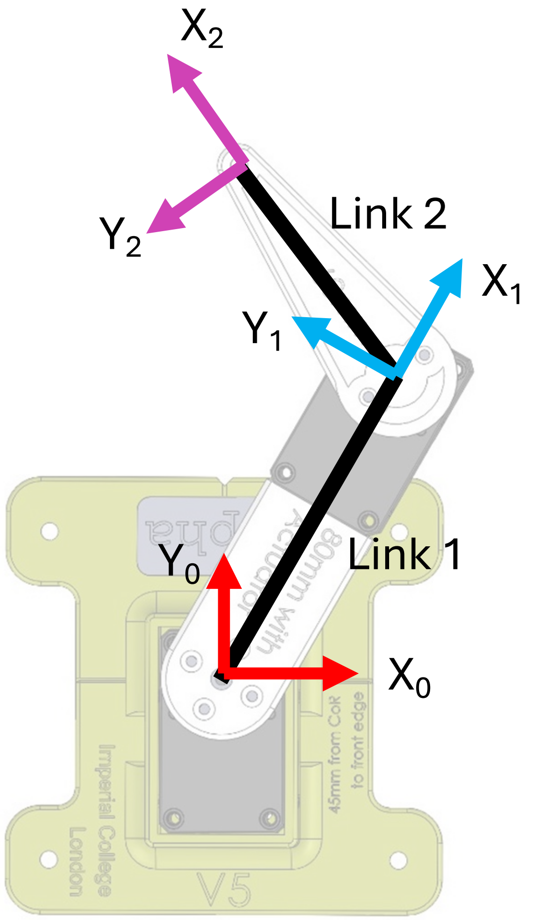
DH Table

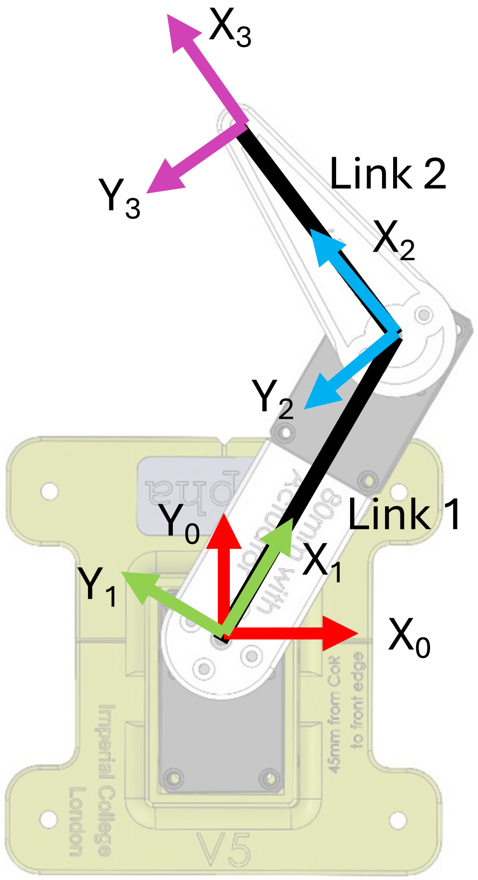
Standard vs Modified

Frame assignment

For the **standard** DH table, the frames are put at the "end" of the links.



For the **modified** DH table, the frames are put at the "head" of the links. Sometimes an additional frame is needed for end-effector.



Standard DH Table

In Lecture 3

i	θ_i	d_i	a_i	α_i

Add-on to the lecture slides
Should be i instead of i-1
Should be in the sequence of θ, d, a, and α

Transformation Sequence:

θ and d are wrt to the previous frame;
a and α are wrt to the current frame.

