Simulation of on-off Binary Transmission through awgn Channel

Bayesian Classification
Simulation and Modeling
Wireless communication

Dr. Md. Imdadul Islam
Professor, Department of Computer Science and Engineering Jahangirnagar University

https://www.juniv.edu/teachers/imdad

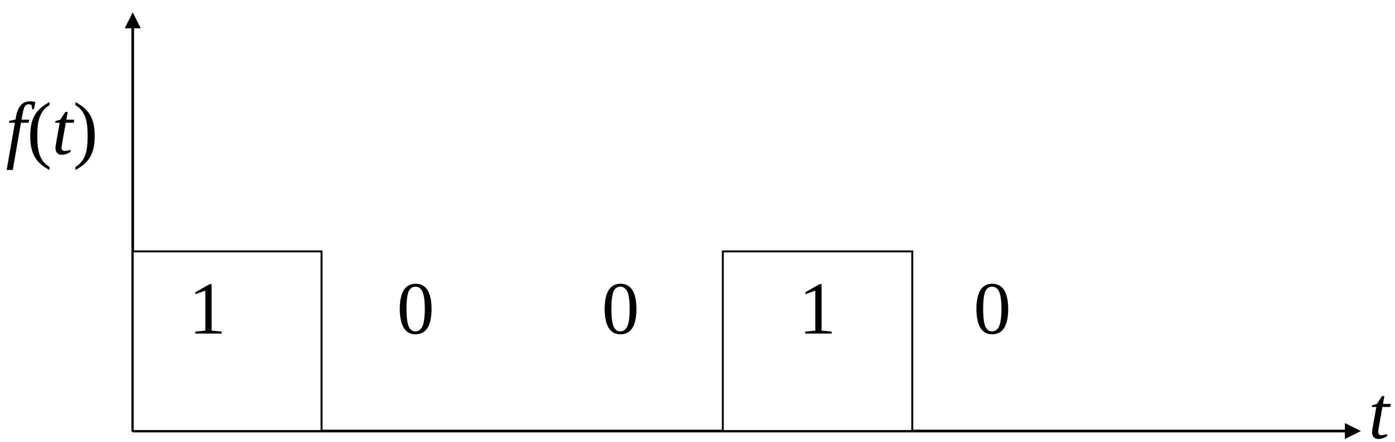
On-Off Keying (OOK)

In OOK binary logic 1 provides carrier wave over the symbol period [0, *T*] and logic 0 turns the carrier off. The received OOK signal is expressed as,

$$r(t) = s(t) + n(t) = A cos(2\pi f_c t) + n(t) \quad \text{for logic 1}$$

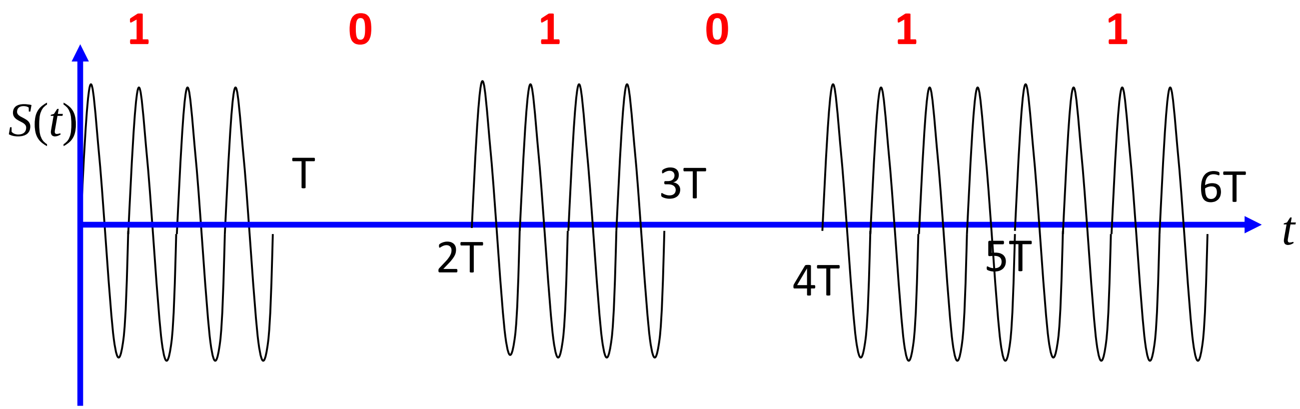
$$r(t) = 0 + n(t) \quad \text{for logic 0}$$

