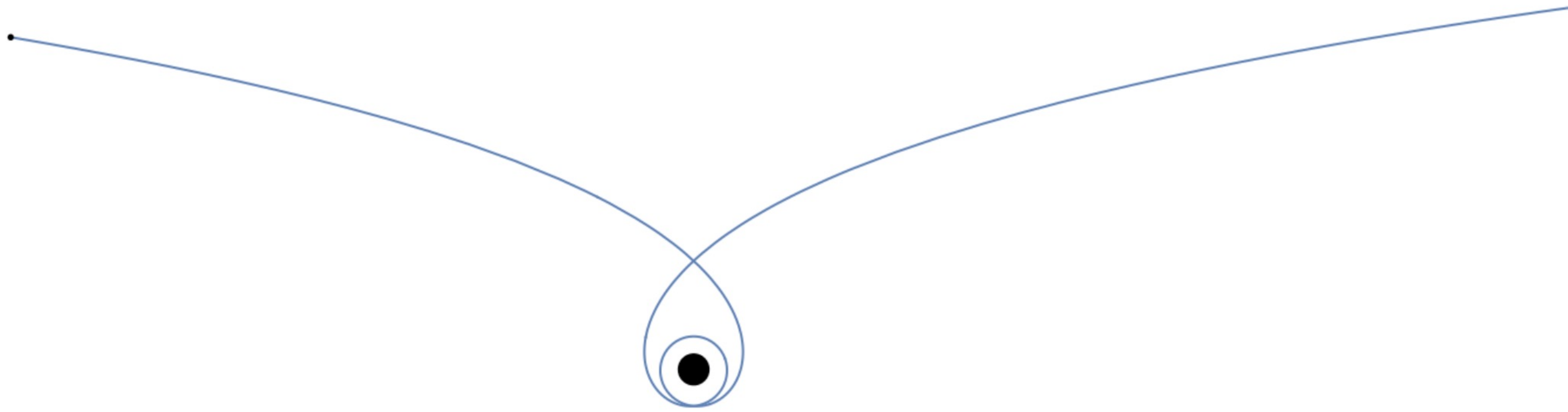
Self-force in hyperbolic black hole encounters



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with Leor Barack

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Capra 25 @ University College Dublin

University of Southampton

CAPRA

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- EMRI scattering motivation.
- Scatter geodesics.
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- Self-force correction to the scatter angle:
  - Post-Minkowskian expansion.
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- Sample results.

Motivation: EMRI scatter orbits

- Exact post-Minkowskian calculations for bound orbits [Damour '19; Bini, Damour & Geralico '20].

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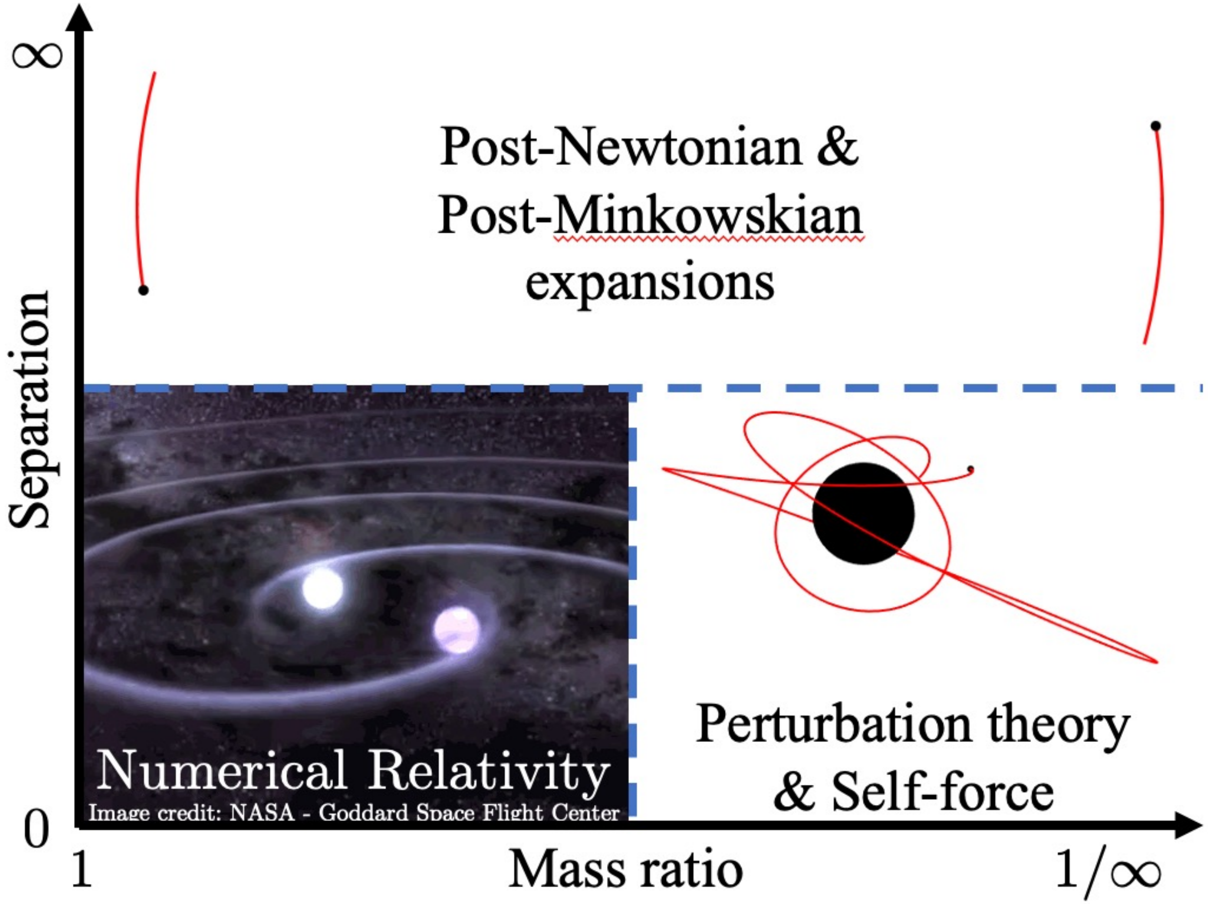
$$2\text{SF} \to 6\text{PM}$$



