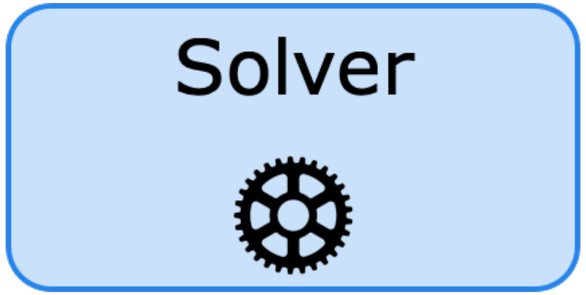
Small Proofs from Congruence Closure

Oliver Flatt, Samuel Coward, Max Willsey,

Zachary Tatlock, Pavel Panchekha

Solvers and Proofs



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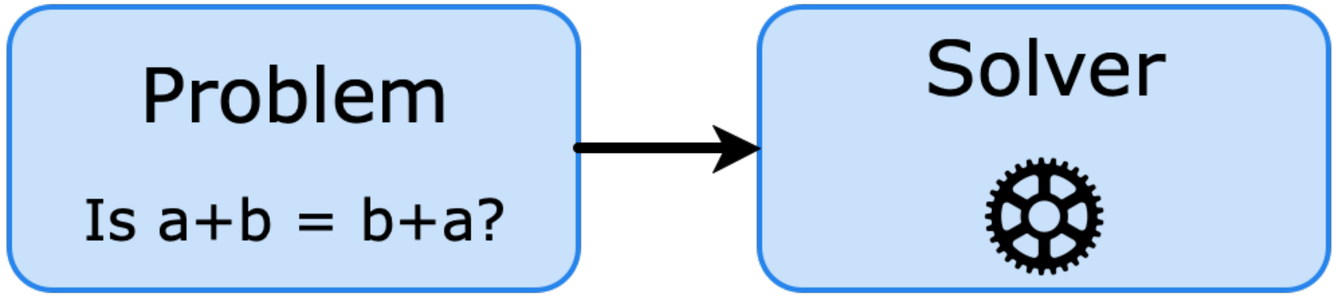

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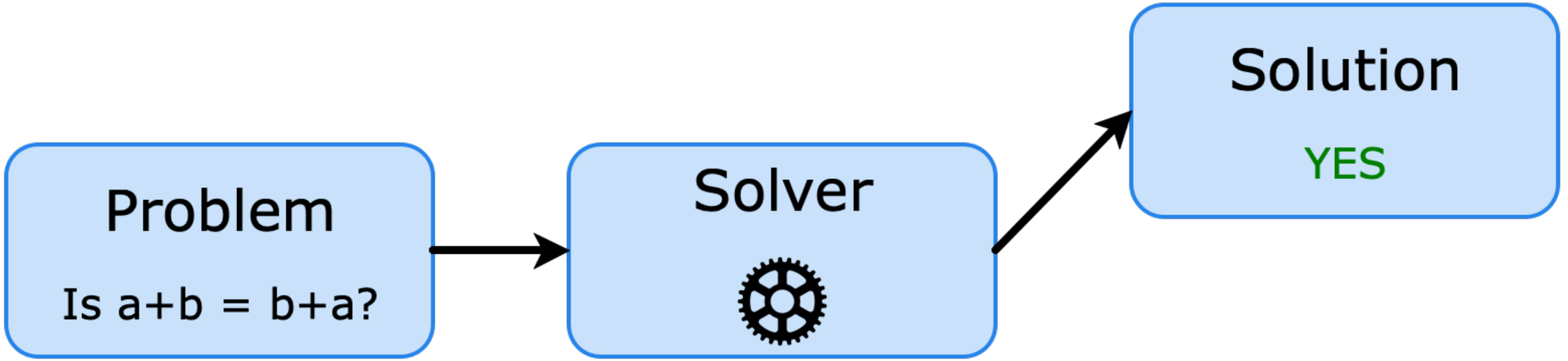

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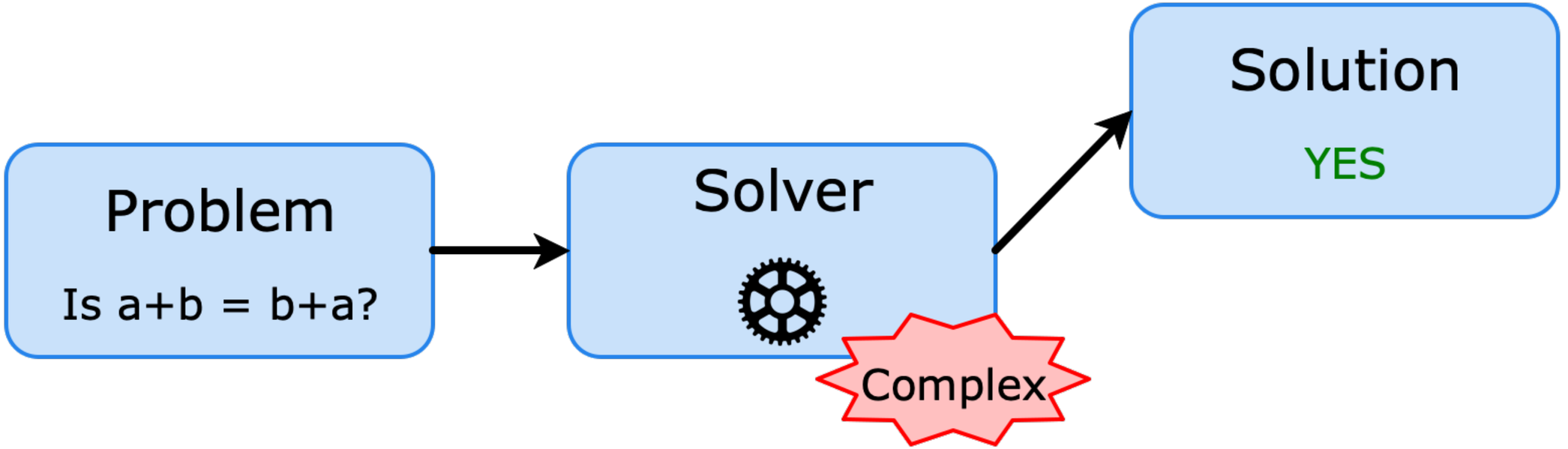

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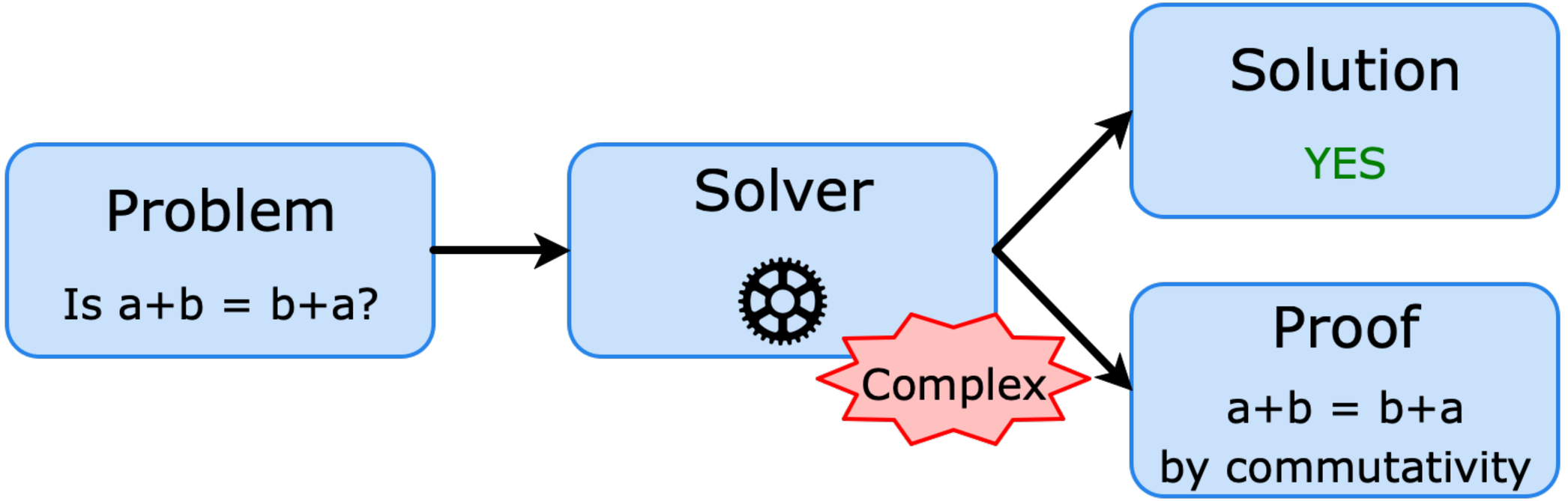

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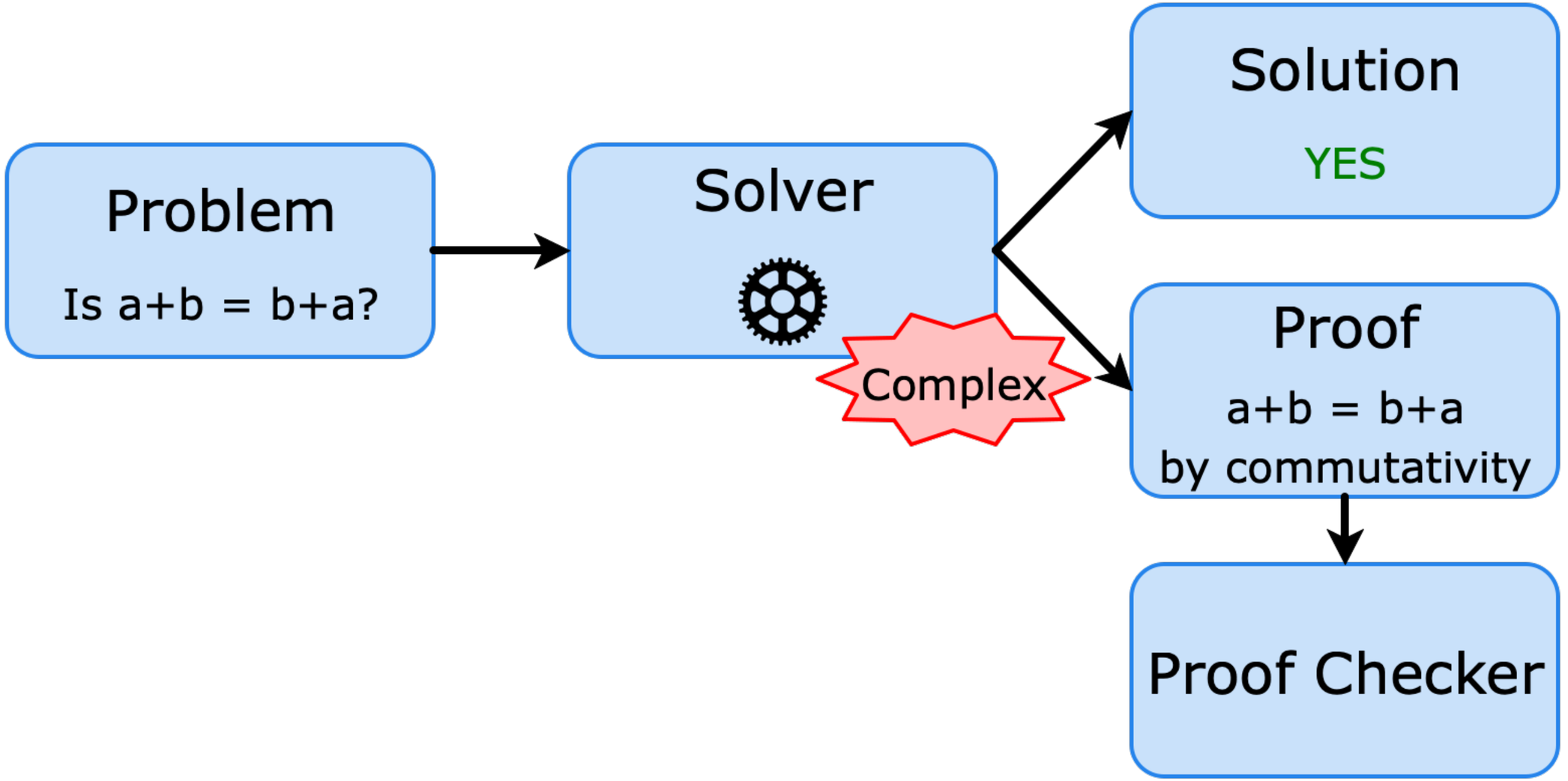

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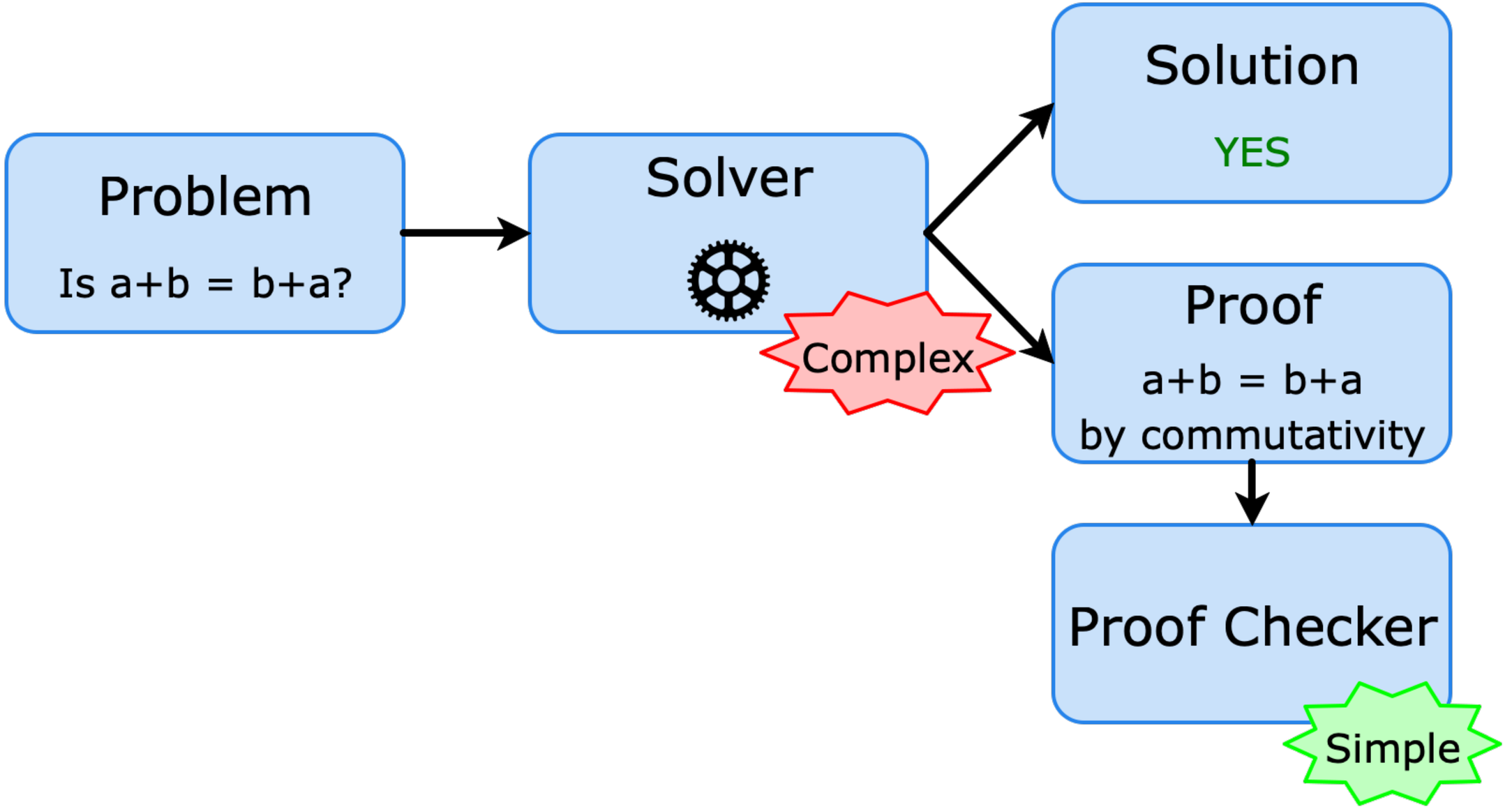

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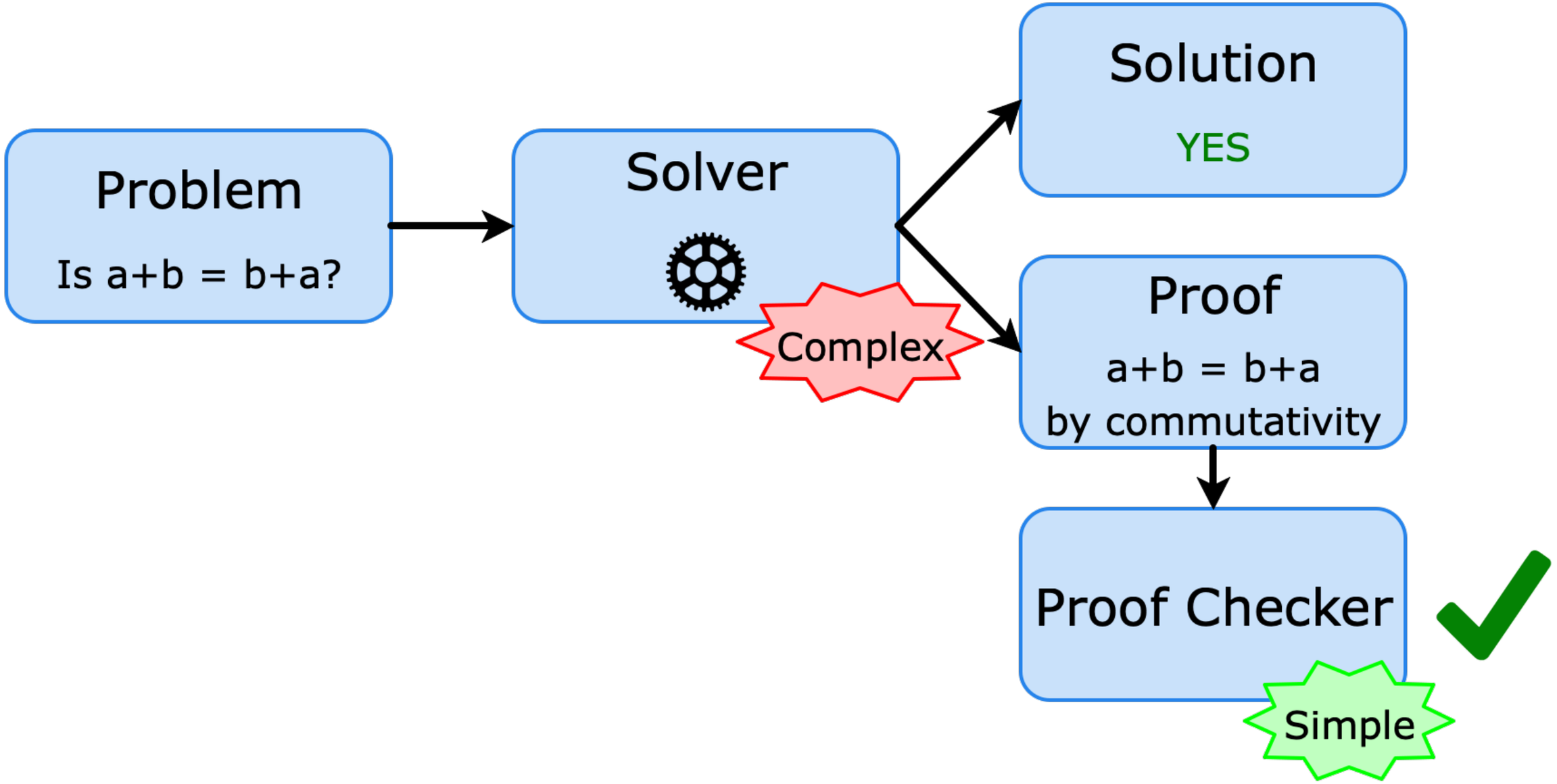

Proofs are Useful

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Checking

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Can we trust the solver?

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Debugging

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How did we prove 0 = 1?

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...And More

Fuzzing

Optimization

Proofs can be Long

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Too Specific

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Slow

Proofs can be Long

This Talk:

Finding **smaller** proofs from congruence closure

Why Congruence Closure?

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Congruence Closure forms the basis of many solvers

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Generates all proofs of **equality**

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Enables **equality saturation**

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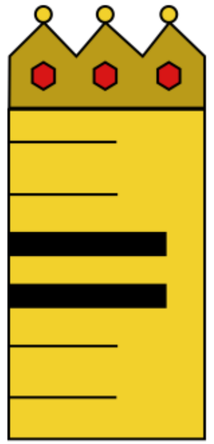

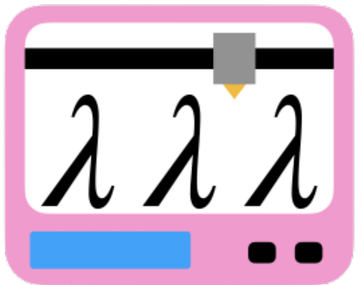

- [x] Motivation
- [ ] Congruence Closure
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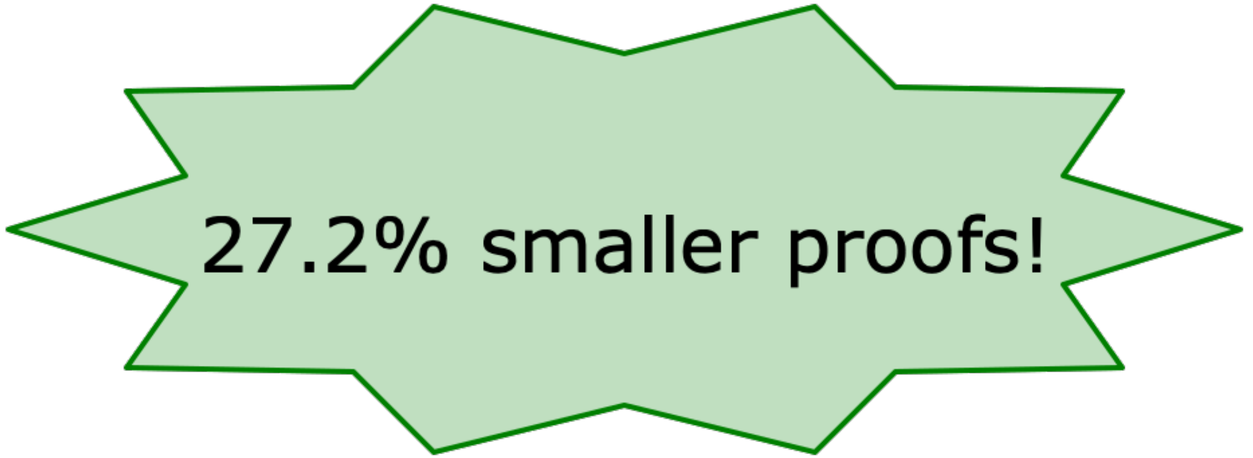

Congruence Closure

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Input: equalities between terms

$a = b$

$f(a) = f(b)$

$b = c$

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stored in an **e-graph** data structure

Ask: is $a = c$?

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stored in an **e-graph** data structure

Ask: is $a = c$?