For binary sequence 1 0 0 1 0, $f(t)$ is shown in fig. below



Amplitude Shift Keying (ASK)

OOK signal now can be expressed as, $s(t) = f(t). Acos(2\pi f_c t)$.
The wave of OOK for 1 0 1 0 1 1 is shown in fig. below

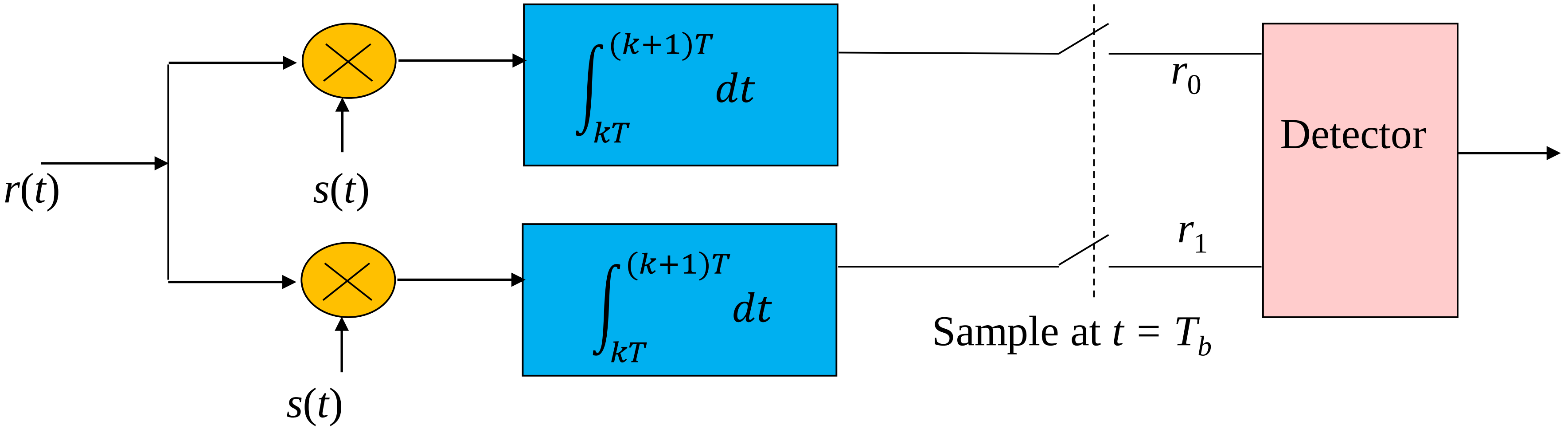


On-Off Binary Signal Transmission

The received waveform,

$$r(t) = \begin{cases} n(t) & \text{when 0 is transmitted} \\ s(t) + n(t) & \text{when 1 is transmitted} \end{cases}$$

In the following figure, input at the detector of receiver is the output of correlator.