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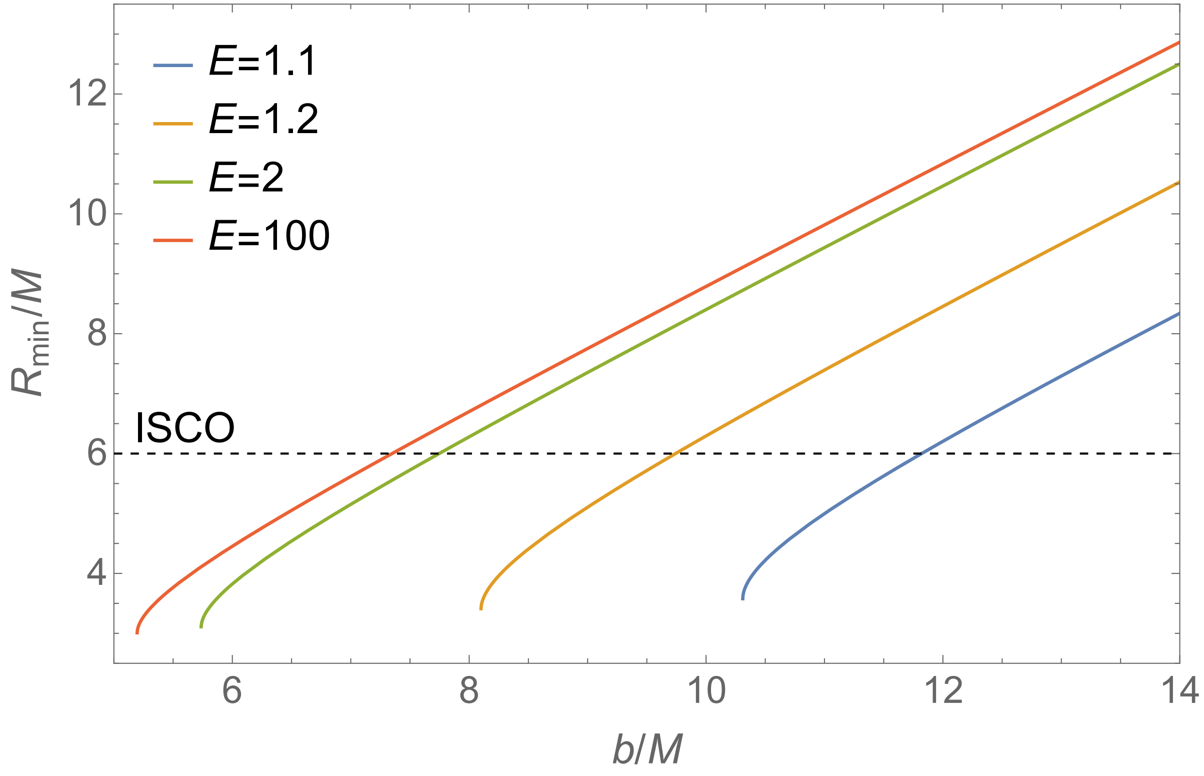

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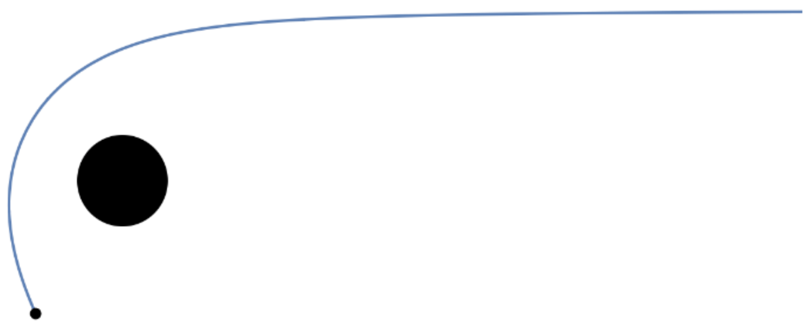

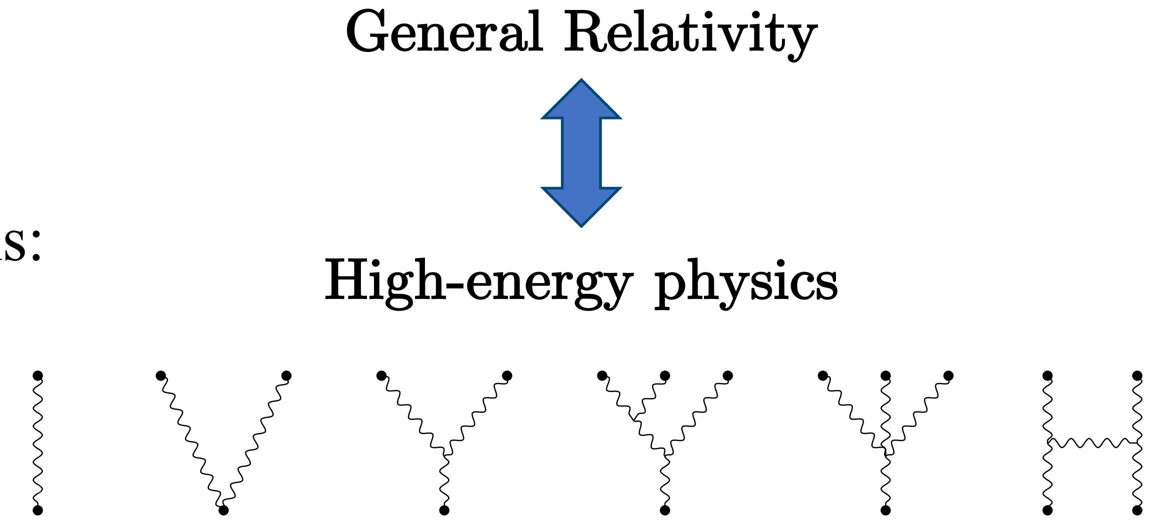

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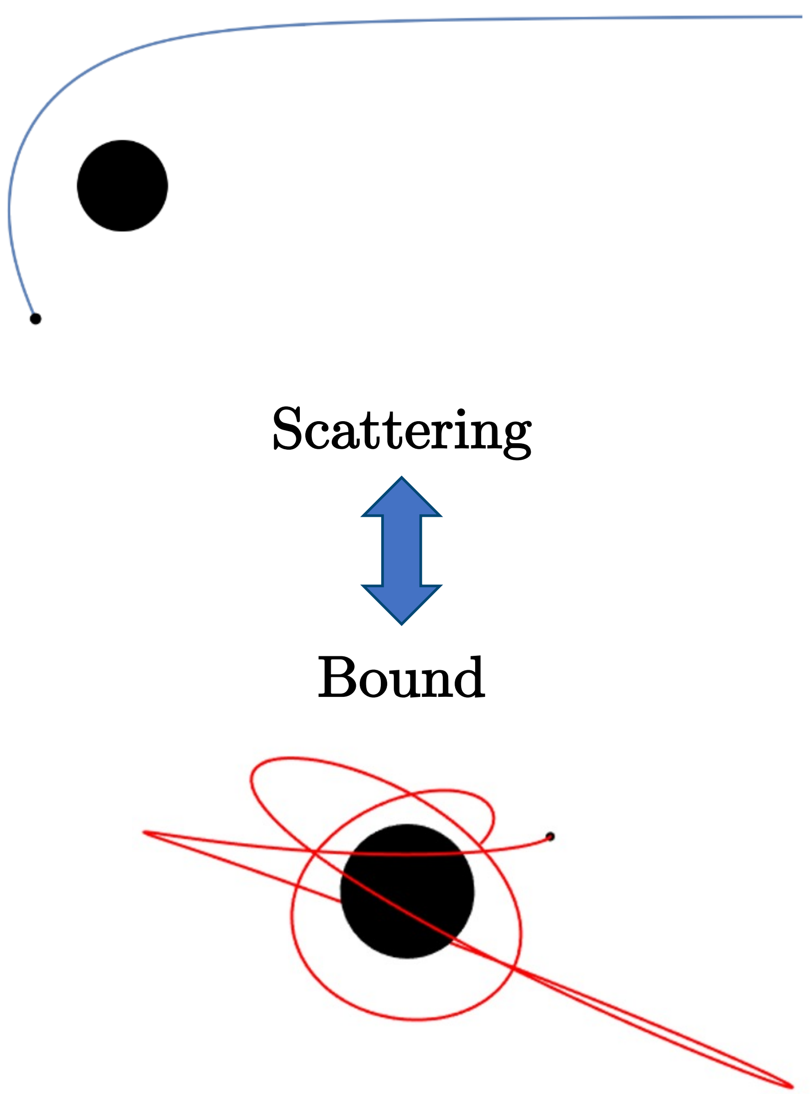
- Calibration of Effective-One-Body (EOB) in the ultra-strong field.
- Comparisons with scatter amplitude calculations:
    - QFT and EFT [Bern et al. '21].
- Dictionary between scatter and bound [Cho et al. '21].
    - Scatter angle ↔ periastron advance.



