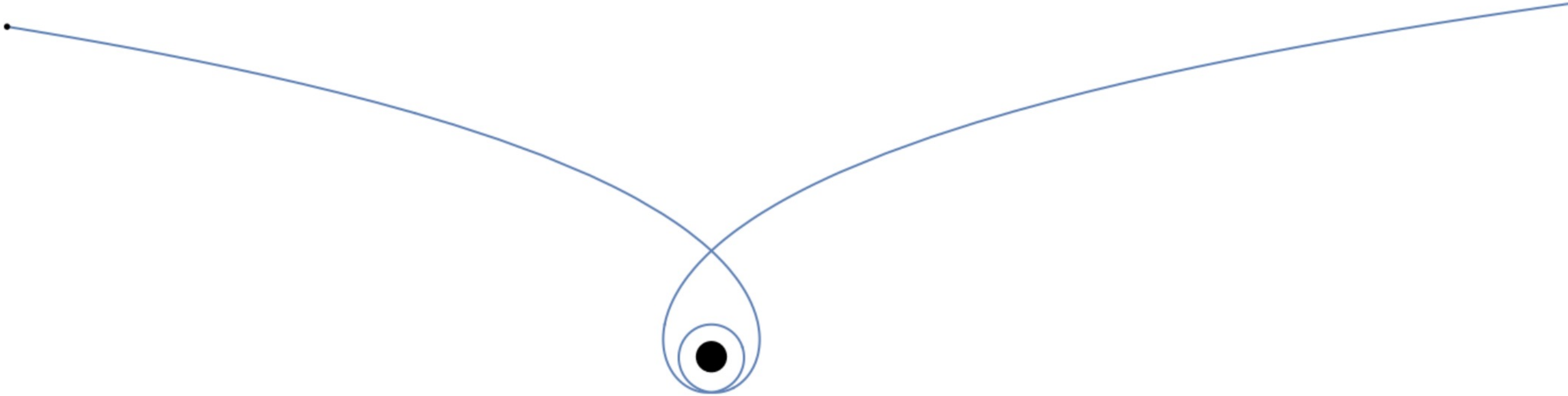
Self-force in hyperbolic black hole encounters



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with Leor Barack

GR23 5th July 2022

University of Southampton

STAG RESEARCH CENTRE

EMRIs: Expanding in the mass ratio

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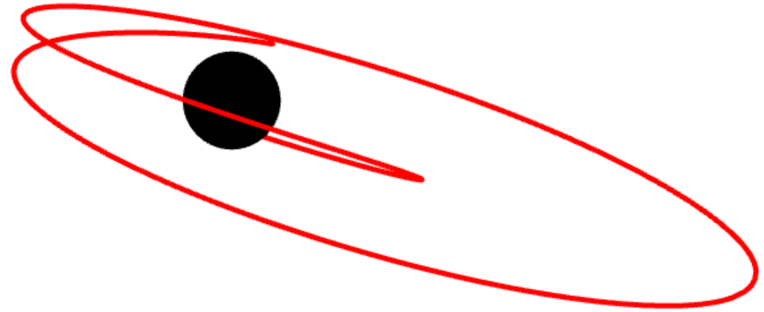
- EMRI: compact object orbiting a supermassive BH with mass ratio:

$$\eta := \frac{m}{M} < 10^{-4}$$



- Expand quantities in the mass ratio:

$$\mathrm{g}_{\alpha\beta} = g_{\alpha\beta} + \eta h^{(1)}_{\alpha\beta} + \eta^2 h^{(2)}_{\alpha\beta} + \dots$$

Schwarzschild/Kerr

- GWs from EMRIs can be detected with upcoming LISA detector.

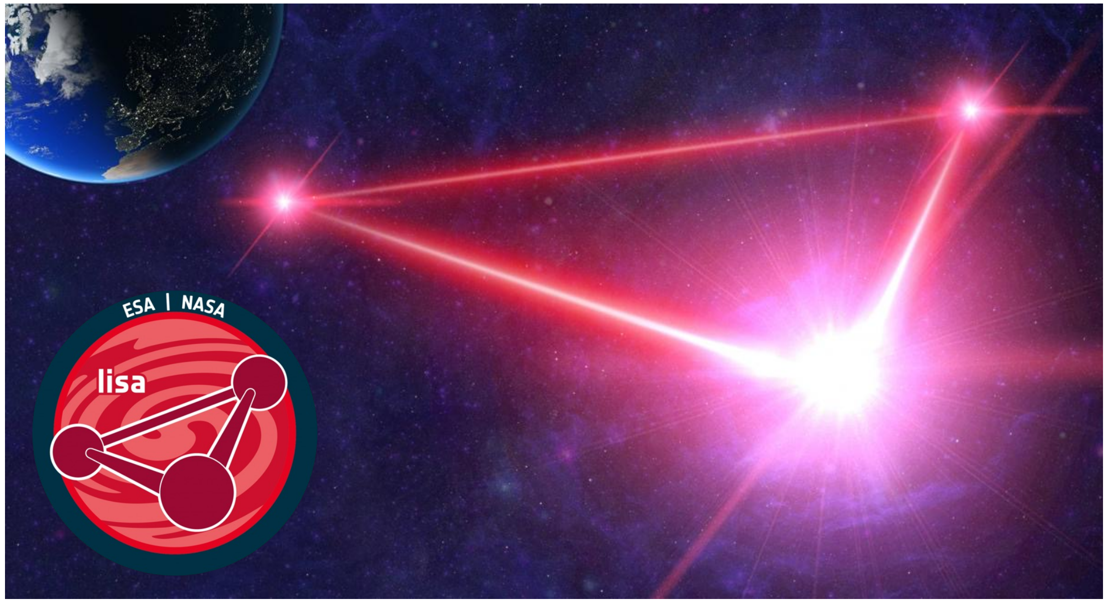
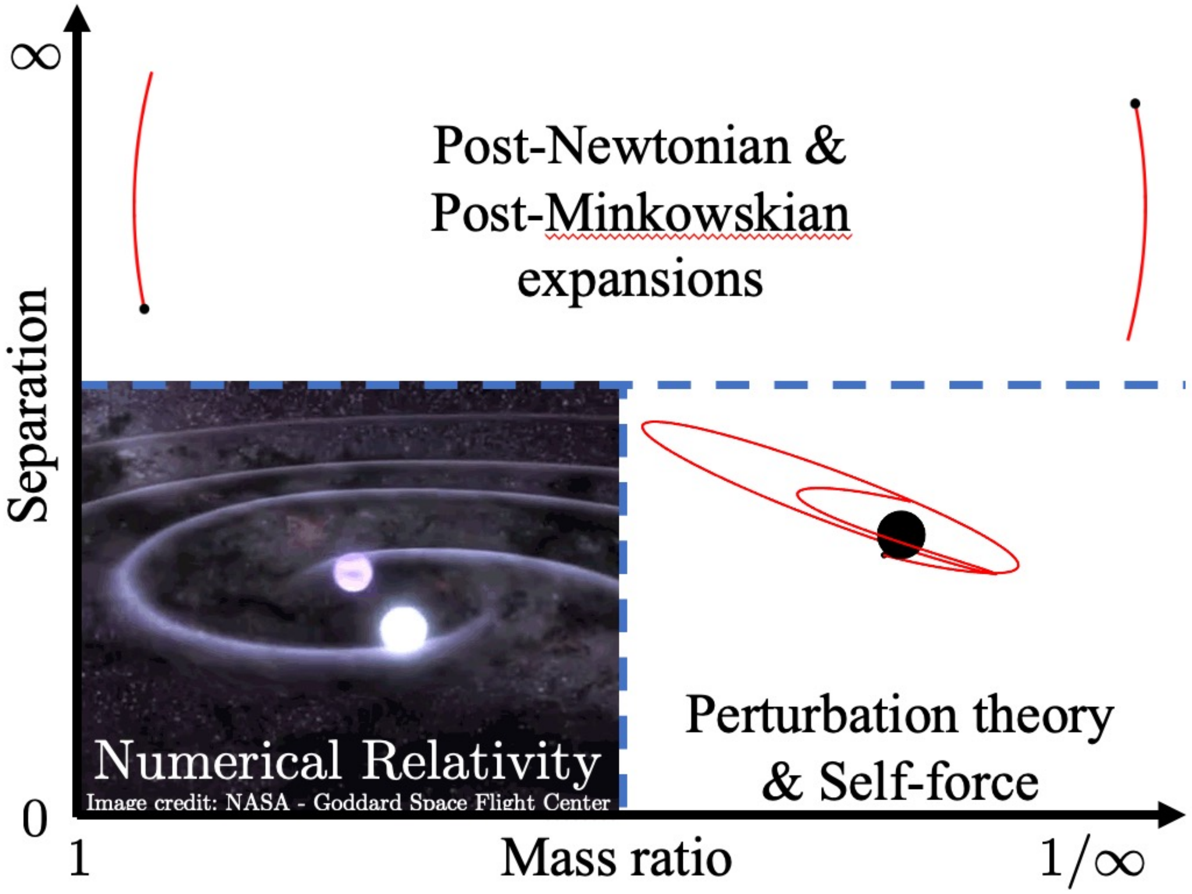


