University of Dublin, Trinity College

JS Engineering: 3C1 Signals & Systems
S1: Linear Time Invariant Systems
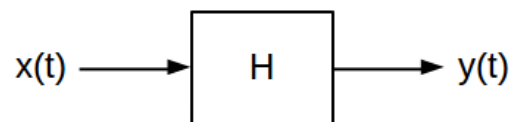
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Contents

1 Abstract

Laboratory S1 introduces concepts of Matlab and sinusoidal signal analysis within LTI systems in order to demonstrate and familiarise the relationships that exist between the various concepts within qualitative analysis.

2 Matlab

In this laboratory, the scripting language *Matlab* was used which gets compiled at its runtime. This makes for a language that is great for rapid-prototyping and extensive functionality within arithmetic, graphing, and graphical user interfaces. During this report, Matlab code snippets will be occasionally shown.

3 Signals

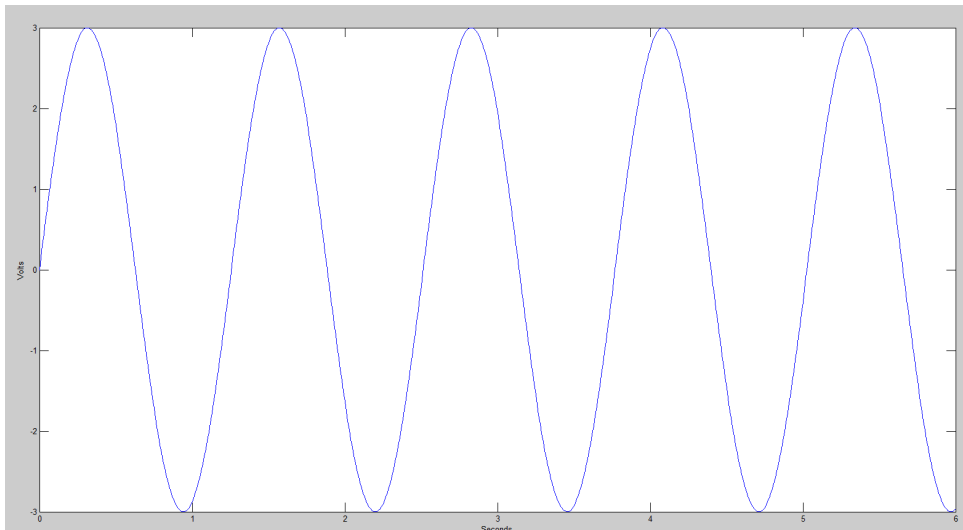## 3.1 Plotting the Deterministic Sine Wave x_1

- Generate a deterministic signal $x_1 = 3sin(5t)$ over the range $0 \leq t \leq 6$ seconds.

```
t = (0:0.01:6); % range 0-6 with steps of 0.01s
x1 = 3 * sin(5 * t);
figure(1); plot(t,x1);
title('Sine Wave Seconds v Voltages');
xlabel('Seconds (s)'); ylabel('Volts (V)');
```


3.1.1 Properties

- Voltage extrema

  - Maximum: 3V
  - Minimum: -3V

- Frequency

- 1.256s for 1 oscillation
- $\frac{1}{1.256} \approx 0.796\text{Hz}$
- Also determined from $\frac{5}{2\pi}$, where 5 refers to the factor before t, which also results in 0.796 Hz

- Periodicity

  - Time for one cycle of the wave is 1.256s.

3.2 Expanding to x_2

- Generate a deterministic signal $x_2 = A sin(\omega_1 t + \phi)$ over the range $0 \leq t \leq 6$ seconds.

```
w1 = 5;
phi = -3;
A = 3;
x2 = A * sin(w1 * t + phi);
hold on;plot(t,x2,'r');title('Sine Wave Seconds v Voltages');
xlabel('Seconds (s)');ylabel('Volts (V)');
```

