Homework 5 & 6

Instructions

1) You are free to discuss the questions with your classmates, but if I notice the same answer is copied all the copiers will get "0" for that question.
2) I prefer typed reports (MS Word, LaTeX etc.) Please be consistent in notations. You can use the following notations:
   - lower case bold for vector ($\boldsymbol{q} = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}^T$)
   - lower case regular for scalar (q_1),
   - capital letter for matrix (C)
   - lower case right subscript/superscript for the reference frame notations (C_{ab}, $\boldsymbol{\omega}_{ab}^{b}$).
3) Provide the algorithms that you coded in the submitted package.
4) Due date is 13th January (Friday) until the midnight (23:59).
5) Submit your reports on ODTUClass.
6) Title of your report file (or zipped package) should be: AE486_2022_Name_Surname_HW5&6

Questions

1) (**60pts**) A satellite's position and velocity vectors in ECI at time t_0 are given as below:

$$\boldsymbol{r} = 2500\boldsymbol{i} + 16000\,\boldsymbol{j} + 4000\boldsymbol{k}\ (\mathrm{km})$$
$$\boldsymbol{v} = -3\boldsymbol{i} - 1\boldsymbol{j} + 5\boldsymbol{k}\ (\mathrm{km/s})$$

   a) **(20 pts)** Find the time since or till the nearest perigee pass for the spacecraft.
   b) **(20 pts)** What will be the position and velocity vector (in ECI) after 1 hour ($t_0 + 1\mathrm{h}$)? Solve using the Kepler's equation.
   c) **(20 pts)** Solve the same question in b) using a numerical integrator. Compare the results.

2) (**20pts**) Assume that your communication spacecraft, which has a mass of 5000kg, will be left to a LEO circular parking orbit at an altitude of 200km. How much propellant do you need to transfer it to geosynchronous mission orbit? ($I_{sp} = 300\mathrm{s}$).

3) **(20pts)** During your observations you noticed that an asteroid is approaching to the Earth. It has an altitude of 100,000 km with a speed of 6 km/s and a flight path angle of -80deg. The flight path angle γ is defined as $\gamma = \dfrac{v_r}{v_\perp}$.
   Will the asteroid hit to the Earth or just fly-by? What is the time for the crash or closest approach?

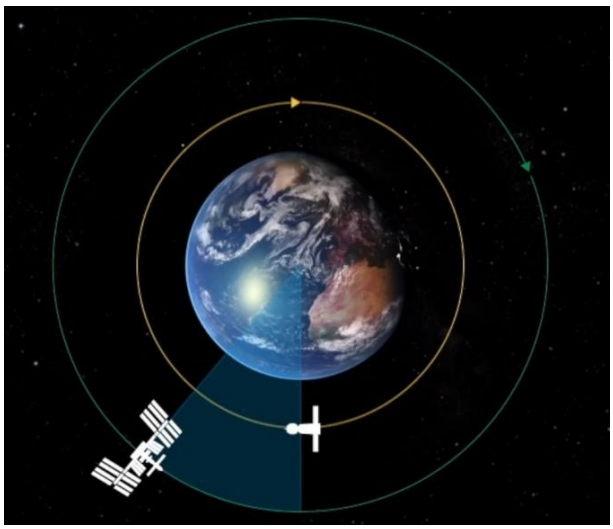
4) **(100pts)** Soyuz spacecraft will meet and dock with the International Space Station (ISS). It is launched into an initial circular parking orbit with an altitude of $h = 200\mathrm{km}$. ISS is also in a circular orbit with an altitude of $h = 450\mathrm{km}$ and both spacecraft have coplanar orbits.

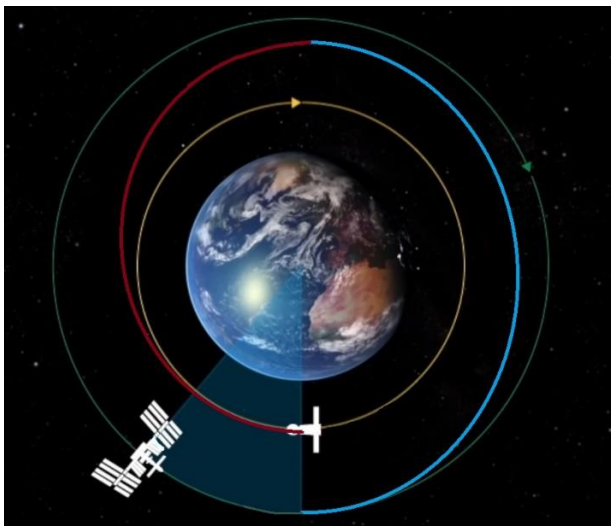
   a) **(30pts)** At t_0 two spacecraft have an alignment as shown in the left figure to the next page. They are both rotating clockwise in their orbits and the ISS is leading the Soyuz with a true anomaly difference of $\Delta\theta = 20\,\mathrm{deg}$. Two spacecraft is to meet with an Hohmann transfer. Calculate the waiting time for necessary alignment and the required amount of Δv.

b) **(30pts**) What would be the total Δv and time if Soyuz immediately starts the Hohmann transfer at t_0 and then meets with the ISS by performing a phasing maneuver?

c) **(40pts**) Actually Soyuz is performing a bi-elliptic transfer to meet the ISS* by following two ellipses whose semi-major axes are smaller than the radius of the orbit of the ISS (see the below figure to the right). If Soyuz is going to start the maneuver at t_0, what will be the required amount of Δv? Compare the elapsed time with those in a) and b).

* See https://www.youtube.com/watch?v=M2_NeFbFcSw&t=595s for the details.





Good luck!
H.E. Soken