The output of the correlator or the input of the detector when 1 is transmitted,

$$r_1 = \int_0^{T_b} r(t)s(t)dt = \int_0^{T_b} \{s(t) + n(t)\}s(t)dt$$

$$= \int_0^{T_b} s^2(t)dt + \int_0^{T_b} s(t)n(t)dt = E + n$$

The input at the detector when 0 is transmitted,

$$r_0 = \int_0^{T_b} r(t)s(t)dt = \int_0^{T_b} \{0 + n(t)\}s(t)dt$$

$$= 0 + \int_0^{T_b} s(t)n(t)dt = 0 + n = n$$

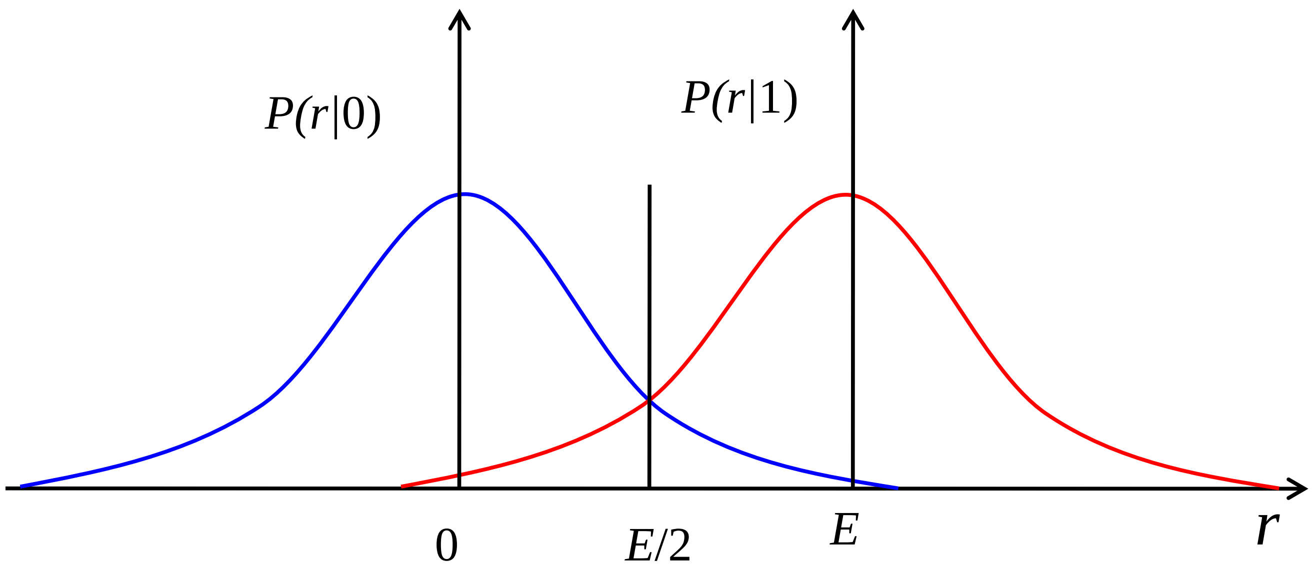
The input at the detector,

$$r = \begin{cases} n & \text{when 0 is transmitted} \\ E + n & \text{when 1 is transmitted} \end{cases}$$

;where n is a zero-mean Gaussian r.v. with variance $\sigma^2 = EN_0/2$ and $N_0/2$ is the psd of awgn.

If above received signal, r is considered as a random variable then the conditional pdf of r,

$$p(r|1) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(r-E)^2/2\sigma^2} \quad \text{when 1 is transmitted}$$