Note that the same plot is overlaid using the hold on command for the plot given a previous one.
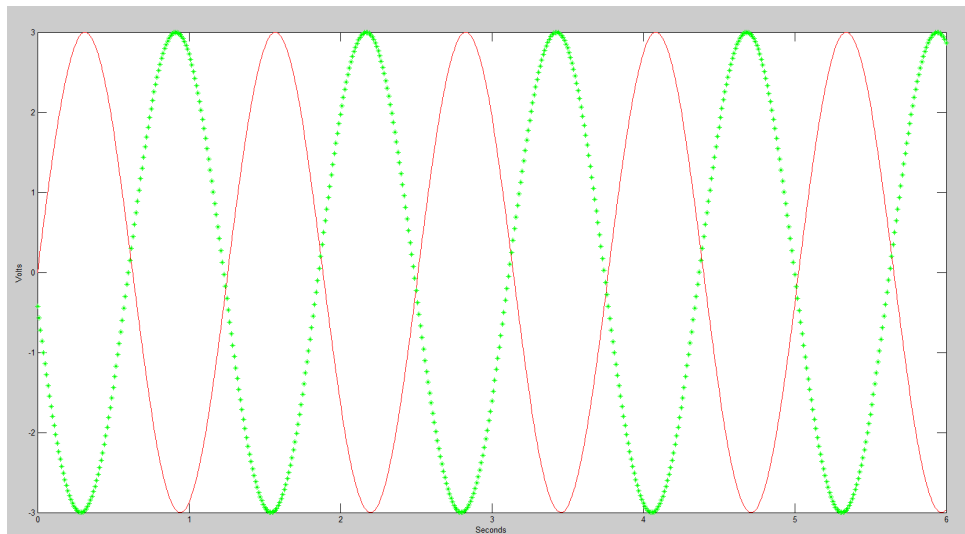
3.2.1 Properties

- Frequency

  - 1.256s for 1 oscillation
  - $\frac{1}{1.256} \approx 0.796\text{Hz}$
  - Also determined from $\frac{5}{2\pi}$, where 5 refers to the factor before t, which also results in 0.796 Hz

- Periodicity

  - Time for one cycle of the wave is 1.256s.

- Differences between x_1 and x_2

  - x_1 is a regular sine wave that has no offsets or lags. x_2 however, is offset such that it has almost become inverted. They two signals share the same amplitude and frequency.

- Is there a phase lag?

  - Yes there is. x_2 contains a lag ϕ that is not present in x_1.

- Is this a delay or an advance?

  - This is a delay with respect to x_1.

3.2.2 Delay

Between x_2 and x_1, there is about 0.657s lag time. I picked one point common to both signals (-3V), and recorded the difference in time between the two signals to reach this.

3.3 Expanding Further to x_3

- Generate a deterministic signal $x_3 = A sin(\omega_1 t + \phi)$ over the range $0 \leq t \leq 6$ seconds.

```
w1 = 5;
phi = -3 + (2 * 3.14);
A = 3;
x3 = A * sin(w1 * t + phi);
hold on;plot(t,x3,'r');title('Sine Wave Seconds v Voltages');
xlabel('Seconds (s)');ylabel('Volts (V)');
```
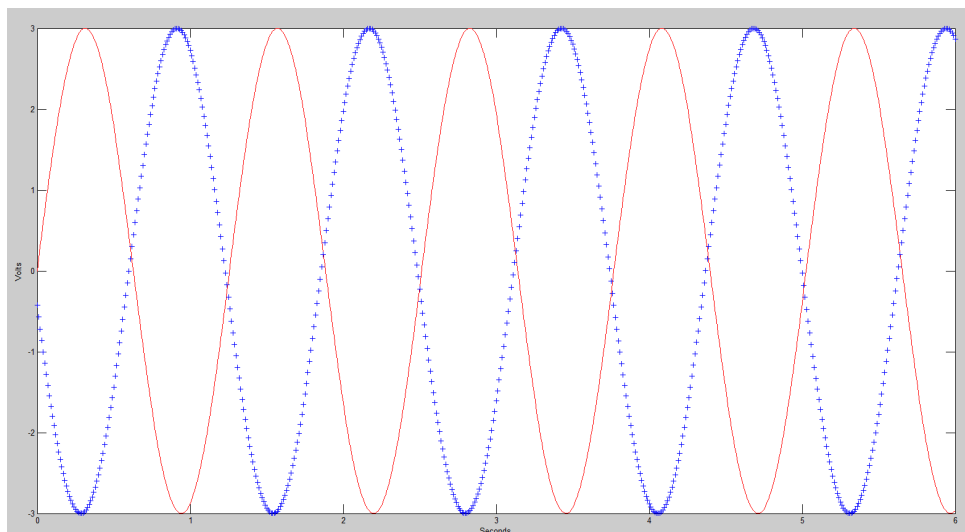

3.3.1 Properties

The function generated by x_3 is identical to x_2. The only difference is that x_3 has been shifted by 2π, thus they are in phase with one another, and overlay accordingly.