The relation is also closed under **congruence**

$\forall\ x, y: x = y \Rightarrow f(x) = f(y)$

E-Graph Example

A graph with **3** kinds of edges

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Inputs:

f(a)

f(b)

a = b

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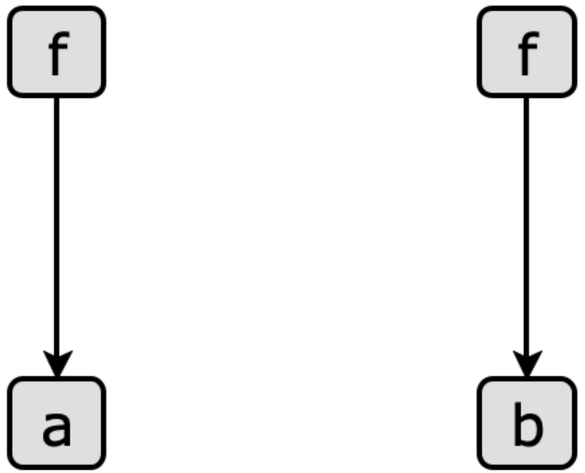

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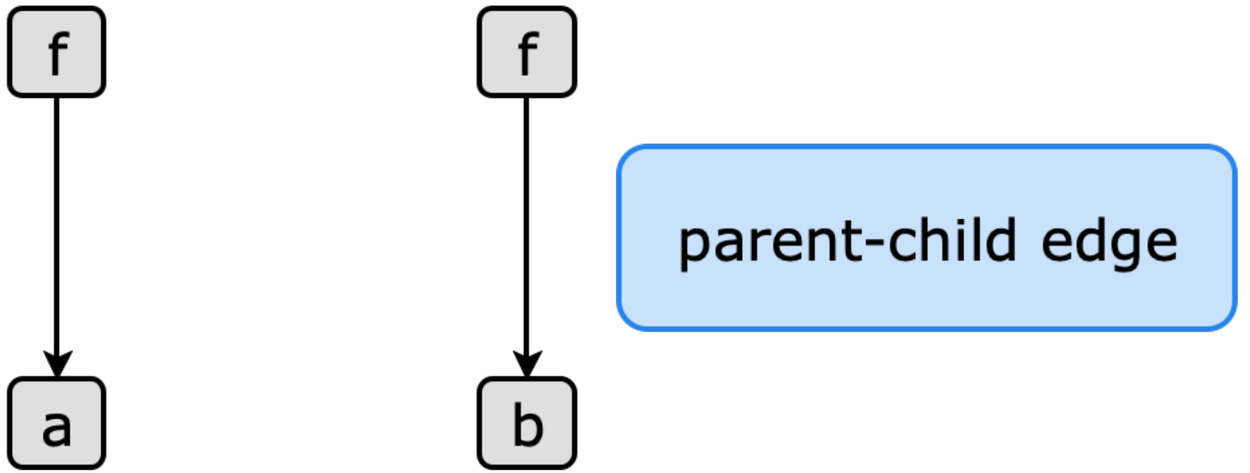

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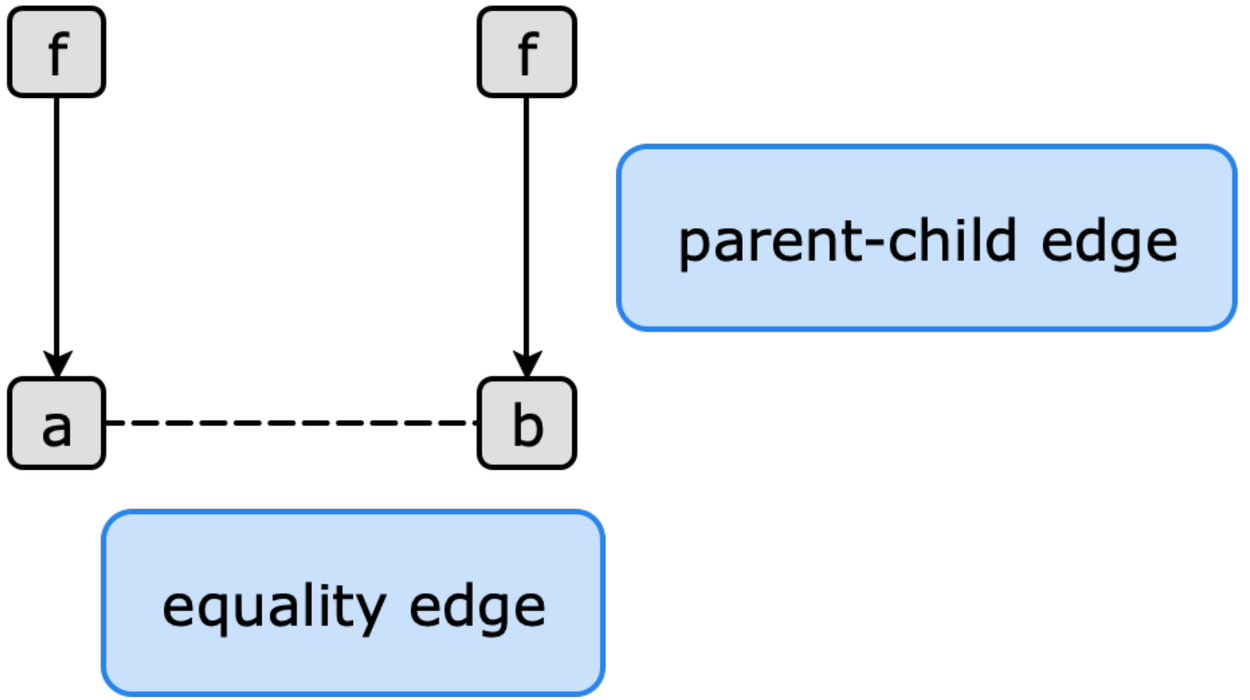

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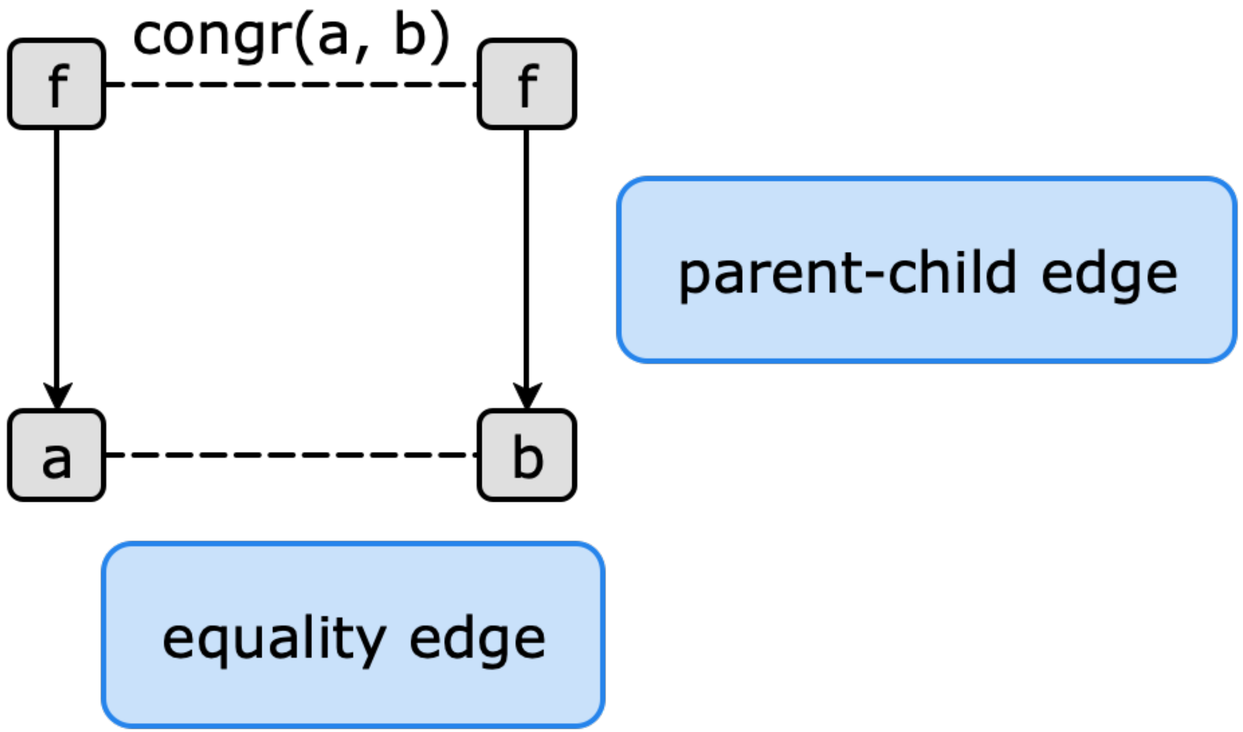

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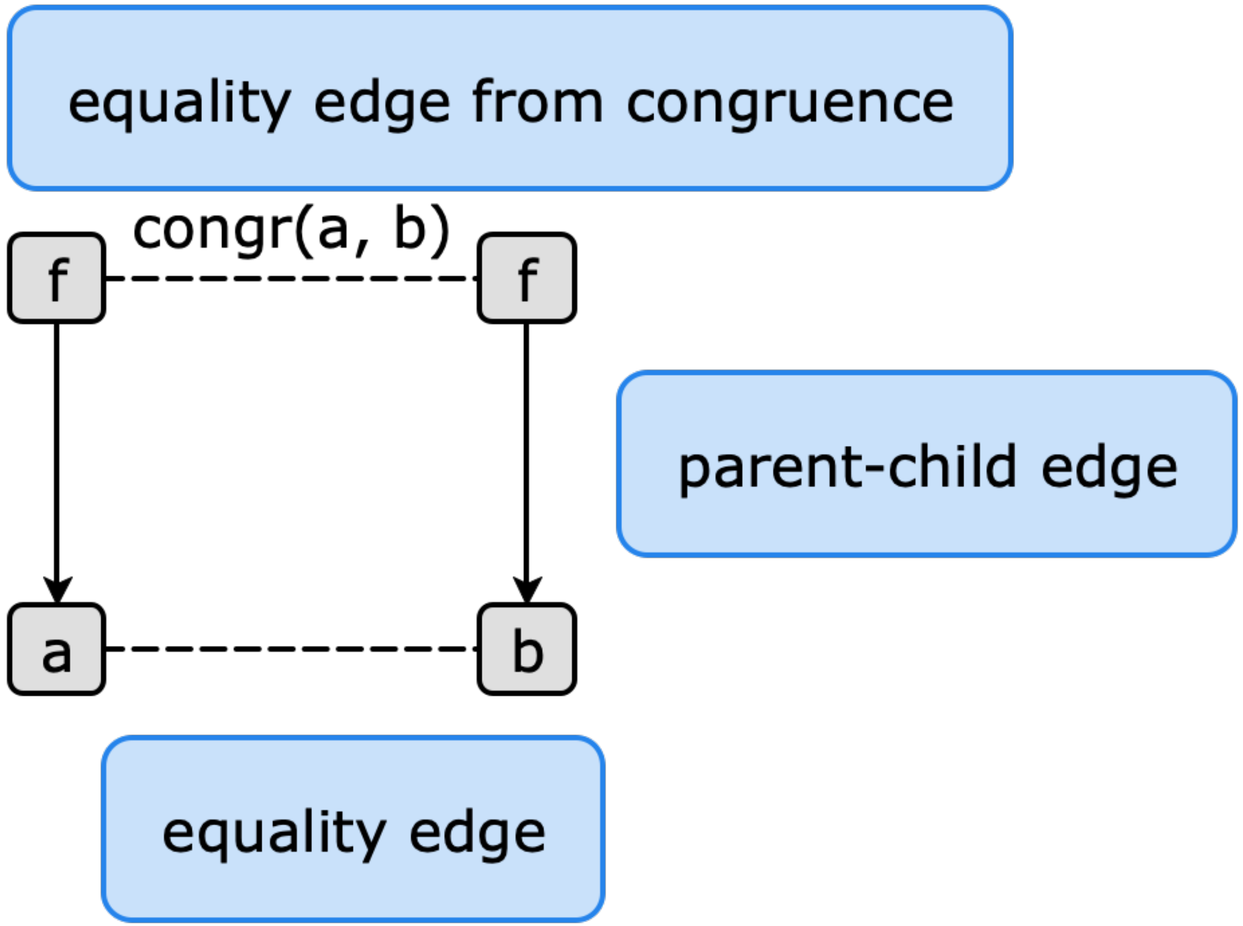

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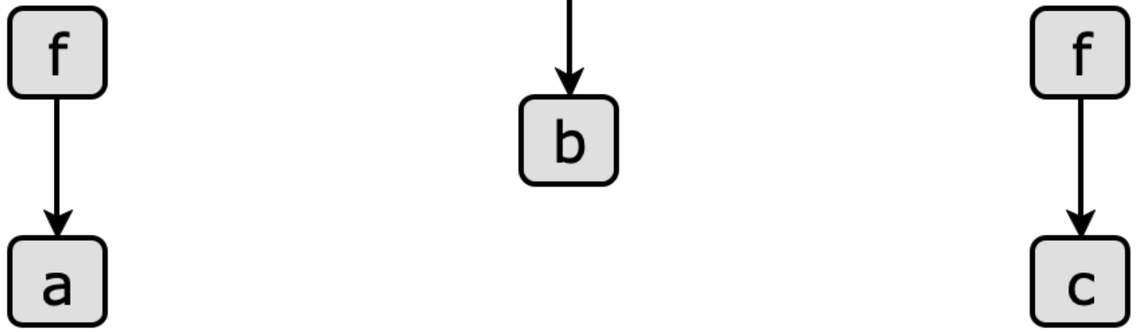

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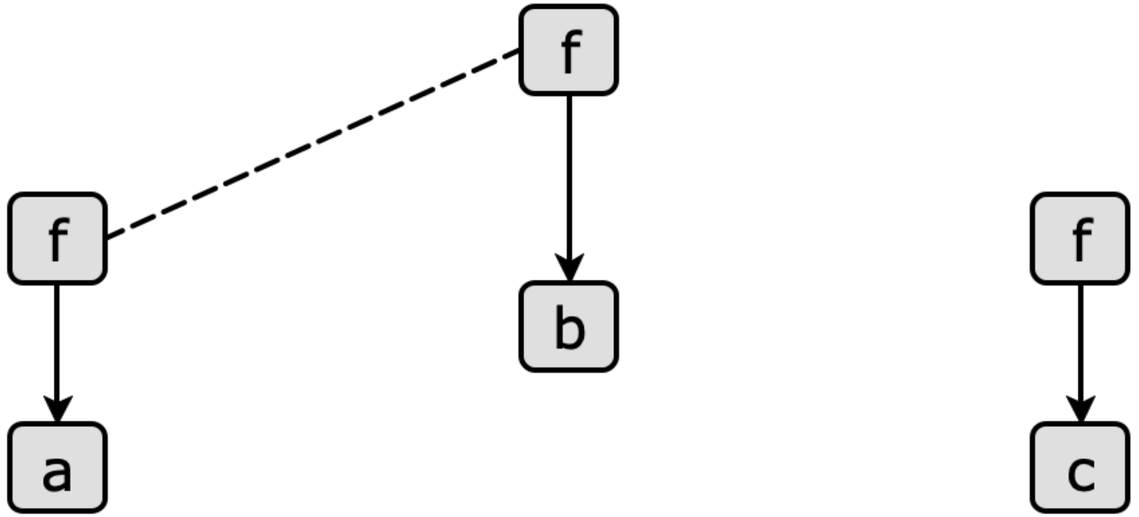

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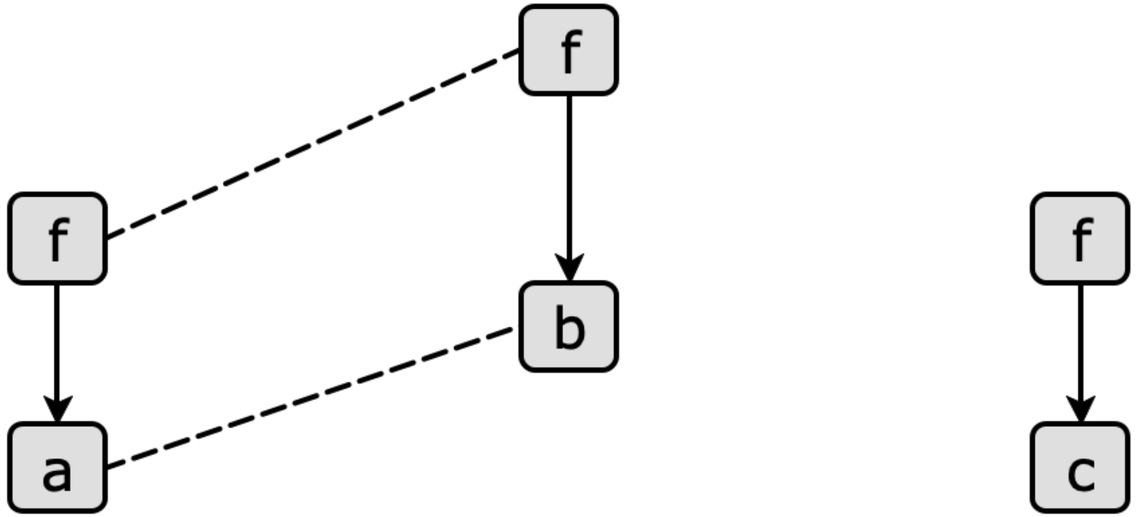

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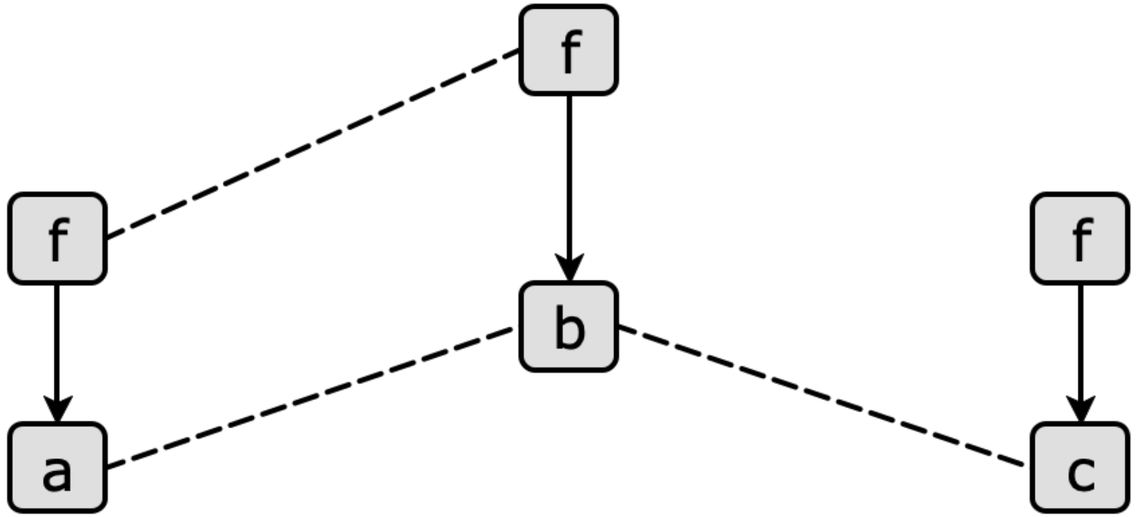

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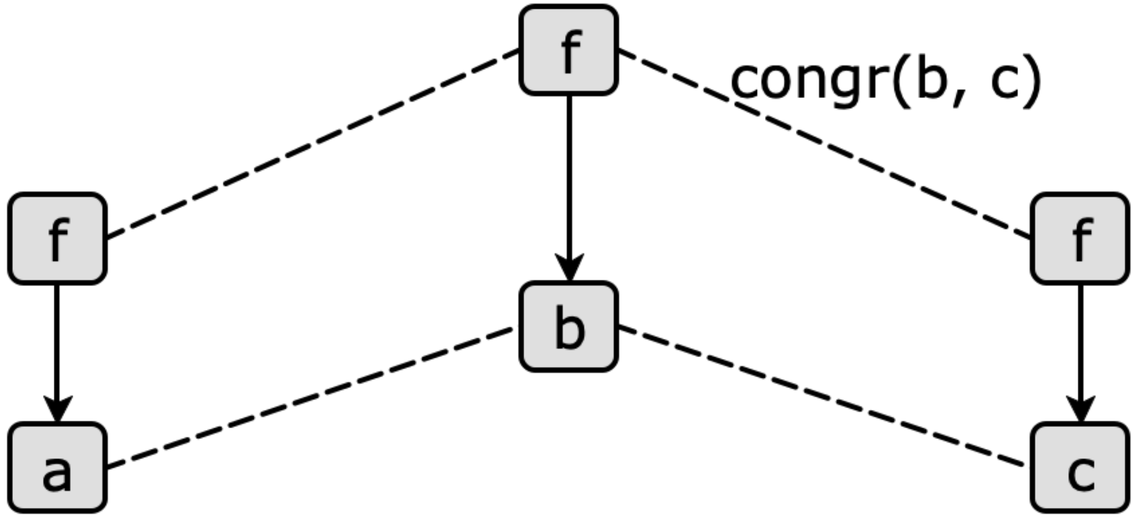

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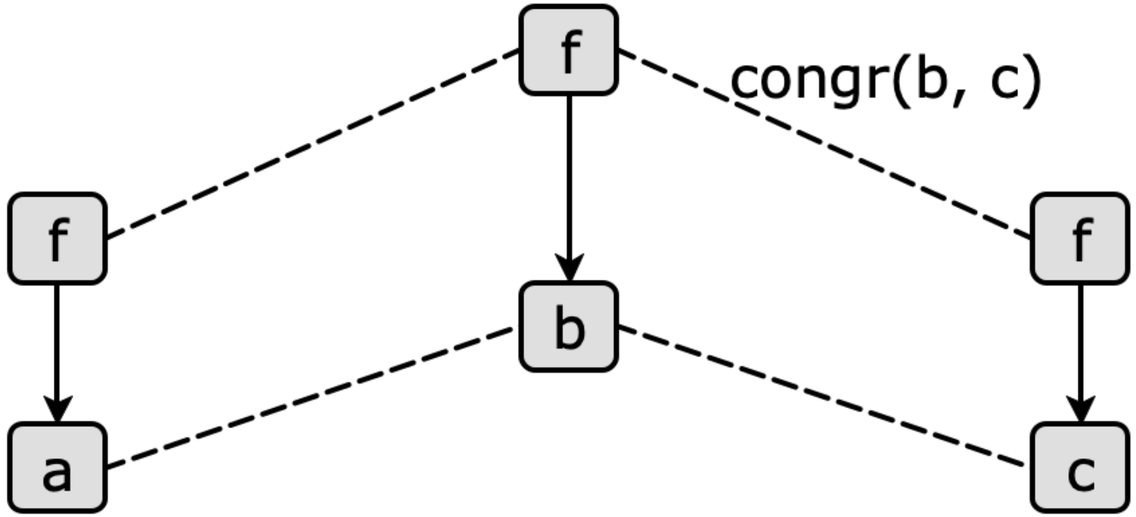

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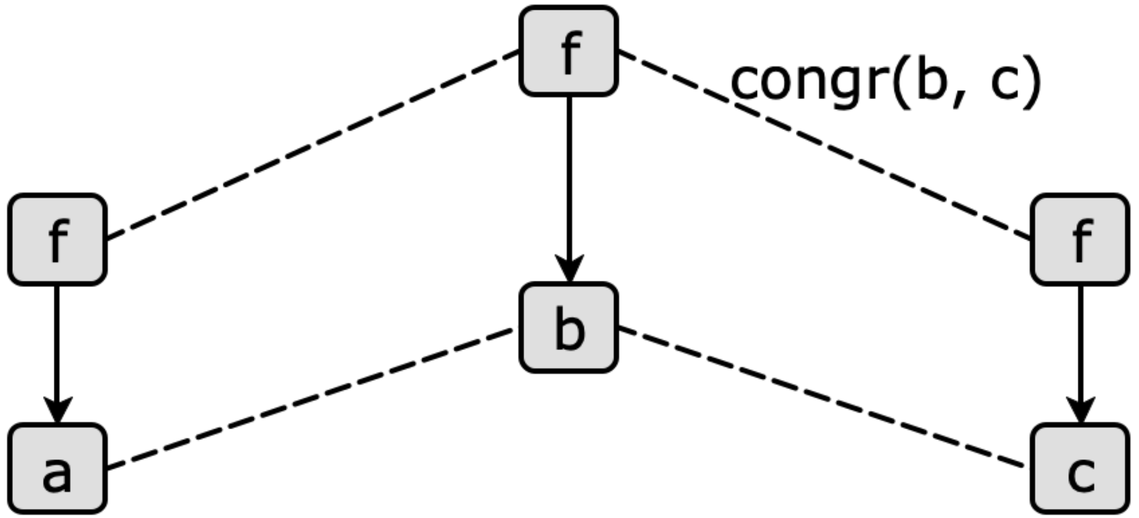

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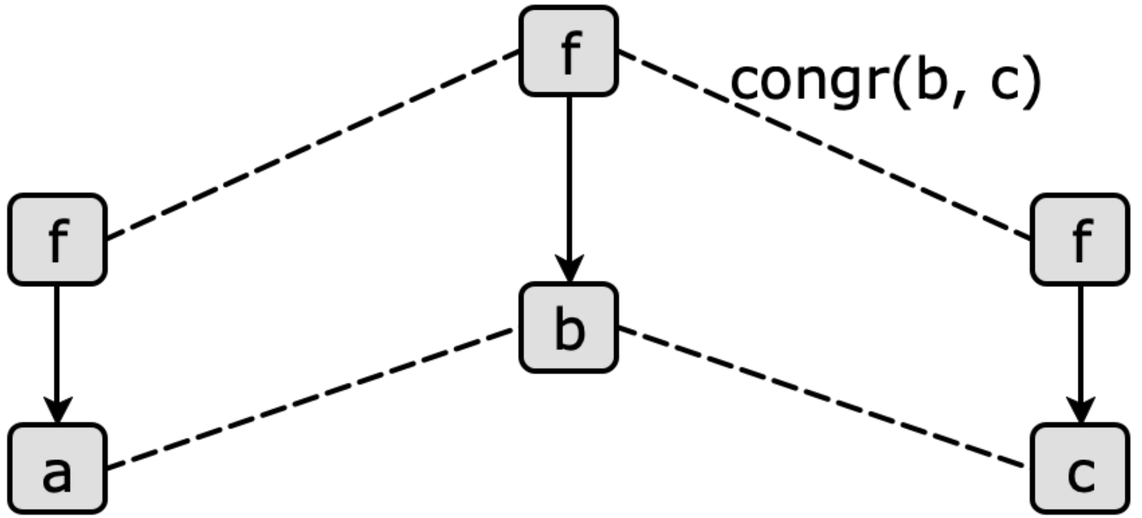

Are a and c equal? Yes!