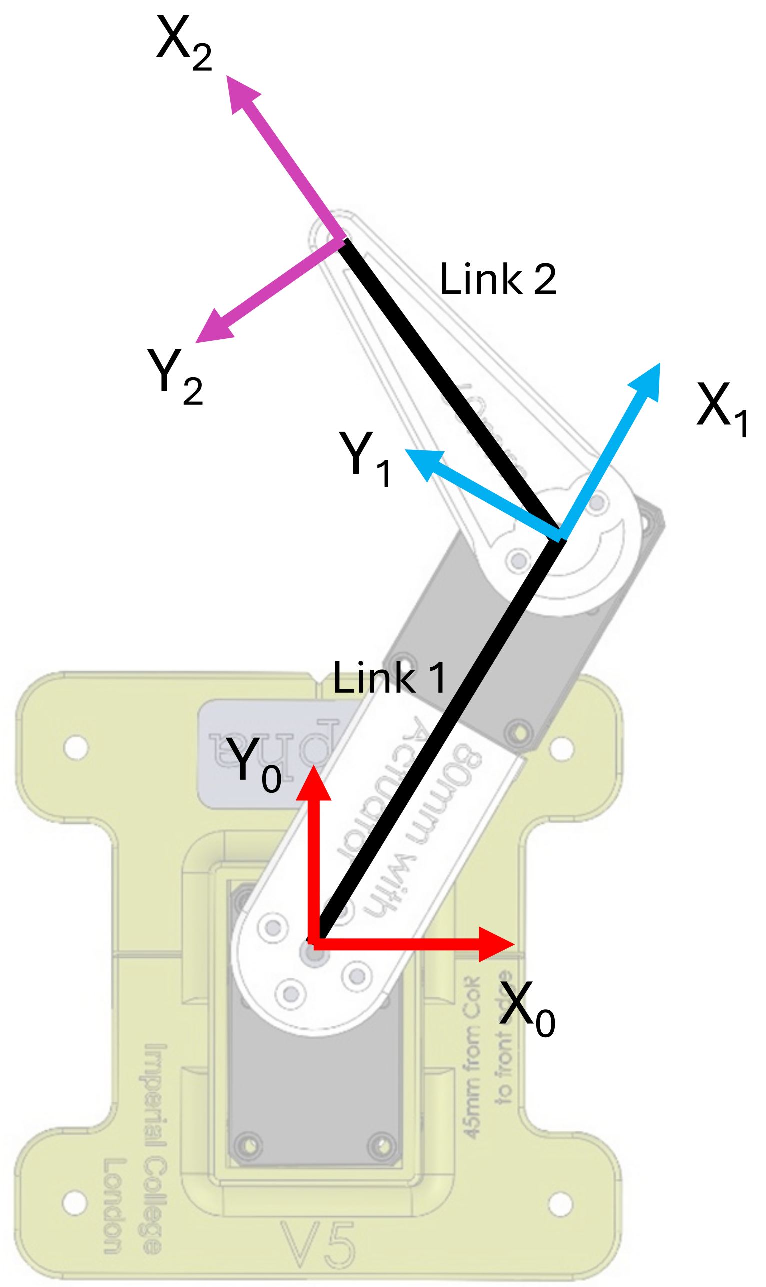
$$^{i-1}T_i = R_{z_{i-1}}(\theta_i) * T_{z_{i-1}}(d_i) * T_{x_i}(a_i) * R_{x_i}(\alpha_i)$$

Now, lets try to fill the table and calculate T

i	θ_i	d_i	a_i	α_i
1				
2				

The frames are put at the “end” of the links.

So, there are two frames.



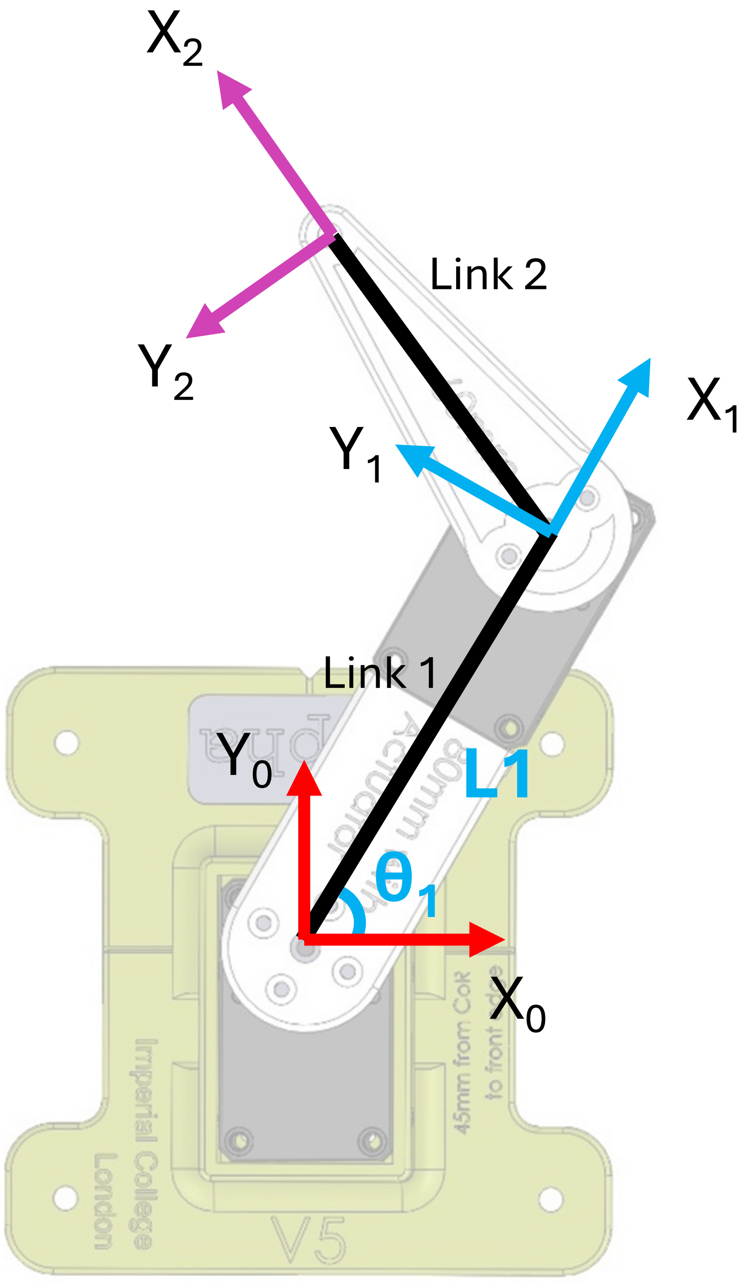
i	θ_i	d_i	a_i	α_i
1	θ_1	0	L1	0
2				

The frames are put at the “end” of the links.