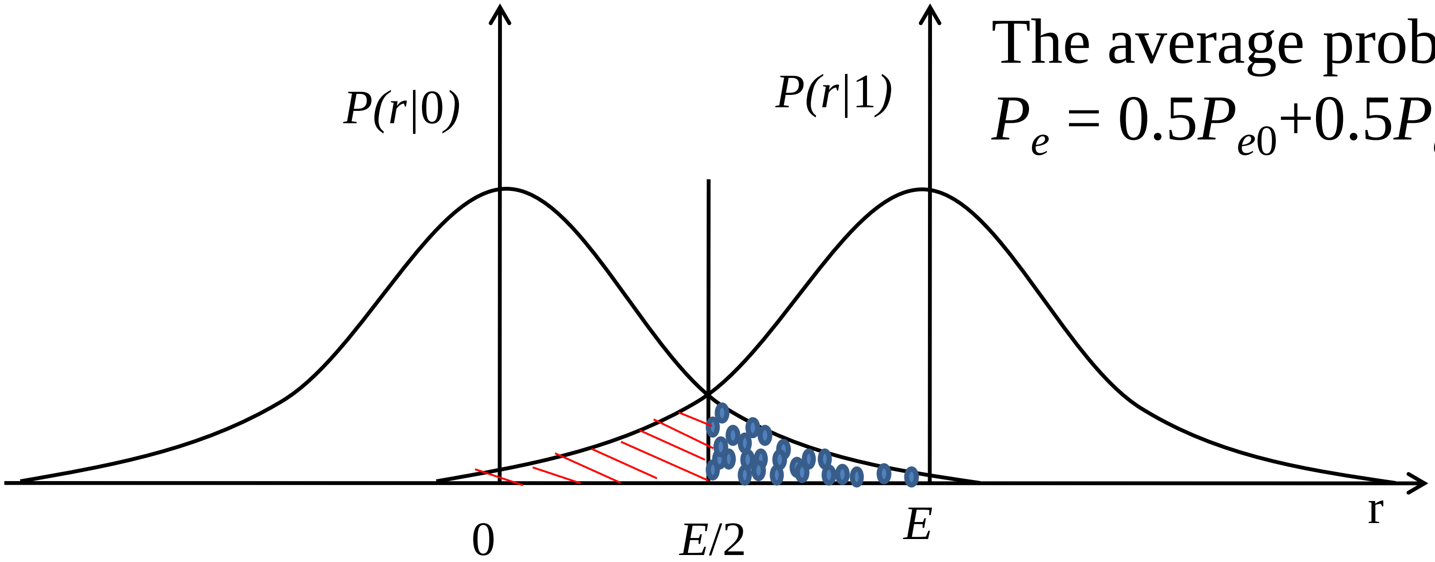
$$p(r|0) = \frac{1}{\sqrt{2\pi}\sigma} e^{-r^2/2\sigma^2} \quad \text{when 0 is transmitted}$$

The probability of error when 0 is transmitted

$$p_{e0} = P(r > E/2) = \int_{E/2}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-r^2/2\sigma^2}\, dr$$

The probability of error when 1 is transmitted

$$p_{e1} = P(r < E/2) = \int_{-\infty}^{E/2} \frac{1}{\sqrt{2\pi}\sigma} e^{-(r-E)^2/2\sigma^2}\, dr$$



The average probability of error,
$P_e = 0.5P_{e0} + 0.5P_{e1}$

$$p_{e0} = P(r > E/2) = \int_{E/2}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-r^2/2\sigma^2}\, dr$$

$$\frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2}\, dt = erf(z) = 1 - erfc(z)$$

$$= \frac{1}{2} - \int_0^{E/2} \frac{1}{\sqrt{2\pi}\sigma} e^{-r^2/2\sigma^2}\, dr$$

$$Q(z) = \frac{1}{2} erfc\left(\frac{z}{\sqrt{2}}\right)$$

Let $\frac{r^2}{2\sigma^2} = t^2$

$\Rightarrow \frac{r}{\sqrt{2}\sigma} = t$

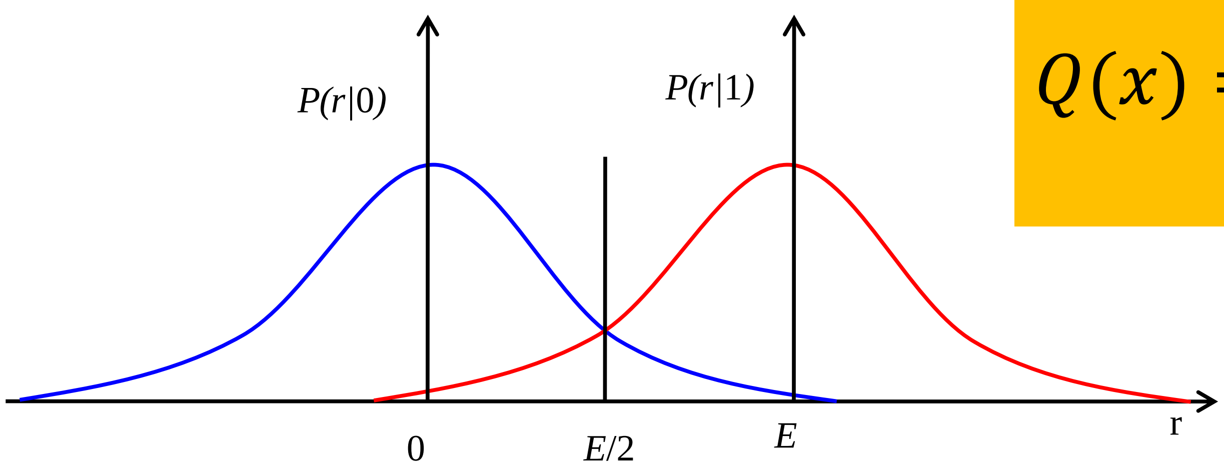
$\Rightarrow \frac{dr}{\sqrt{2}\sigma} = dt$

