Introduction to Artificial intelligence – HW1

Ofri Kleinfeld 302893680

Heuristic function choice:

The heuristic function we will choose to implement is the heuristic shown in class, known as "Manhattan distance".

The heuristic scores a game state by summing the "Manhattan distance" of every panel, from its current position on the board, to its goal position on the board.

Formally we will denote:

$N - the\ board\ size$

$x_i, y_i - the\ \boldsymbol{current}\ board\ cordinats\ of\ panel\ i, for\ a\ givan\ state\ S$

$gx_i, gy_i - the\ \boldsymbol{goal}\ board\ cordinats\ of\ panel\ i\ (its\ coordinates\ in\ the\ goal\ state)$

So "Manhattan distance" score will be:

$$Manhattan\_distance(S) = \sum_{i=1}^{i=N-1} |x_i - gx_i| + |y_i - gy_i|$$

Heuristic function Characteristics:

1) The Manhattan distance heuristic is consistent.
   Proof:
   In each step of the algorithm, we change the position of exactly one panel.
   Hence our heuristic score can improve only be 1.
   So, it holds that $h(n) - h(n') \leq 1$, for state n and a successor state n'.
   Since all the possible steps cost the same, and their cost equal exactly 1,
   We get $h(n) - h(n') \leq 1 = c(n, n')$ and the heuristic is consistent.

2) The Manhattan distance heuristic is admissible/under estimate.
   Proof:
   We will show that h(n) is always not higher than the real cost h*(n).
   The heuristic score will be equal exactly to the true cost from a given state to the goal state **only if we have a sequence of valid actions**, **where for each action the heuristic score doesn't increase.**
   For example, we can refer to the following state:

1	2	3
4	5	
7	8	6

   Here h(n) = 1 and indeed we need a sequence of 1 action (UP), which decreases the heuristic function.
   If we don't have a sequence of actions that doesn't increase the heuristic function, our estimated cost will be greater from the actual cost – because we will have to increase the estimated cost, before we will be able to decrease it.
   For example, denote the following state:

2	1	3
4	5	6
7	8	

   For this state h(n) = 2, but every step taken from this state will increase h(n). In order to switch the 1 and 2 panels we have to first move other panels from their goal position and hence increase the heuristic score.