So, there will be two frames.

Frame 0 to 1:

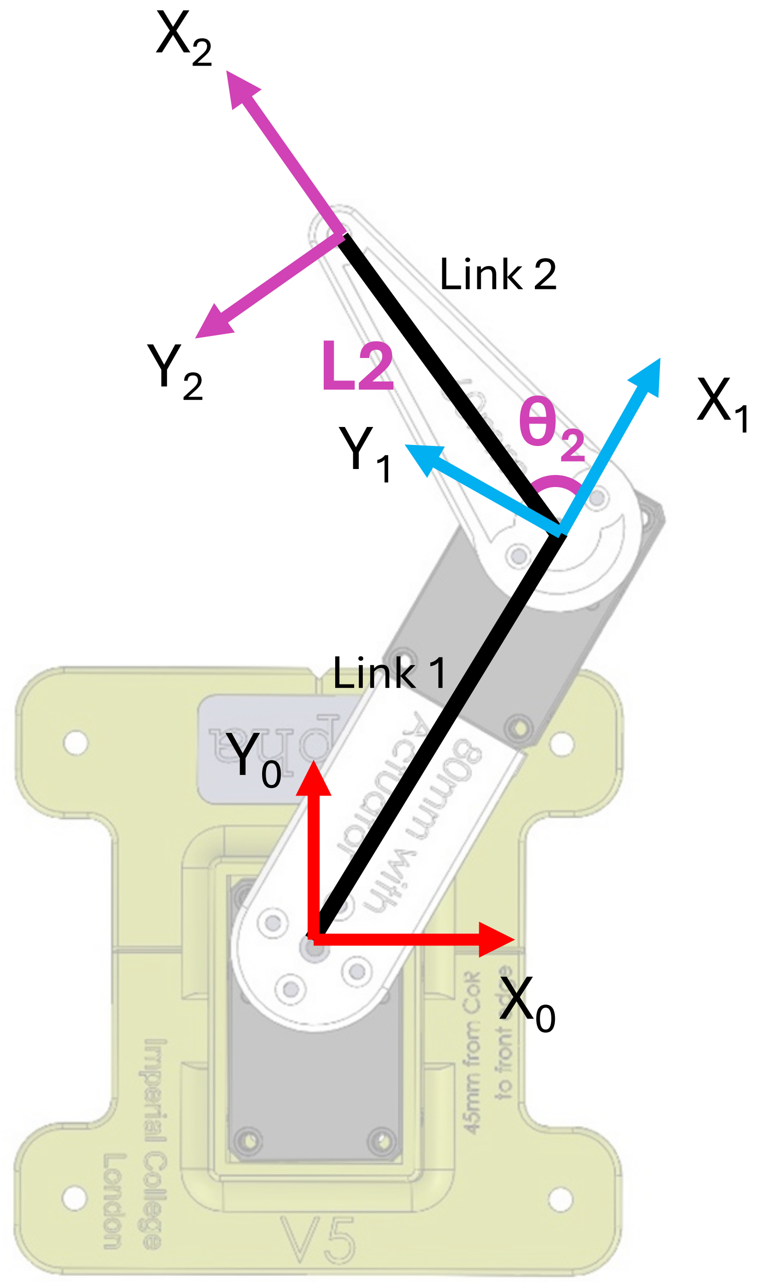
- θ_1 wrt to Z_0, no offset d
- L1 wrt to X_1, no joint twist α



i	θ_i	d_i	a_i	α_i
1	θ_1	0	L1	0
2	θ_2	0	L2	0

Frame 1 to 2:

- θ_2 wrt to Z_1 , no offset d
- L2 wrt to X_2, no joint twist α



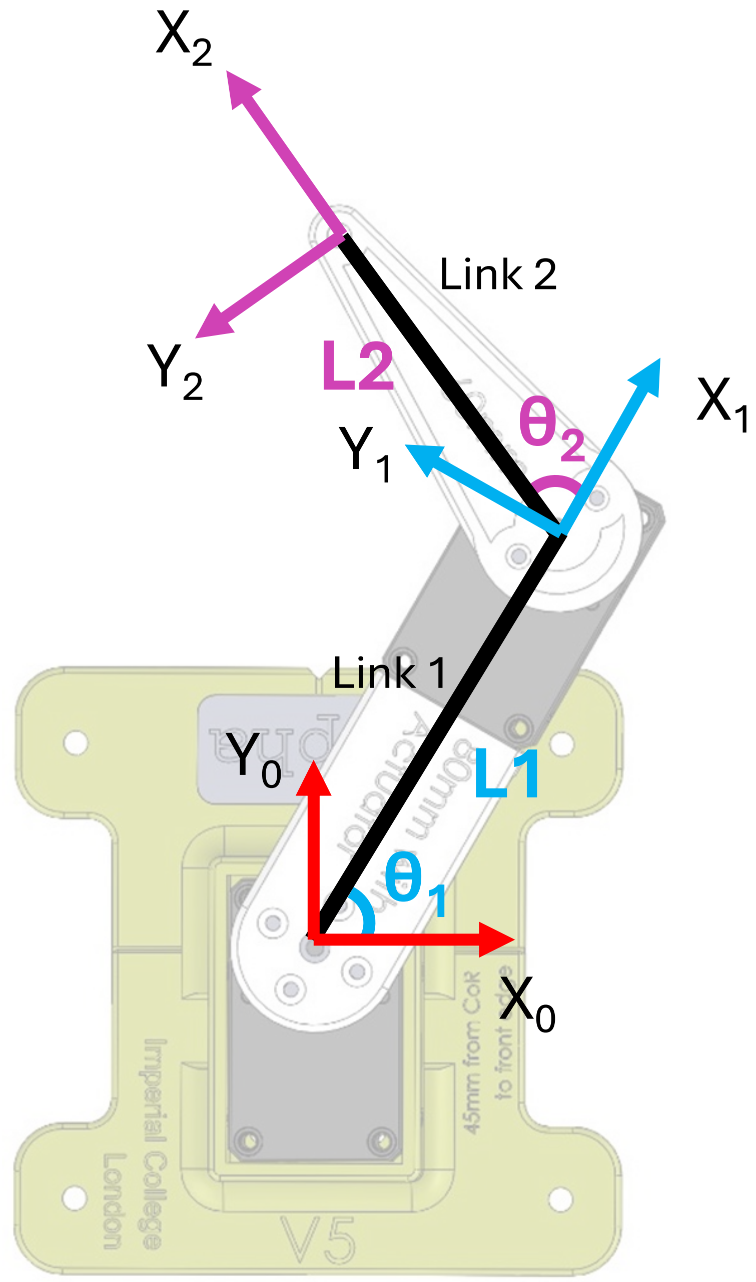
i	θ_i	d_i	a_i	α_i
1	θ_1	0	L1	0
2	θ_2	0	L2	0

Recall:

$$^{i-1}T_i = R_{z_{i-1}}(\theta_i) * T_{z_{i-1}}(d_i) * T_{x_i}(a_i) * R_{x_i}(\alpha_i)$$

$$^{i-1}T = \begin{bmatrix} c\theta_i & -s\theta_i c\alpha_i & s\theta_i s\alpha_i & a_i c\theta_i \\ s\theta_i & c\theta_i c\alpha_i & -c\theta_i s\alpha_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

You can try to validate the equation from the top to the bottom.