Image credit: NASA/JPL-Caltech/NASAEA/ESA/CXC/STScl/GSFCSVS/S.Barke

Motivation: EMRI scatter orbits

- Exact post-Minkowskian calculations for bound orbits [Damour ’19; Bini, Damour & Geralico ’20].



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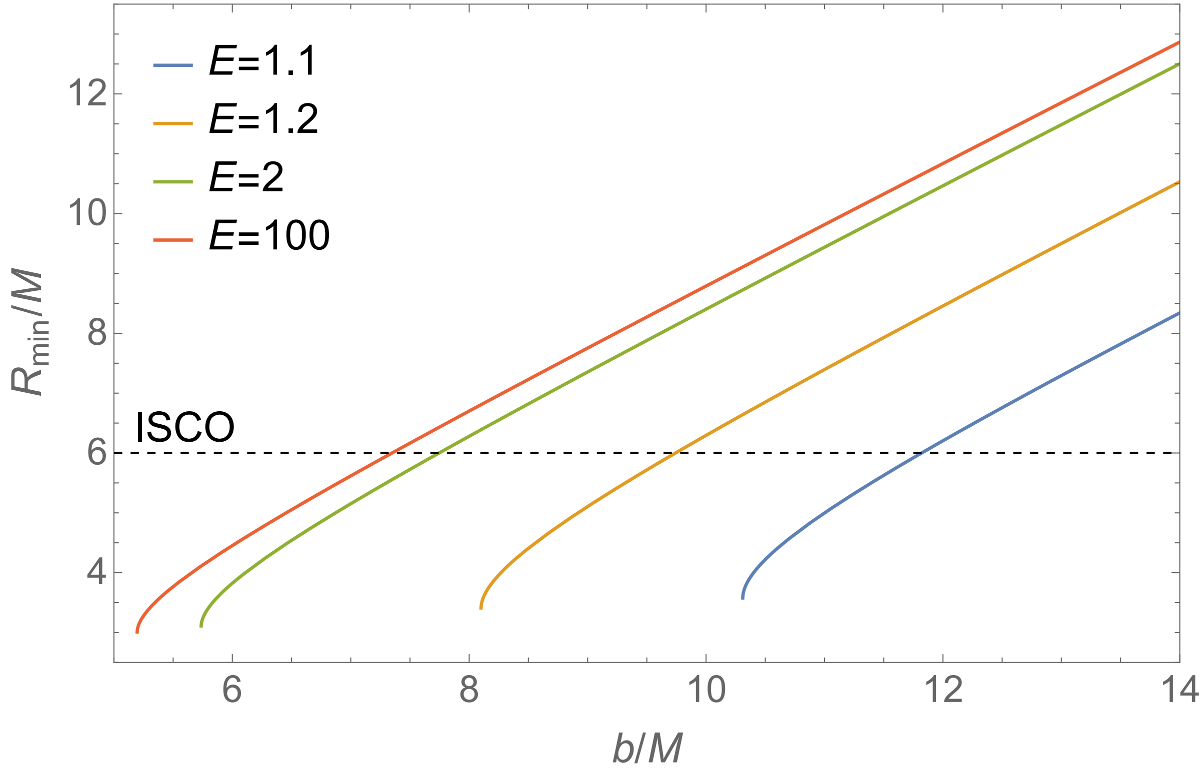
- Calibration of Effective-One-Body (EOB) in the ultra-strong field.



