# Forecasting Methods

# Problem Set VI

### Problem Set for Exercise 6

For feedback to your solutions of the problem set, please hand the R script file in by January 15th, 2023. Please send them by email to *abigail.asare@uol.de* with the subject “Forecasting: Problem Set VI”.

## Question 1

Update the level

$\ell_t = \alpha y_t + (1-\alpha)\ell_{t-1}$

Forecast one step ahead

$\hat{y}_{t+1} = \ell_t$

- If parameter $\alpha = 0.83$
- if the level $\ell_t = 510.31$ level
- calculate the following:

| Year | time t | obs $y_t$ | level $\ell_t$ | Forecast ŷ_t | |
|---|---|---|---|---|---|
| 2006 | 0 | | 510.31 | — | |
| 2007 | 1 | 488.89 | 510.31 | 510.31 | → 0.83×488.89+(1−0.83)×510.31 = 492,53 |
| 2008 | 2 | 509.87 | 510,31 | 492,53 | → 0,83×509.87+(1−0,83)×492,53 = 506,92 |
| 2009 | 3 | 456.72 | 492,53 | 506,92 | → 0,83×456,72+(1−0,83)×506,92 = 465,25 |
| 2010 | 4 | 473.82 | 506,92 | 465,25 | → 0,83×473,82+(1−0,83)×465,25 = 472.36 |
| 2011 | 5 | 525.95 | 465,25 | 472,36 | → 0,83×525.95+(1−0,83)×472,36 = 516,84 |
| 2012 | 6 | 549.83 | 472,36 | 516,84 | → 0,83×516,84+(1−0,83)×516,84 = 544,22 |
| 2013 | 7 | 542.34 | 516,84 | 544,22 | → 0,82×544,22+(1−0,83)×544,22 = 542,65 |
| | h | | 544,22 | 542,66 | |
| 2014 | 1 | | 542,66 | 542,66 | |
| 2015 | 2 | | 542,66 | 542,66 | |
| 2016 | 3 | | 542,66 | 542,66 | |
| 2017 | 4 | | 542,66 | 542,66 | |
| 2018 | 5 | | 542,66 | 542,66 | |

## Question 2

- If parameter $\alpha = 0.7$ and $\beta = 0.8$
- if $\ell_t$ = level and $b_t$ = trend
- calculate the following:

| time<br>t | obs<br>$y_t$ | level<br>$\ell_t$ | trend<br>$b_t$ | Forecast<br>ŷ_t |
|---|---|---|---|---|
| 0 | - | 148,112.60 | 436.57 | - |
| 1 | 160,217.99 | | | |
| 2 | 143,538.70 | | | |
| 3 | 148,158.37 | | | |
| 4 | 139,589.44 | | | |
| 5 | 147,395.12 | | | |
| 6 | 161,243.67 | | | |
| h | | | | |
| 1 | | | | |
| 2 | | | | |
| 3 | | | | |

Level

→ $(0.7 \times 160217.99) + (1+0.7)(148112,60 + 436.57)$

→ Holt's linear trend method updates both the level and trend at each time step.

→ update the level: $\ell_t = \alpha y_t + (1+\alpha)(\ell_{t-1} + b_{t-1})$

→ update the trend: $b_t = \beta(\ell_t - \ell_{t-1}) + (1+\beta) b_{t-1}$

→ $\hat{y}_{t+h} = \ell_t + h b_t$ (for h steps ahead)

## Question 3

- If parameter $\alpha = 0.306$ , $\beta = 0.0003$ and $\gamma = 0.426$
- if $\ell_t$ = level , $b_t$ = trend and $s_t$ = additive seasonality
- calculate the following:

| Year | Quarter | time t | obs $y_t$ | level $\ell_t$ | trend $b_t$ | season $s_t$ | Forecast ŷ_t |
|---|---|---|---|---|---|---|---|
| 2004 | Q1 | -3 | | | | 9.70 | |
| 2004 | Q2 | -2 | | | | -9.31 | |
| 2004 | Q3 | -1 | | | | -1.69 | |
| 2004 | Q4 | 0 | | 32.26 | 0.70 | 1.31 | |
| 2005 | Q1 | 1 | 42.21 | | | | |
| 2005 | Q2 | 2 | 24.65 | | | | |
| 2005 | Q3 | 3 | 32.67 | | | | |
| 2005 | Q4 | 5 | 37.26 | | | | |
| 2006 | Q1 | 6 | 73.26 | | | | |
| 2006 | Q2 | 7 | 47.70 | | | | |
| 2006 | Q3 | 8 | 61.10 | | | | |
| 2006 | Q4 | 9 | 66.06 | | | | |
| | | h | | | | | |
| 2007 | Q1 | 1 | | | | | 70.56 |
| 2007 | Q2 | 2 | | | | | 52.57 |
| 2007 | Q3 | 3 | | | | | 59.86 |
| 2007 | Q4 | 4 | | | | | 66.61 |
| 2008 | Q1 | 5 | | | | | 73.97 |
| 2008 | Q2 | 6 | | | | | 55.79 |
| 2008 | Q3 | 7 | | | | | 62.68 |
| 2008 | Q4 | 8 | | | | | 69.4 |

→ Level: $\ell_t = \alpha(y_t - s_{t-L}) + (1-\alpha)(\ell_{t-1} + b_{t-1})$

→ Trend: $b_t = \beta(\ell_t - \ell_{t-1}) + (1-\beta) b_{t-1}$

→ Seasonality = $s_t = \gamma(y_t - \ell_{t-1}) + (1-\gamma) s_{t-L}$

→ Forecast = $\hat{y}_{t+m} = \ell_t + m b_t + s_{t-L+1+(m-1) \bmod L}$ (for m steps ahead)

## Question 4

- If parameter $\alpha = 0.0441$ , $\beta = 0.030$ and $\gamma = 0.002$
- if $\ell_t$ = level ,$b_t$ = trend and $s_t$ = multiplicative seasonality
- calculate the following:

| Year | Quarter | time t | obs $y_t$ | level $\ell_t$ | trend $b_t$ | season $s_t$ | Forecast ŷ__t |
|---|---|---|---|---|---|---|---|
| 2004 | Q1 | -3 | | | | 1.24 | |
| 2004 | Q2 | -2 | | | | 0.77 | |
| 2004 | Q3 | -1 | | | | 0.96 | |
| 2004 | Q4 | 0 | | 32.26 | 0.70 | 1.02 | |
| 2005 | Q1 | 1 | 42.21 | | | | |
| 2005 | Q2 | 2 | 24.65 | | | | |
| 2005 | Q3 | 3 | 32.67 | | | | |
| 2005 | Q4 | 5 | 37.26 | | | | |
| 2006 | Q1 | 6 | 73.26 | | | | |
| 2006 | Q2 | 7 | 47.70 | | | | |
| 2006 | Q3 | 8 | 61.10 | | | | |
| 2006 | Q4 | 9 | 66.06 | | | | |
| | | h | | | | | |
| 2007 | Q1 | 1 | | | | | |
| 2007 | Q2 | 2 | | | | | |
| 2007 | Q3 | 3 | | | | | |
| 2007 | Q4 | 4 | | | | | |
| 2008 | Q1 | 5 | | | | | |
| 2008 | Q2 | 6 | | | | | |
| 2008 | Q3 | 7 | | | | | |
| 2008 | Q4 | 8 | | | | | |

→ Level: $\ell_t = \alpha \frac{y_t}{s_{t-s}} + (1-\alpha)(\ell_{t-1} + b_{t-1})$

→ Trend: $b_t = \beta(\ell_t - \ell_{t-1}) + (1-\beta) b_{t-1}$

→ Seasonal: $s_t = \gamma \frac{y_t}{\ell_t} + (1-\gamma) s_{t-s}$

→ Forecast: $\hat{y}_{t+h|t} = (\ell_t + h b_t) s_{t-s+h_m}$

↳ remainder of h/s

seasonal period