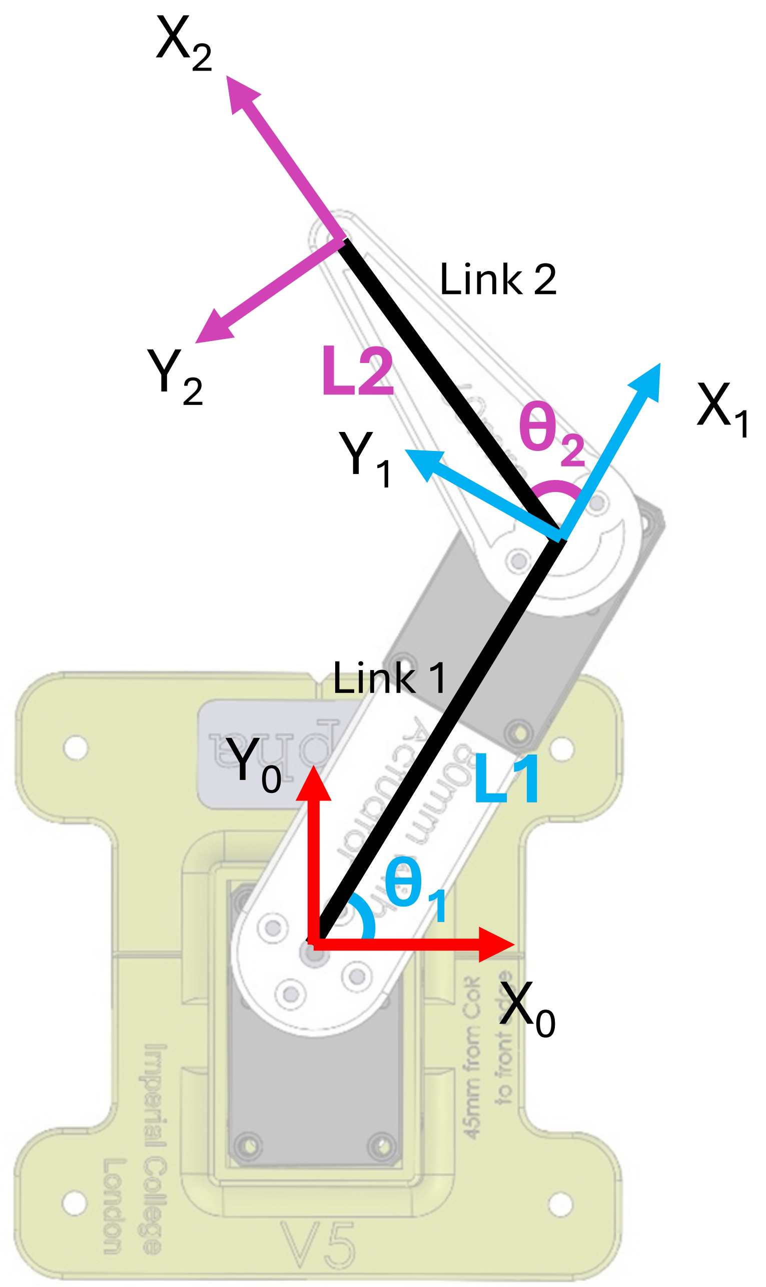


i	θ_i	d_i	a_i	α_i
1	θ_1	0	L1	0
2	θ_2	0	L2	0

$$
{}^2T_0 = {}^1T_0 * {}^2T_1 = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & l_1 c\theta_1 \\ s\theta_1 & c\theta_1 & 0 & l_1 s\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & l_2 c\theta_2 \\ s\theta_2 & c\theta_2 & 0 & l_2 s\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
= \begin{bmatrix} c\theta_1 c\theta_2 - s\theta_1 s\theta_2 & -c\theta_1 s\theta_2 - s\theta_1 c\theta_1 & 0 & l_1 c\theta_1 + l_2[c\theta_1 c\theta_2 - s\theta_1 s\theta_2] \\ s\theta_1 c\theta_2 + c\theta_1 s\theta_2 & -s\theta_1 s\theta_2 + c\theta_1 c\theta_2 & 0 & l_1 s\theta_1 + l_2[s\theta_1 c\theta_2 + c\theta_1 s\theta_2] \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
= \begin{bmatrix} c(\theta_1+\theta_2) & -s(\theta_1+\theta_2) & 0 & l_1 c\theta_1 + l_2 c(\theta_1+\theta_2) \\ s(\theta_1+\theta_2) & c(\theta_1+\theta_2) & 0 & l_1 s\theta_1 + l_2 s(\theta_1+\theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$



Modified DH Table

Not in Lecture 3

***** What we used *****

i	α_{i-1}	a_{i-1}	θ_i	d_i

Transformation Sequence:

α and a are wrt to the previous frame;
θ and d are wrt to the current frame.

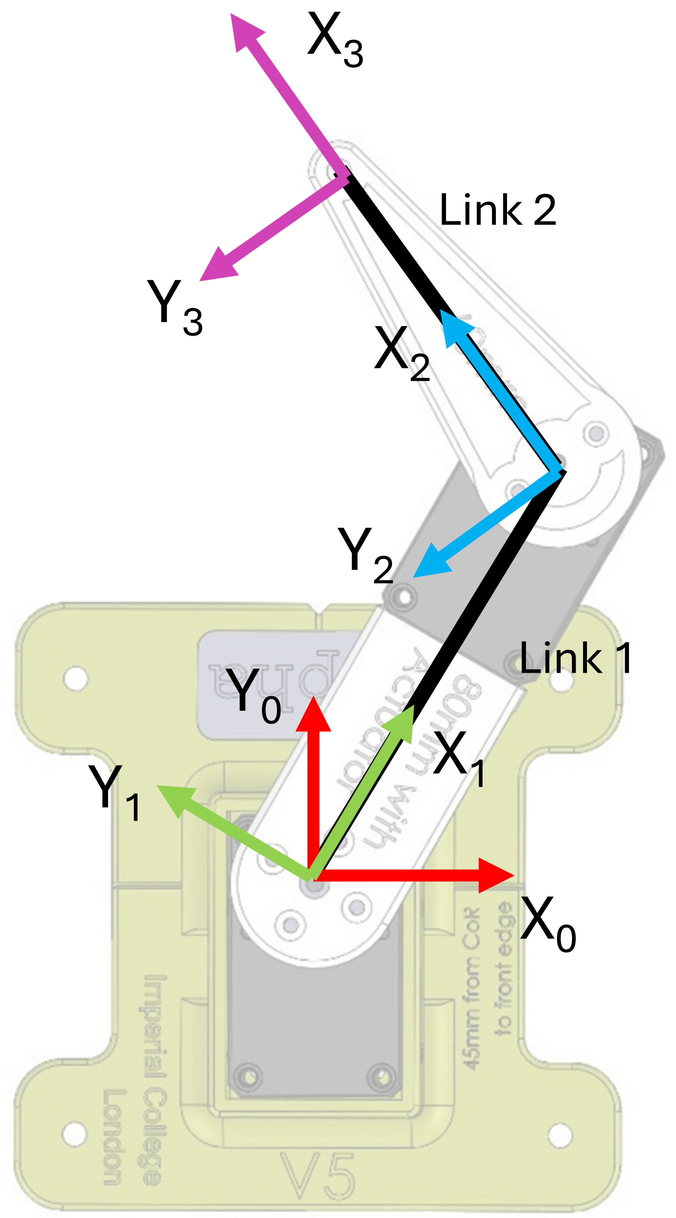
$$^{i-1}T_i = R_{x_{i-1}}(\alpha_{i-1}) * T_{x_{i-1}}(a_{i-1}) * R_{z_i}(\theta_i) * T_{z_i}(d_i)$$

