6828 \ (km) \tag{4.3}$$

Then the period and transfer time is,

$$T_t = \frac{2\pi}{\sqrt{\mu}} a_t^{3/2} = 5461.5 \ (s) \tag{4.4}$$

$$t = \frac{T}{2} = 2730.8 \ (s) \tag{4.5}$$

Also, we have to calculate the period of the ISS orbit.

$$T_{iss} = \frac{2\pi}{\sqrt{\mu}} r_{iss}^{3/2} = 5615 \ (s) \tag{4.6}$$

Then, the true anomaly angle traveled by the ISS during the transfer is,

$$\Delta\theta_{iss} = t\frac{2\pi}{T_{iss}} = 3.0557 \ (rad) \tag{4.7}$$

Then, true anomaly difference between two spacecraft after the transfer is,

$$\Delta\theta = 20\frac{\pi}{180} - (\pi - \Delta\theta_{iss}) = 0.2632 \ (rad) \tag{4.8}$$

In order to dock the two spacecraft, we have to wait for necessary alignment to eliminate this 15.08 deg difference. To calculate the waiting time for necessary alignment, we will calculate the relative angular velocity between the two spacecraft.

$$\omega_{rel} = \frac{2\pi}{T_{pr}} - \frac{2\pi}{T_{iss}} = 6.44 \times 10^{-5}\ (rad/s) \tag{4.9}$$

Then, the necessary time for to eliminate the true anomaly difference between two spacecraft is,

$$t_{req} = \frac{\Delta\theta}{\omega_{rel}} = 4087.2\ (s) \tag{4.10}$$

To find the required amount of Δv, we can calculate,

$$v_{pr} = \sqrt{\frac{\mu}{r_{pr}}} = 7.7843\ (km/s) \tag{4.11}$$

$$v_{iss} = \sqrt{\frac{\mu}{r_{iss}}} = 7.6405\ (km/s) \tag{4.12}$$

To obtain velocity values for the perigee and the apogee points of the transfer orbit, we need to find the angular momentum of the orbit first,

$$h_t = \sqrt{2\mu}\sqrt{\frac{r_{t_p} r_{t_a}}{r_{t_p} + r_{t_a}}} = 51681\ (km^2/s) \tag{4.13}$$

Then, transfer orbit velocity at the perigee point,

$$v_{t_1} = \frac{h_t}{r_{t_p}} = 7.8566\ (km/s) \tag{4.14}$$

Transfer orbit velocity at the apogee point,

$$v_{t_2} = \frac{h_t}{r_{t_a}} = 7.5689\ (km/s) \tag{4.15}$$

To calculate required amount of Δv,

$$\Delta v_1 = |v_{t_1} - v_{pr}| = 0.0722\ (km/s) \tag{4.16}$$

$$\Delta v_2 = |v_{iss} - v_{t_2}| = 0.0716\ (km/s) \tag{4.17}$$

Then,

$$\Delta v_{hohmann} = \Delta v_1 + \Delta v_2 = 0.1438\ (km/s) \tag{4.18}$$

4.2 Using Hohmann Transfer Directly then Using Phasing Maneuver

When we use Hohmann Transfer directly, there will be a 0.2632 radian difference in true anomaly between two spacecraft which is known from previous question.

Now, we will calculate the angular velocity of the ISS orbit to calculate the period of the phasing angle,

$$\omega_{iss} = \frac{2\pi}{T_{iss}} = 0.0011\ (rad/s) \tag{4.19}$$

Then,

$$\Delta T = \frac{0.2632}{\omega_{iss}} = 235.2041\ (s) \tag{4.20}$$

Finally, the period of the phasing angle is,

$$T_{ph} = T_{iss} - \Delta T = 5379.8\ (s) \tag{4.21}$$

Now, we have to obtain the apogee velocity of the phasing angle. For this we will calculate semi-major axis of the phasing orbit,

$$a_{ph} = \left(\frac{T_{ph}\sqrt{\mu}}{2\pi}\right)^{\frac{2}{3}} = 6636\ (km) \tag{4.22}$$

Then, the perigee radius,

$$r_{ph_p} = 2a_{ph} - r_{iss} = 6443.9\ (km) \tag{4.23}$$

Now, we will obtain angular momentum of the orbit,

$$h_{ph} = \sqrt{2\mu}\sqrt{\frac{r_{ph_p} r_{iss}}{r_{ph_p} + r_{iss}}} = 51409\ (km^2/s) \tag{4.24}$$

Now, we can find the apogee velocity,

$$v_{ph_a} = \frac{h_{ph}}{r_{ISS}} = 7.5291\ (km/s) \tag{4.25}$$

The required amount of Δv for phasing maneuver is,

$$\Delta v_{ph} = 2|v_{ISS} - v_{ph_a}| = 0.2227\ (km/s) \tag{4.26}$$

The total amount of Δv is,

$$\Delta v_{hohmann+ph} = \Delta v_{ph} + \Delta v_{hohmann} = 0.3665\ (km/s) \tag{4.27}$$

4.3 Using Bi-Elliptic Transfer

In order to dock two spacecraft at $\theta = 0$ true anomaly angle in the ISS orbit, Soyuz must complete one revolution on the bi-elliptic orbits when the ISS complete the remain part of the revolution.