4 Linear Time Invariant Systems

4.1 LTI Systems

4.1.1 Input vs Output

- Input v Output

    - Both of the input and output signals of system 1 are sine waves. The output takes a few cycles to reach a steady state with amplitude varying accordingly.

4.1.2 Differences between Signals

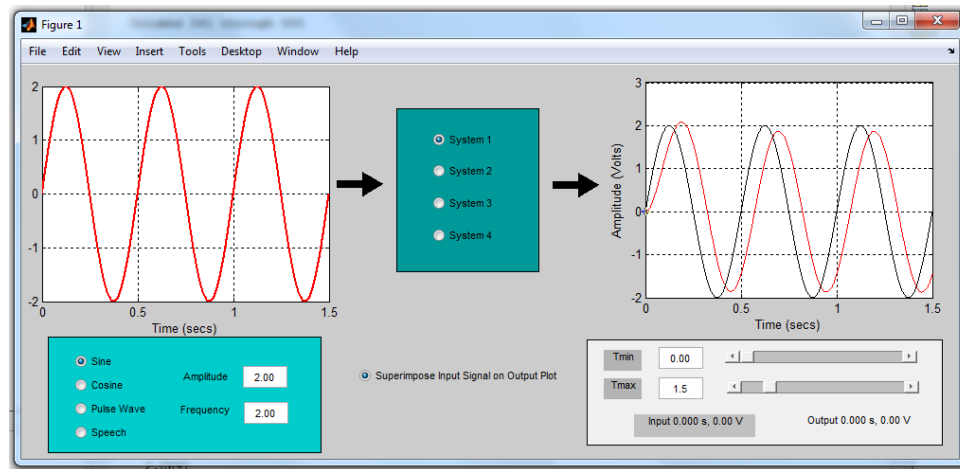- What are the differences between the input and output signals using system 1?

    - The difference between the input and output lies with an approximate 0.08s delay with respect to the input. The frequencies remain the same though, while the amplitude initially overshoots and then decays to a steady state amplitude that is lower than the input's amplitude voltage after some cycles have elapsed.

- What is the same?

    - The sine transient input has remained the same.

### 4.1.3 2x Amplitude Factors

- What is the corresponding increase in the output signal amplitude given that its initial transients have decayed?

    - The amplitude of the output signal is approximately 3.71V where the input signal's amplitude is 4V.

- How long does the system take to settle into a steady state response?

    - After approximately 0.325s the system has entered a steady state.

4.1.4 Relationship between Factors

- Given input amplitudes $0.5, 1.0$ ... 2.5, measure the corresponding output amplitudes

    - Given the range of amplitudes, the following table and graph show the linear effect of amplitude vs output:

Input (V)	Output (V)
0.5	0.47
1.0	0.93
1.5	1.4
2.0	1.87
2.5	2.34

Table 1: Input vs Output Voltages
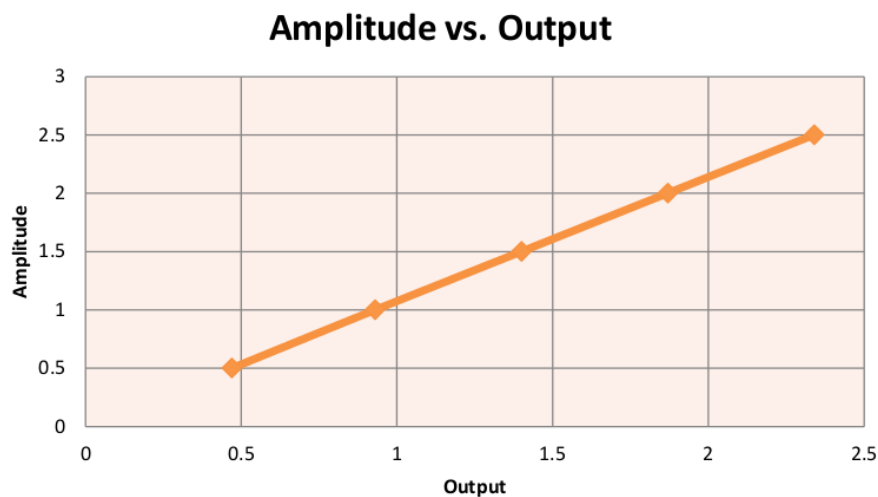
4.1.5 Pulse Wave Transients
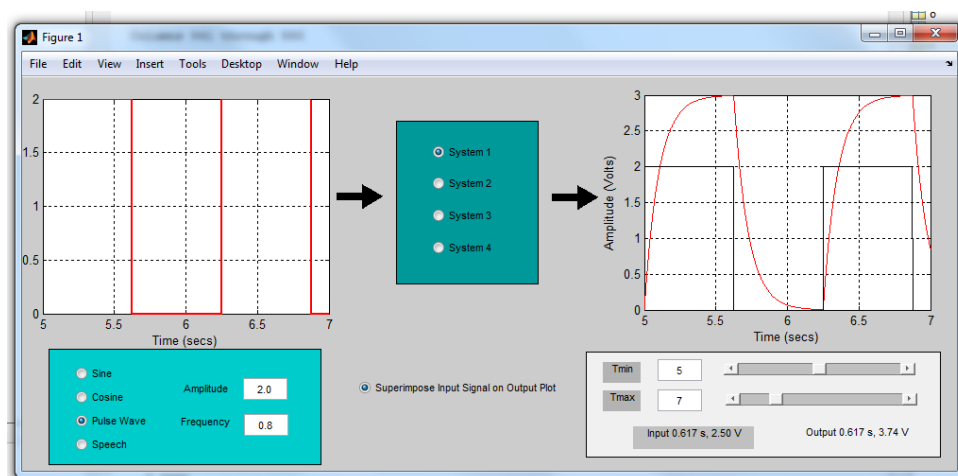
- Switch to the Pulse Wave and set its frequency to 0.8Hz noting the output amplotude to be the maximum value of the output pulse after the initial transients have decayed.

Input (V)	Output (V)
0.5	0.75
1.0	1.5
1.5	2.25
2.0	3.0
2.5	3.75

Table 2: Input vs Output Voltages

Amplitude vs. Output
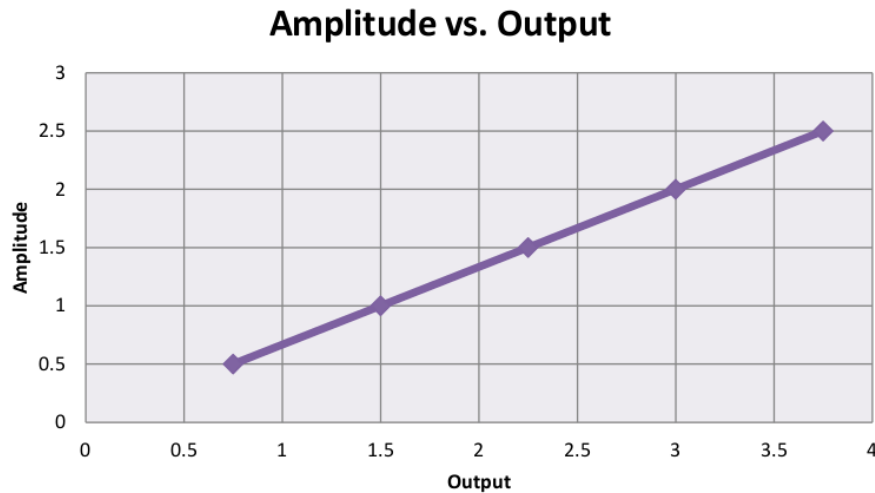
4.1.6 Cosine Waves in System 1
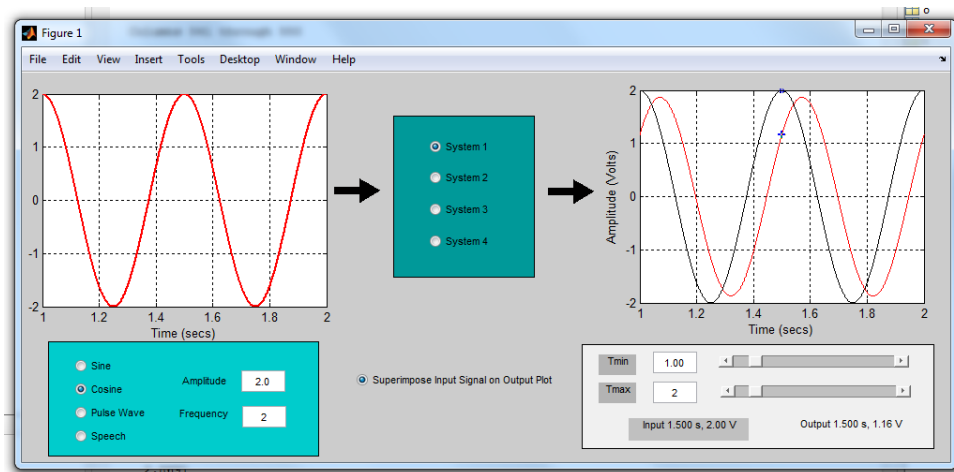
- What is the difference between the input and output signals using system 1?