Now, lets try to fill the table and calculate T

i	α_{i-1}	a_{i-1}	θ_i	d_i
1				
2				
3				

The frames are put at the “head” of the links

So, there are 2 frames + 1 frame for the end effector.



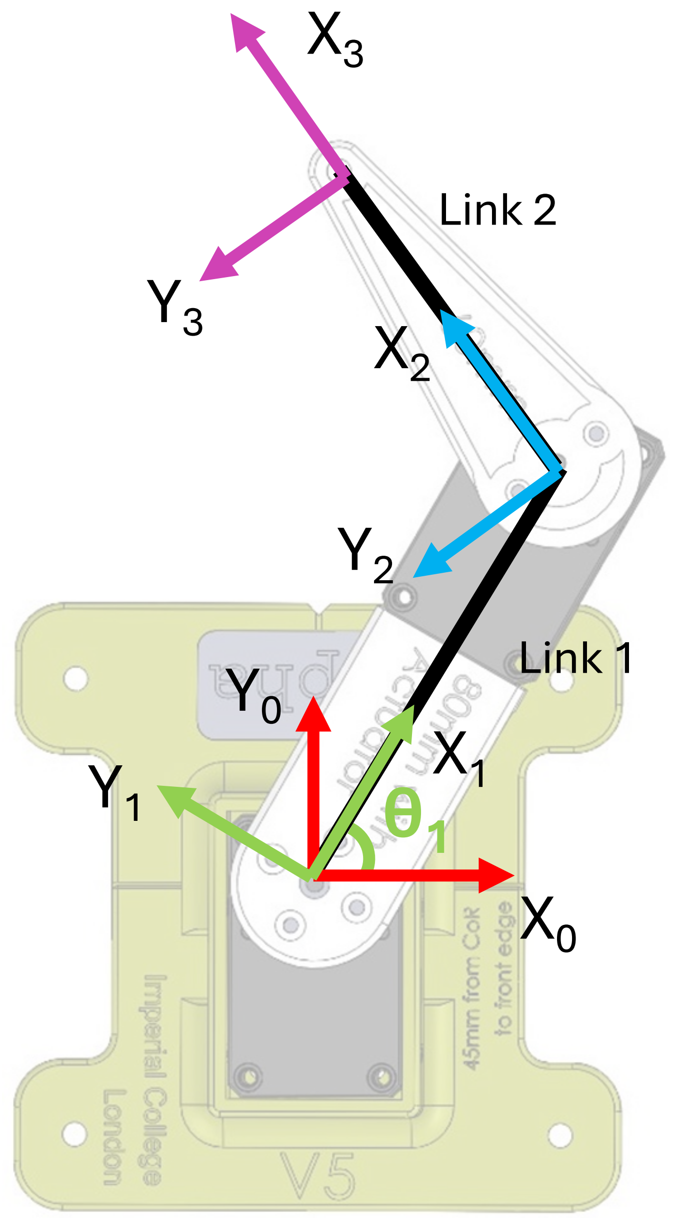
i	α_{i-1}	a_{i-1}	θ_i	d_i
1	0	0	θ_1	0
2				
3				

The frames are put at the "head" of the links

So, there are 2 frames + 1 frame for the end effector.