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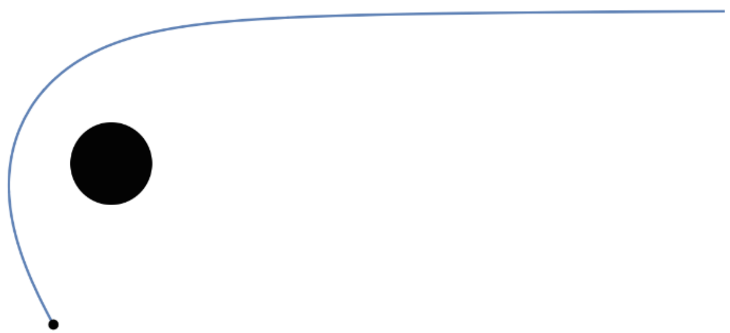
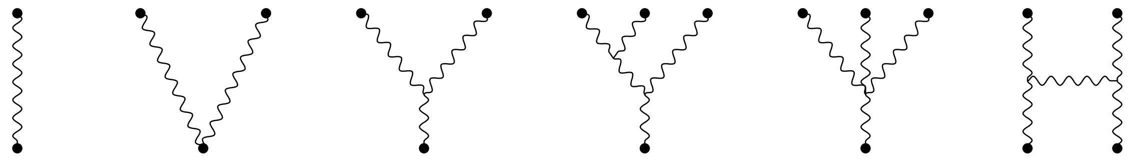


General Relativity

High-energy physics



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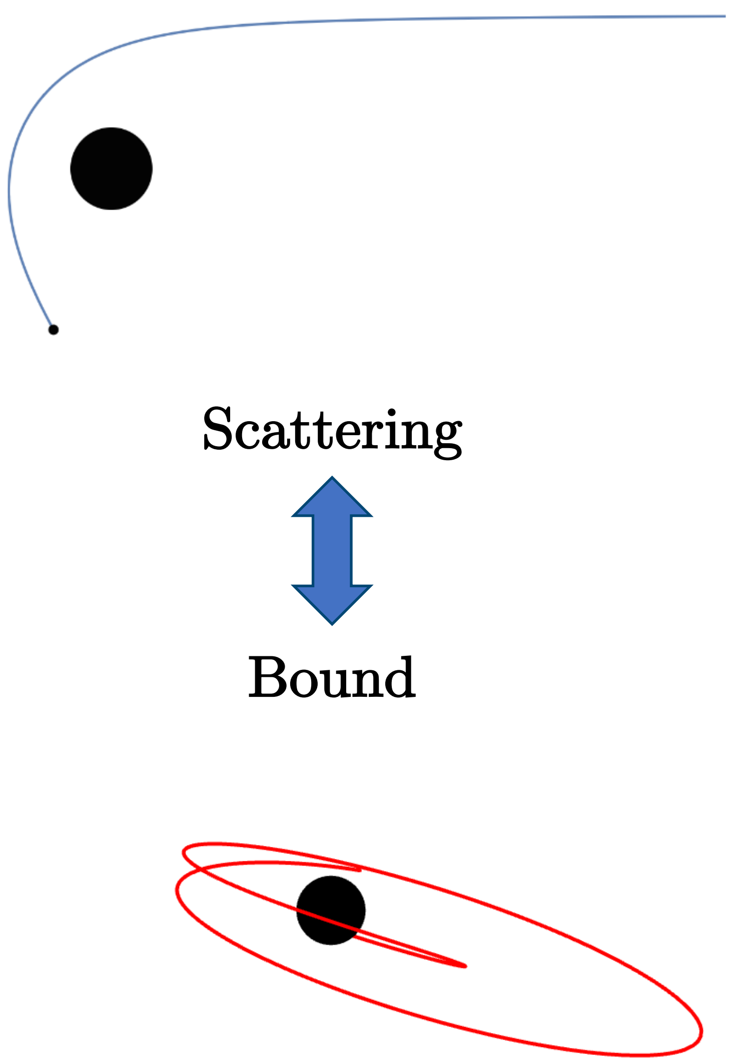
- Dictionary between scatter and bound [Cho et al. '21].
    - Scatter angle ↔ periastron advance.



Scattering geodesics

Energy and angular momentum:

$$E > 1 \qquad\qquad L > L_{\mathrm{crit}}(E)$$

Velocity at infinity and impact parameter:

$$v_\infty := \left.\frac{dr}{dt}\right|_{r\to\infty} \qquad\qquad b := \lim_{r\to\infty} r\sin|\varphi(r) - \varphi(\infty)|$$

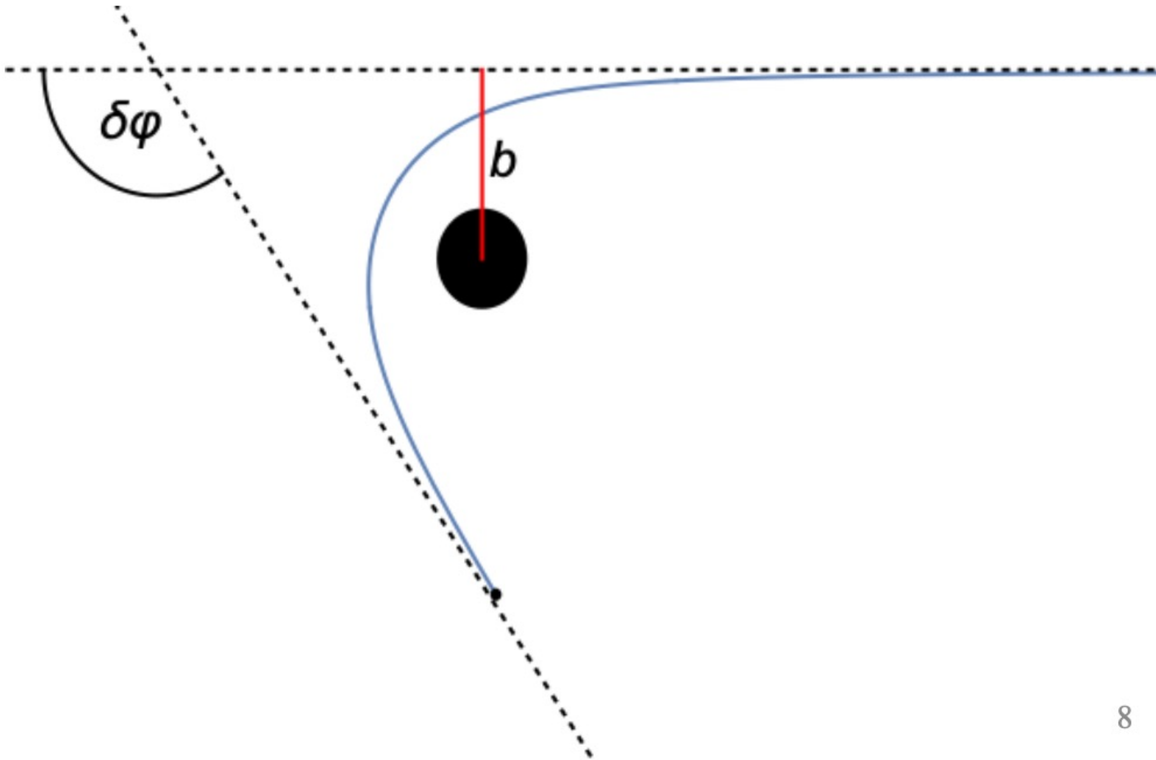
Geodesics:

$$\frac{dt}{d\tau} = \frac{\boxed{E}\,r}{r-2M} \qquad\qquad \frac{d\varphi}{d\tau} = \frac{\boxed{L}}{r^2}$$

$$\left(\frac{dr}{d\tau}\right)^2 = \boxed{E^2} - V(\boxed{L};r)$$

Scatter angle:

$$\delta\varphi := \int_{-\infty}^{\infty} \frac{d\varphi}{dt}dt - \pi$$



Self-forced equations of motion

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Equations of motion:

$$\dot{E} = -\eta F_t \qquad \dot{L} = \eta F_\varphi$$

Integrate:

$$E = E_\infty - \eta \int_{-\infty}^{\tau} F_t \, d\tau \qquad L = L_\infty + \eta \int_{-\infty}^{\tau} F_\varphi \, d\tau$$

Geodesic

Can split self-force into conservative and dissipative pieces:

$$F_\alpha^{\text{cons}}(r, \dot{r}) = -F_\alpha^{\text{cons}}(r, -\dot{r}) \qquad F_\alpha^{\text{diss}}(r, \dot{r}) = F_\alpha^{\text{diss}}(r, -\dot{r}) \qquad \alpha = t, \varphi$$

Dissipative self-force removes energy and angular momentum from the system.

Self-force correction to the scatter angle

Scatter angle as a radial integral:

$$\delta\varphi = \sum_{\pm} \int_{r_p^\pm}^{\infty} \frac{\dot\varphi^\pm}{\dot r^\pm} dr - \pi = \sum_{\pm} \int_{r_p^\pm}^{\infty} \frac{H^\pm(r; E, L)}{\sqrt{r - r_p^\pm}} dr - \pi$$

Substitute in time-varying E and L [OL & Barack, in prep]:

$$\delta\varphi = \delta\varphi^{(0)} + \eta\delta\varphi^{(1)}$$

$$\boxed{\delta\varphi^{(1)} = \sum_{\pm} \int_{R_{\min}}^{\infty} \left[\mathcal{G}_E^\pm(r; E_\infty, L_\infty)\boxed{F_t^\pm} - \mathcal{G}_L^\pm(r; E_\infty, L_\infty)\boxed{F_\varphi^\pm}\right] dr}$$

Can split into conservative and dissipative pieces on outgoing leg:

$$\delta\varphi^{(1)}_{\text{cons}} = \int_{R_{\min}}^{\infty} \left[\mathcal{G}_E^{\text{cons}} F_t^{\text{cons}} - \mathcal{G}_L^{\text{cons}} F_\varphi^{\text{cons}}\right] dr \qquad \delta\varphi^{(1)}_{\text{diss}} = \int_{R_{\min}}^{\infty} \left[\mathcal{G}_E^{\text{diss}} F_t^{\text{diss}} - \mathcal{G}_L^{\text{diss}} F_\varphi^{\text{diss}}\right] dr$$

Scalar field evolution scheme

Decompose scalar field:

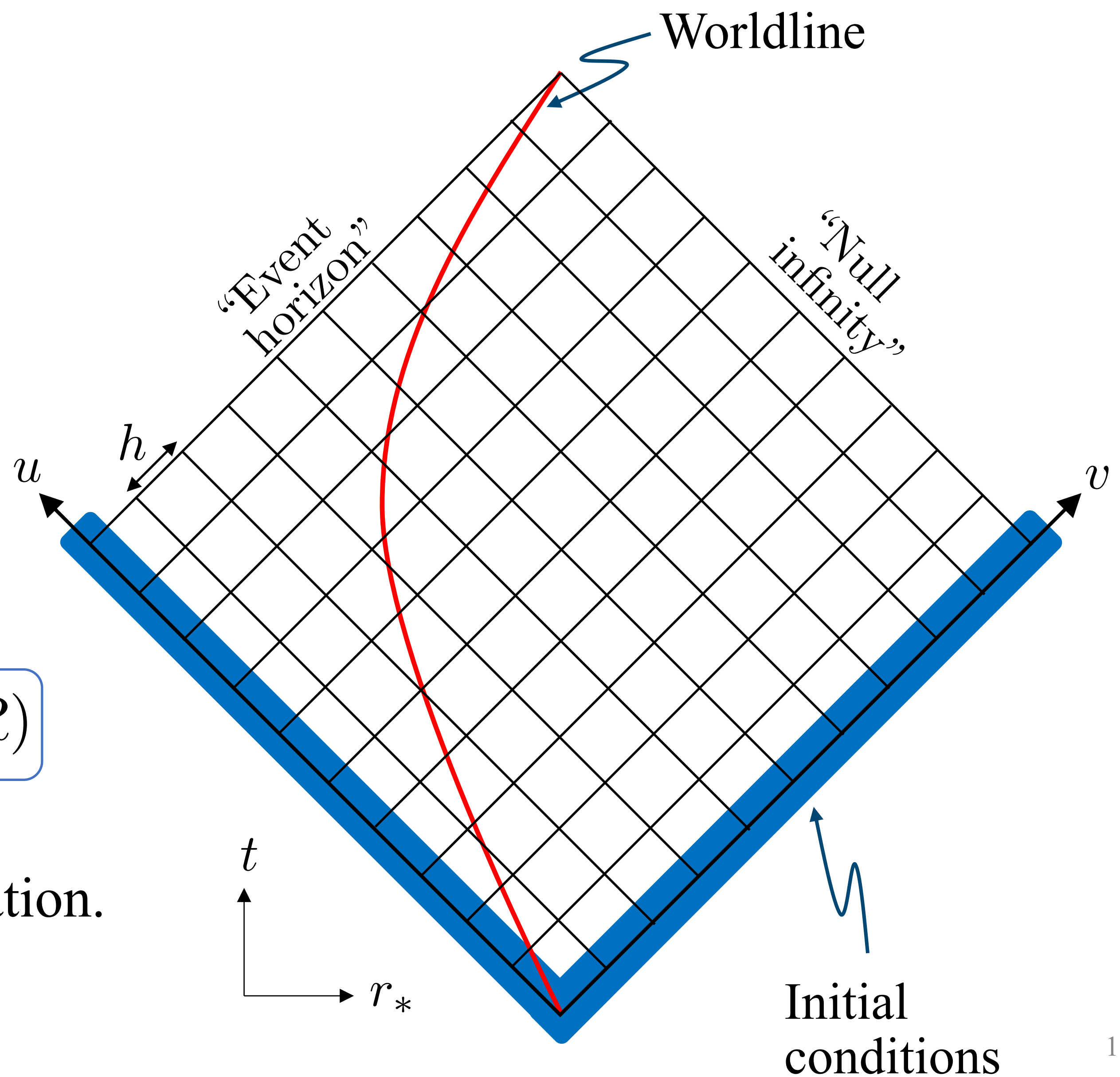
$$\Phi = \frac{2\pi q}{r} \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \psi_{\ell m}(t,r) Y_{\ell m}(\theta,\varphi)$$

