

# ADVANCED METHODS IN NLP

## EXERCISE #1 SOLUTION

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## 1 Basics

(a) Indeed, for every dimension  $i$ :

$$\text{softmax}(\mathbf{x} + c)_i = \frac{e^{x_i + c}}{\sum_j e^{x_j + c}} = \frac{e^c e^{x_i}}{\sum_j e^c e^{x_j}} = \frac{e^{x_i}}{\sum_j e^{x_j}} = \text{softmax}(\mathbf{x})_i$$

which implies that  $\text{softmax}(\mathbf{x} + c) = \text{softmax}(\mathbf{x})$ .  $\square$

(c) Let us compute the gradient:

$$\begin{aligned} \frac{\partial \sigma}{\partial x} &= \frac{\partial (1 + e^{-x})^{-1}}{\partial x} \\ &= -(1 + e^{-x})^{-2} (-e^{-x}) = e^{-x} (1 + e^{-x})^{-2} \\ &= \left( \frac{1}{\sigma(x)} - 1 \right) \sigma^2(x) \\ &= \sigma(x) (1 - \sigma(x)) \end{aligned}$$

## 2 Word2vec

(a) Note that  $y$  is an one-hot vector and therefore  $\text{CE}(y, \hat{y})$  is dependant only in  $\hat{y}_o = p(o|c)$ , so we get:

$$\text{CE}(y, \hat{y}) = - \sum_o y_o \cdot \log(\hat{y}_o) = - \log(\hat{y}_o) = -u_o^T v_c + \log \left( \sum_{w=1}^W \exp(u_w^T v_c) \right)$$

Now we derive:

$$\begin{aligned} \frac{\partial J_{\text{softmaxCE}}}{\partial v_c} &= \frac{\partial \left[ -u_o^T v_c + \log \left( \sum_{w=1}^W \exp(u_w^T v_c) \right) \right]}{\partial v_c} \\ &= -u_o + \left( \sum_{w=1}^W \exp(u_w^T v_c) \right)^{-1} \sum_{w=1}^W \exp(u_w^T v_c) \cdot u_w \\ &= -u_o + \sum_{w=1}^W \underbrace{\frac{\exp(u_w^T v_c)}{\sum_{j=1}^W \exp(u_j^T v_c)}}_{=p(w|c)} \cdot u_w \\ &= -u_o + \sum_{w=1}^W p(w|c) \cdot u_w \end{aligned}$$

(b) As in (a):

$$\begin{aligned}
\frac{\partial J_{\text{softmaxCE}}}{\partial u_o} &= -v_c + \left( \sum_{w=1}^W \exp(u_w^T v_c) \right)^{-1} \exp(u_o^T v_c) \cdot v_c \\
&= -v_c + \frac{\exp(u_o^T v_c)}{\underbrace{\sum_{j=1}^W \exp(u_j^T v_c)}_{=p(o|c)}} \cdot v_c \\
&= -v_c (1 - p(o|c))
\end{aligned}$$

$$\begin{aligned}
\frac{\partial J_{\text{softmaxCE}}}{\partial u_{w \neq o}} &= \left( \sum_{j=1}^W \exp(u_j^T v_c) \right)^{-1} \exp(u_w^T v_c) \cdot v_c \\
&= -v_c \cdot p(w|c)
\end{aligned}$$

(c) Using 1(c) and the fact that:

$$\begin{aligned}
\sigma(x)(1 - \sigma(x)) &= \sigma(x) \left( \frac{1}{1 - \sigma(x)} \right)^{-1} \\
&= \sigma(x) \left( 1 + \frac{\sigma(x)}{1 - \sigma(x)} \right)^{-1} \\
&= \sigma(x) (1 + e^x)^{-1} \\
&= \sigma(x) \sigma(-x)
\end{aligned}$$

we get:

$$\begin{aligned}
\frac{\partial J_{\text{negsample}}}{\partial v_c} &= \frac{\partial \left( -\log \sigma(u_o^T v_c) - \sum_{k=1}^K \log \sigma(-u_k^T v_c) \right)}{\partial v_c} \\
&= -\sigma^{-1}(u_o^T v_c) \cdot \frac{\partial \sigma(u_o^T v_c)}{\partial v_c} - \sum_{k=1}^K \sigma^{-1}(-u_k^T v_c) \cdot \frac{\partial \sigma(-u_k^T v_c)}{\partial v_c} \\
&= -\sigma^{-1}(u_o^T v_c) \cdot \sigma(u_o^T v_c) \cdot \sigma(-u_o^T v_c) \cdot u_o \\
&\quad - \sum_{k=1}^K \sigma^{-1}(-u_k^T v_c) \cdot \sigma(-u_k^T v_c) \cdot \sigma(u_k^T v_c) \cdot (-u_k) \\
&= -\sigma(-u_o^T v_c) \cdot u_o + \sum_{k=1}^K \sigma(u_k^T v_c) \cdot u_k
\end{aligned}$$

$$\begin{aligned}
\frac{\partial J_{\text{negsample}}}{\partial u_o} &= \frac{\partial \left( -\log \sigma(u_o^T v_c) - \sum_{k=1}^K \log \sigma(-u_k^T v_c) \right)}{\partial u_o} \\
&= -\sigma^{-1}(u_o^T v_c) \cdot \frac{\partial \sigma(u_o^T v_c)}{\partial u_o} \\
&= \sigma^{-1}(u_o^T v_c) \sigma(u_o^T v_c) \sigma(-u_o^T v_c) \cdot v_c \\
&= -\sigma(-u_o^T v_c) \cdot v_c \\
&= -(1 - \sigma(u_o^T v_c)) \cdot v_c \\
&= (\sigma(u_o^T v_c) - 1) \cdot v_c
\end{aligned}$$

$$\begin{aligned}
\frac{\partial J_{\text{negsample}}}{\partial u_k (\neq o)} &= \frac{\partial \left( -\log \sigma(u_o^T v_c) - \sum_{k=1}^K \log \sigma(-u_k^T v_c) \right)}{\partial u_k} \\
&= -\sigma^{-1}(-u_k^T v_c) \cdot \frac{\partial \sigma(-u_k^T v_c)}{\partial u_k} \\
&= -\sigma^{-1}(-u_k^T v_c) \sigma(-u_k^T v_c) \sigma(u_k^T v_c) \cdot (-v_c) \\
&= \sigma(u_k^T v_c) \cdot v_c
\end{aligned}$$

(d) Let  $F \in \{J_{\text{softmaxCE}}, J_{\text{negsample}}\}$  so we actually found the gradients in previous sections:

$$\begin{aligned}
\frac{\partial J_{\text{skipgram}}(w_c, v_c)}{\partial U} &= \sum_{-m \leq j \neq 0 \leq m} \frac{\partial F(w_{c+j}, v_c)}{\partial U} \\
\frac{\partial J_{\text{skipgram}}(w_c, v_c)}{\partial v_c} &= \sum_{-m \leq j \neq 0 \leq m} \frac{\partial F(w_{c+j}, v_c)}{\partial v_c} \\
\frac{\partial J_{\text{skipgram}}(w_c, v_c)}{\partial v_{j \neq c}} &= 0
\end{aligned}$$