Frame 0 to 1

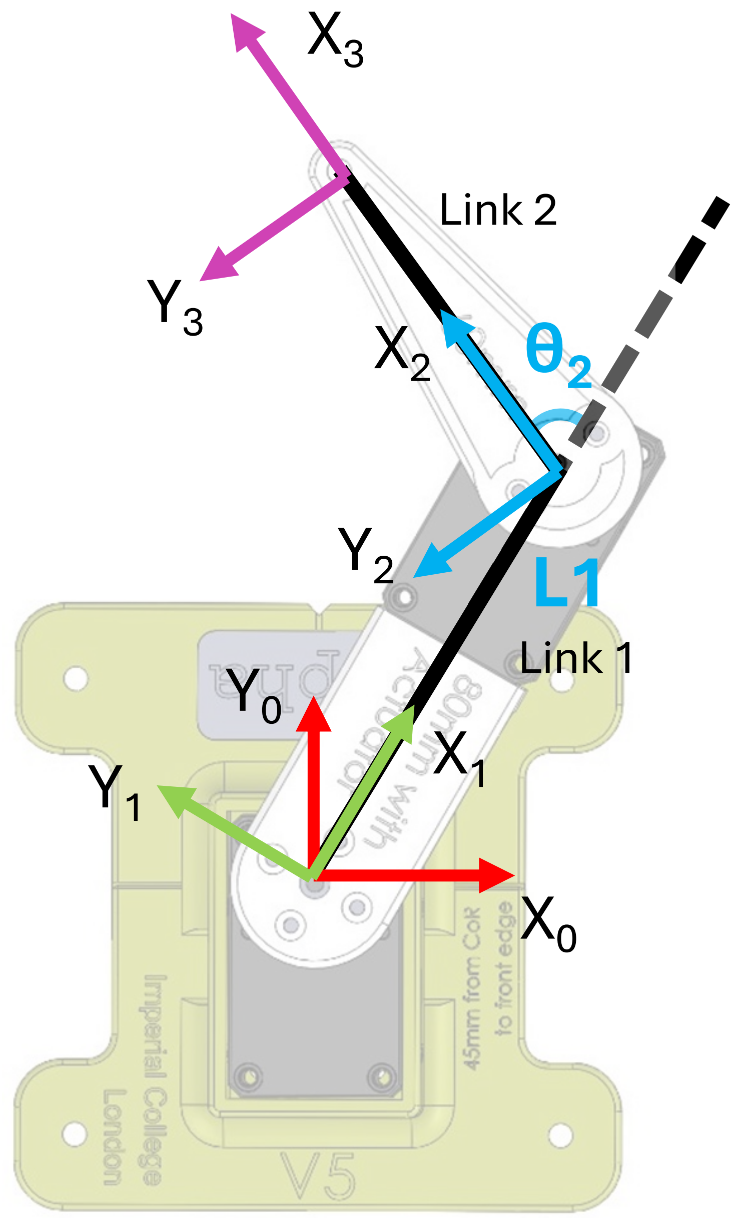
- Overlap with the world frame, no a and no joint twist α
- θ_1 wrt to Z_1 , no offset d



i	α_{i-1}	a_{i-1}	θ_i	d_i
1	0	0	θ_1	0
2	0	L1	θ_2	0

Frame 1 to 2

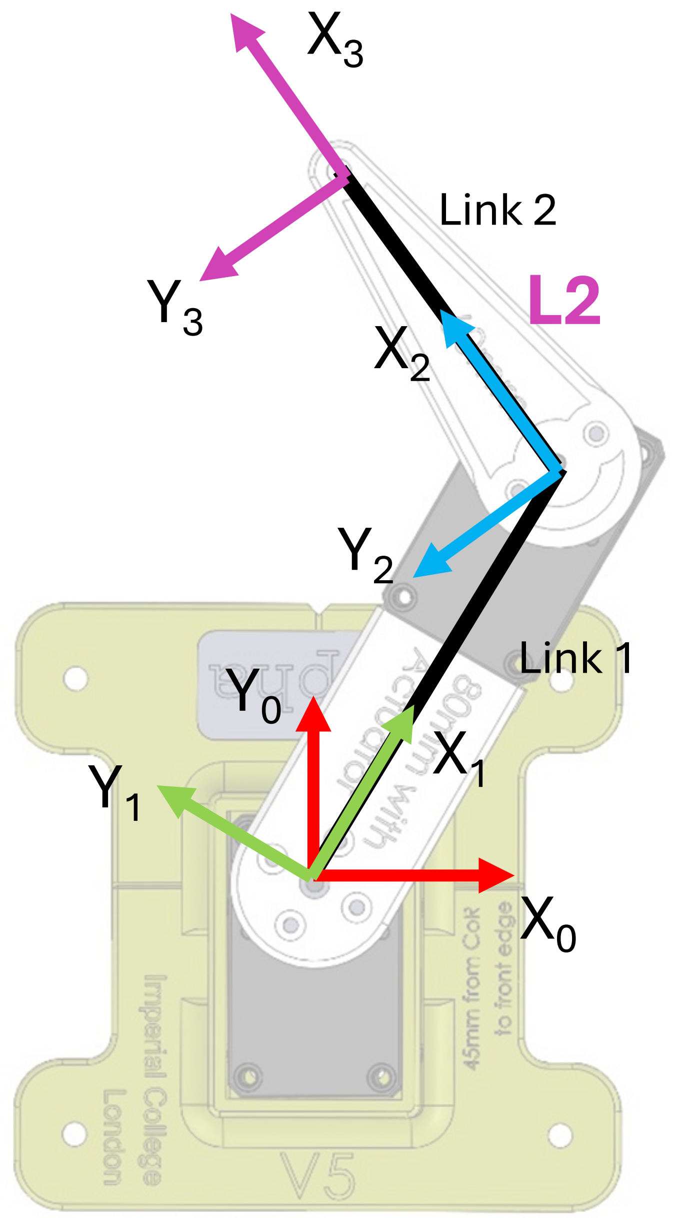
- L1 wrt X_1, no joint twist α
- θ_2 is wrt to Z_2, no offset d