Are f(a) and a? No!

Are f(a) and f(c)? Yes!

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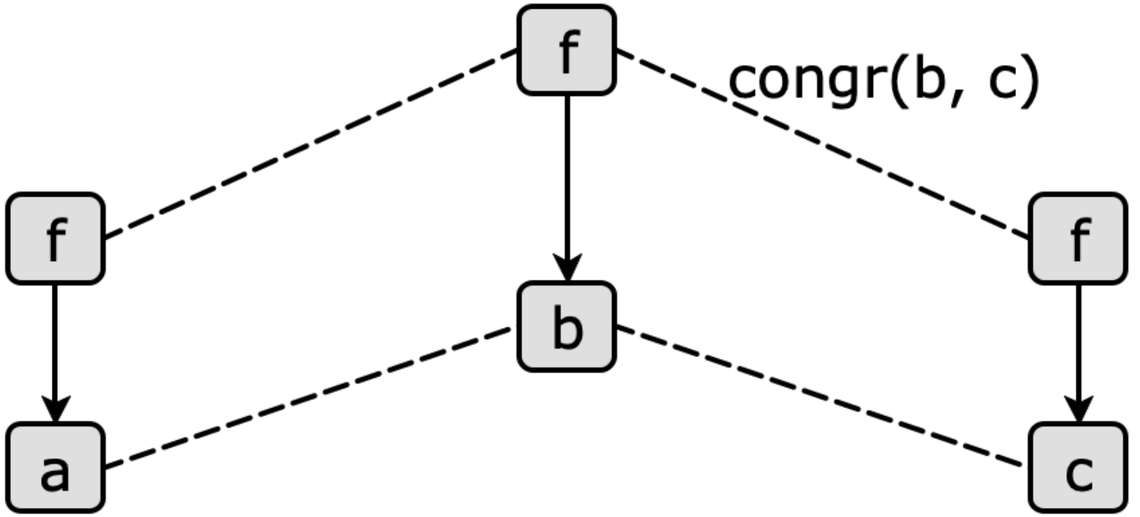

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Key idea: equality edges form equivalence classes of terms

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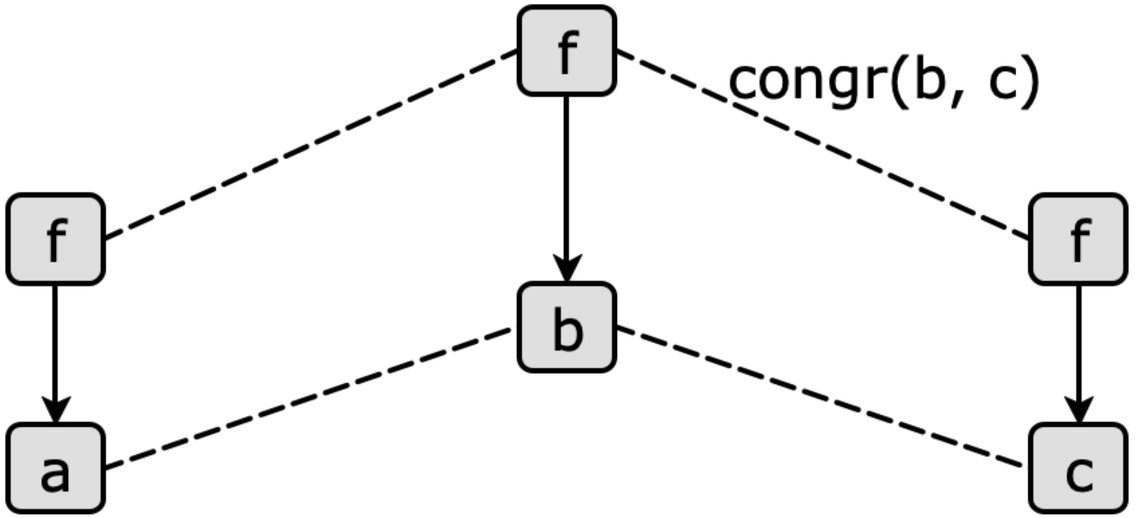

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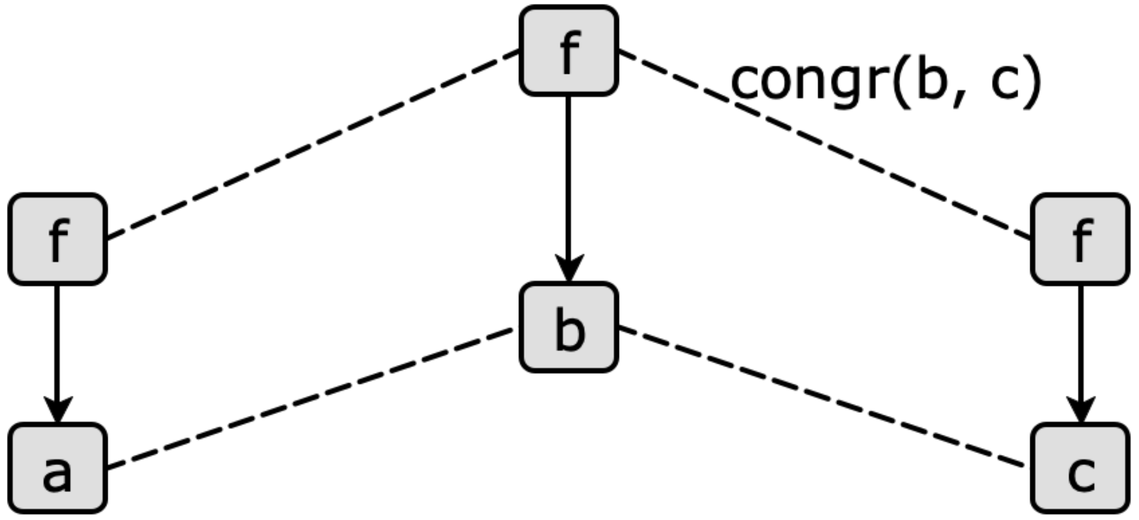

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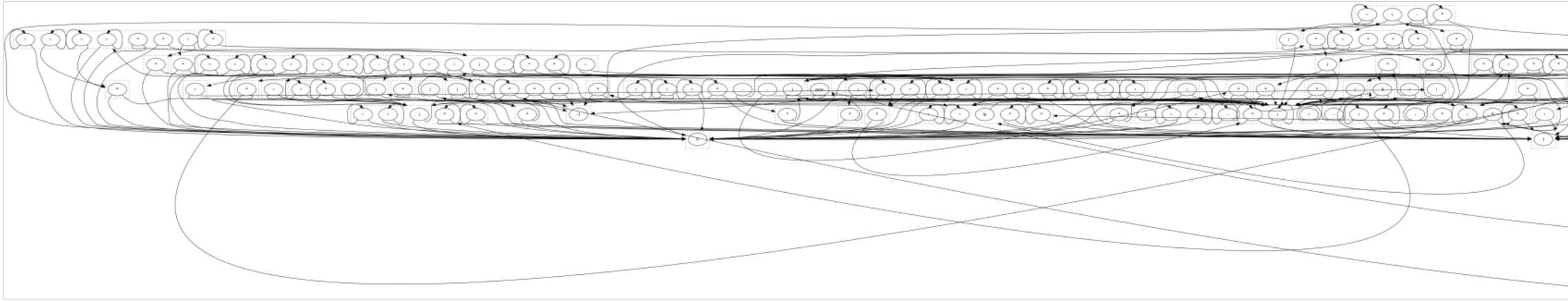

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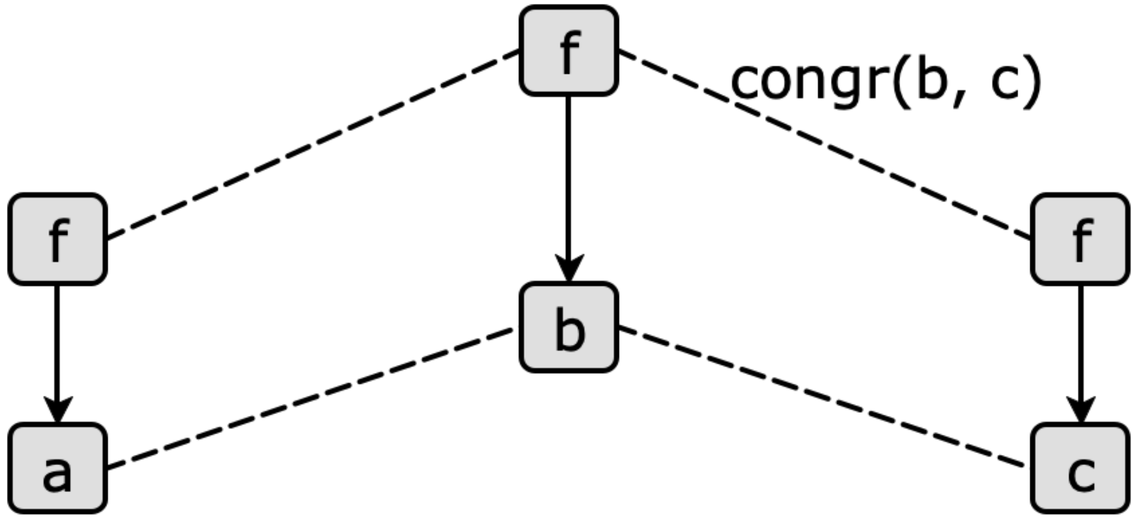

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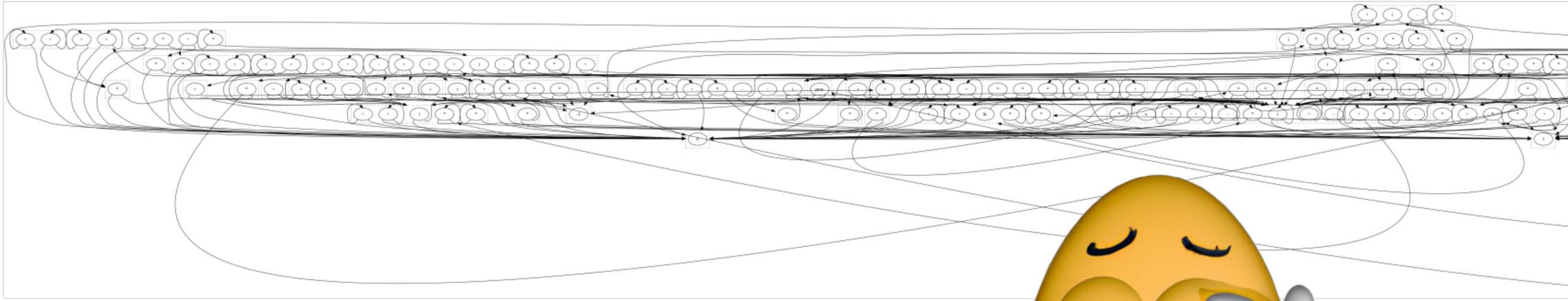
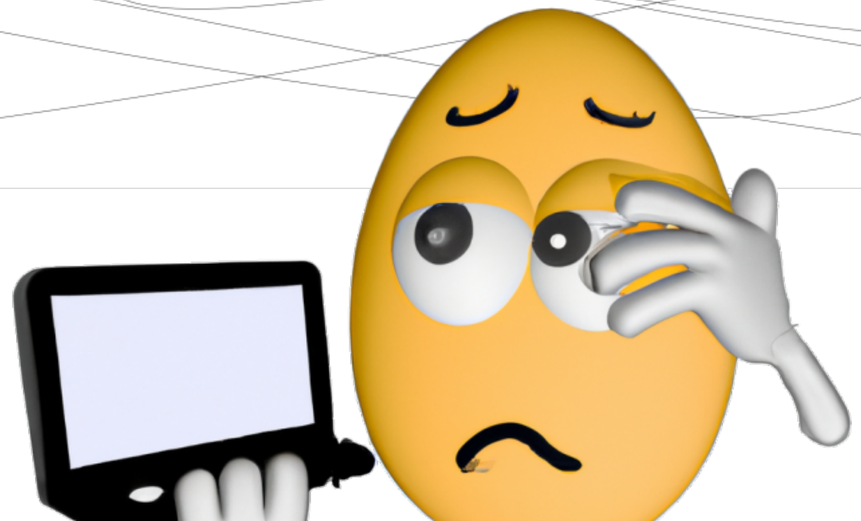
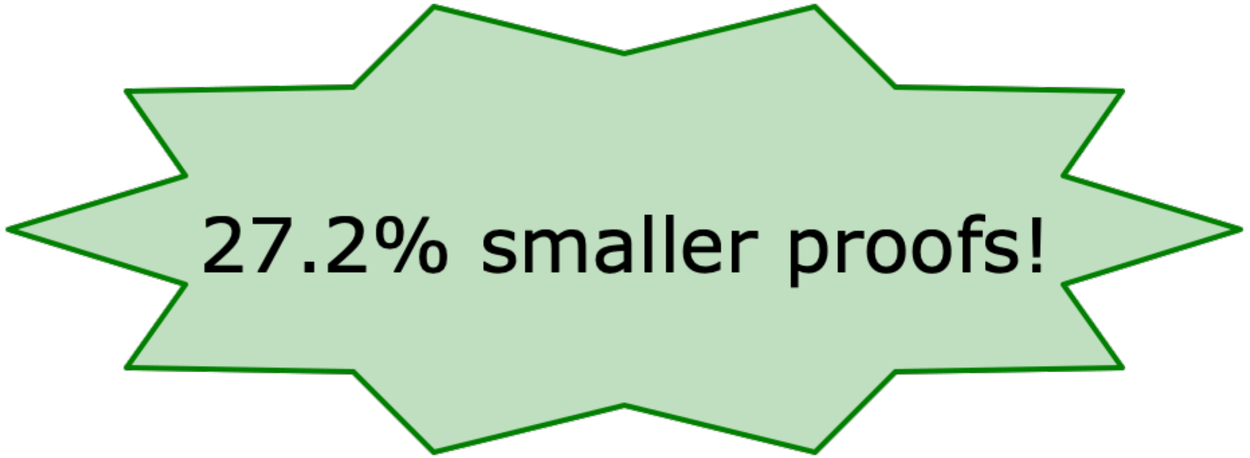




- [x] Motivation
- [x] Congruence Closure
- [ ] Proofs from Congruence Closure
- [ ] Finding Small Proofs



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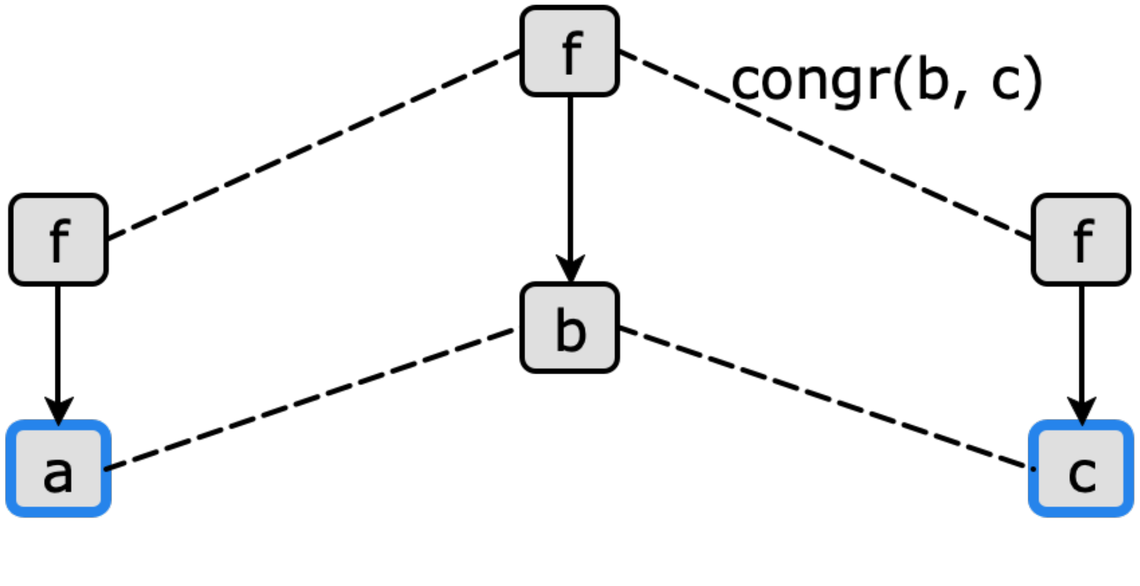

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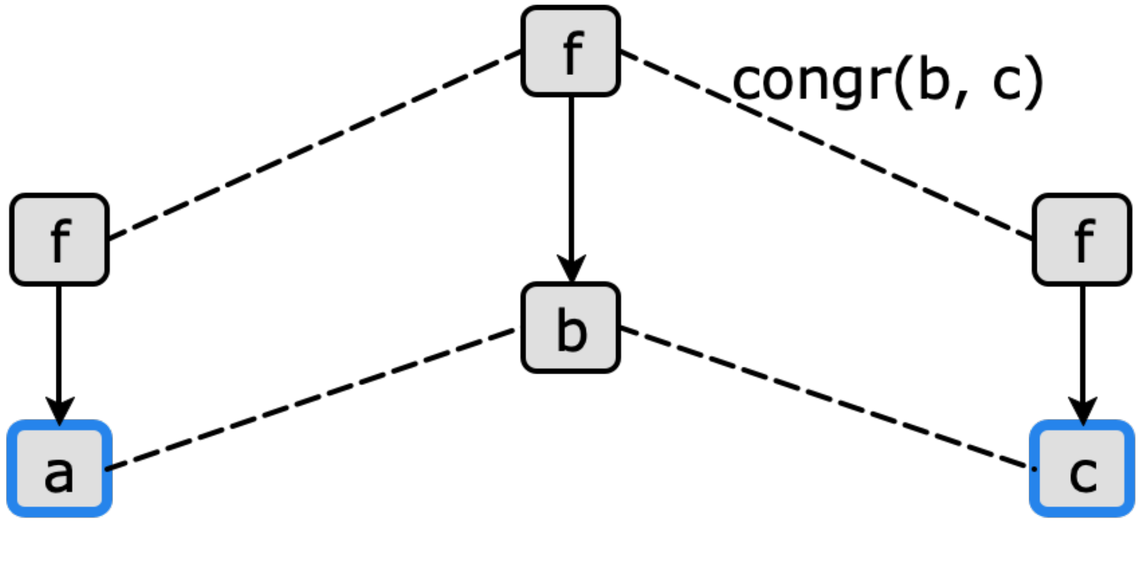

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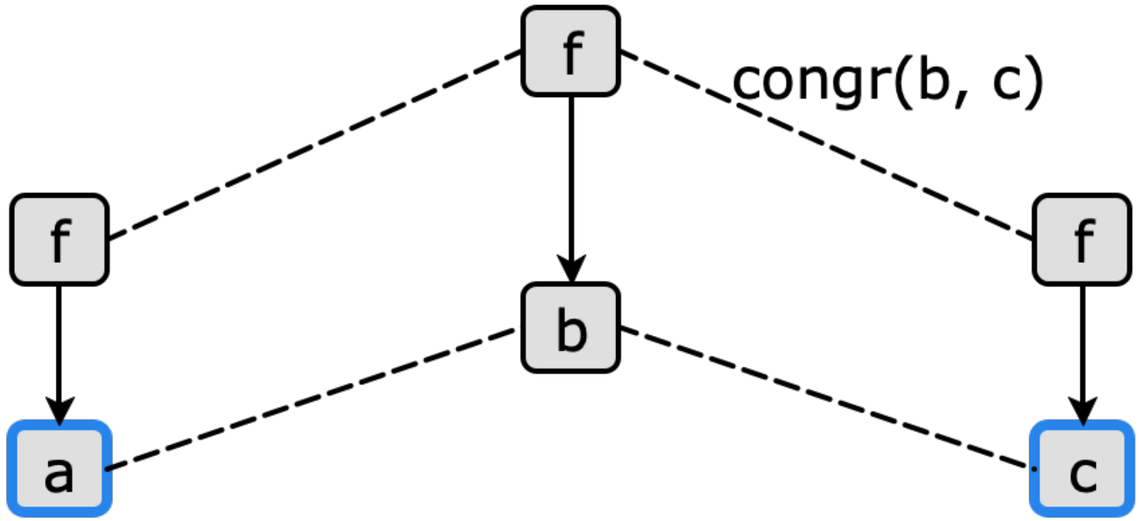

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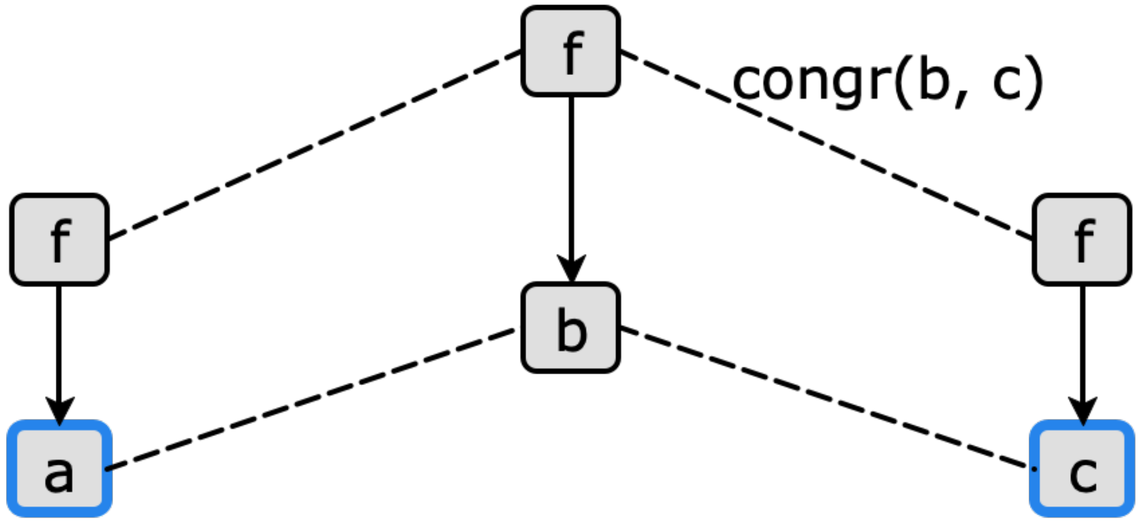

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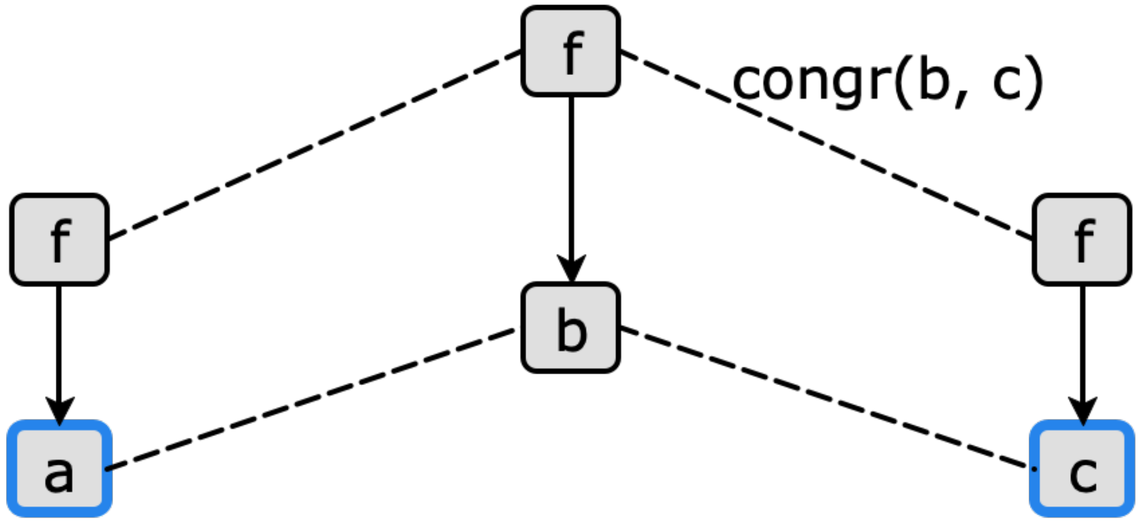

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done!