$\Rightarrow dr = \sqrt{2}\sigma dt$

$$P_{e0} = \frac{1}{2} - \int_0^{\frac{E}{2\sqrt{2}\sigma}} \frac{1}{\sqrt{2\pi}\sigma} e^{-t^2} \sqrt{2}\sigma dt$$

$$= \frac{1}{2} - \frac{1}{\sqrt{\pi}} \int_0^{\frac{E}{2\sqrt{2}\sigma}} e^{-t^2}\, dt = \frac{1}{2} erfc\left(\frac{E}{2\sqrt{2}\sigma}\right)$$

$$= Q\left(\frac{E}{2\sigma}\right)$$

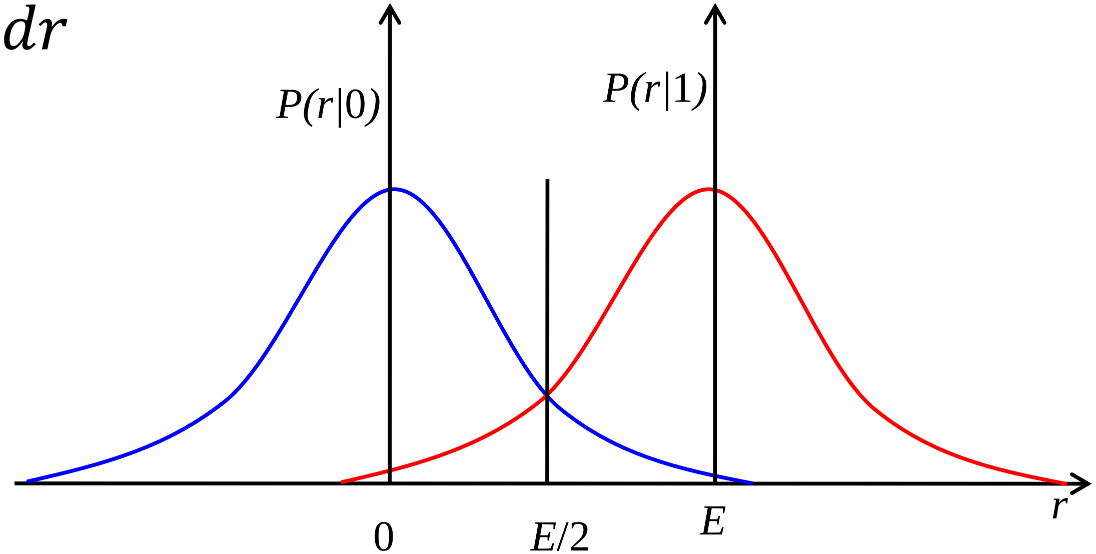


$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-u^2} du$$

$$p_{e1} = P(r < E/2) = \int_{-\infty}^{E/2} \frac{1}{\sqrt{2\pi}\sigma} e^{-(r-E)^2/2\sigma^2}\, dr$$

$$= \frac{1}{2} - \int_{E/2}^{E} \frac{1}{\sqrt{2\pi}\sigma} e^{-(r-E)^2/2\sigma^2}\, dr$$

$$\frac{(r-E)^2}{2\sigma^2} = t^2$$
$$\Rightarrow \frac{r-E}{\sqrt{2}\sigma} = t$$
$$\Rightarrow \frac{dr}{\sqrt{2}\sigma} = dt$$
$$\Rightarrow dr = \sqrt{2}\sigma dt$$