    – The difference between the outputa dn the intput signals is that the output signal's amplitude value is again higher than the input signal's one, where the amplitude of the output signal is now $\approx 1.87V$ and the input is $2V$.

- What is the same?

    – The output signal is shifted to the right of the input signal by ≈ 0.06s, as seen in the images.

4.1.7 Mathematical Approximation

- For both the sine and cosine functions with frequency 2Hz and amplitude 2 under system 1, approximate expressions for input and output signals.

  - Input sine
    - $x(t) = 2sin(4\pi t)$

  - Input cosine
    - $x(t) = 2cos(4\pi t)$

  - Output sine
    - $y(t) = 1.87sin(12.5t)$

  - Output cosine
    - $y(t) = 1.86cos\left(4\pi + \frac{\pi}{2}\right)$

4.1.8 Classification of Systems

- Classify systems 2, 3 & 4 as LTI or non-LTI over a wide range.

  - System 2 is LTI
  - System 3 is LTI
  - System 4 is LTI

Within the given systems, time was adjusted accordingly in order to get a variety of values obtained from the input and output sine and cosine waves. Using the data marker tool, it was possible to retrieve the necessary values for gathering data.

	Sine	
	In (V)	Out (V)
1.0	0	-1.5
1.1	1.9	0.6
1.2	1.1	1.85
1.3	-1.2	0.45
1.4	-1.9	1.55
1.5	0	-1.5

Table 3: Sine System

The same is true for the cosine values, whereby the same method was used again with the change from sine to cosine in the settings of the linear systems.

	Cosine	
	In (V)	Out (V)
1.0	2	-1.16
1.1	0.61	1.75
1.2	-1.61	-0.07
1.3	-1.64	-1.79
1.4	0.72	-0.94
1.5	2	1.16

Table 4: Cosine System

4.2 Gain & Phase as a Function of a Frequency

4.2.1 Completed Sine Input Table for System 1

|	Input			Output		
Frequency (rad/sec)	Frequency (Hz)	Amplitude (x)	Frequency (rad/sec)	Amplitude (y)	Phase Lag (rad)	Gain (y/x)
1	0.159	1	1	1.27	0.01	1.27
5	0.796	1	5	0.45	0.04	0.45
8	1.273	1	8	0.29	0.03	0.29
10	1.592	1	10	0.24	0.025	0.24
12	1.909	1	12	0.20	0.02	0.20
15	2.387	1	15	0.16	0.014	0.16
20	3.183	1	20	0.12	0.012	0.12
30	4.775	1	30	0.08	0.008	0.08
50	7.958	1	50	0.05	0.004	0.05
100	15.915	1	100	0.02	0.003	0.02

Table 5: LTI inputs vs outputs

4.2.2 Plotted Graphs

- Plot a graph of Gain vs Frequency (rad/sec) & Phase (rad) vs Frequency (rad/sec)