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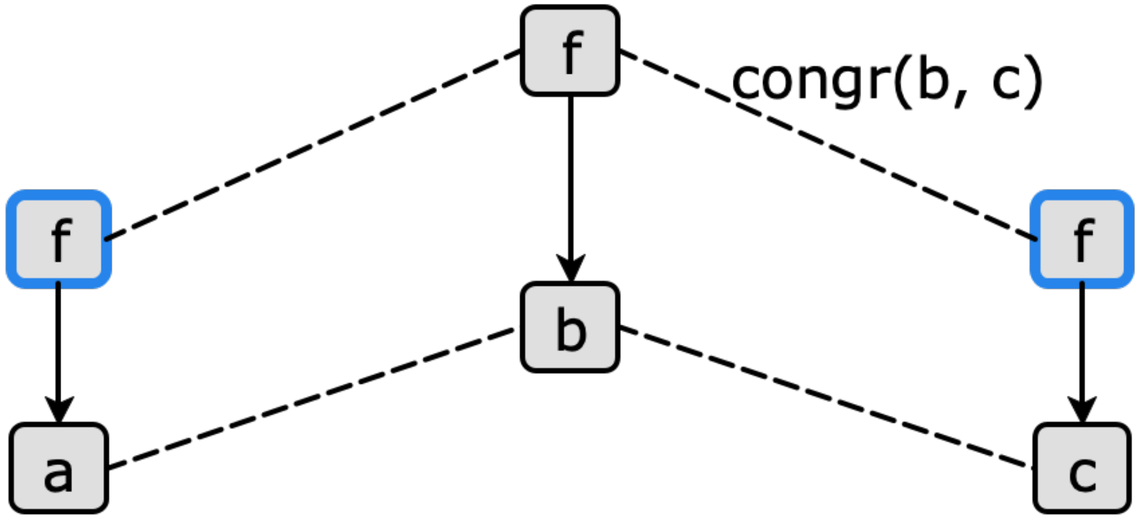
b = c

a = c unnecessary



Prove f(a) and f(c) are equal:

Inputs:

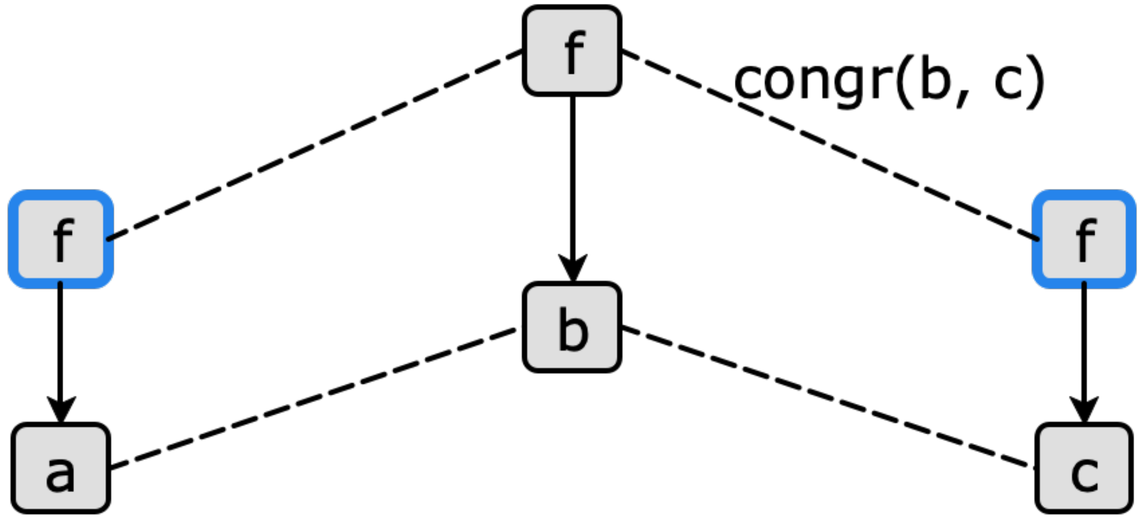
f(c)

f(a) = f(b)

a = b

b = c

a = c unnecessary



Prove f(a) and f(c) are equal:

f(a) = f(b)

Inputs:

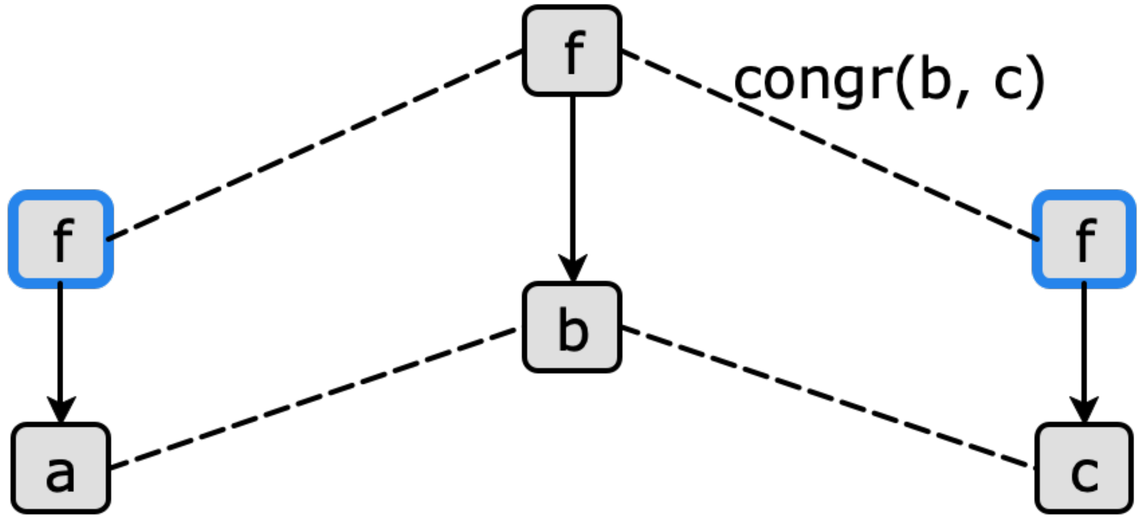
f(c)

f(a) = f(b)

a = b

b = c

a = c unnecessary



Prove f(a) and f(c) are equal:

f(a) = f(b)

Prove f(b) = f(c) by congruence:

Inputs:

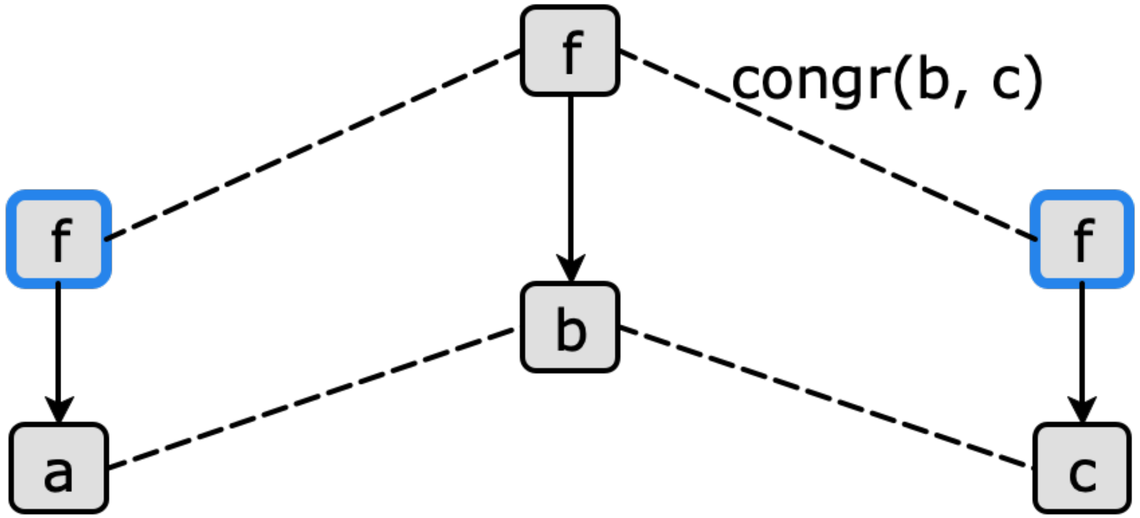
f(c)

f(a) = f(b)

a = b

b = c

a = c unnecessary



Prove f(a) and f(c) are equal:

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Inputs:

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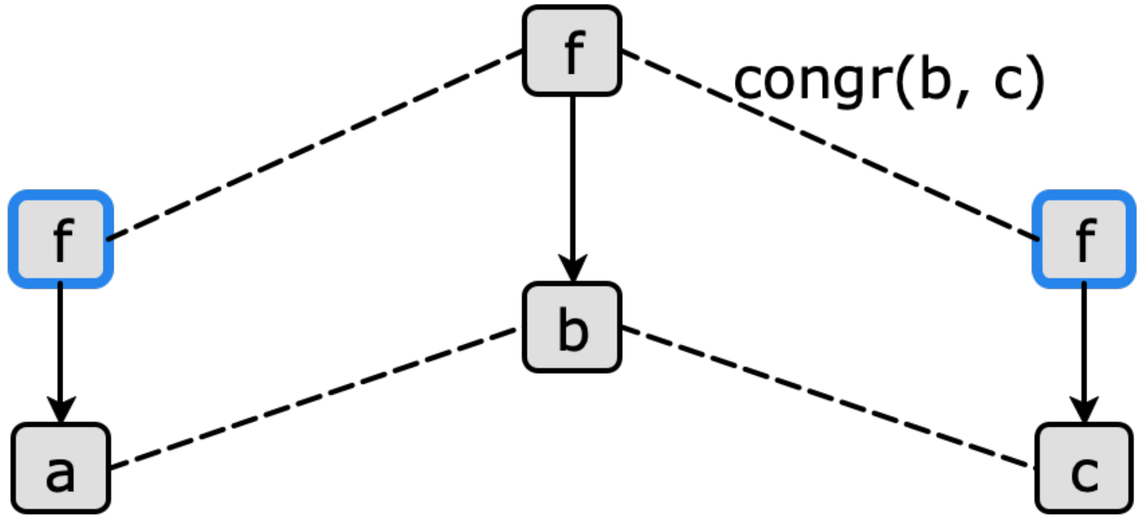
f(a) = f(b)

a = b

b = c

a = c unnecessary



Prove f(a) and f(c) are equal:

f(a) = f(b)

Prove f(b) = f(c) by congruence:

b = c

done!

Inputs:

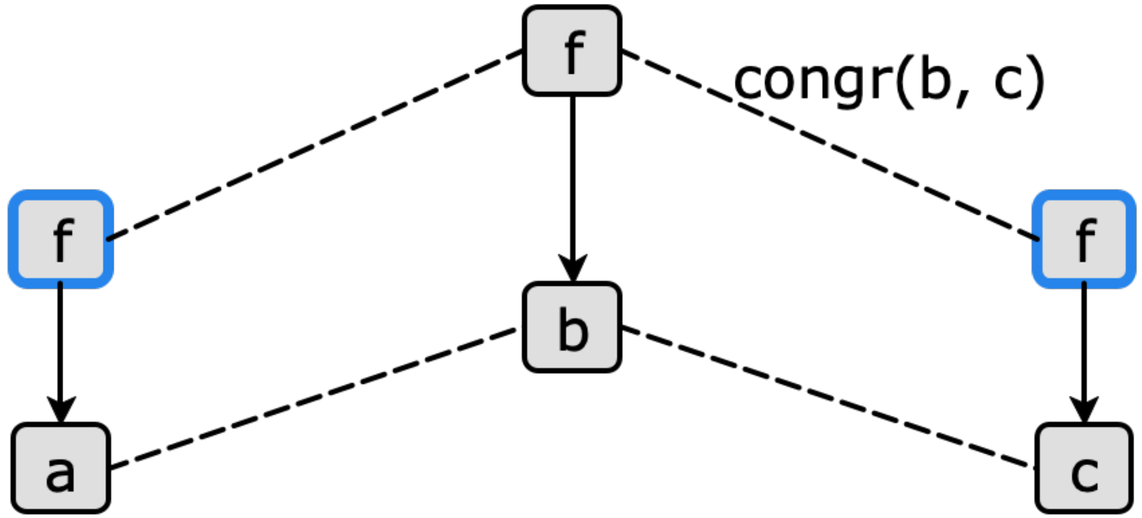
f(c)

f(a) = f(b)

a = b

b = c

a = c unnecessary



Prove f(a) and f(c) are equal:

f(a) = f(b)

Prove f(b) = f(c) by congruence:

b = c

done!

Prove f(a) and f(c) are equal:

f(a) = f(b)

Prove f(b) = f(c) by congruence:

b = c

done!

We define **proof size** as the number of **unique** equalities in the proof

Prove f(a) and f(c) are equal:

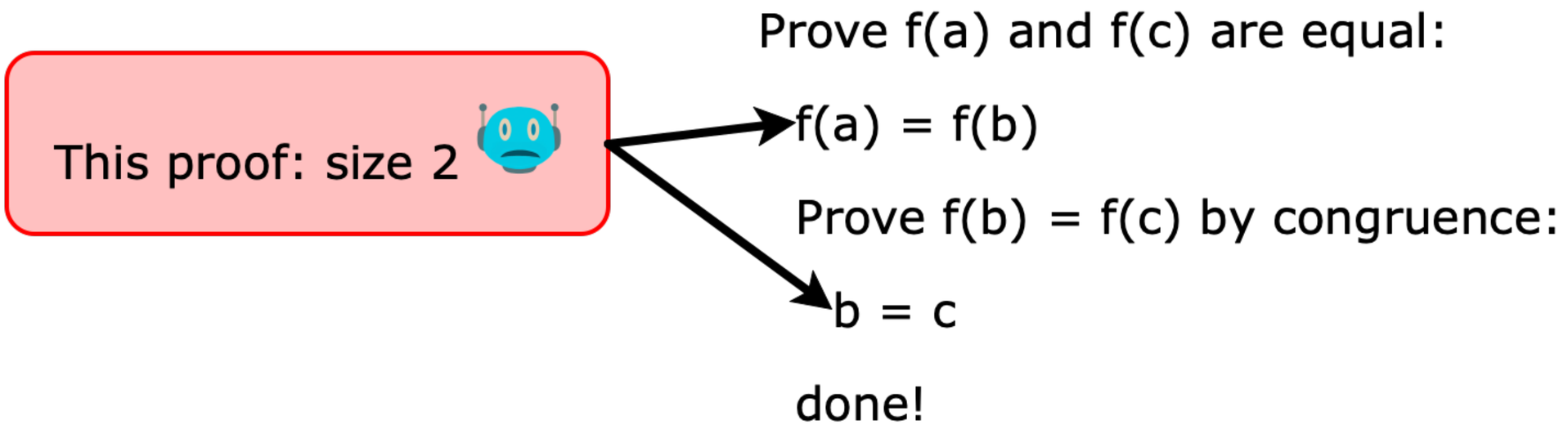
f(a) = f(b)

Prove f(b) = f(c) by congruence:

b = c

done!

We define **proof size** as the number of **unique** equalities in the proof



We define **proof size** as the number of **unique** equalities in the proof

This proof: size 2 

We can do better!

Prove f(a) and f(c) are equal:

f(a) = f(b)

Prove f(b) = f(c) by congruence:

b = c

done!

Leveraging Additional Equalities

Inputs:

f(c)

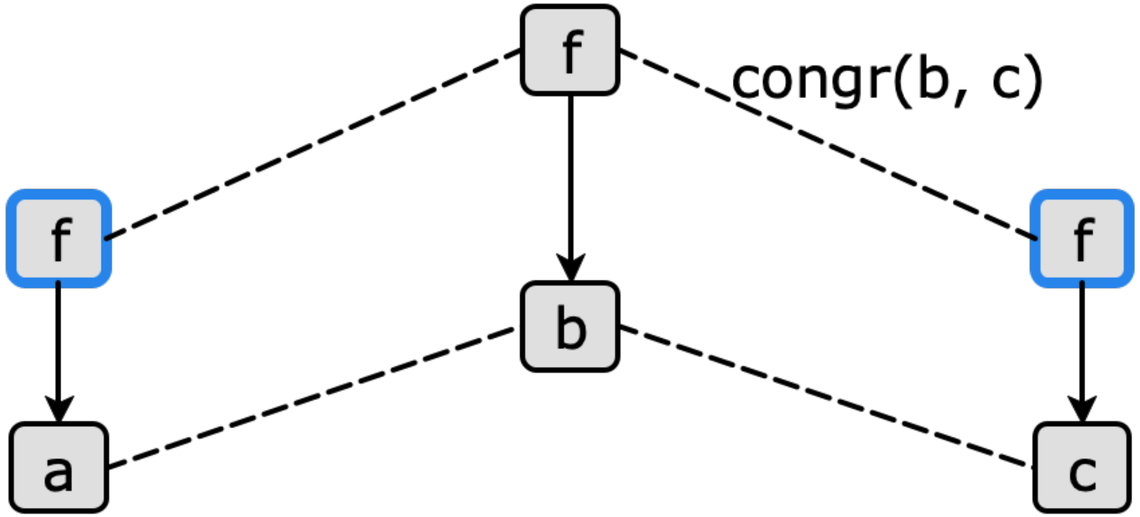
f(a) = f(b)

a = b

b = c

a = c

unnecessary



Leveraging Additional Equalities

Inputs:

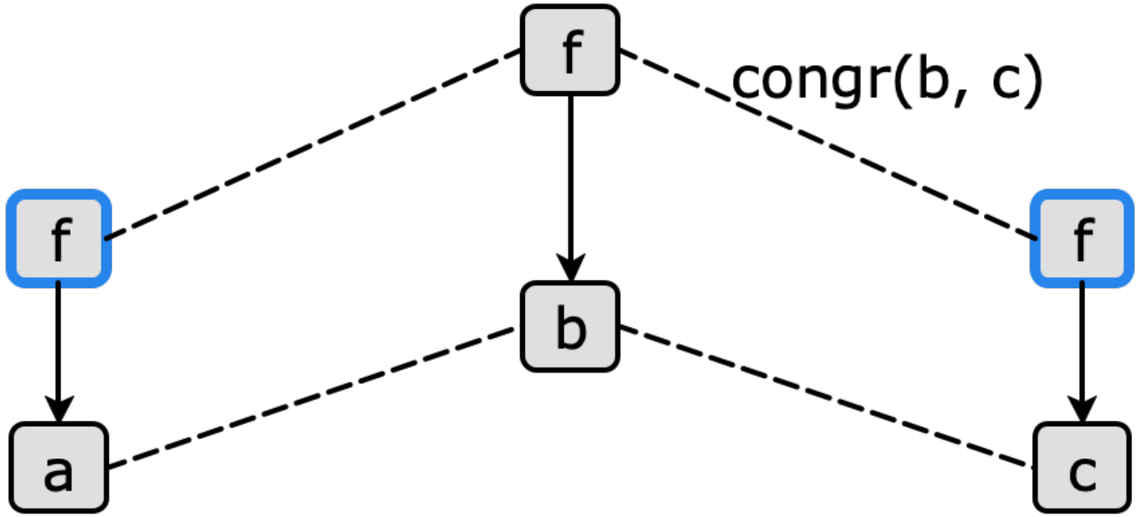
f(c)

f(a) = f(b)

a = b

b = c

a = c useful



Leveraging Additional Equalities

Inputs:

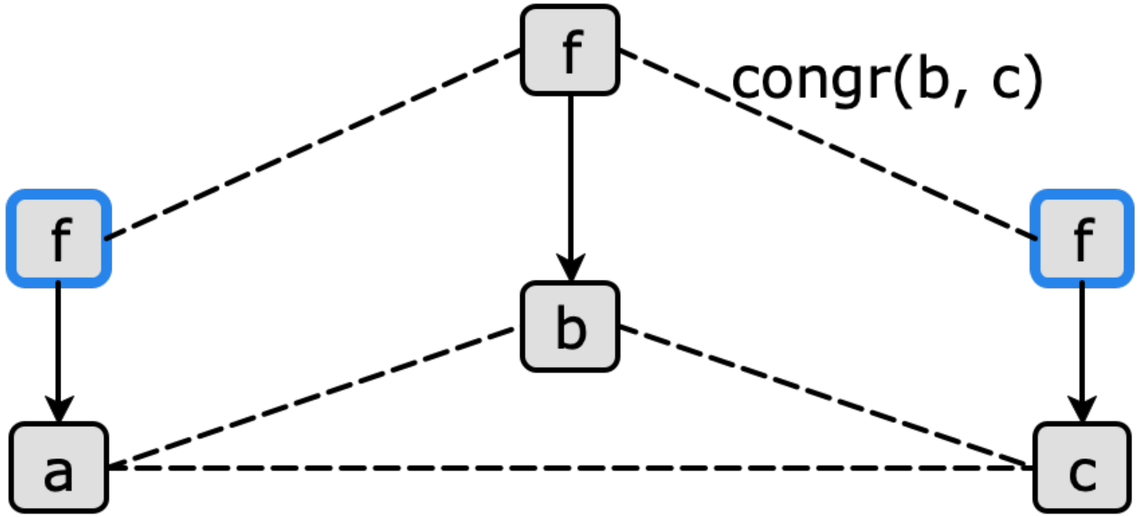
f(c)

f(a) = f(b)

a = b

b = c

a = c useful



Leveraging Additional Equalities

Inputs:

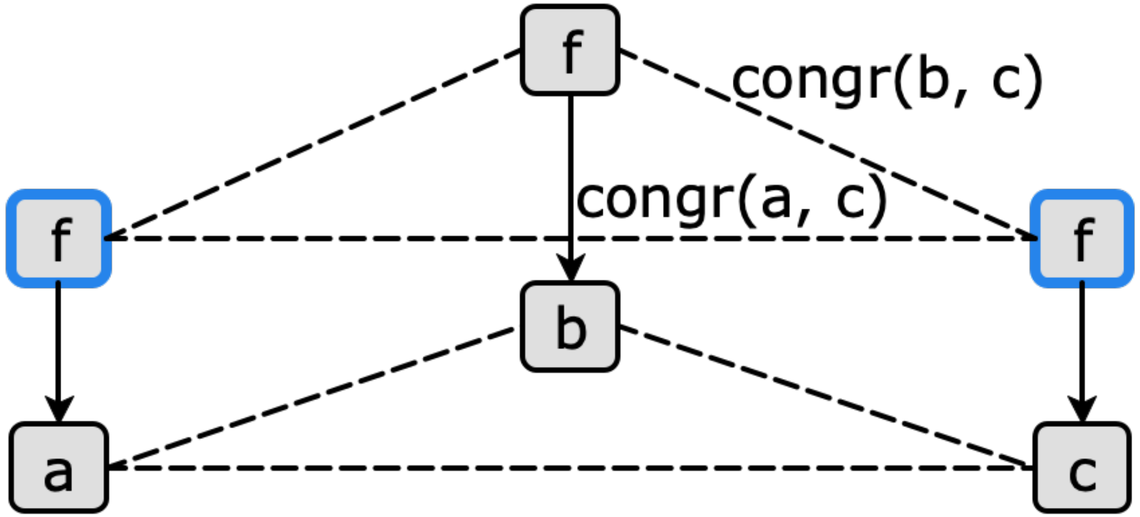
f(c)

f(a) = f(b)

a = b

b = c

a = c useful



Leveraging Additional Equalities

Inputs:

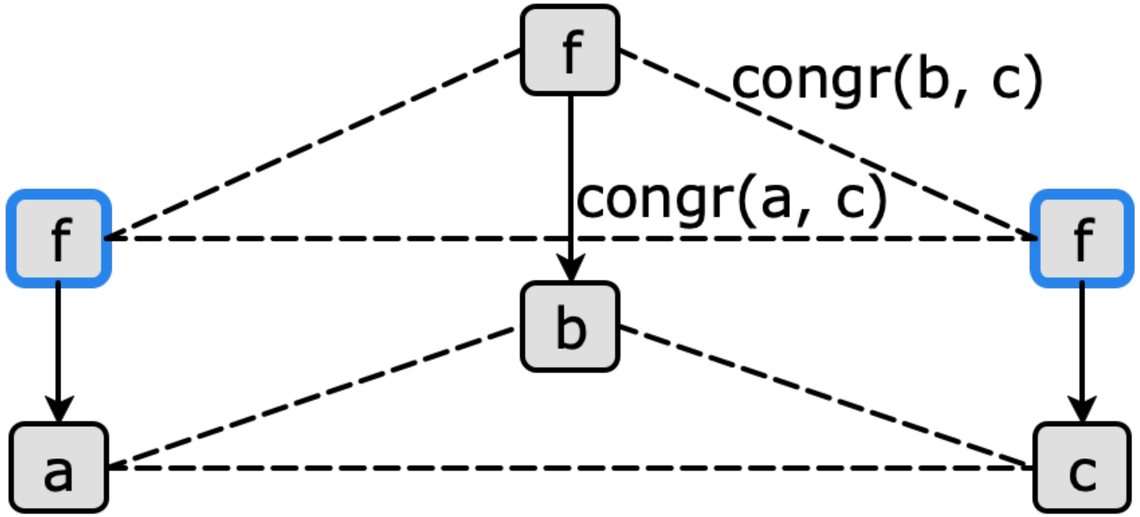
f(c)

f(a) = f(b)

a = b

b = c

a = c useful



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f(c)

f(a) = f(b)

a = b

b = c

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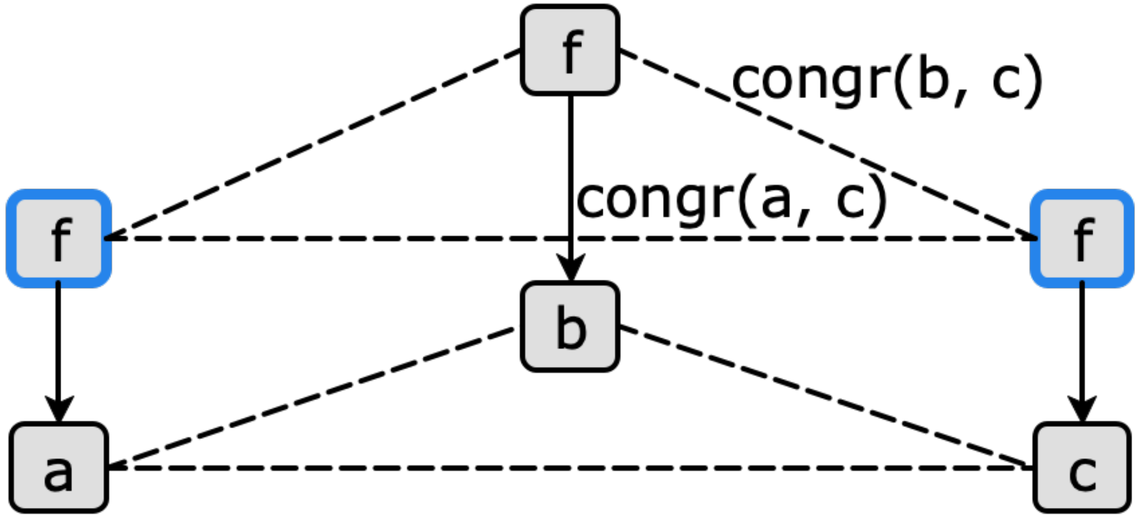


Prove f(a) and f(c) are equal:

Prove f(a) = f(c) by congruence:

Leveraging Additional Equalities

Inputs:

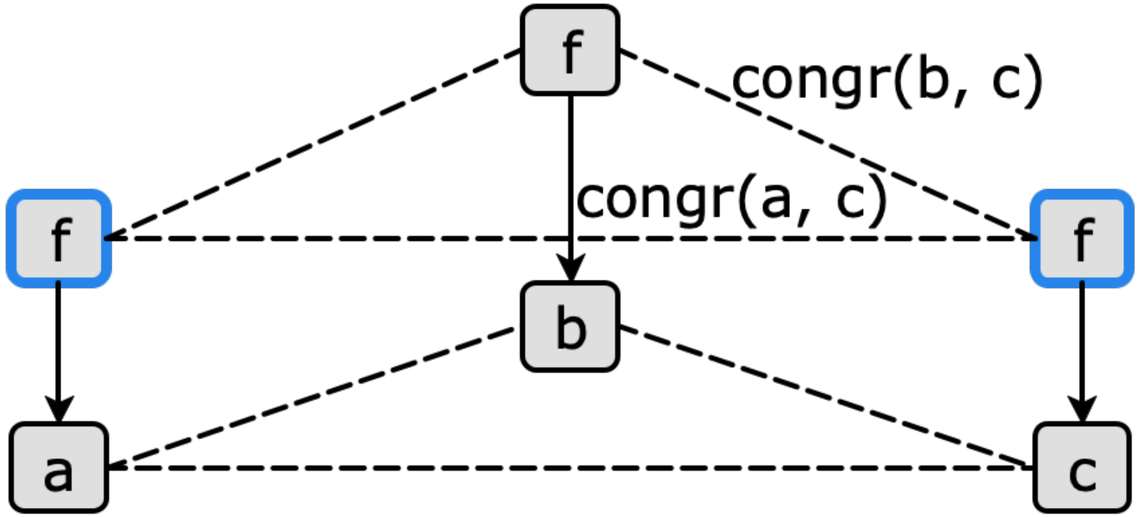
f(c)

f(a) = f(b)

a = b

b = c

a = c useful



Prove f(a) and f(c) are equal:

Prove f(a) = f(c) by congruence:

a = c

Leveraging Additional Equalities

Inputs:

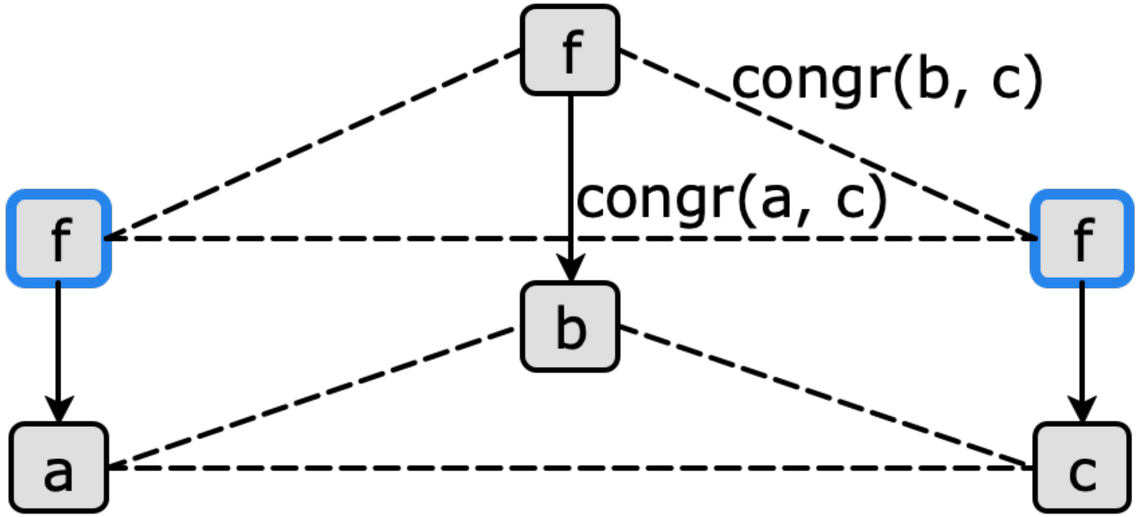
f(c)

f(a) = f(b)

a = b

b = c

a = c useful



Prove f(a) and f(c) are equal:

Prove f(a) = f(c) by congruence:

a = c

done!

Leveraging Additional Equalities

Inputs:

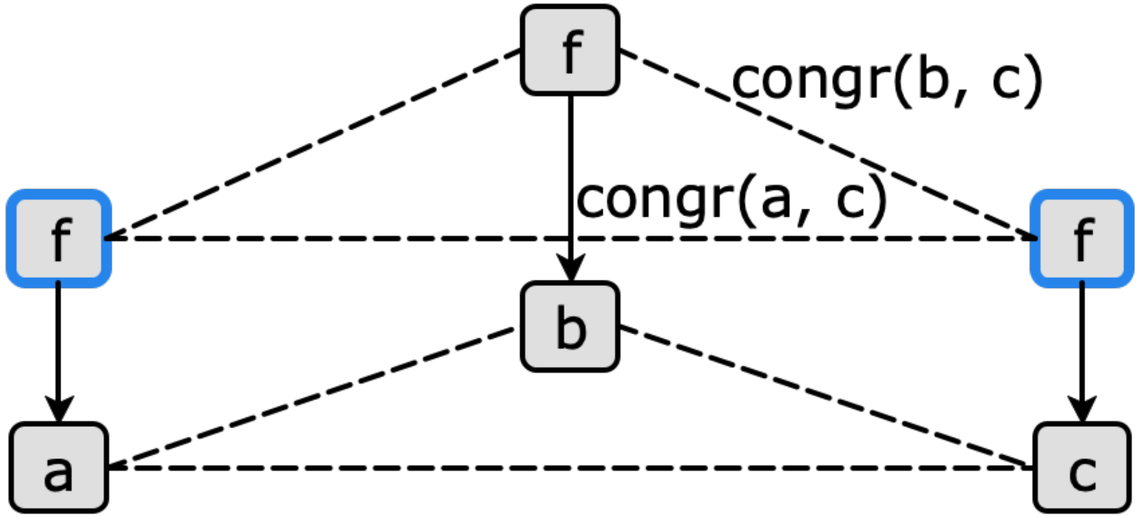
f(c)

f(a) = f(b)

a = b

b = c

a = c useful



Prove f(a) and f(c) are equal:

Prove f(a) = f(c) by congruence:

a = c

done!

Proof size: 1

Leveraging Additional Equalities

Inputs:

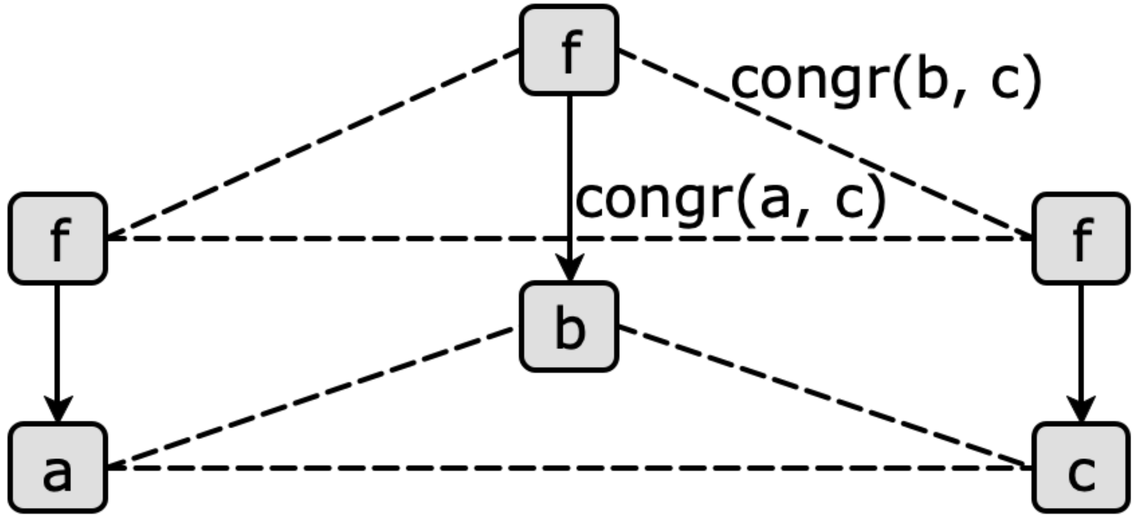
f(c)

f(a) = f(b)

a = b

b = c

a = c useful



Prove f(a) and f(c) are equal:

Prove f(a) = f(c) by congruence:

a = c

done!

Proof size: 1

Leveraging Additional Equalities

Inputs:

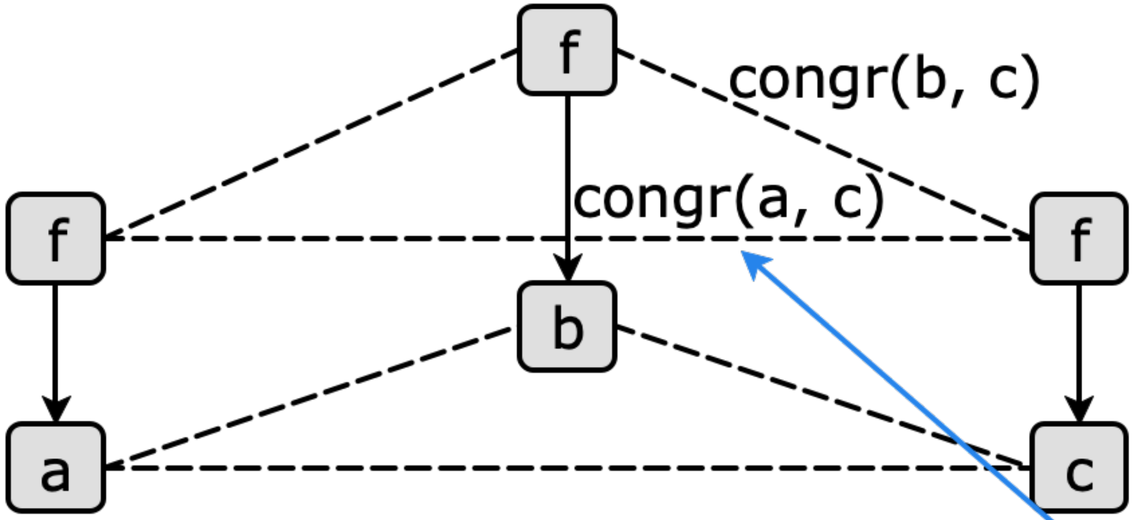
f(c)

f(a) = f(b)

a = b

b = c

a = c useful



Key idea:

Try alternate path

new!

Prove f(a) and f(c) are equal:

Prove f(a) = f(c) by congruence:

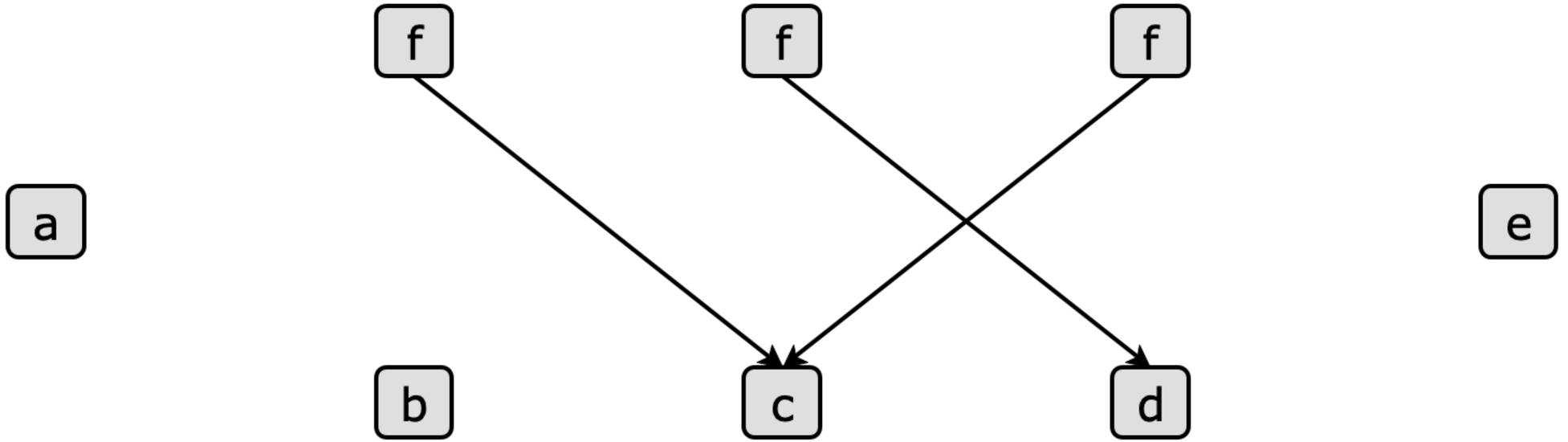
a = c

done!

Proof size: 1

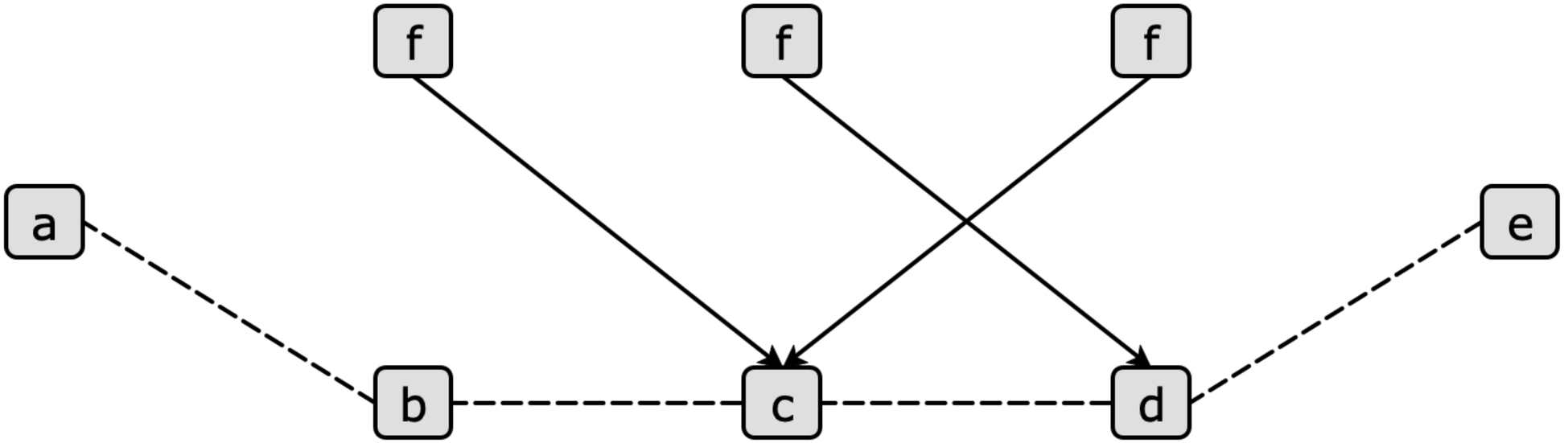
The Crux of The Problem

Prove a = e



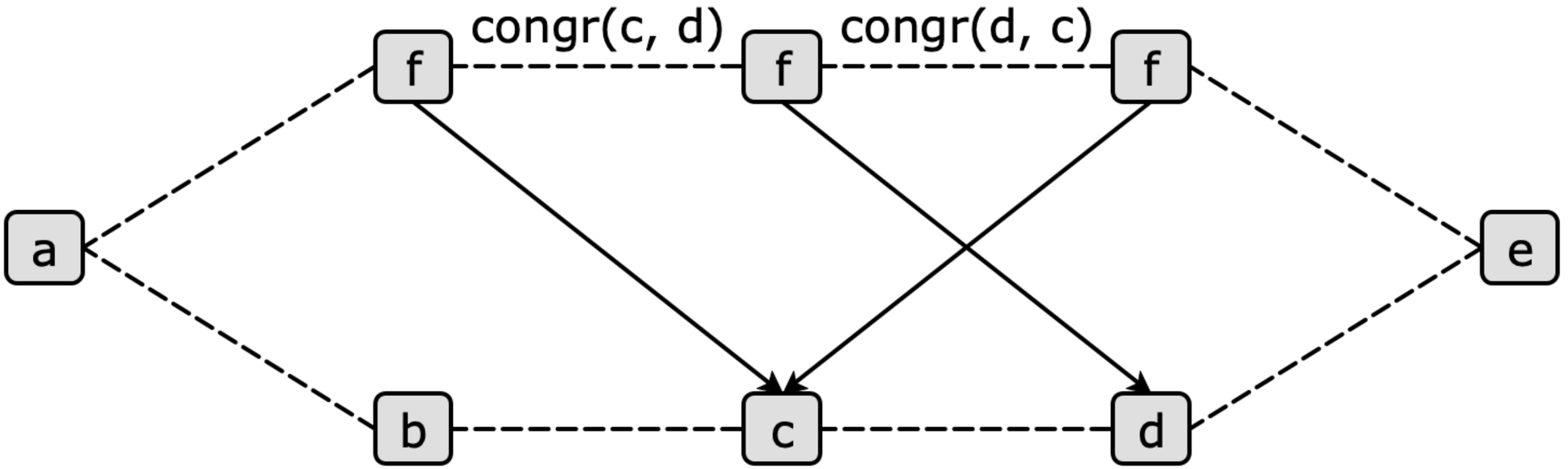
The Crux of The Problem

Prove a = e



The Crux of The Problem

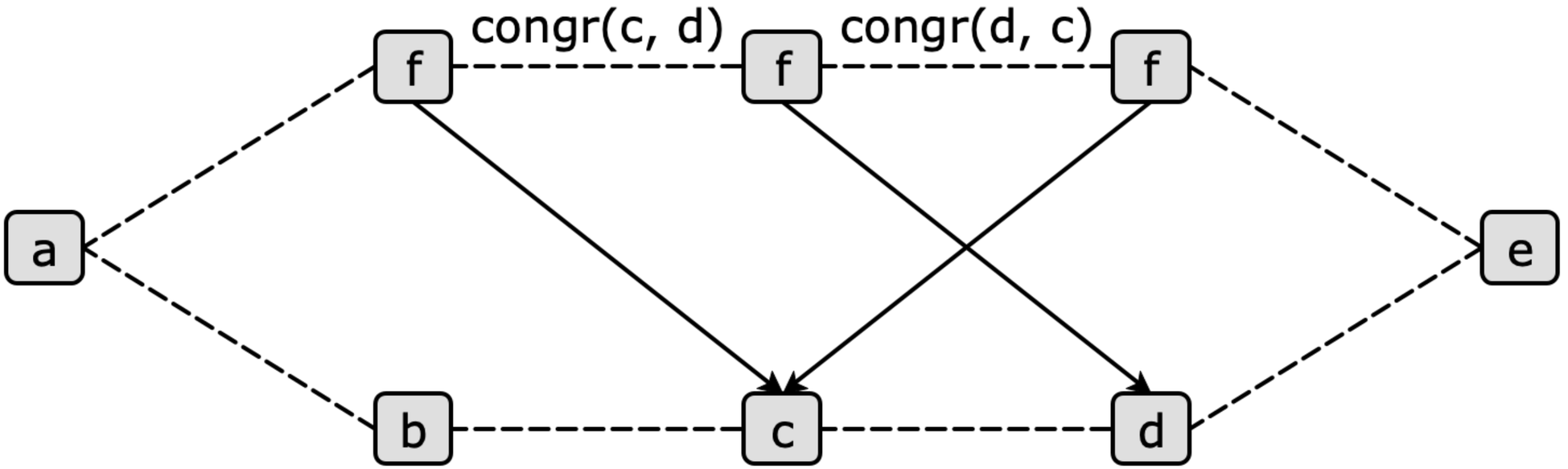
Prove a = e



The Crux of The Problem

Prove a = e

Bottom: size 4

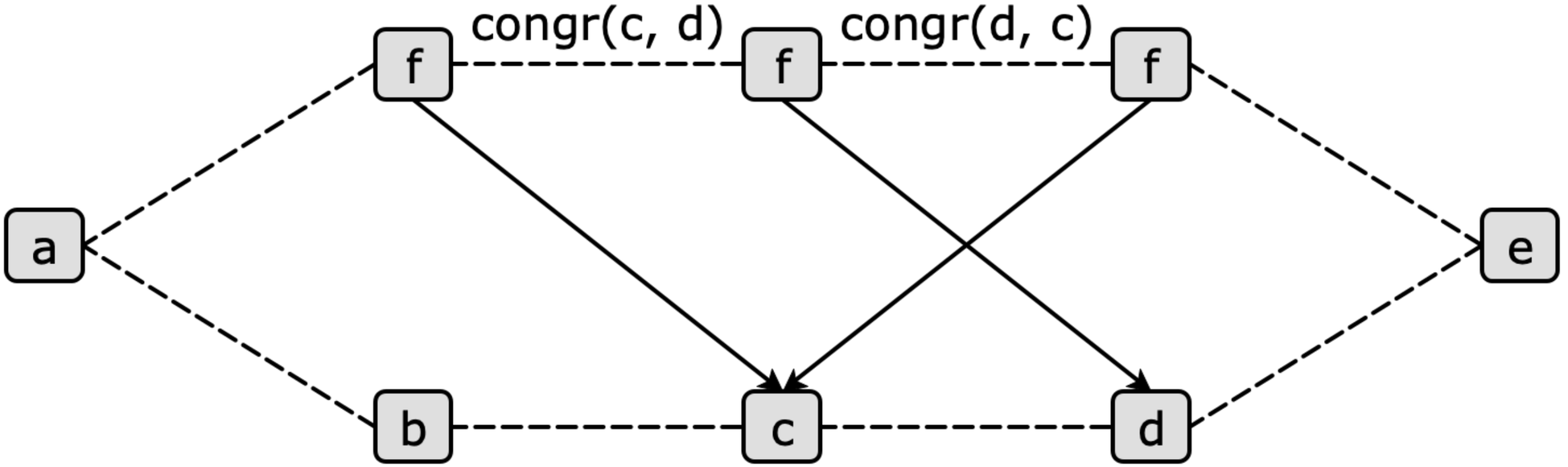


The Crux of The Problem

Prove a = e

Bottom: size 4

Top: ???