$$P_{e1} = \frac{1}{2} - \int_{\frac{-E}{2\sqrt{2}\sigma}}^{0} \frac{1}{\sqrt{2\pi}\sigma} \sqrt{2}\sigma e^{-t^2}\, dt$$

$$= \frac{1}{2} - \int_{\frac{-E}{2\sqrt{2}\sigma}}^{0} \frac{1}{\sqrt{\pi}} e^{-t^2}\, dt = \frac{1}{2} - \int_{0}^{\frac{-E}{2\sqrt{2}\sigma}} \frac{1}{\sqrt{\pi}} e^{-t^2}\, dt = \frac{1}{2} erfc\left(\frac{-E}{2\sqrt{2}\sigma}\right) = \frac{1}{2} erfc\left(\frac{E}{2\sqrt{2}\sigma}\right)$$

$$= Q\left(\frac{E}{2\sigma}\right)$$

$P_e = 0.5P_{e0} + 0.5P_{e1}$

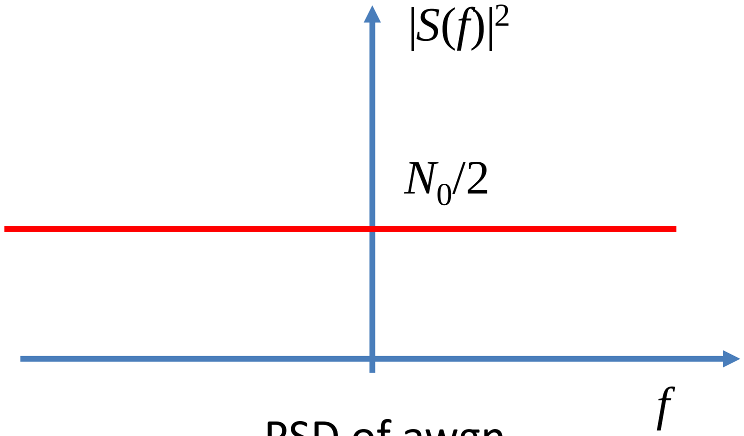
$$P_e = \frac{1}{2}Q\left(\frac{E}{2\sigma}\right) + \frac{1}{2}Q\left(\frac{E}{2\sigma}\right) = Q\left(\frac{E}{2\sigma}\right)$$

Here the variance of zero-mean Gaussian random variable *n* is,

$$\sigma^2 = \frac{EN_0}{2}$$

$$\therefore P_e = Q\left(\frac{E}{2}\sqrt{\frac{2}{EN_0}}\right) = Q\left(\sqrt{\frac{E}{2N_0}}\right)$$

$$= Q\left(\sqrt{\frac{SNR}{4}}\right)$$



PSD of awgn

PSD of awgn is $N_0/2$, therefore $E/(N_0/2)$ is the SNR

```
%Matlab code
M=20000; %Number of bits used for simulation against each SNR
for k=1:8,  % loop for SNR
SNR=2+k*2; %The value of SNR in dB
tx=randi(2, M, 1)-1;
rx=awgn(tx,SNR);

e(k)=0;  %initialization of error
for i=1:M, %for loop of error
if tx(i)==1;
if rx(i)<=0.5;
e(k)=e(k)+1;
end
end
```

```
if tx(i)==0;
if rx (i)>=0.5;
e(k)=e(k)+1;
end
end
end %for loop of error
end  %loop for SNR
```

```
pe=e/M; %probability of error
SNR=4:2:18;
SNR_a=10.^(SNR/10); %absolute value of SNR
pb=qfunc(sqrt(SNR_a/4)); %Theoretical Pb

plot(SNR,pe,'r>:',SNR,pb,'bs:', 'LineWidth', 2, 'MarkerSize',10)
legend('simulation','theory')
xlabel('SNR in db')
ylabel('Pe')
grid on;
```

$SNR\_dB=10\log_{10}(SNR\_a)$