Scattering geodesics

Energy and angular momentum:

$$E > 1 \qquad\qquad L > L_{\text{crit}}(E)$$

Velocity at infinity and impact parameter:

$$v_\infty := \left.\frac{dr}{dt}\right|_{r\to\infty} \qquad b := \lim_{r\to\infty} r \sin|\varphi(r) - \varphi(\infty)|$$

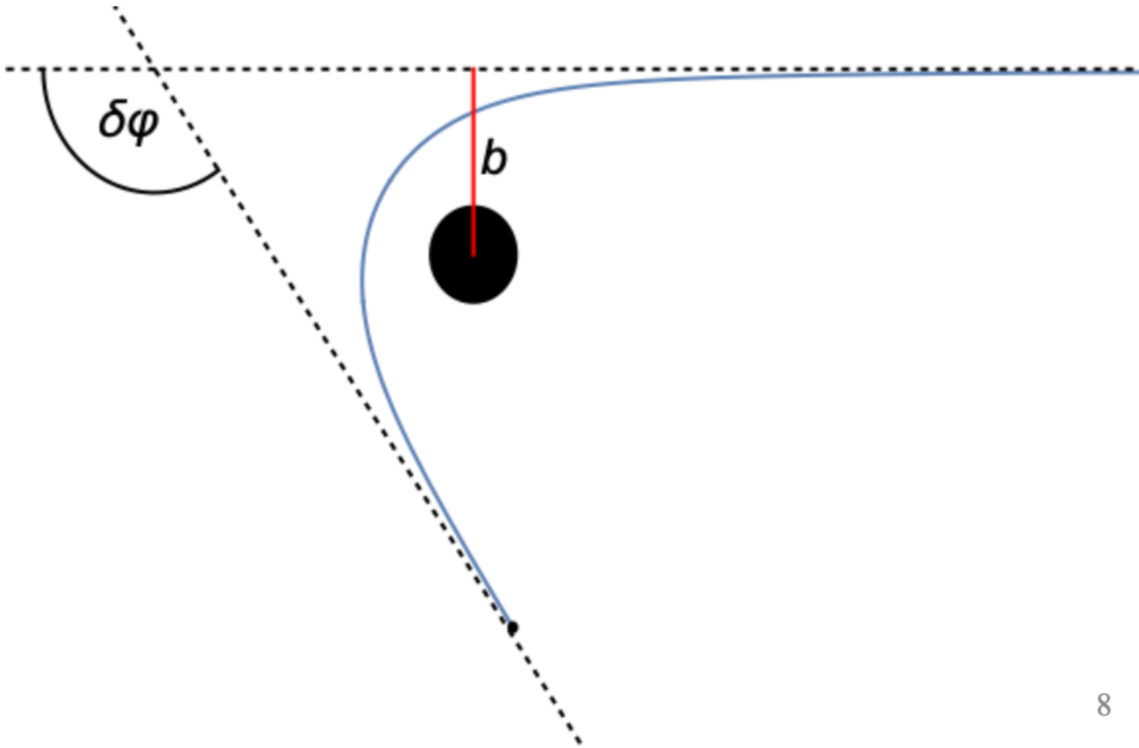
Geodesics:

$$\frac{dt}{d\tau} = \frac{E\,r}{r - 2M} \qquad\qquad \frac{d\varphi}{d\tau} = \frac{L}{r^2}$$

$$\left(\frac{dr}{d\tau}\right)^2 = E^2 - V(L;r)$$

Scatter angle:

$$\delta\varphi := \int_{-\infty}^{\infty} \frac{d\varphi}{dt} dt - \pi$$



Self-forced equations of motion

Expand in the mass ratio for fixed $\{v_\infty, b\}$:

$$\dot{E} = -\eta F_t \qquad \dot{L} = \eta F_\varphi \qquad \eta := \frac{\mu}{M}$$

$$E = E_\infty - \eta \int_{-\infty}^{\tau} F_t \, d\tau \qquad L = L_\infty + \eta \int_{-\infty}^{\tau} F_\varphi \, d\tau$$

Geodesic

Expand other quantities:

$$r_p = R_{\min} + \eta\, r_p^{(1)} + \mathcal{O}(\eta^2)$$

Perturb the defining equation for the periastron:

$$E(r_p)^2 - V(L; r_p) = 0 \quad \rightarrow \quad 2E_\infty \int_{-\infty}^{\tau_p} F_t \, d\tau = \left.\frac{\partial V(r,L)}{\partial r}\right|_0 r_p^{(1)} + \left.\frac{\partial V(r,L)}{\partial L}\right|_0 \int_{-\infty}^{\tau_p} F_\varphi \, d\tau$$

Rearrange for $r_p^{(1)}$.

Self-force correction to the scatter angle

Scatter angle as a radial integral:

$$\delta\varphi = \sum_{\pm} \int_{r_p^{\pm}}^{\infty} \frac{\dot{\varphi}^{\pm}}{\dot{r}^{\pm}} dr - \pi = \sum_{\pm} \int_{r_p^{\pm}}^{\infty} \frac{H^{\pm}(r; E, L)}{\sqrt{r - r_p^{\pm}}} dr - \pi$$

Perturb integral [details in upcoming paper]:

$$\delta\varphi = \delta\varphi^{(0)} + \eta\delta\varphi^{(1)}$$

$$\boxed{\delta\varphi^{(1)} = \sum_{\pm} \int_{R_{\min}}^{\infty} \left[\mathcal{G}_E^{\pm}(r; E_\infty, L_\infty) \boxed{F_t^{\pm}} - \mathcal{G}_L^{\pm}(r; E_\infty, L_\infty) \boxed{F_\varphi^{\pm}}\right] dr}$$

Can split into conservative and dissipative pieces on outgoing leg:

$$\delta\varphi^{(1)}_{\mathrm{cons}} = \int_{R_{\min}}^{\infty} \left[\mathcal{G}_E^{\mathrm{cons}} F_t^{\mathrm{cons}} - \mathcal{G}_L^{\mathrm{cons}} F_\varphi^{\mathrm{cons}}\right] dr \qquad \delta\varphi^{(1)}_{\mathrm{diss}} = \int_{R_{\min}}^{\infty} \left[\mathcal{G}_E^{\mathrm{diss}} F_t^{\mathrm{diss}} - \mathcal{G}_L^{\mathrm{diss}} F_\varphi^{\mathrm{diss}}\right] dr$$

PM self-force correction to the scatter angle

Post-Minkowskian weak-field approximation: $$\frac{M}{v_\infty^2 b} \ll 1$$

Expand in M/b : $$R_{\text{min}} = b - \frac{M}{v_\infty^2} + O(b^{-1})$$

Leading order correction to the scatter angle:

$$\delta\varphi^{(1)}_{\text{cons PM}} = \frac{2}{v_\infty^2 E_\infty^2} \int_b^\infty \left[(b/v_\infty) F_t^{\text{cons}} + F_\varphi^{\text{cons}} \right] \frac{r\,dr}{r^2 - b^2}$$

$$\delta\varphi^{(1)}_{\text{diss PM}} = -\frac{2M}{b^2 E_\infty^2 v_\infty^4} \int_b^\infty \left[(b/v_\infty)\left(3v_\infty^2 - 1\right) F_t^{\text{diss}} + \left(v_\infty^2 + 1\right) F_\varphi^{\text{diss}} \right] \frac{r\,dr}{\sqrt{r^2 - b^2}}$$

PM self-force correction to the scatter angle

Check our formula in the PM regime.