$q:$ Scalar charge

1+1D scalar wave equation:

$$\boxed{\psi_{,uv} + V(\ell;r)\psi = S_\psi(\ell;x_p^\mu)\,\delta(r-R)}$$

Evolve finite-difference version of 1+1D equation.



Scalar self-force

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Equations of motion:

$$E(\tau) = E_\infty - \epsilon \int_{-\infty}^{\tau} F_t \, d\tau \qquad L(\tau) = L_\infty + \epsilon \int_{-\infty}^{\tau} F_\varphi \, d\tau \qquad \epsilon = \frac{q^2}{mM}$$

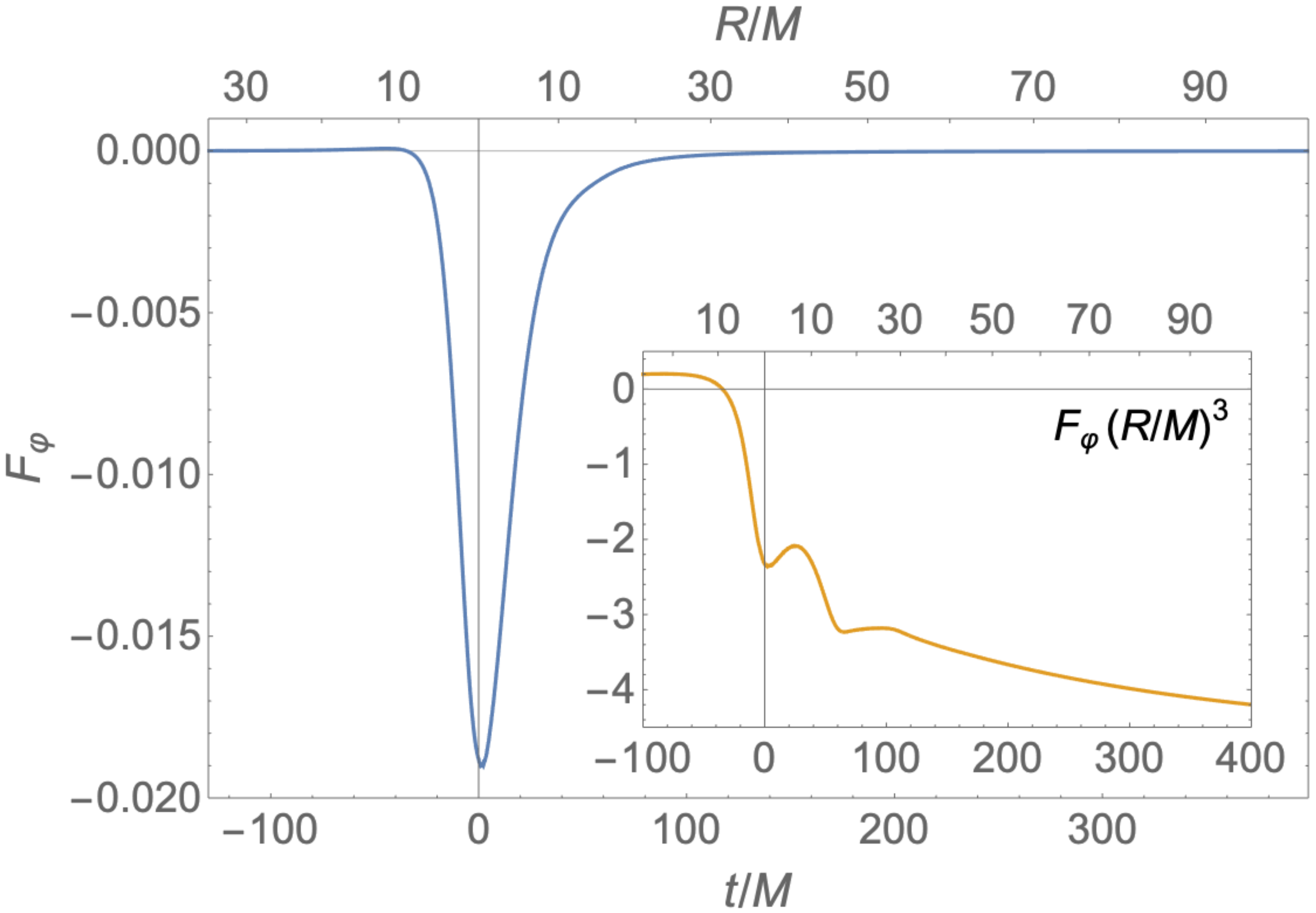
Derivatives of scalar field:

$$\tilde{F}_\mu := q \, \nabla_\mu \Psi \Big|_{x_p}$$

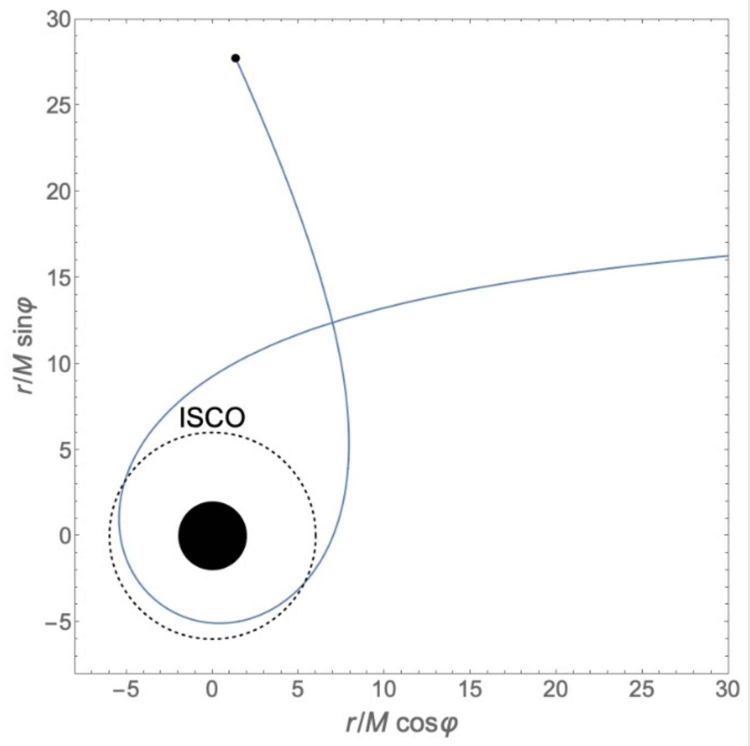
Projection orthonormal to four-velocity:

$$F_\mu = \left(\delta^\nu{}_\mu + u^\nu u_\mu\right) \tilde{F}_\nu \qquad u^\mu := \frac{dx_p^\mu}{d\tau}$$