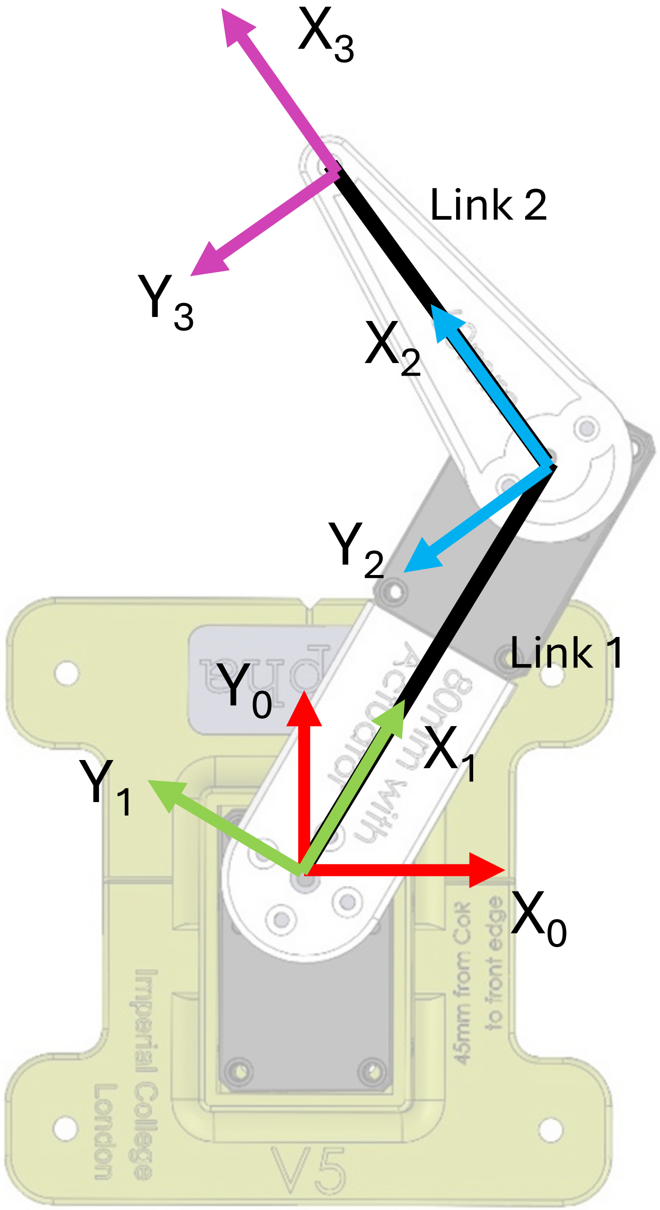
i	α_{i-1}	a_{i-1}	θ_i	d_i
1	0	0	θ_1	0
2	0	L1	θ_2	0
3	0	L2	0	0

Frame 2 to 3 – End effctor

- L2 wrt X_2, no joint twist α
- No joint angle θ, no offset d



i	α_{i-1}	a_{i-1}	θ_i	d_i
1	0	0	θ_1	0
2	0	L1	θ_2	0
3	0	L2	0	0



Recall:

$$^{i-1}T_i = R_{x_{i-1}}(\alpha_{i-1}) * T_{x_{i-1}}(a_{i-1}) * R_{z_i}(\theta_i) * T_{z_i}(d_i)$$

$$^{i-1}T_i = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -d_i s\alpha_{i-1} \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & d_i c\alpha_{i-1} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

You can try to validate the equation from the top to the bottom.

i	α_{i-1}	a_{i-1}	θ_i	d_i
1	0	0	θ_1	0
2	0	L1	θ_2	0
3	0	L2	0	0

$$^{i-1}T_i = R_{x_{i-1}}(\alpha_{i-1}) * T_{x_{i-1}}(a_{i-1}) * R_{z_i}(\theta_i) * T_{z_i}(d_i)$$

$$= \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & l_1 \\ s\theta_2 & c\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c(\theta_1+\theta_2) & -s(\theta_1+\theta_2) & 0 & l_1 c\theta_1 + l_2 c(\theta_1+\theta_2) \\ s(\theta_1+\theta_2) & c(\theta_1+\theta_2) & 0 & l_1 s\theta_1 + l_2 s(\theta_1+\theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$