Substitute in leading-order PM gravitational self-force [Gralla & Lobo '22]:

$$\delta\varphi^{(1)}_{\text{cons PM}} = \boxed{\frac{7\pi}{4}\left(\frac{M}{b}\right)^2} + \boxed{O\left(\frac{M}{b}\right)^3} \qquad \delta\varphi^{(1)}_{\text{diss PM}} = \boxed{O\left(\frac{M}{b}\right)^3}$$

Substitute in leading-order PM scalar self-force [Gralla & Lobo '22]:

$$\delta\varphi^{(1)}_{\text{cons PM}} = \boxed{-\frac{\pi}{4}\left(\frac{M}{b}\right)^2} + \boxed{O\left(\frac{M}{b}\right)^3} \qquad \delta\varphi^{(1)}_{\text{diss PM}} = \boxed{O\left(\frac{M}{b}\right)^3}$$

Leading-order 2PM contributions purely conservative and independent of v_∞.

Leading-order terms have opposite signs.

Both GSF and SSF corrections agree with [Gralla & Lobo '22].

Scalar field evolution scheme

Decompose scalar field:

$$\Phi = \frac{2\pi q}{r} \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \psi_{\ell m}(t,r) Y_{\ell m}(\theta,\varphi)$$

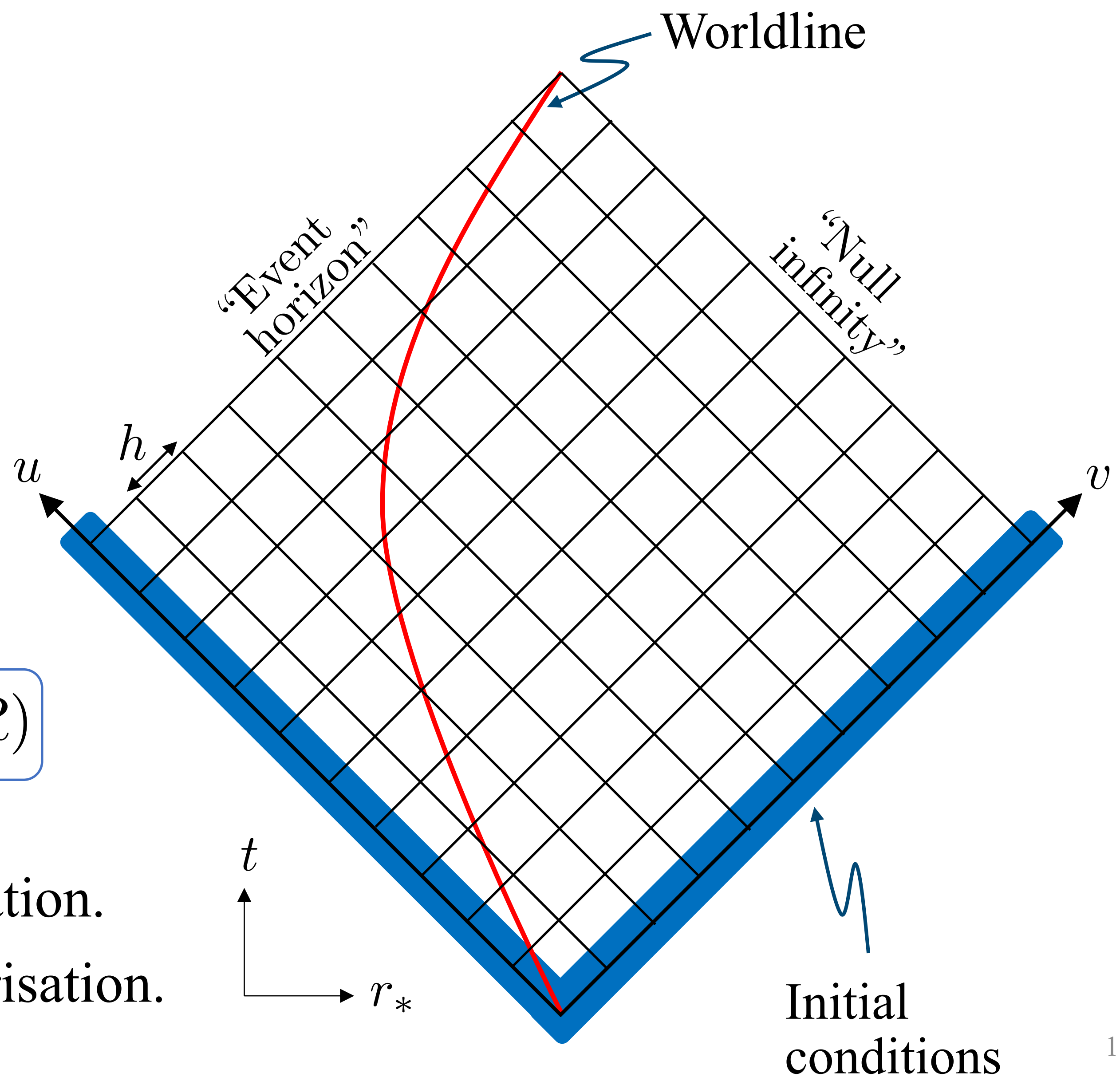
$q:$ Scalar charge

1+1D scalar wave equation:

$$\boxed{\psi_{,uv} + V(\ell;r)\psi = S_\psi(\ell;x_p^\mu)\,\delta\,(r-R)}$$

Evolve finite-difference version of 1+1D equation.

Extract regular field/SF via mode-sum regularisation.



Scalar self-force and mass correction

Equation of motion:

$$\frac{d}{d\tau}(\mu u^\alpha) = q\nabla^\alpha\Phi^R$$

u^α : 4-velocity

Φ^R : Regular field

Tangent to u^α :

$$\frac{d\mu}{d\tau} = -q\frac{d\Phi^R}{d\tau}$$

Integrate:

$$\mu(\tau) = \mu_0 - q\Phi^R(\tau)$$

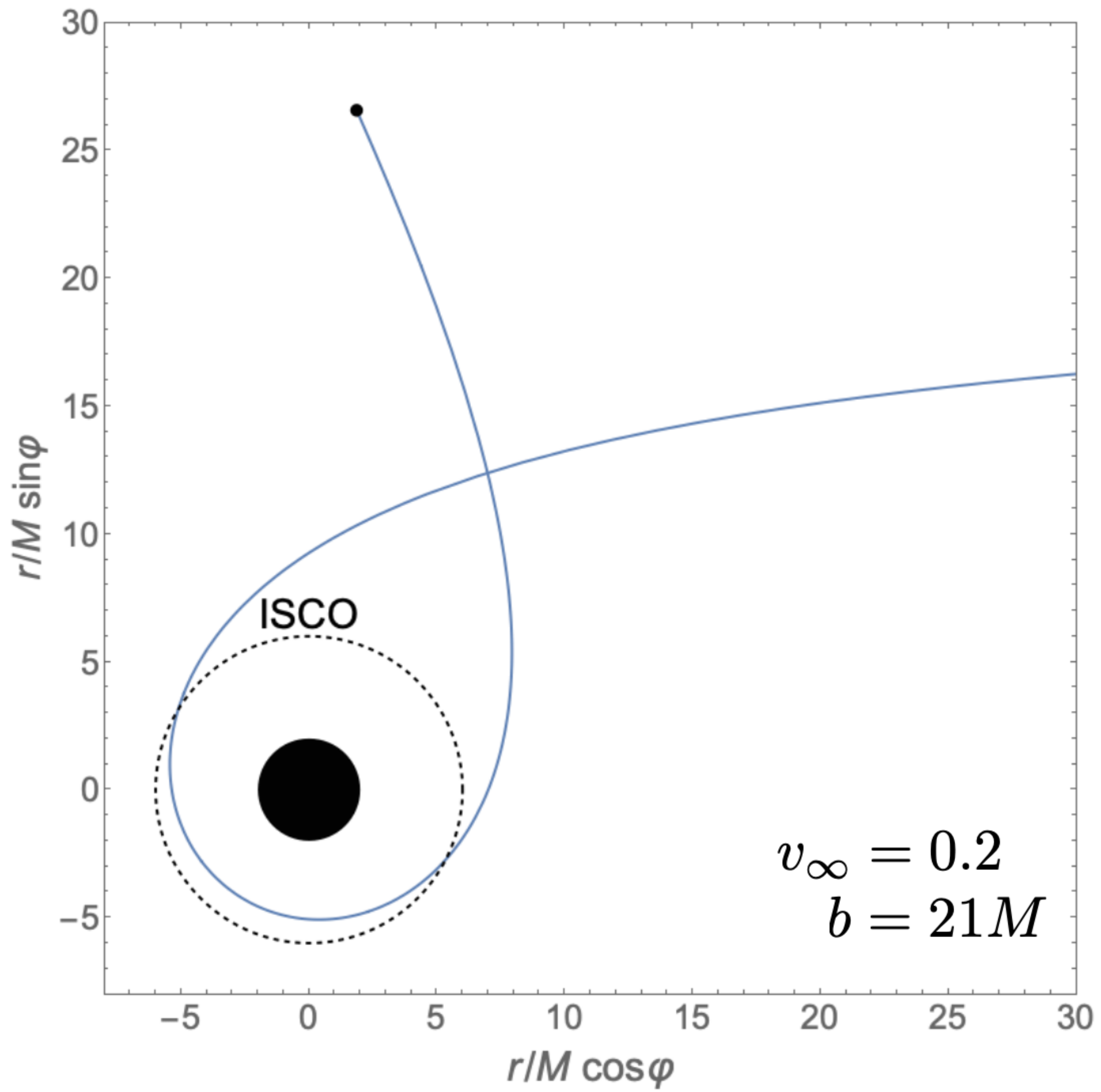
Rest mass

Orthogonal to u^α :

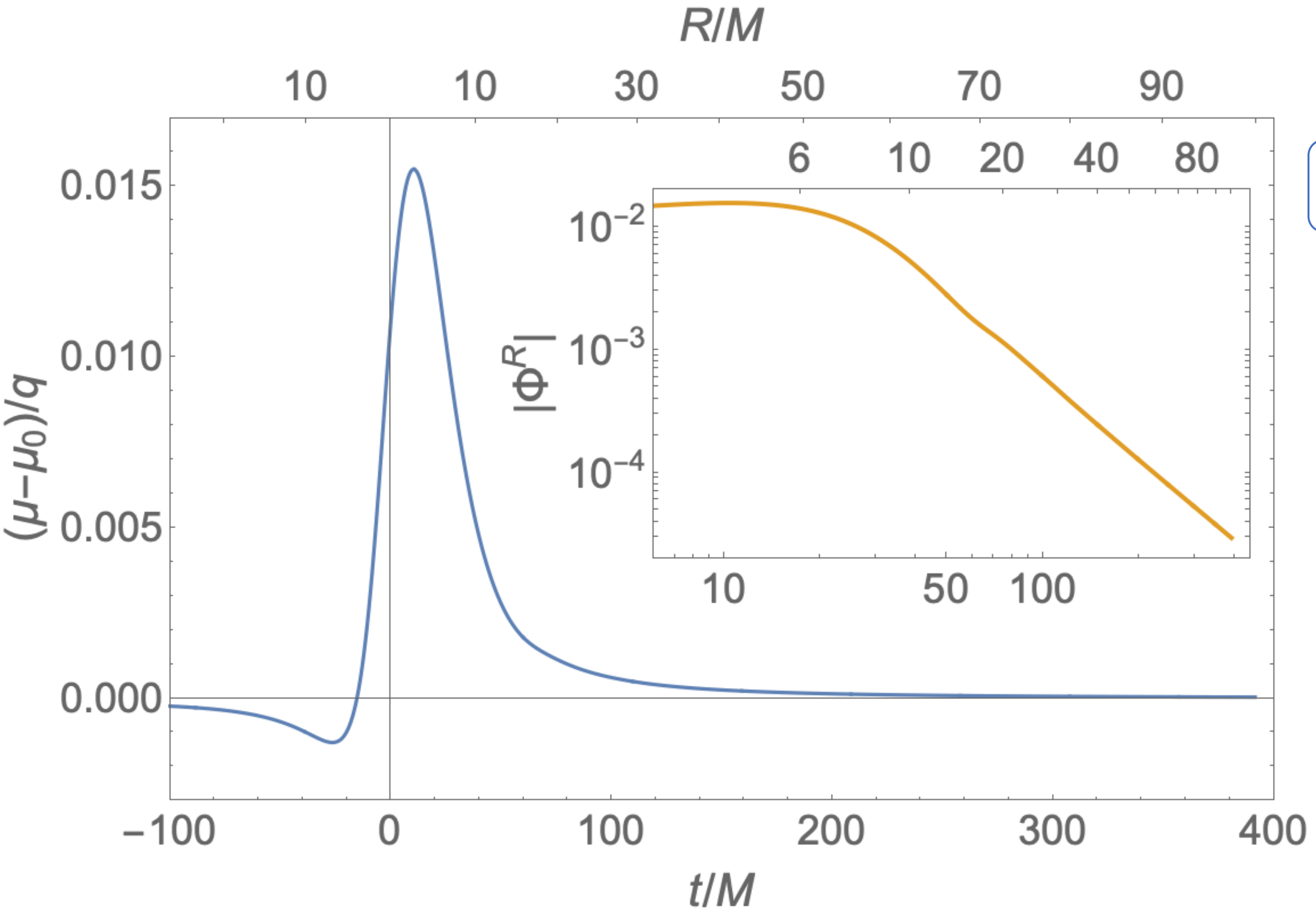
$$\mu\frac{du^\alpha}{d\tau} = q(\delta^\alpha_\beta + u^\alpha u_\beta)\nabla^\beta\Phi^R =: \mu\epsilon F^\alpha$$