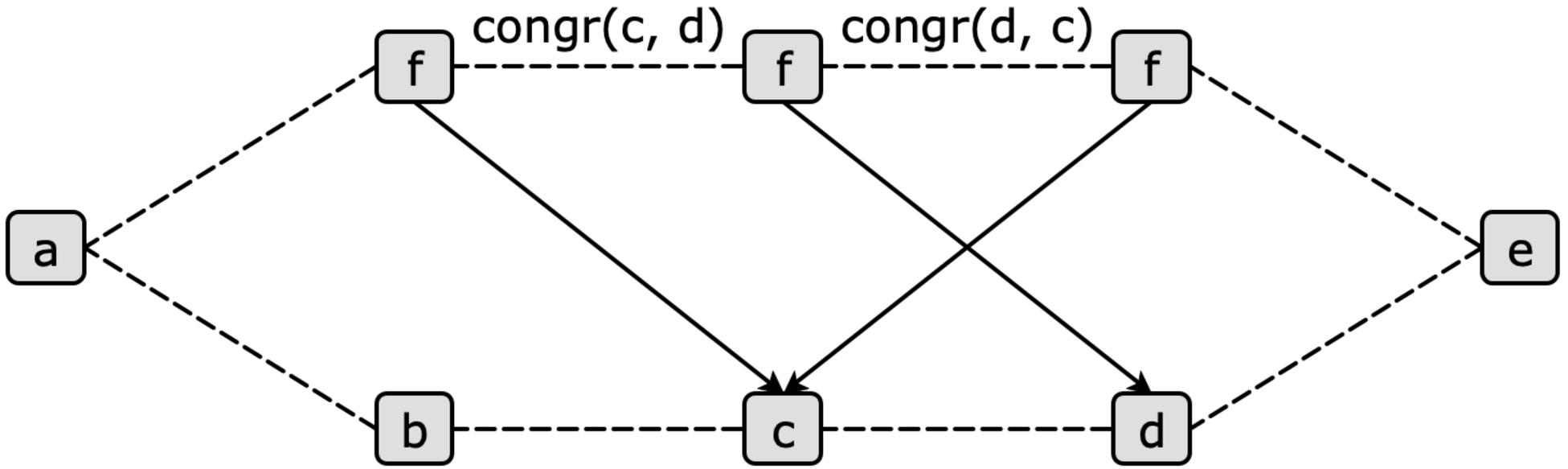


The Crux of The Problem

Prove a = e

Bottom: size 4

Top: size 3



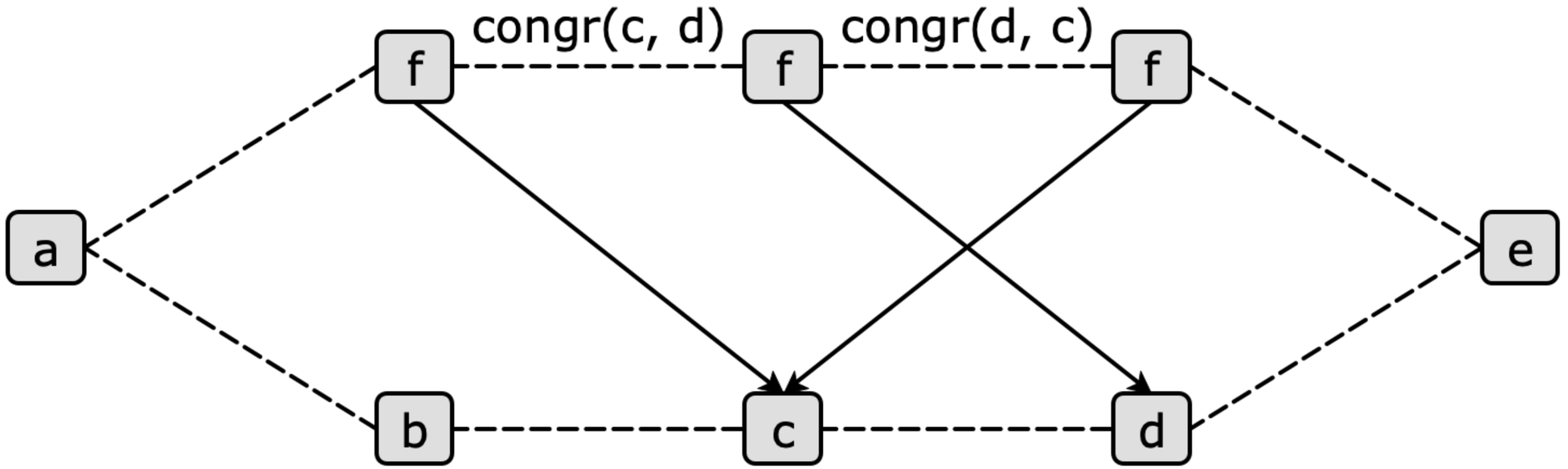
The Crux of The Problem

Prove a = e

Bottom: size 4

Top: size 3

Proof re-use is free

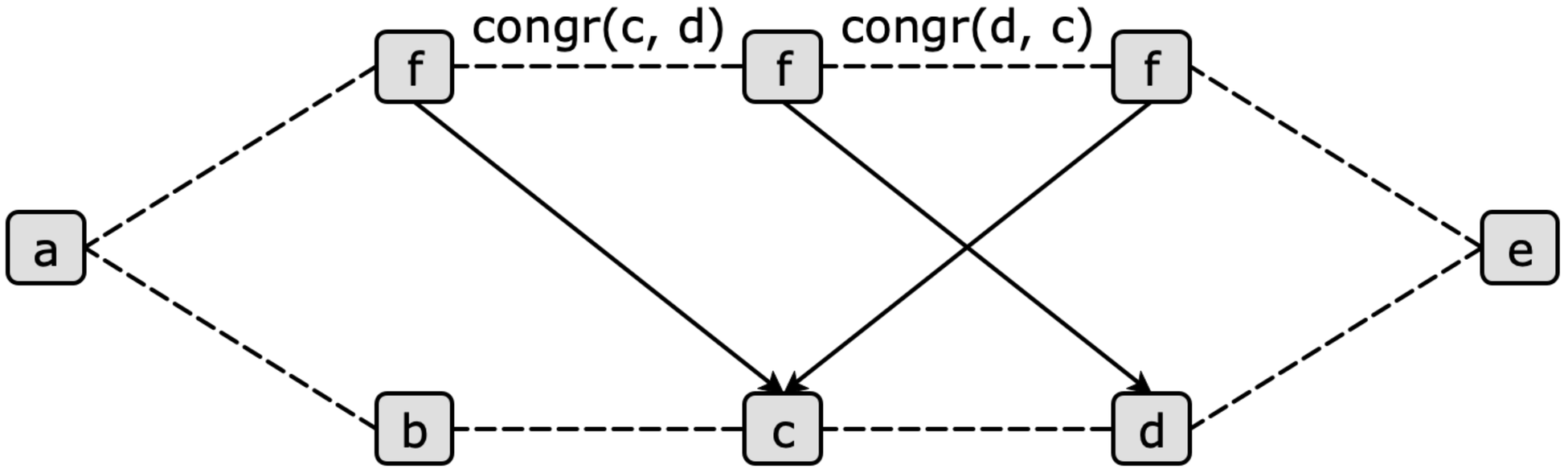


The Crux of The Problem

Prove a = e

Bottom: size 4

Top: size 3



Proof re-use is free

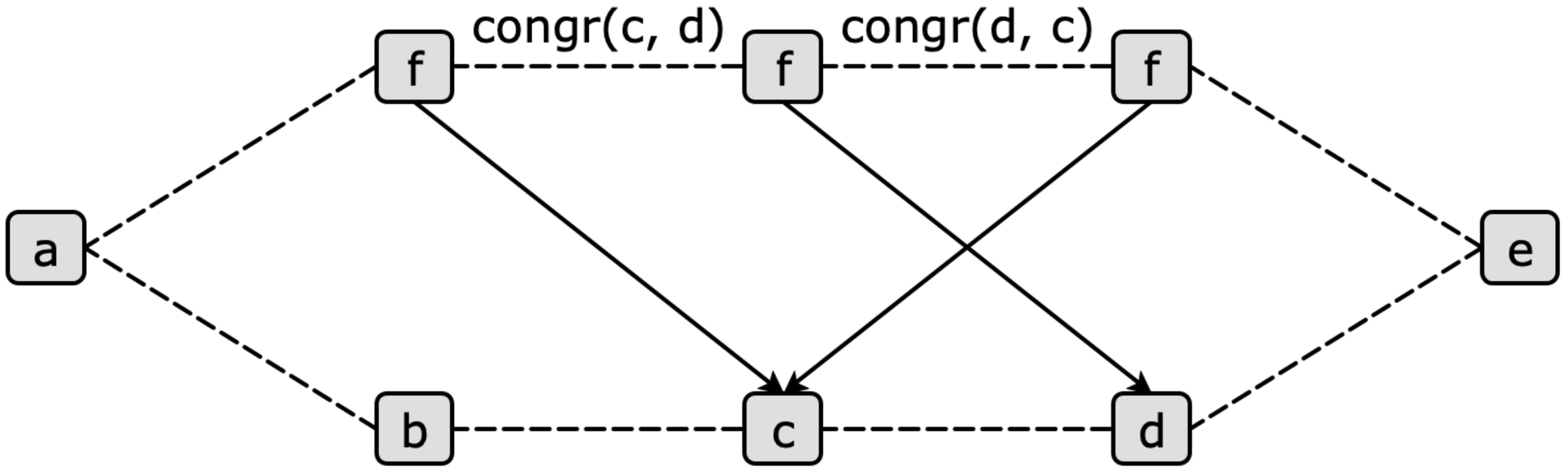
Subproofs can be complex

The Crux of The Problem

Prove a = e

Bottom: size 4

Top: size 3



Proof re-use is free

Subproofs can be complex

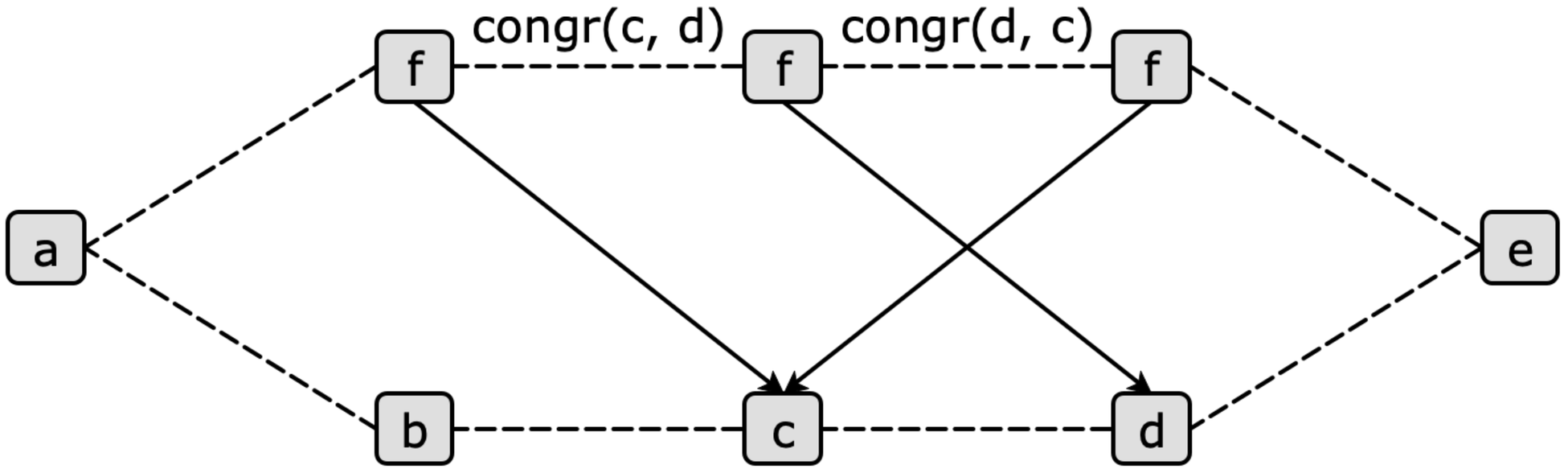
Exponential paths in graph

The Crux of The Problem

Prove a = e

Bottom: size 4

Top: size 3



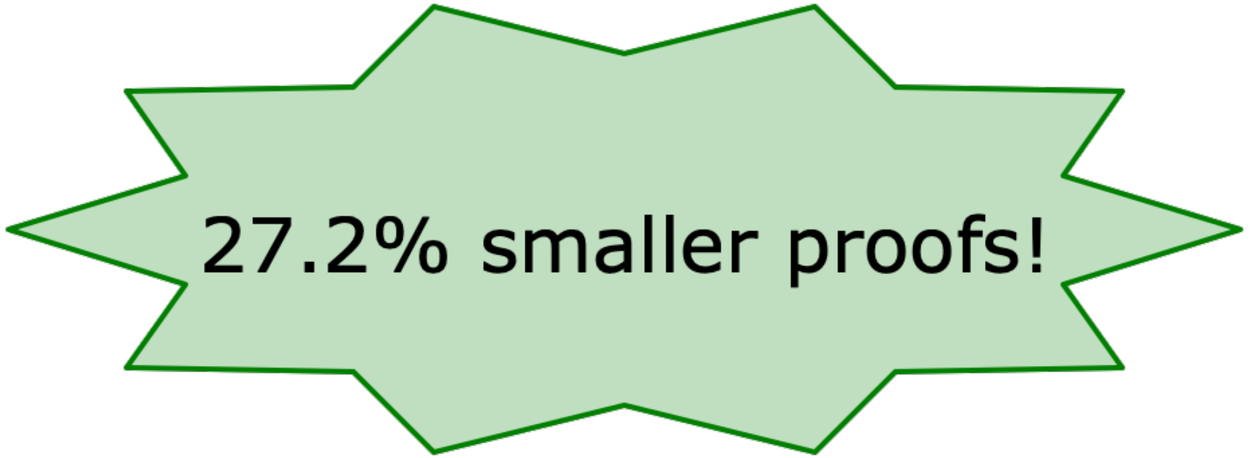
Proof re-use is free

Subproofs can be complex

Exponential paths in graph

NP-Hard Problem (Fellner Et al.)

- [x] Motivation
- [x] Congruence Closure
- [x] Proofs from Congruence Closure
- [ ] Finding Small Proofs



Idea: Shortest Path?

Inputs:

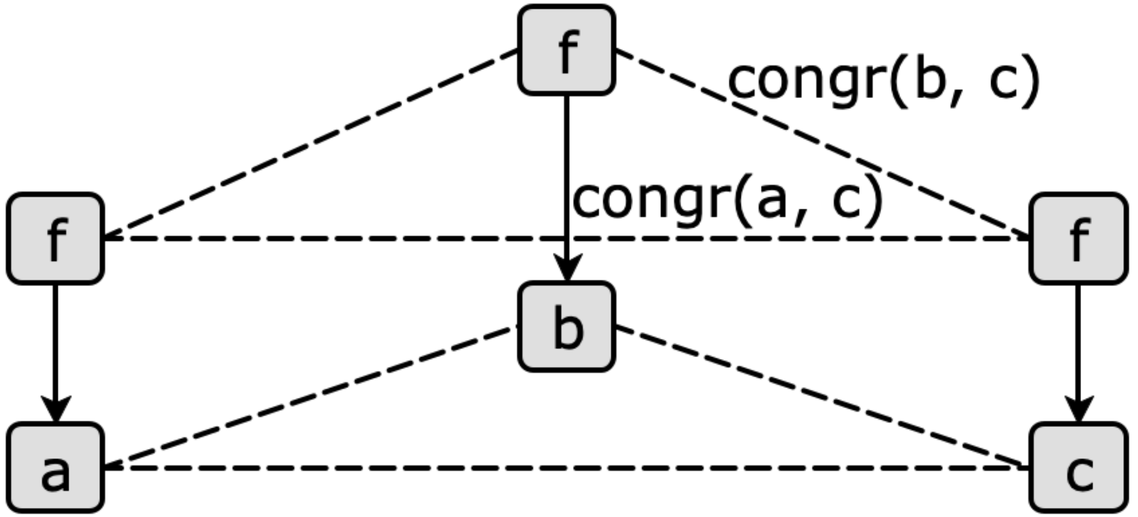
f(c)

f(a) = f(b)

a = b

b = c

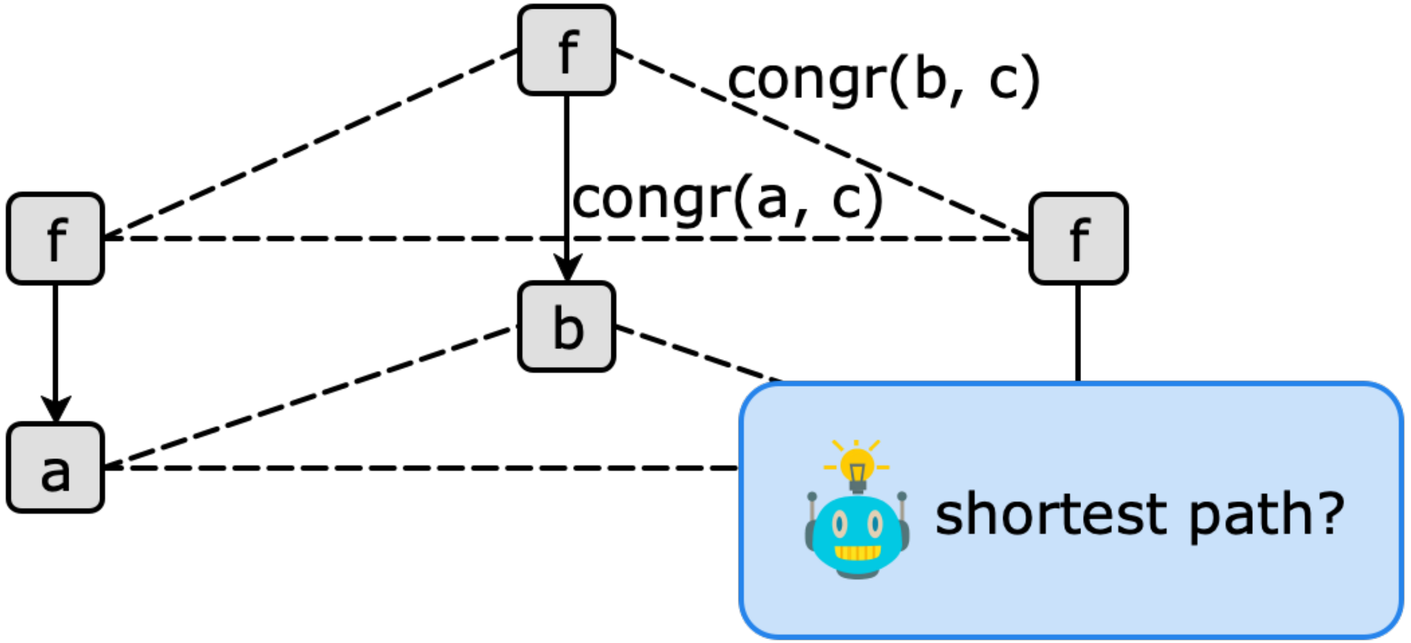
a = c useful



Idea: Shortest Path?

Inputs:

- f(c)
- f(a) = f(b)
- a = b
- b = c
- a = c useful



Idea: Shortest Path?

Inputs:

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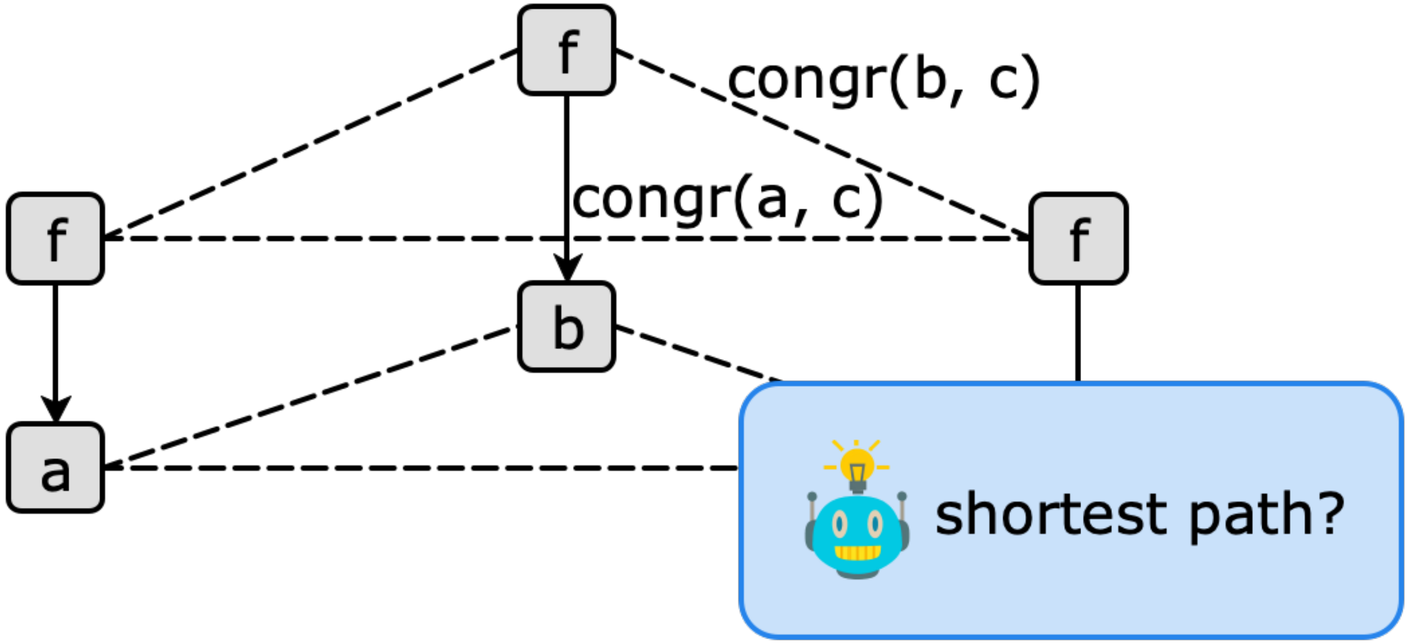
- f(a) = f(b)
- a = b
- b = c
- a = c useful



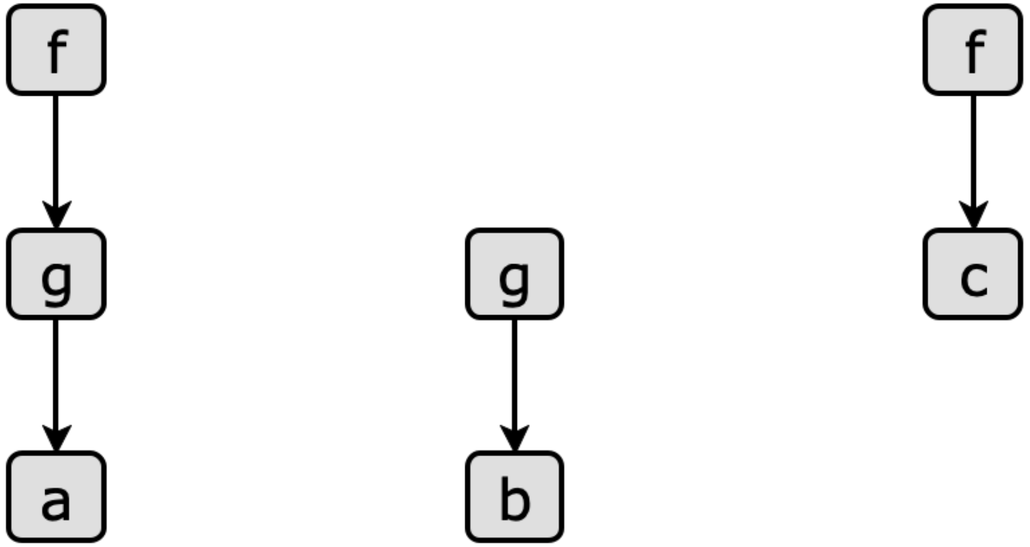
Problem: how big are congruence edges?