Scalar self-force results

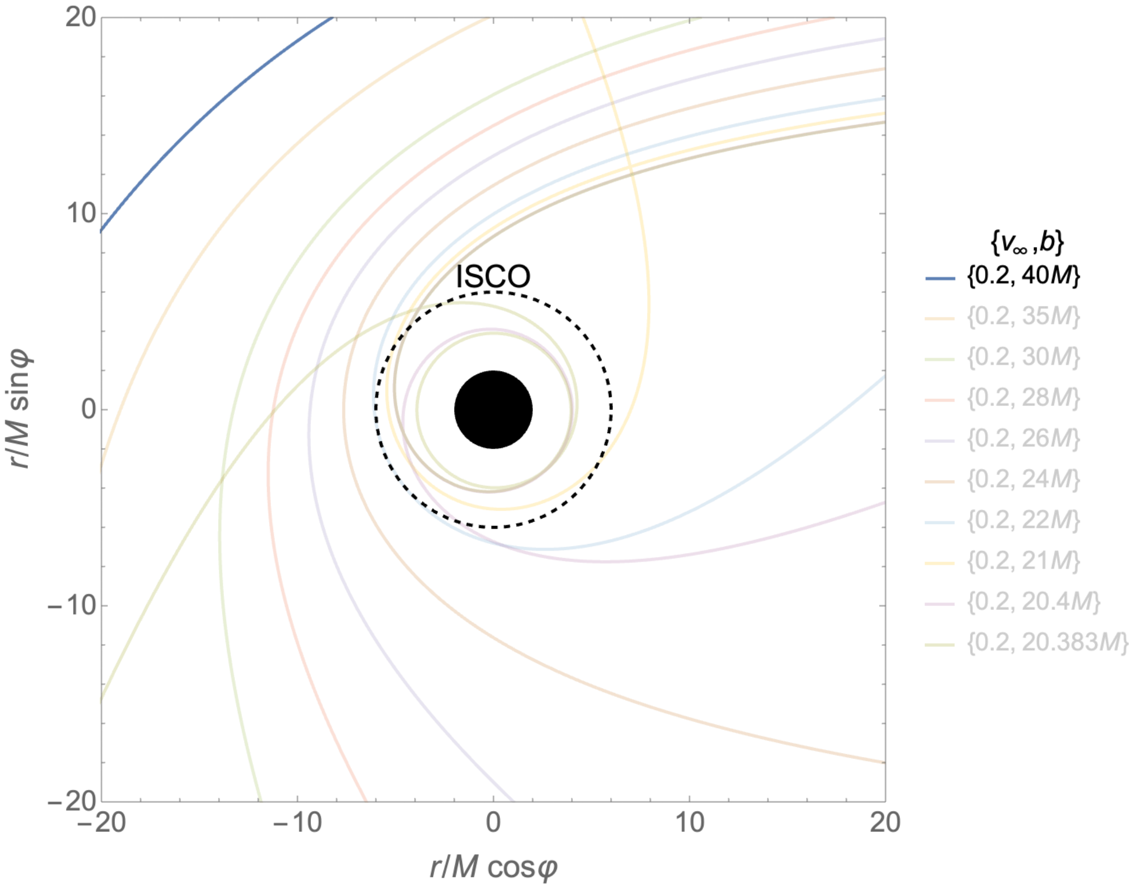


$v_\infty = 0.2$
$b = 21M$

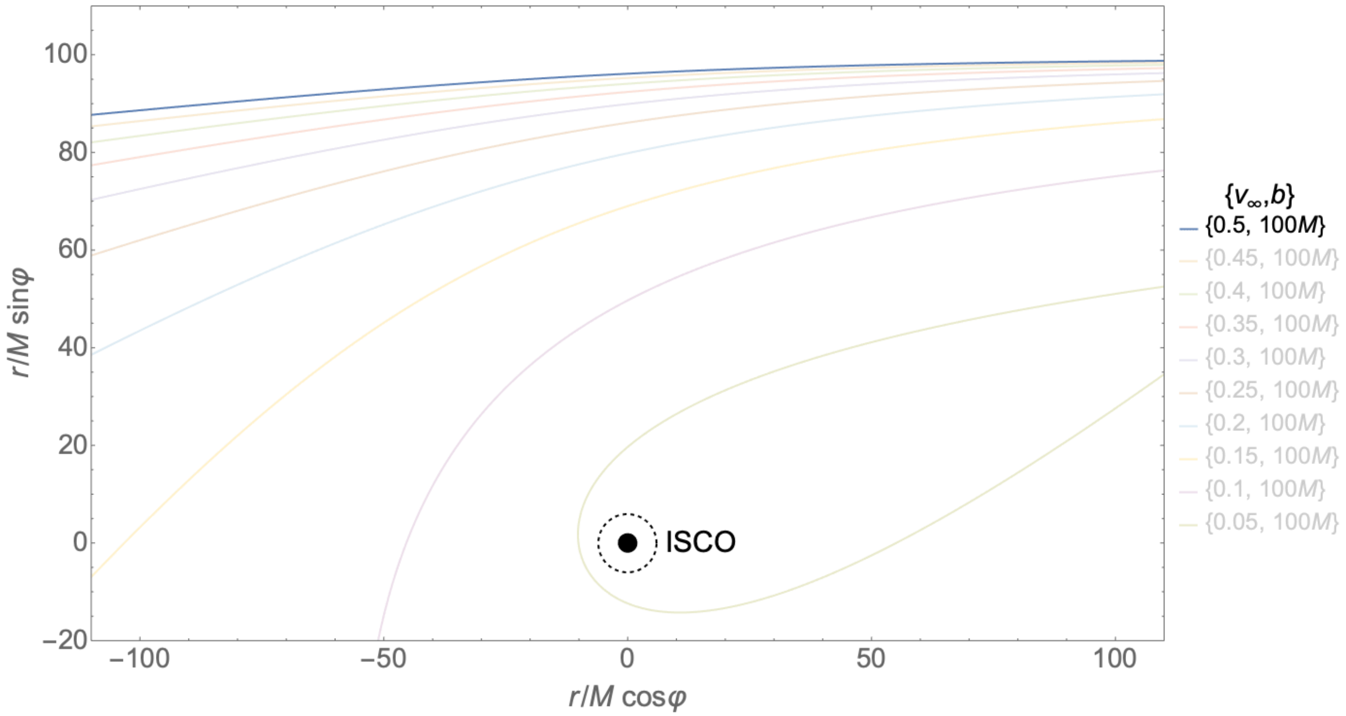


Sample scatter orbits

$$v_\infty = 0.2$$

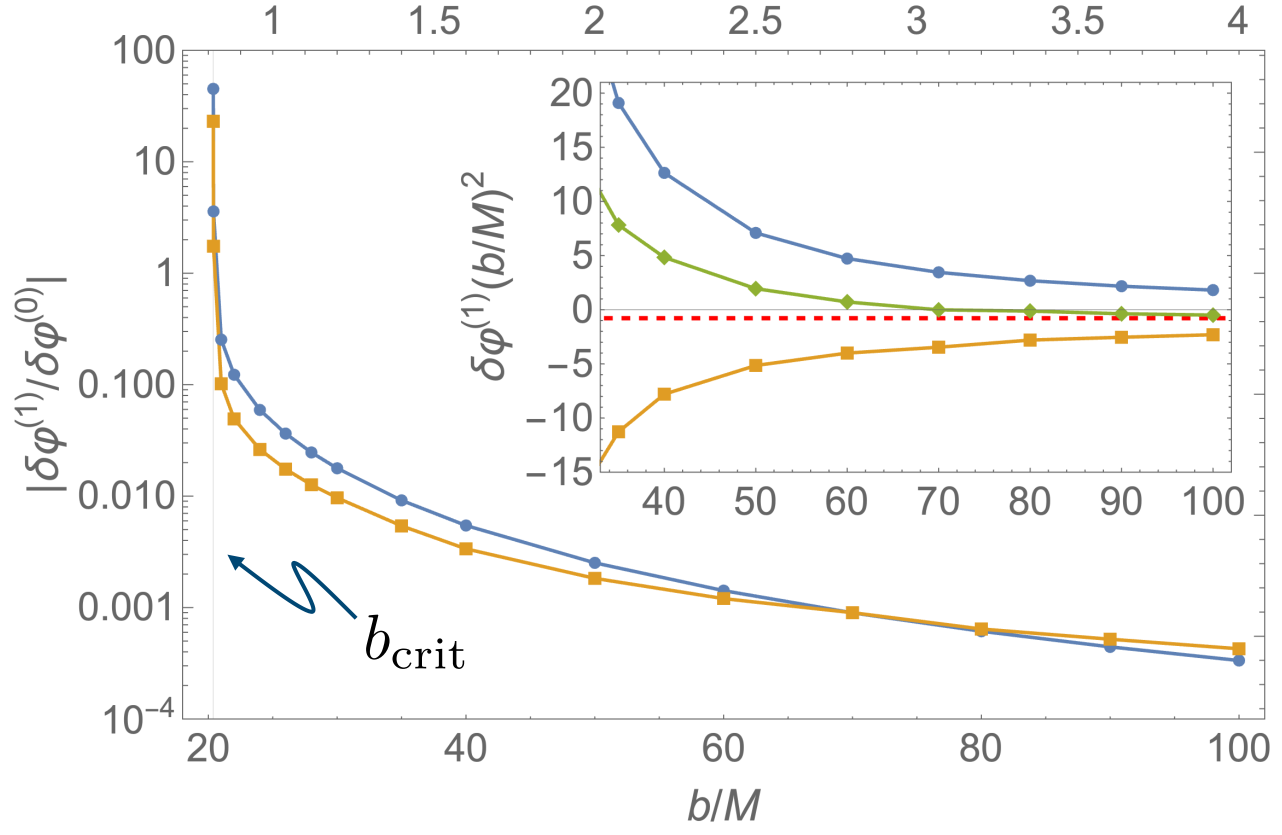


$$b = 100M$$



Scalar self-force correction to $\delta\varphi$

$v_\infty = 0.2$



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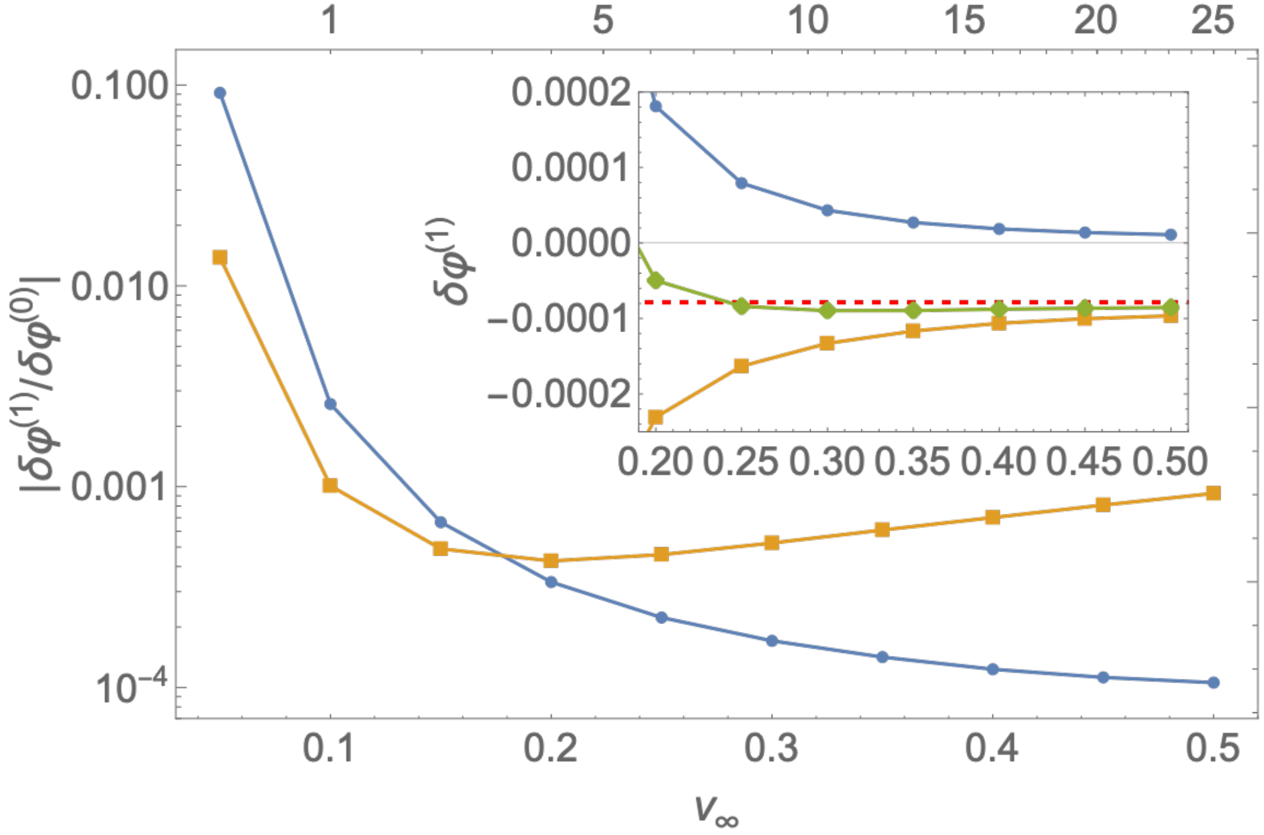
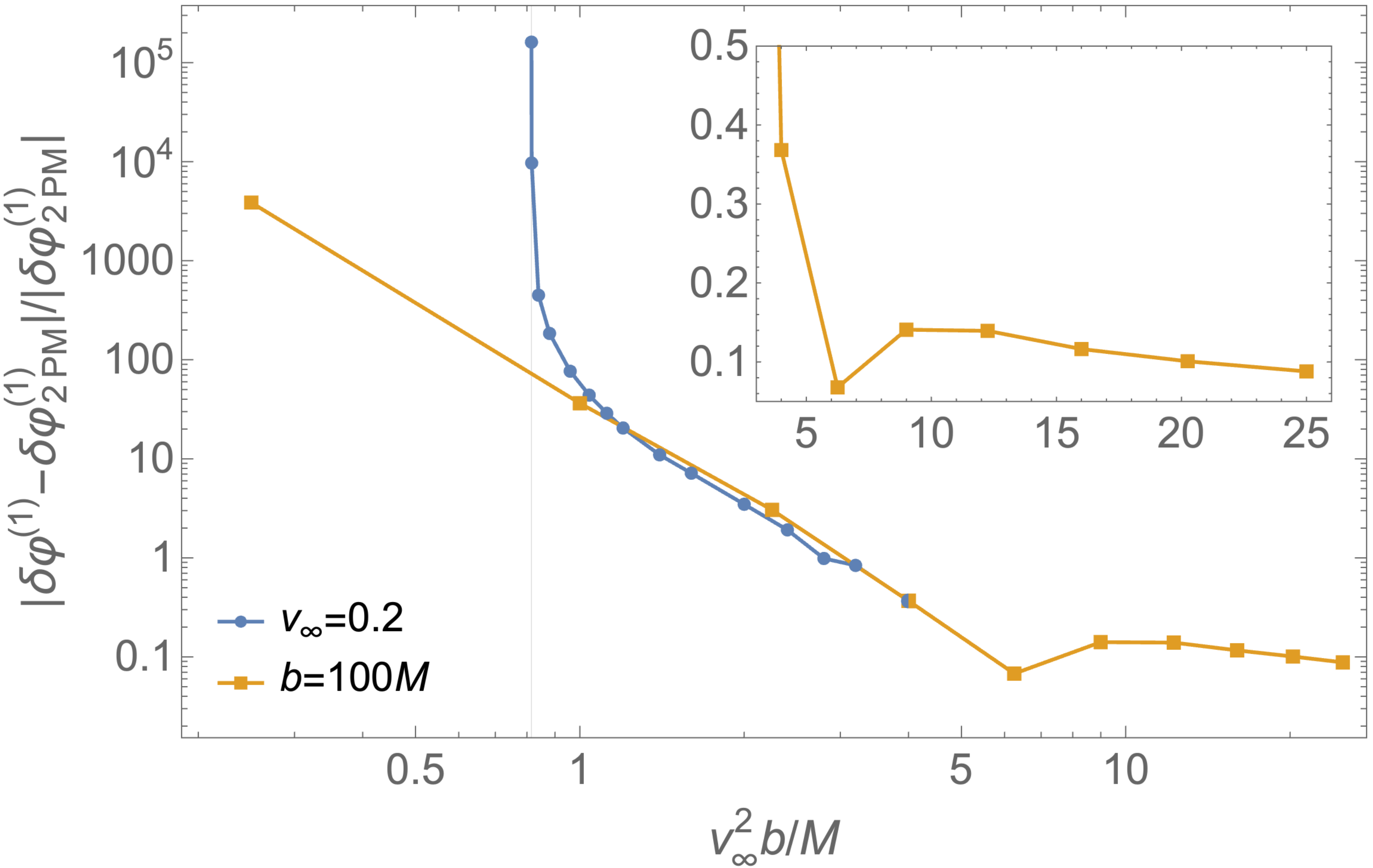


$$\boxed{\delta\varphi = \delta\varphi^{(0)} + \epsilon\,\delta\varphi^{(1)}}$$

$$\boxed{\delta\varphi^{(1)}_{\rm 2PM} = -\frac{\pi}{4}\left(\frac{M}{b}\right)^2}$$ [Gralla & Lobo '22]

Comparison to post-Minkowsian results



Future work

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- Gravitational self-force for hyperbolic orbits:
  - Gravitational self-force correction to the scatter angle.
- Interfaces with other two-body GR models:
  - Informing other models – PM and EOB.
  - Comparisons with other models – PM, PN, NR, QFT and EFT.
- Extension to a Kerr background and second-order self-force.

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GIFs available at www.oliverlong.info/gifs