$$\epsilon := \frac{q^2}{\mu M} \ll 1$$

Sample hyperbolic orbit

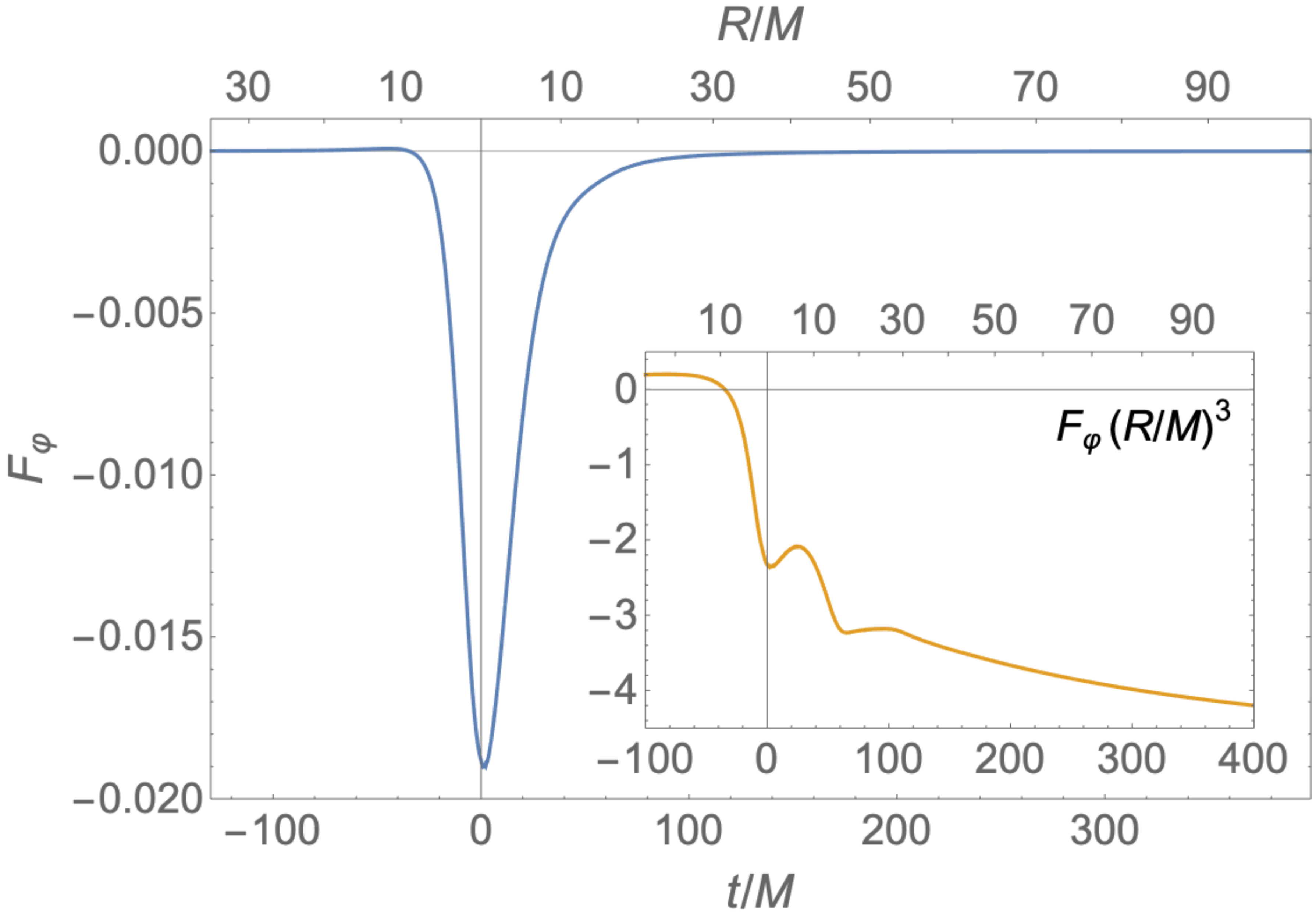


Change in mass results

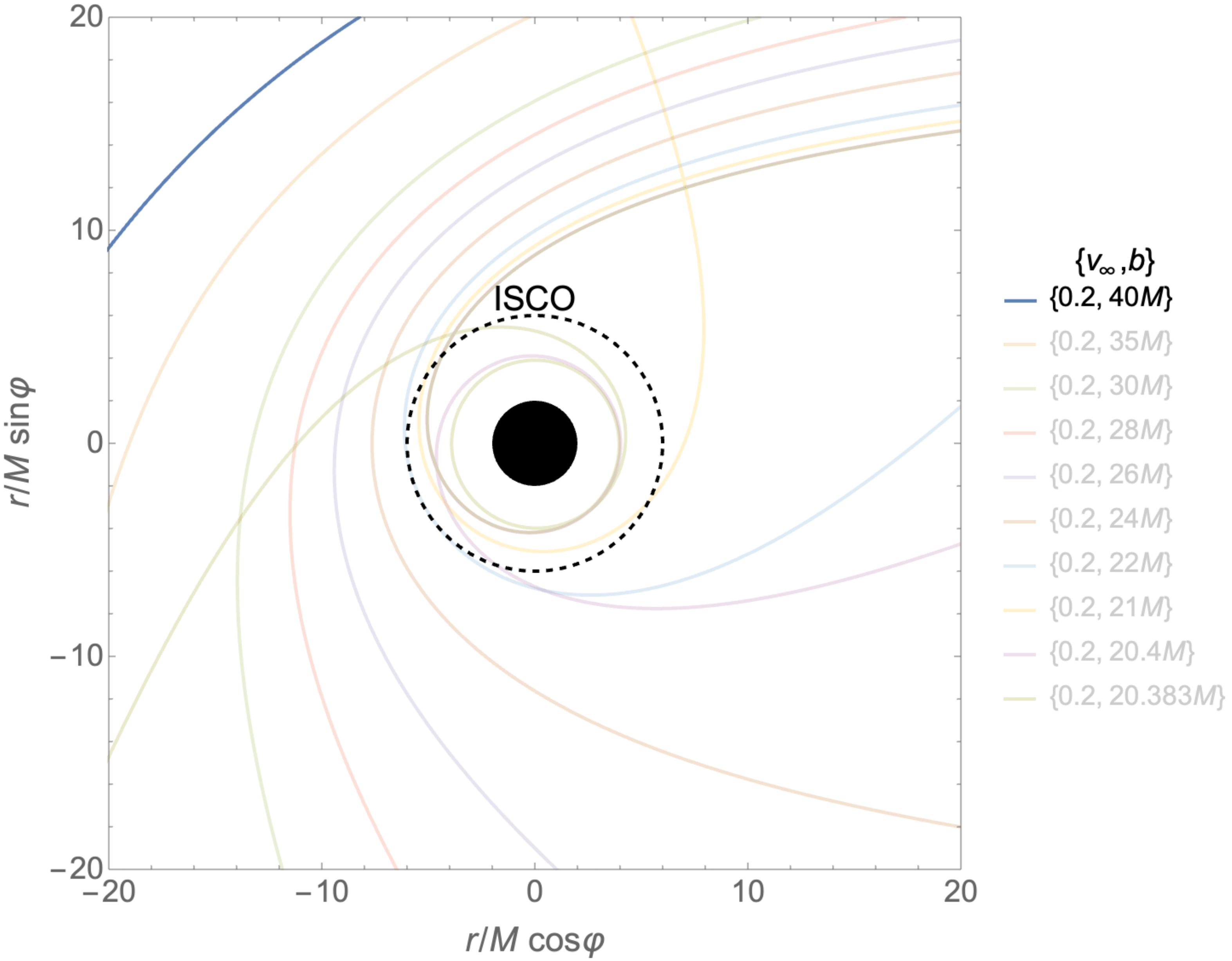


$$\mu = \mu_0 - q\Phi^R$$

Scalar self-force results

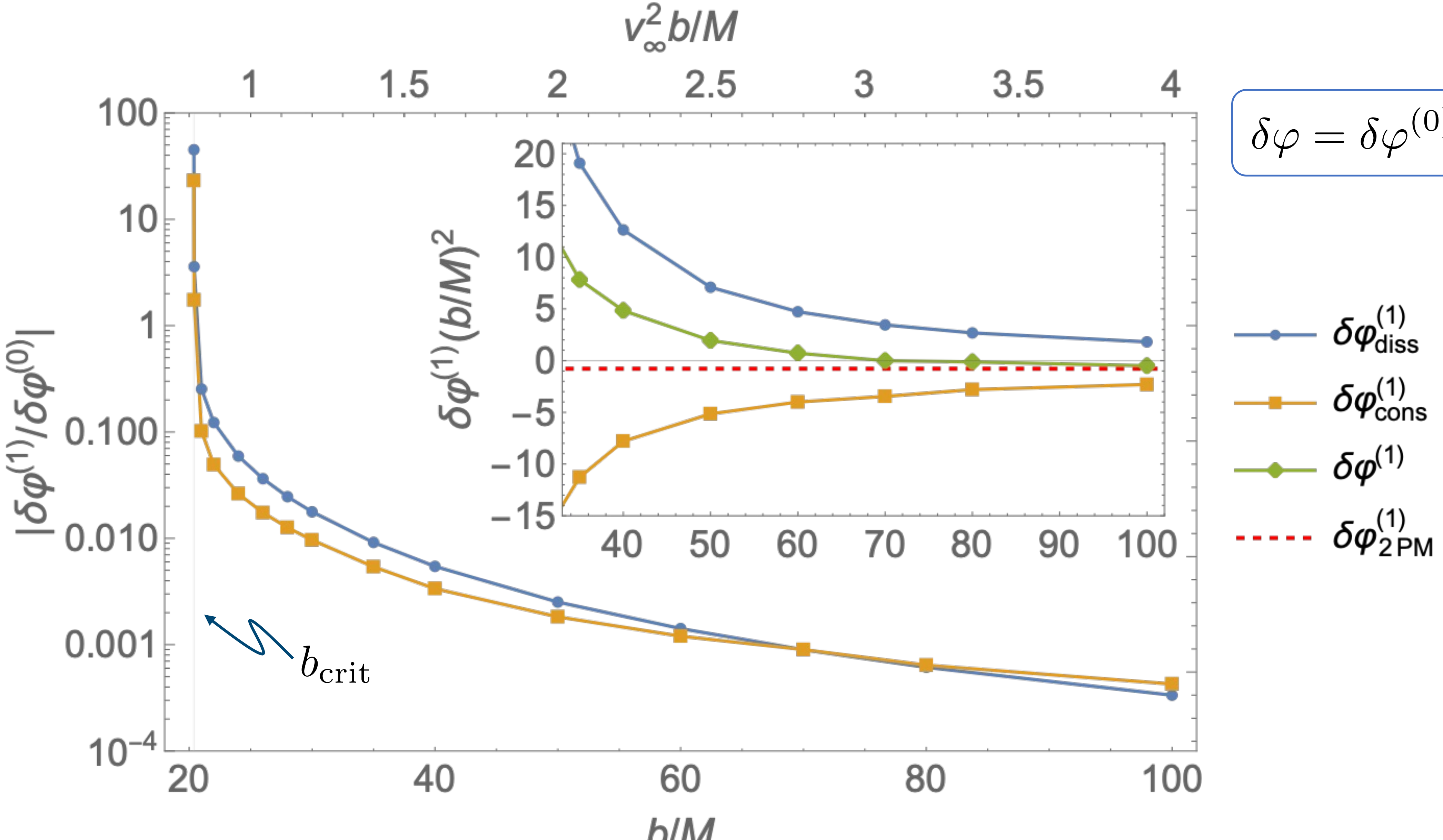


Sample scatter orbits: $v_\infty = 0.2$

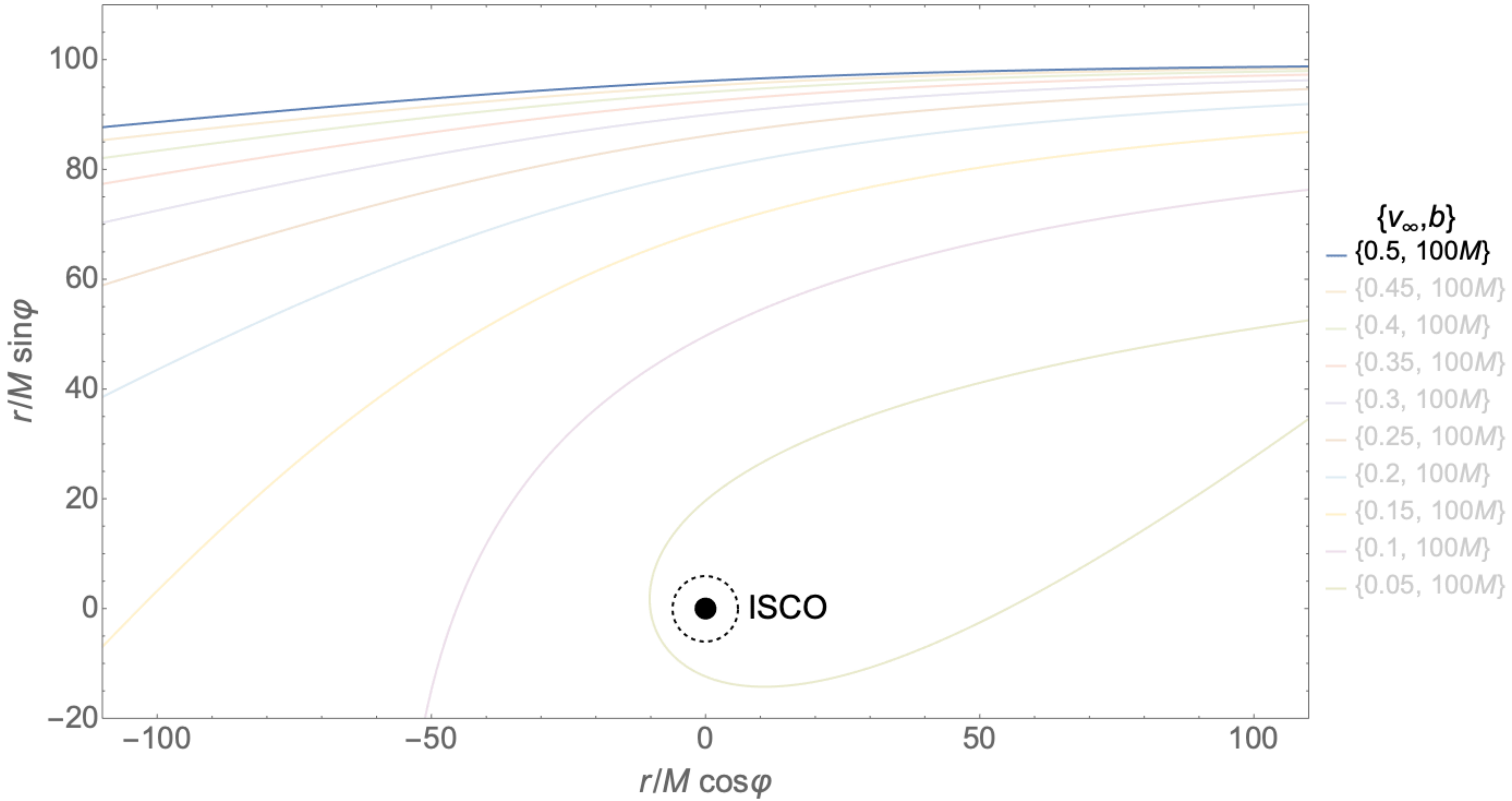


Scalar self-force correction to $\delta\varphi : v_\infty = 0.2$