Proof Size Estimation

Inputs:

a = b

g(b) = c

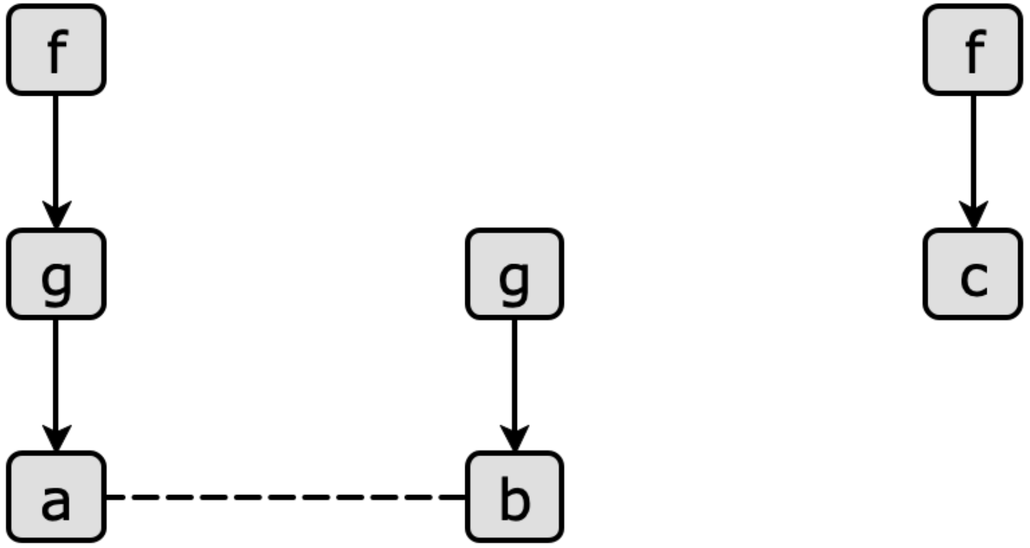


Proof Size Estimation

Inputs:

a = b

g(b) = c

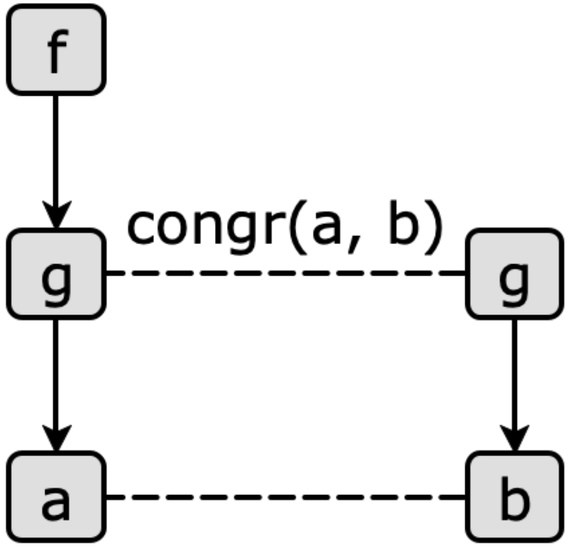


Proof Size Estimation

Inputs:

a = b

g(b) = c

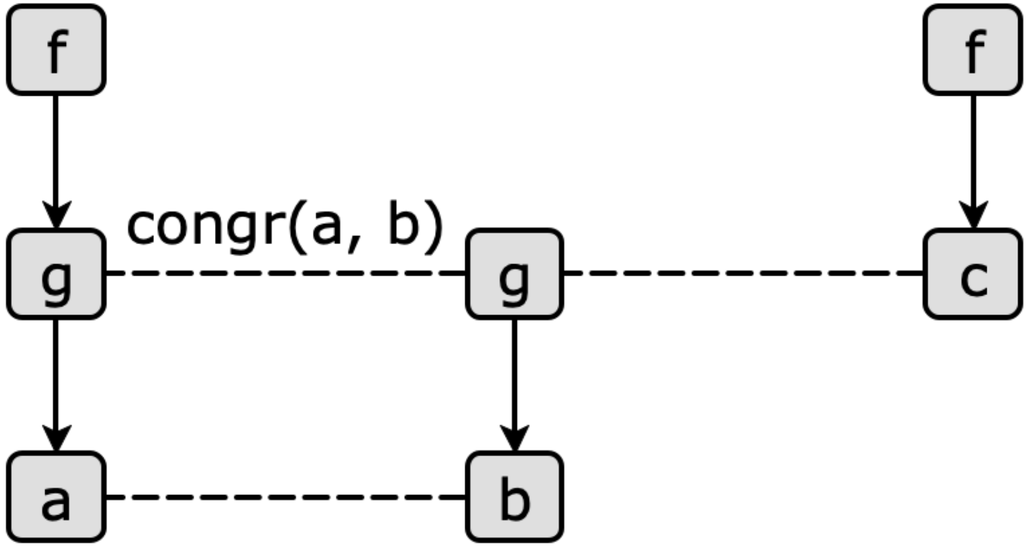


Proof Size Estimation

Inputs:

a = b

g(b) = c

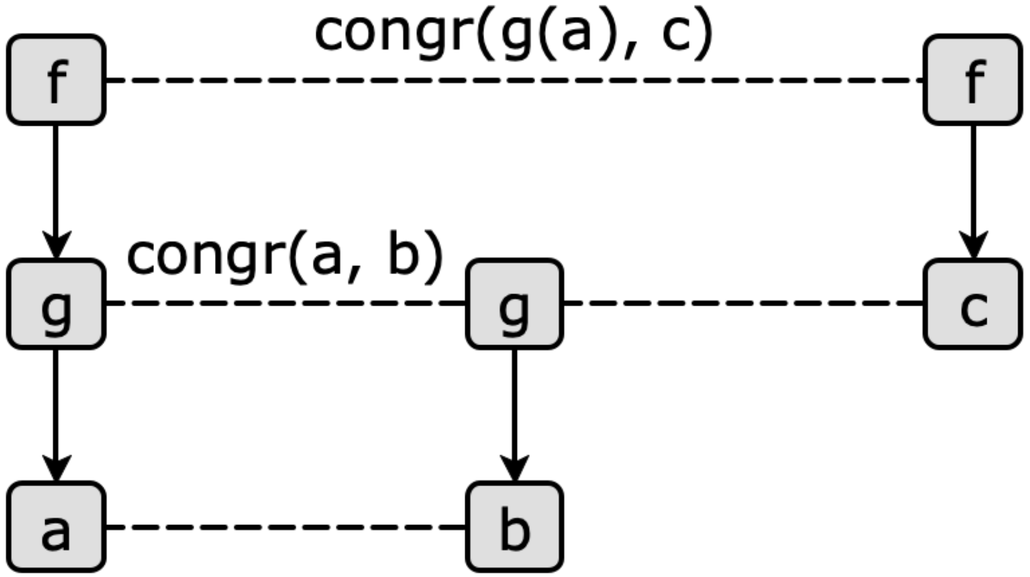


Proof Size Estimation

Inputs:

a = b

g(b) = c

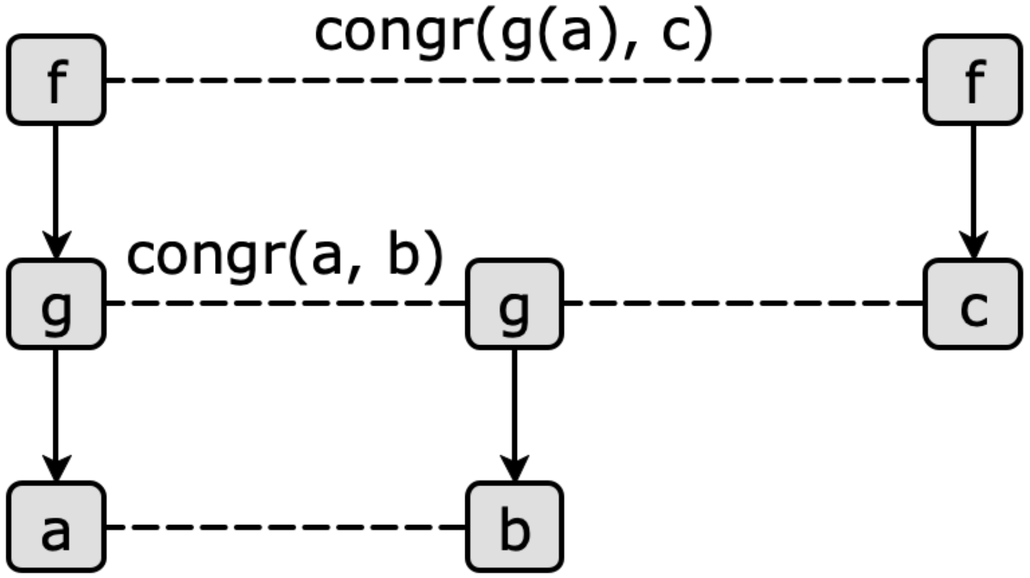


Proof Size Estimation

Inputs:

a = b

g(b) = c



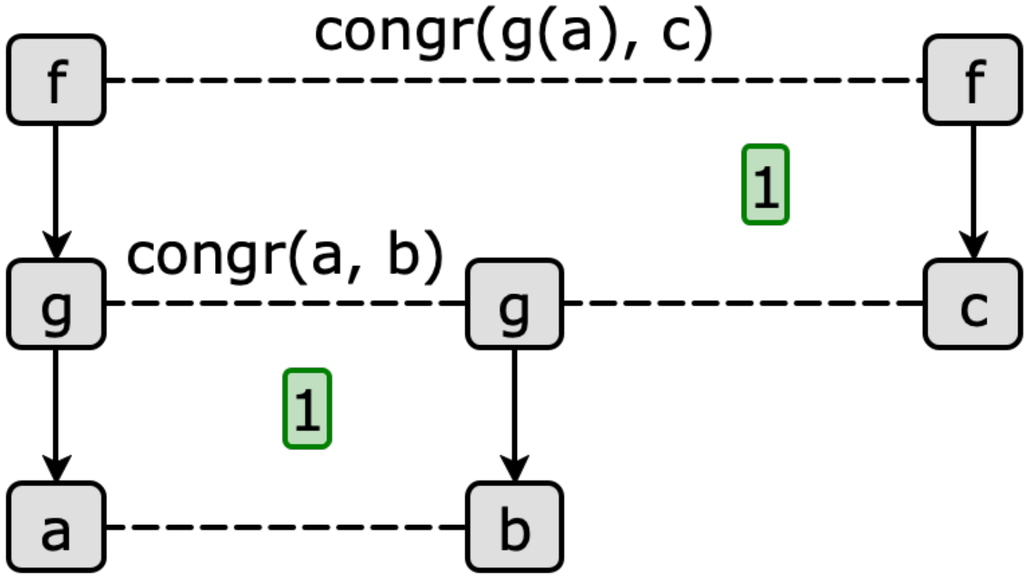
Key idea: compute estimates bottom-up

Proof Size Estimation

Inputs:

a = b

g(b) = c



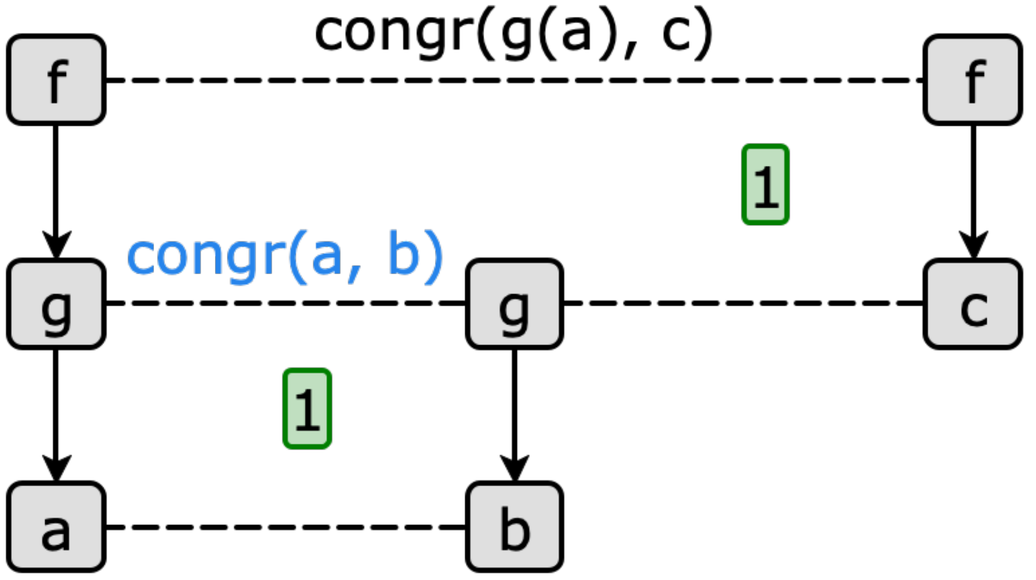
Key idea: compute estimates bottom-up

Proof Size Estimation

Inputs:

a = b

g(b) = c



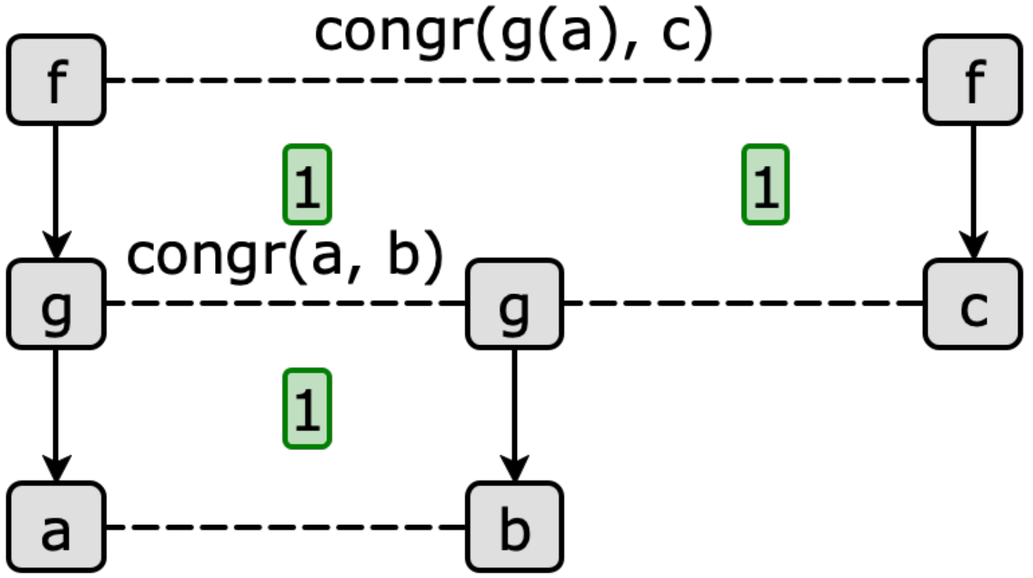
Key idea: compute estimates bottom-up

Proof Size Estimation

Inputs:

a = b

g(b) = c



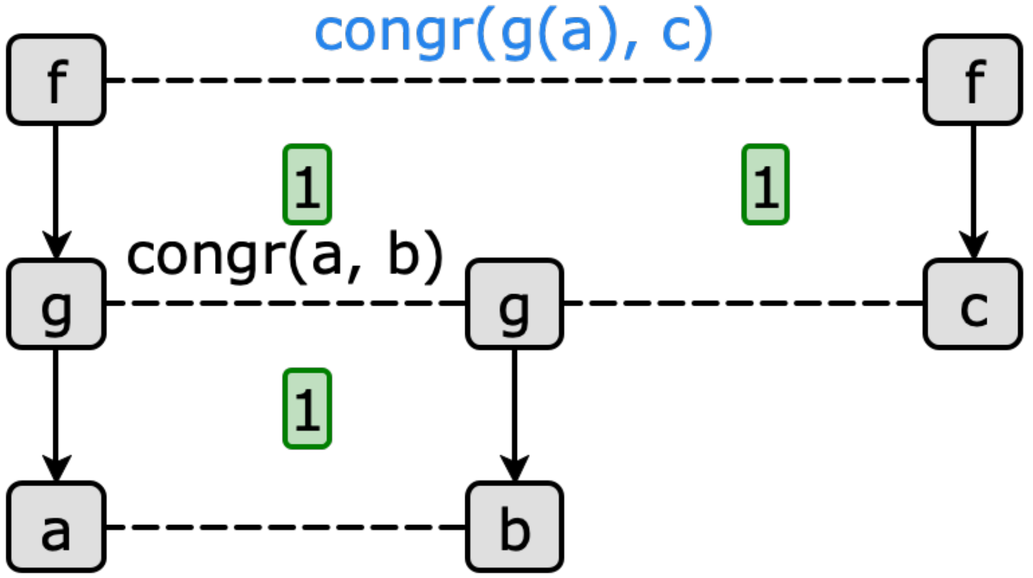
Key idea: compute estimates bottom-up

Proof Size Estimation

Inputs:

a = b

g(b) = c



Key idea: compute estimates bottom-up

Proof Size Estimation

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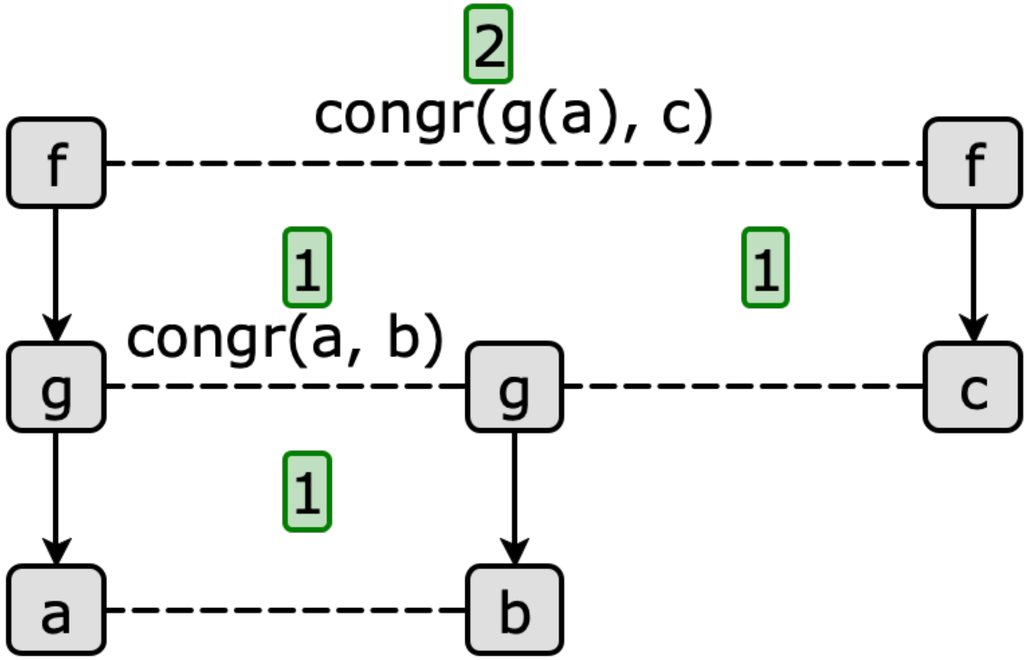

Key idea: compute estimates bottom-up

Proof Size Estimation

Inputs:

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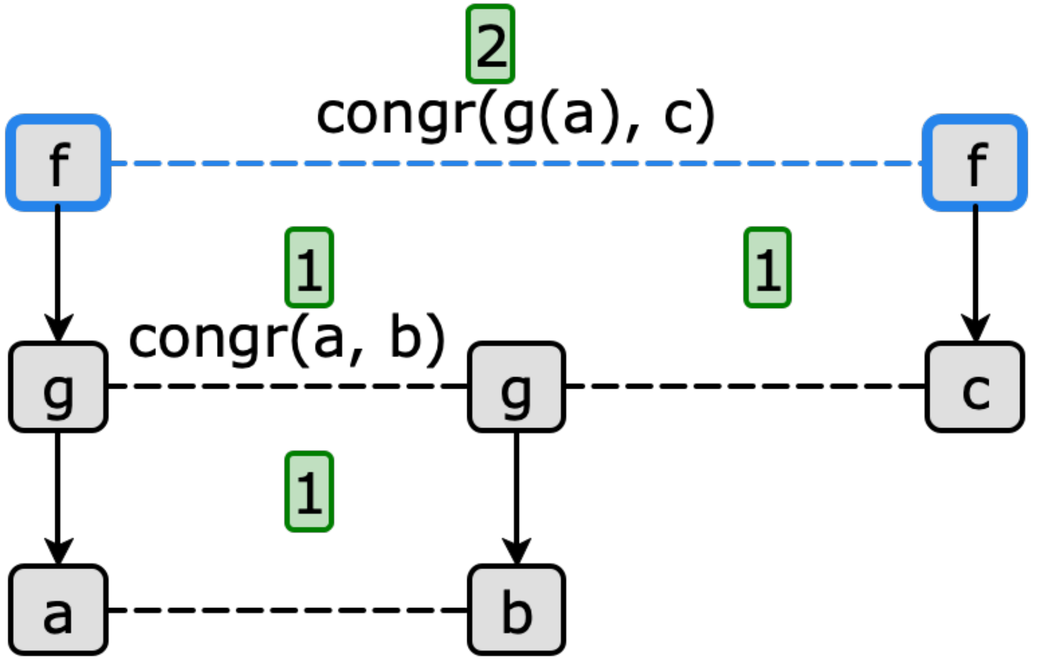
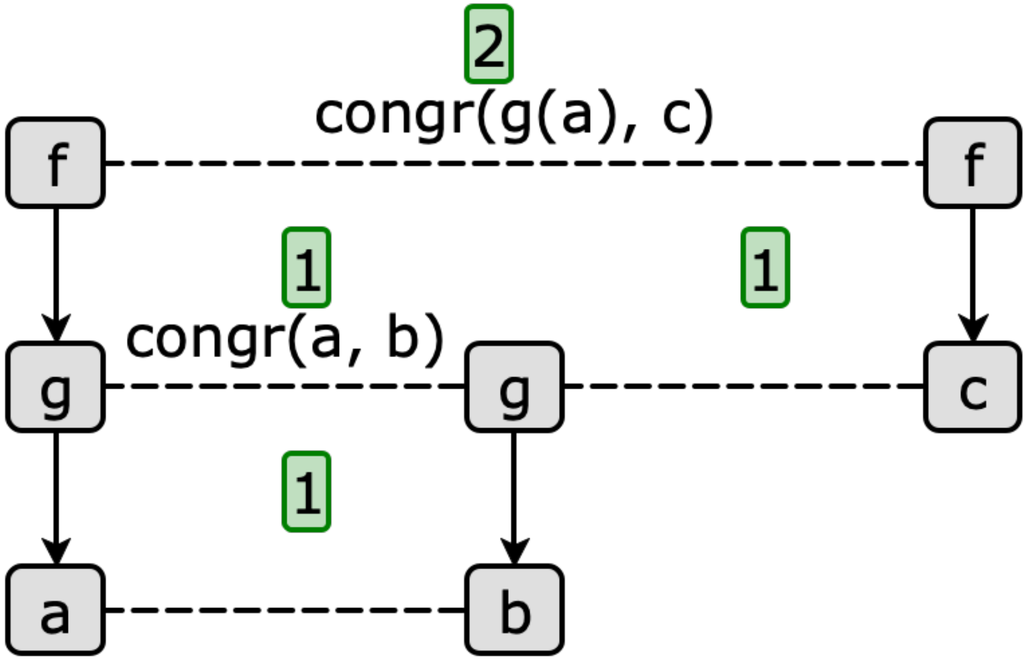
g(b) = c