  - Both of the graphs exhibit an exponential decay. As can be also seen, there is an imperfect curve within my graph due to sampling errors from points I have chosen, therefore my results also have an approximate error within them.
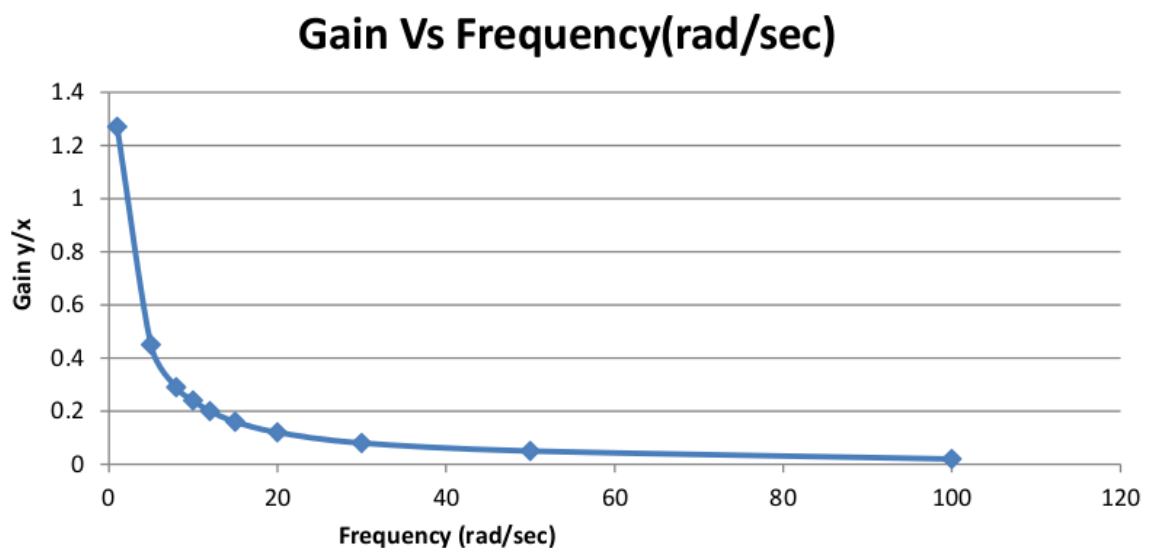
Phase (rad) Vs. Frequency(rad/sec)
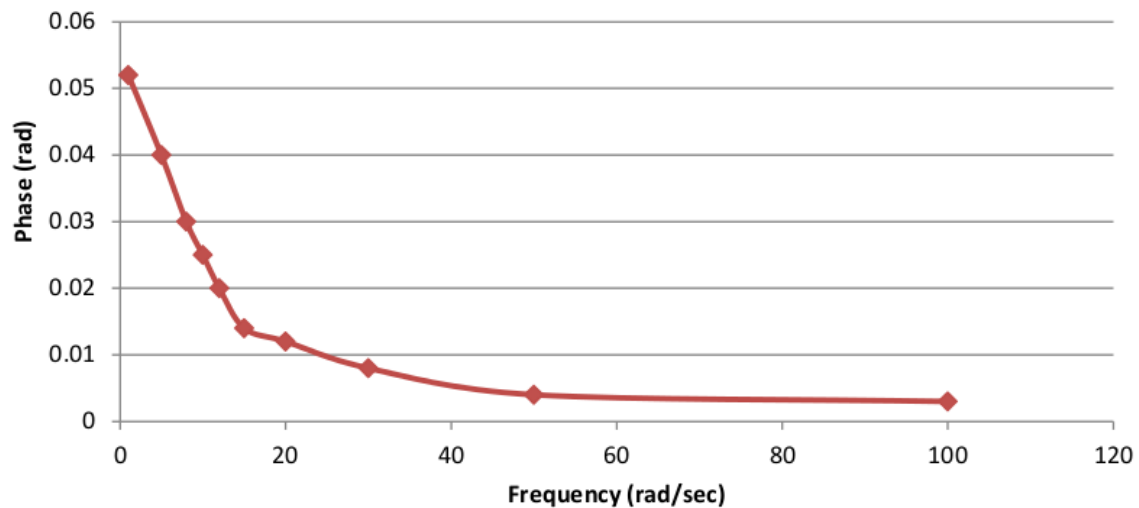
4.2.3 Frequency Properties
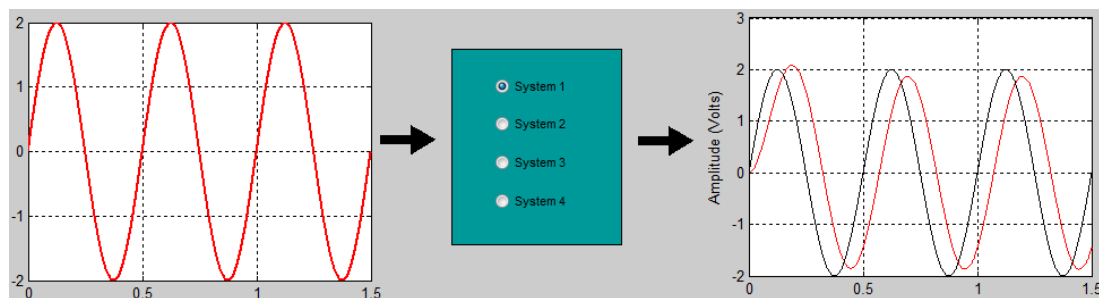
- Is the effect of the system the same at all frequencies?