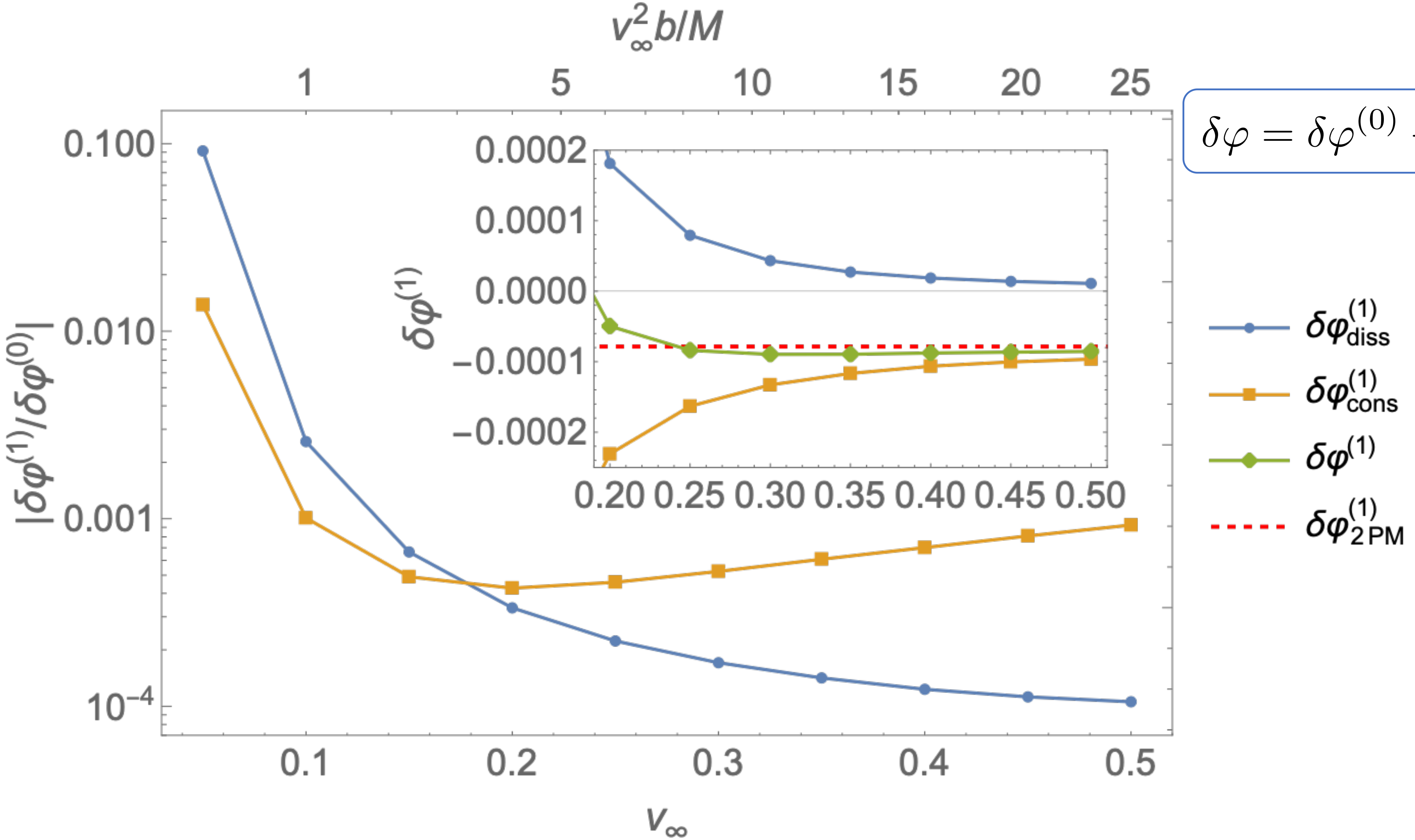
$$\delta\varphi = \delta\varphi^{(0)} + \epsilon\,\delta\varphi^{(1)}$$



Sample scatter orbits: $b = 100M$

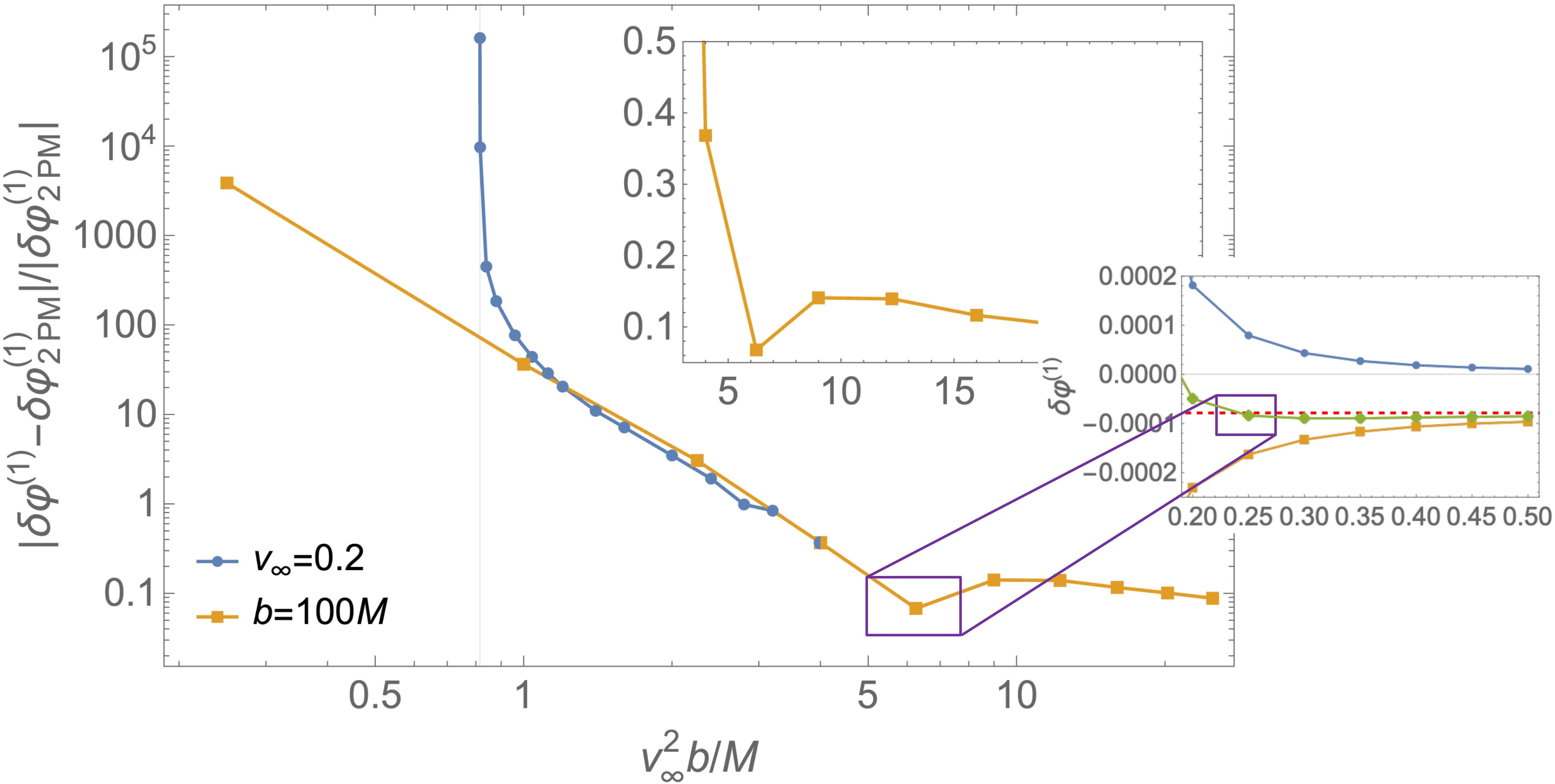


Scalar self-force correction to $\delta\varphi : b = 100M$



$$\boxed{\delta\varphi = \delta\varphi^{(0)} + \epsilon\,\delta\varphi^{(1)}}$$

Comparison to post-Minkowsian results



Future work

- Gravitational self-force for hyperbolic orbits:
  - Spectral code with hyperboloidal slicing (see Rodrigo Panosso Macedo's talk)
  - Gravitational self-force correction to the scatter angle.
- Interfaces with other two-body GR models:
  - Informing other models – PM and EOB.
  - Comparisons with other models – PM, PN, NR, QFT and EFT.
- Extension to a Kerr background and second-order self-force.

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GIFs available at www.oliverlong.info/gifs