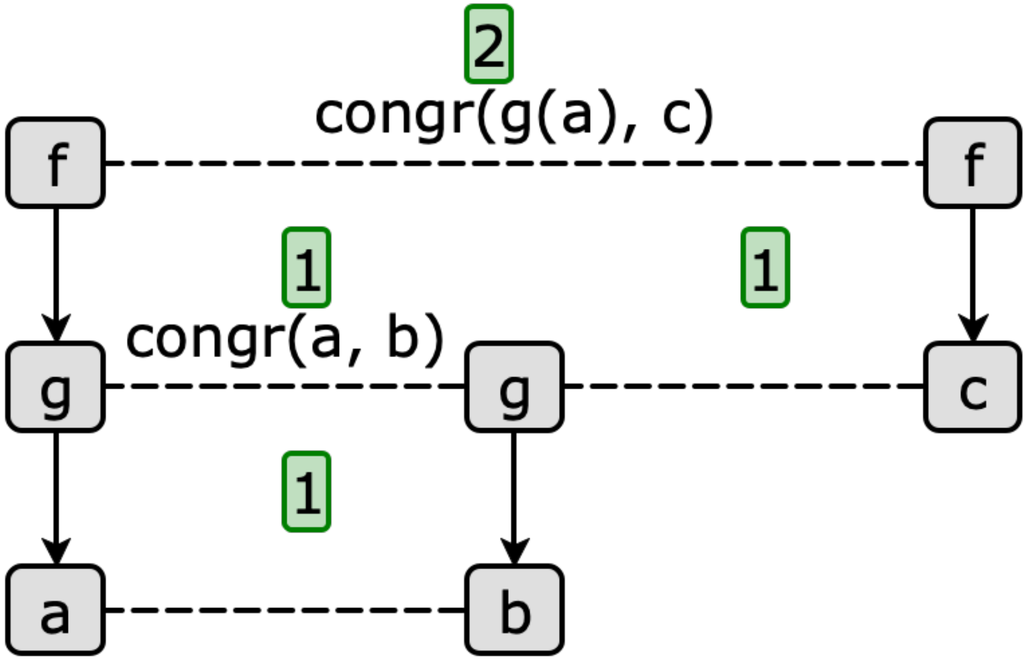
Key idea: compute estimates bottom-up

Putting it All Together



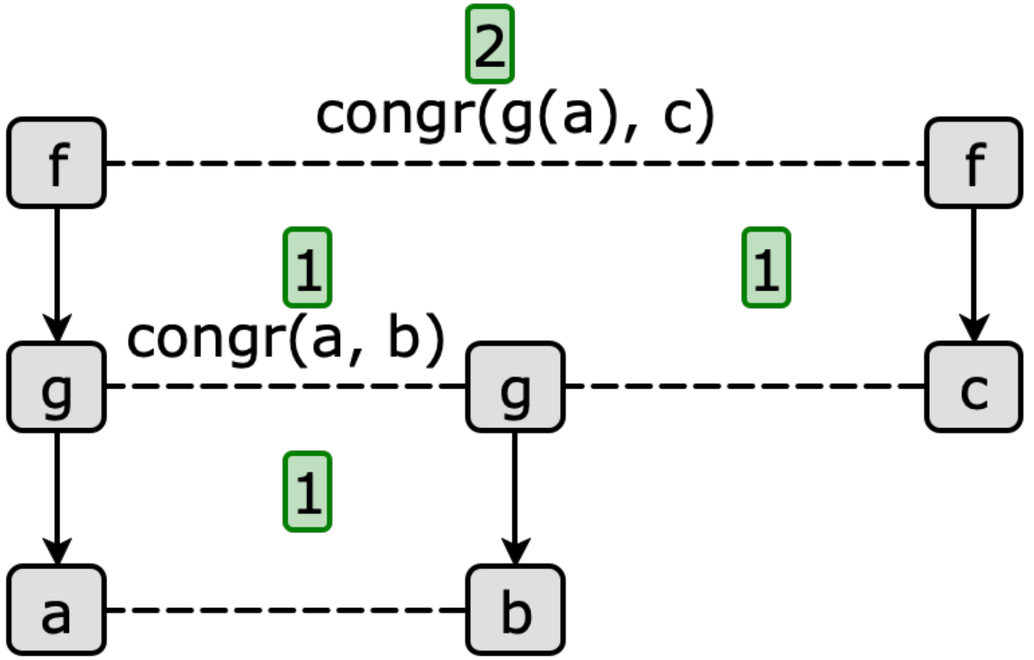
Putting it All Together

1. Compute size estimates



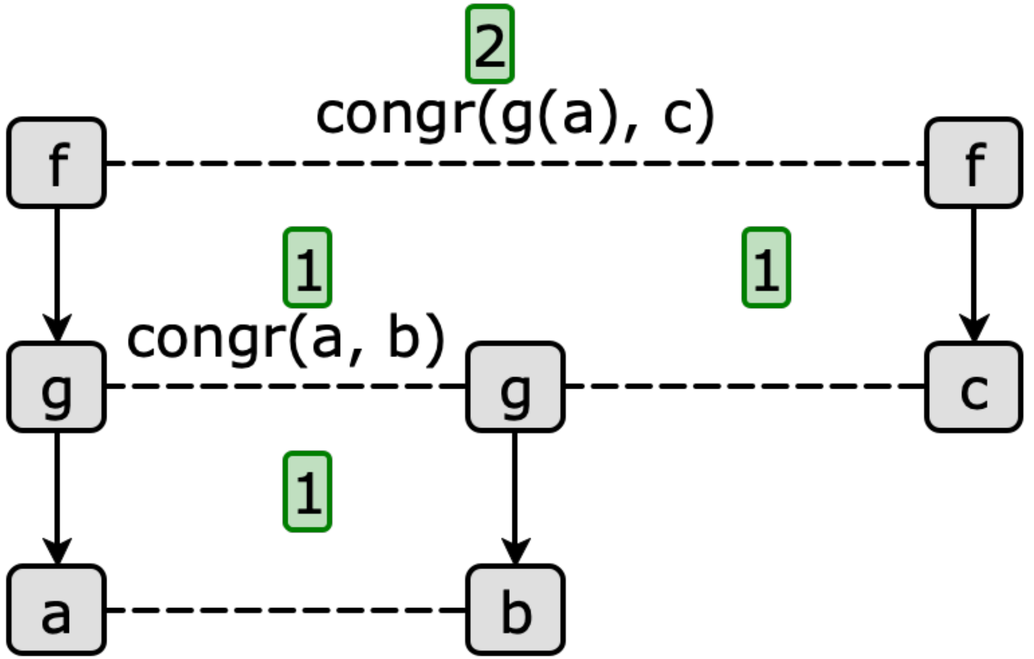
Putting it All Together

1. Compute size estimates
2. Find shortest path

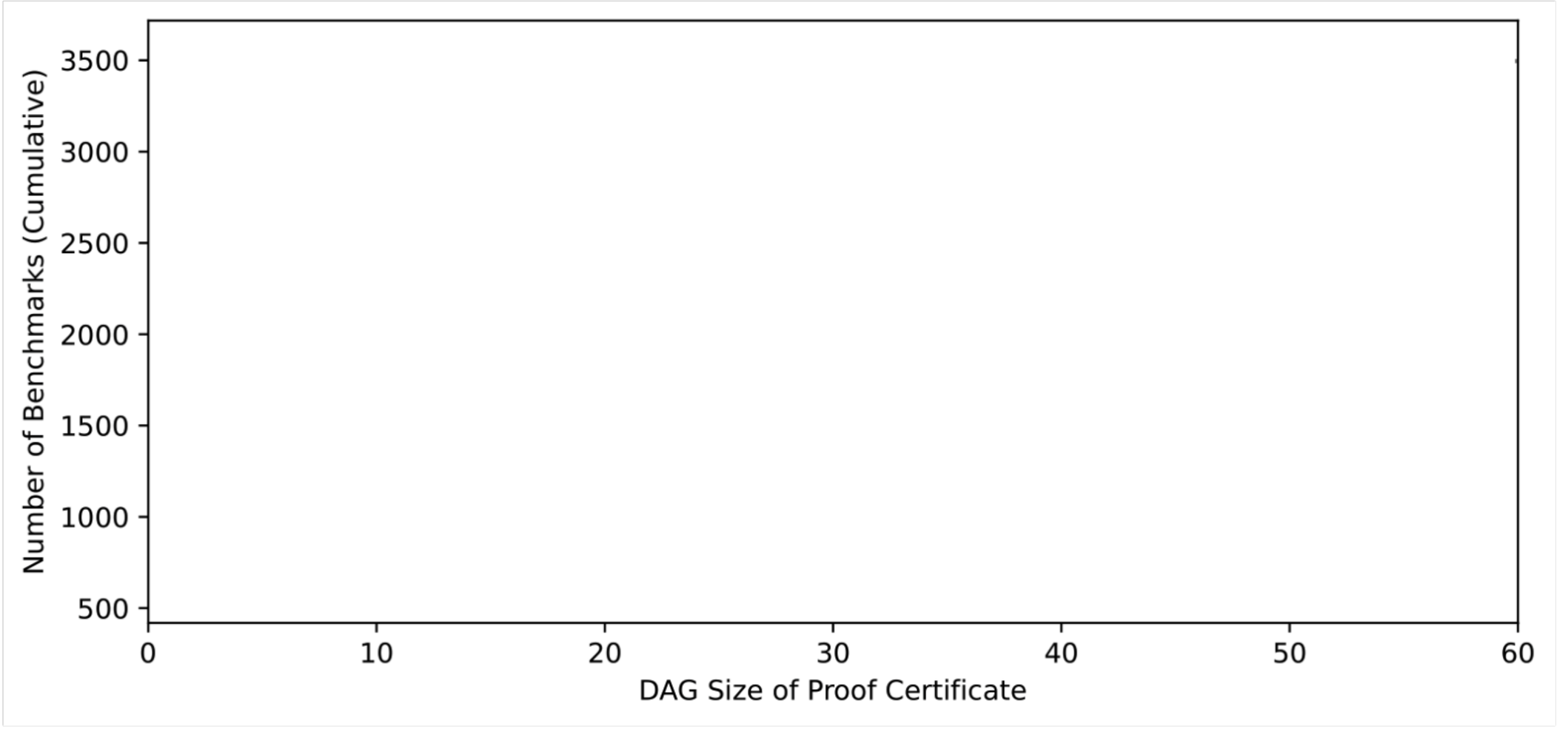


Putting it All Together

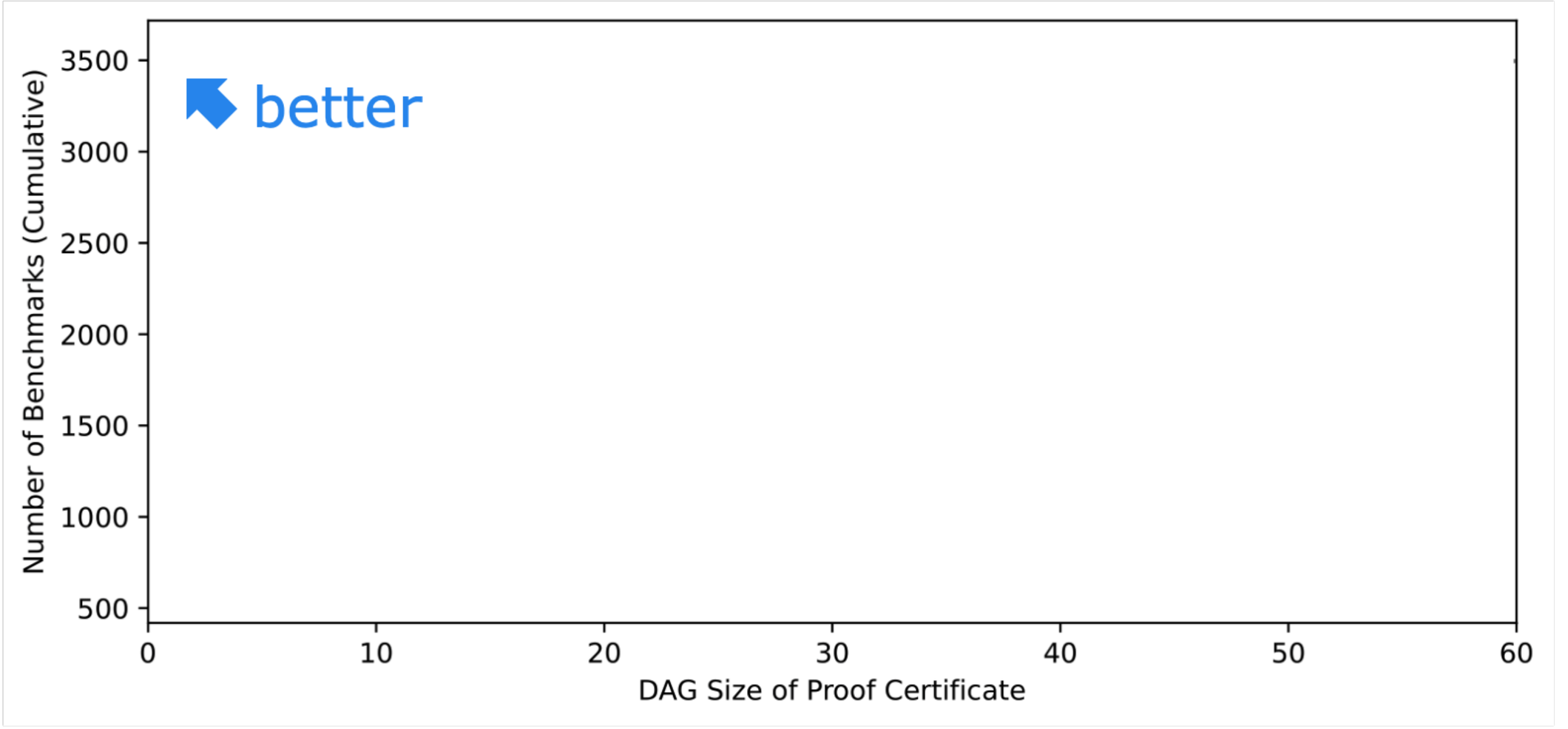
1. Compute size estimates
2. Find shortest path
3. Output extracted proof



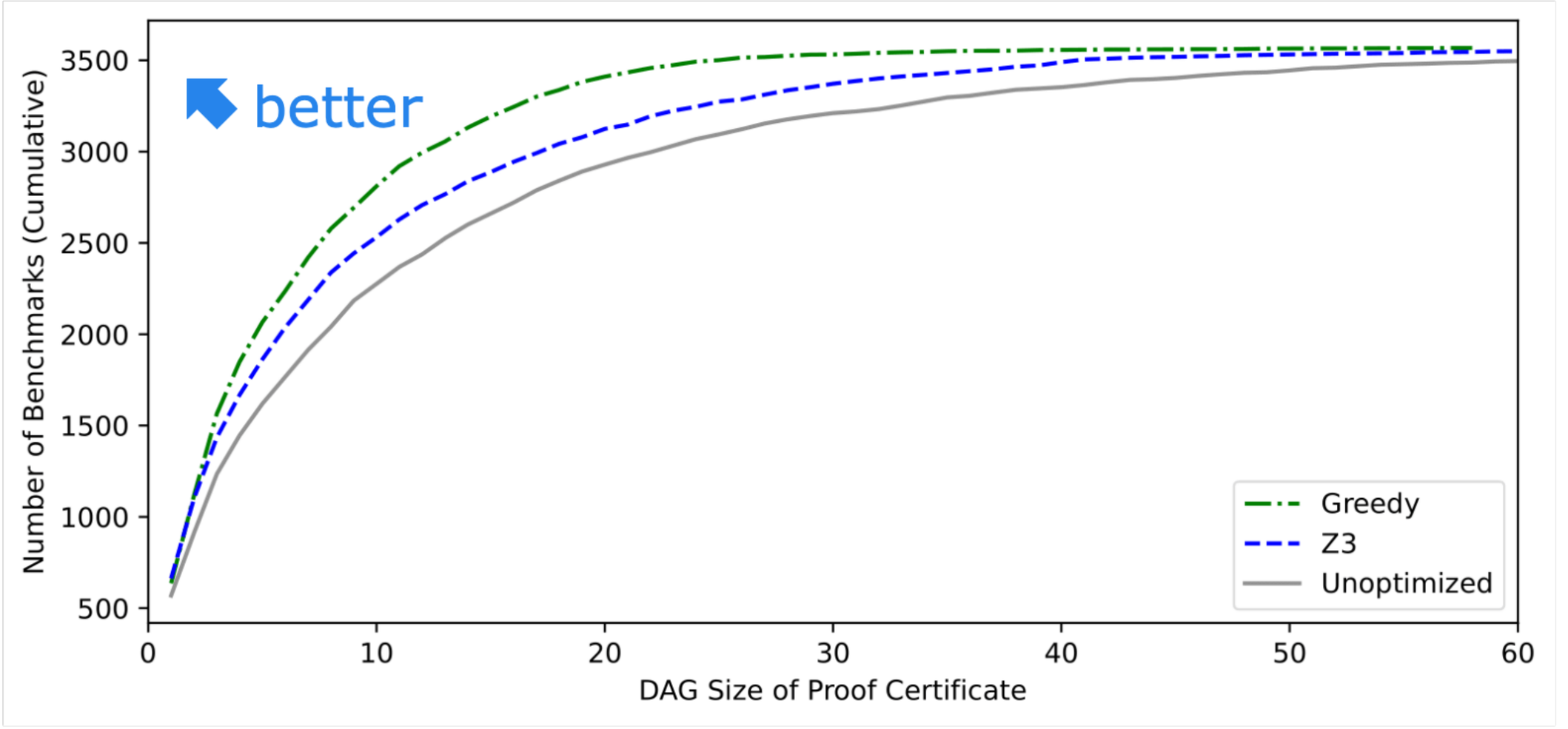
Results


Number of Benchmarks (Cumulative)
3500
3000
2500
2000
1500
1000
500
0
10
20
30
40
50
60
DAG Size of Proof Certificate

Results


better
Number of Benchmarks (Cumulative)
3500
3000
2500
2000
1500
1000
500
0
10
20
30
40
50
60
DAG Size of Proof Certificate

Results


better
Number of Benchmarks (Cumulative)
3500
3000
2500
2000
1500
1000
500
0
10
20
30
40
50
60
DAG Size of Proof Certificate
Greedy
Z3
Unoptimized

Intel Case Study

Multi-operation circuit optimization and translation validation with egg 

Intel Case Study

Multi-operation circuit optimization and translation validation with egg 

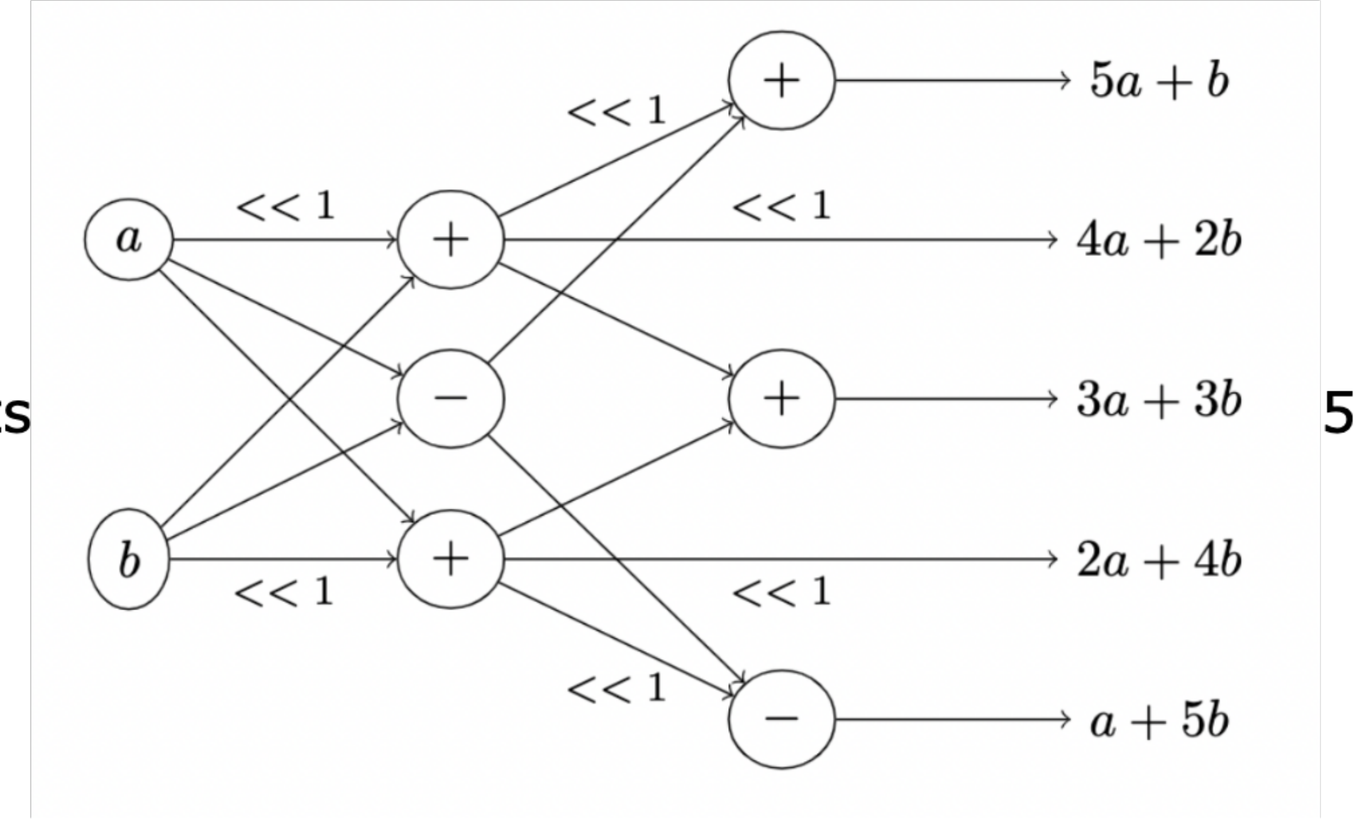
4.7 hours -> 2.3 hours

Intel Case Study

Multi-operation circuit optimization and translation validation with egg

4.7 hours -> 2.3 hours

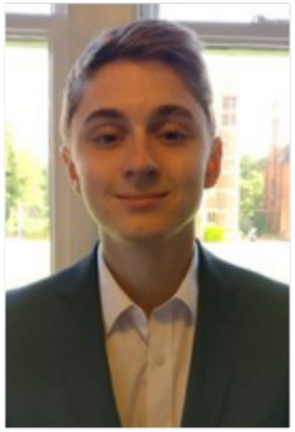
2 inputs



5 outputs

Team and Acknowledgments


Oliver Flatt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Max Willsey


Zachary Tatlock


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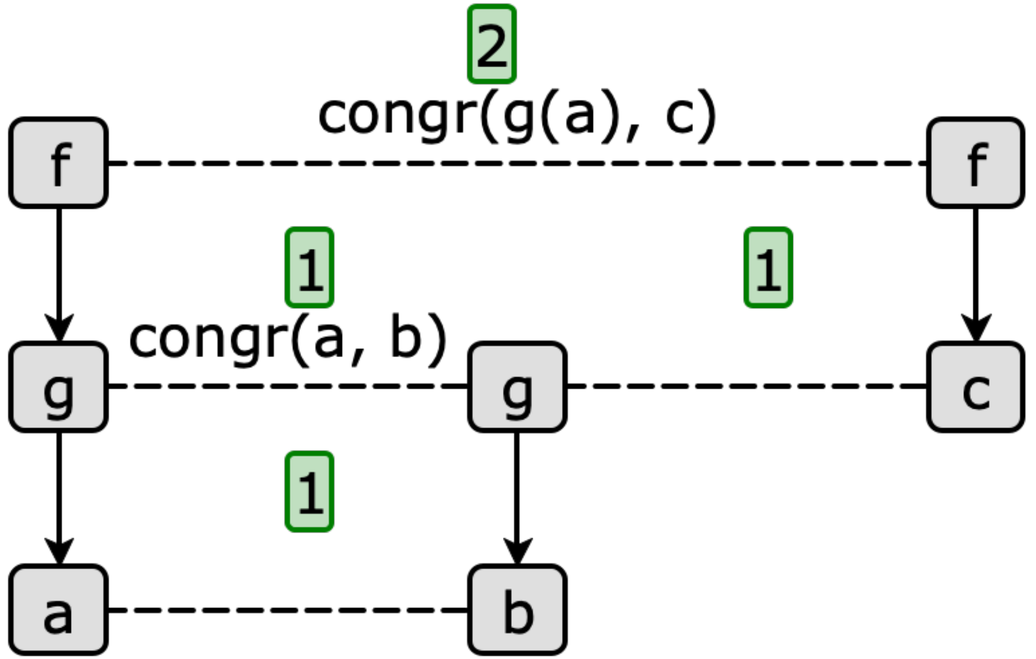
Leonardo de Moura (Microsoft)

Questions?

1. Compute size estimates
2. Find shortest path
3. Output extracted proof



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