  - The effects of the system are different with respect to the frequency provided.

- How does the system behaviour change with frequency?

  - It appears that the system reacts differently depending on the frequency provided. Given a low frequency, gain tended to be higher than 1. Given a high frequency, gain tended to be lower than 1. The table of results was formed from setting the input amplitude at a constant value of 1, and the frequency was adjusted in order to allow us to measure the changes in amplitude of the output with different values of frequency.

4.2.4 Discussion

- Discuss the significance of this plot with respect to the effect of system 1 on the Pulse Wave and Speech Signal.

  - The Phase vs Frequency plot shows an exponential decay in the phase lag in radians per second which pertains to tending to a phase lag of 0, which would entail *phase quadrature*. The significance of this is that this relatinship extends both graphs. As we can see from the first graph, there appears to be an inverse relationship appearing between Gain and Frequency. For low values of frequency, the output amplitude was actually larger than the input, causing a gain value that was greater than 1. The

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graph begins to level off for higher frequencies, and seems to begin to approach a zero gain value. The proportion at which the gain decreases slows down as frequency is increased.

5 Discussion

1. What does frequency and phase mean with respect to a pure sinusoidal signal?

   - Frequency refers to the number of oscillations per second that occur within the signal generated.
   - Phase shift refers to an offset through mathematics that is observed graphically by seeing a *delay* in the signal from the phase ϕ.

2. How does an LTI system affect a pure sinosoid?

   - A sinosoidal input gives rise to a sinosoidal output with the same frequency. The amplitude however, is modified in the process.

3. What is the definition of an LTI system?

   - An LTI system exhibits two behaviours:
     - Time Invariance - The system must always behave the same between any two trials in time given that their starting conditions and inputs remain the same.
     - Additive Superposition - By exciting the system via input $H\{a_1x_1(t)+a_2x_2(t)\}$,the output should return $a_1y_1(t) + a_2y_2(t))$ through linearity.

4. What do the terms *phase shift* and *gain* mean?

   - Phase Shift is a mathematical offset that is observed graphically by horizontal shifting through the characteristic equation $y = Asin(B(x - C)) + D$ which can be applied to all trigonometric functions.
   - Gain is the ratio of the output with respect to the input. As it is unitless, it describes the factor at which a signal has been boosted or reduced with respect to its input.

5. How are the step and impulse response of system characterised?

   - Step response is characterised through two key theorems within a given LTI system, *Initial Value Theorem* and *Final Value Theorem*, which correspond to the simplifications $y_\gamma(0^+) = H(\infty)$ and $y_\gamma(\infty) = H(0)$.
   - Impulse response is characterised through a short-duration time-domain signal. For continuous-time systems, this is the *Dirac-delta function*, $\delta(t)$, while for discrete-time systems, this is the *Kronecker-delta function*, $\delta[n]$; where $h(t)$ or $h[n]$ is defined as the output signal that results when an impulse is applied to the system's input, such that

$$x[n] = \sum_{k=0}^{\infty} x[k]\delta[n - k]$$

$$y[n] = \sum_{k=0}^{\infty} x[k]\delta[n - k]$$

6 Bibliography

References

[1] Alan V. Oppenheim & Alan S. Willsky, *Signals and System, 2nd Edition*, Prentice Hall (1996)

[2] Martin Schetzen, *Linear Time-Invariant Systems, 1st Edition*, Wiley-IEEE Press (2002)

[3] João P. Hespanha *Linear Systems Theory*, Princeton University Press (2009)