So, required time for transfer is,

$$t_{req} = \frac{340\frac{\pi}{180}}{\omega_{iss}} = 5303.1\ (s) \tag{4.28}$$

Then we know,

$$T = \frac{2\pi}{\sqrt{\mu}} a^{3/2} \tag{4.29}$$

$$a = \frac{r_a + r_p}{2} \tag{4.30}$$

We will merge the equations and get,

$$t_{req} = \frac{T_1}{2} + \frac{T_2}{2} \tag{4.31}$$

$$t_{req} = \frac{\pi}{\sqrt{\mu}} \left(\left(\frac{r_{pr} + r_{mid}}{2} \right)^{3/2} + \left(\frac{r_{mid} + r_{iss}}{2} \right)^{3/2} \right) \tag{4.32}$$

r_{mid} distance is the distance where the second engine burn happens. We solve the merged equation and find the r_{mid} distance iteratively.

$$r_{mid} = 6442.1\ (km) \tag{4.33}$$

Now, we know the radius of the orbits. We can calculate angular momentum and velocity values of the orbits.

The angular momentum of the first elliptic orbit for transfer is,

$$h_1 = \sqrt{2\mu} \sqrt{\frac{r_{mid} r_{pr}}{r_{mid} + r_{pr}}} = 50937\ (km^2/s) \tag{4.34}$$

Then, the velocities of the orbit at perigee and apogee points.

$$v_{1_p} = \frac{h_1}{r_{pr}} = 7.7436\ (km/s) \tag{4.35}$$

$$v_{1_a} = \frac{h_1}{r_{mid}} = 7.9069\ (km/s) \tag{4.36}$$

The angular momentum of the second elliptic orbit is,

$$h_2 = \sqrt{2\mu}\sqrt{\frac{r_{iss}r_{mid}}{r_{iss} + r_{mid}}} = 51405\ (km^2/s) \tag{4.37}$$

Again, the velocities of the orbit at perigee and apogee points.

$$v_{2_p} = \frac{h_2}{r_{mid}} = 7.9796\ (km/s) \tag{4.38}$$

$$v_{2_a} = \frac{h_2}{r_{iss}} = 7.5286\ (km/s) \tag{4.39}$$

Now, we have all the required velocities for the maneuver and can calculate the required amount of Δv.

Δv for first burn,

$$\Delta v_1 = |v_{1_p} - v_{pr}| = 0.0407\ (km/s) \tag{4.40}$$

Δv for second burn,

$$\Delta v_2 = |v_{2_p} - v_{1_a}| = 0.0726\ (km/s) \tag{4.41}$$

Δv for third burn,

$$\Delta v_3 = |v_{iss} - v_{2_a}| = 0.1119\ (km/s) \tag{4.42}$$

Finally, total required amount of Δv,

$$\Delta v_{bi-elliptic} = \Delta v_1 + \Delta v_2 + \Delta v_3 = 0.2252\ (km/s) \tag{4.43}$$

Also, if the ISS completes one more revolution next of its remaining revolution, r_{mid} distance should be placed outside of the radius of the ISS. This time, required time for transfer is,

$$t_{req_2} = \frac{(340)\frac{\pi}{180}}{\omega_{iss}} + T_{iss} = 10918\ (s) \tag{4.44}$$

And the required r_{mid} distance is,

$$r_{mid_2} = 14571\ (km) \tag{4.45}$$

Lastly, total required amount of Δv for this case,

$$\Delta v_{bi-elliptic_2} = 2.6822\ (km/s) \tag{4.46}$$

Clearly seen that, if the r_{mid} distance is placed the outside of the radius of the ISS, efficiency of the maneuver is dramatically decreasing.

We calculated three different Δv for three different approaches. They were,

$$\Delta v_{hohmann} = 0.1438 \ (km/s) \tag{4.47}$$

$$\Delta v_{hohmann+ph} = 0.3665 \ (km/s) \tag{4.48}$$

$$\Delta v_{bi-elliptic} = 0.2252 \ (km/s) \tag{4.49}$$

We can say that, in our case, the Hohmann Transfer method is a more efficient option when compared to the Bi-Elliptic transfer method. The Hohmann Transfer method requires less amount of Δv, to achieve a transfer between the two orbits. Additionally, using a Phasing Maneuver can significantly increase the amount of Δv required, making the transfer less efficient. Therefore, the use of a Phasing Maneuver should be carefully considered.