Home Work 4

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Definition 1. *Let C be the set of all circles in a plane, and let R be a relation on C defined as: for any two circles a and b, aRb if circles a and b intersect. We will examine the properties of reflexivity, symmetry, and transitivity for relation R.*

Theorem 1. *The relation R on set C is symmetric but not reflexive or transitive.*

Proof. We will analyze each property separately:

1. Reflexivity: For a relation to be reflexive, it must satisfy aRa for all circles $a \in C$. However, a circle does not intersect itself by definition, because all points on a circle are equidistant from its center, and self-intersection would require two distinct points on the circle to coincide. Therefore, the relation R is not reflexive.

2. Symmetry: For a relation to be symmetric, it must satisfy $aRb \implies bRa$ for all circles $a, b \in C$. If circle a intersects circle b at some point P, then circle b must also intersect circle a at the same point P. Thus, the intersection relationship is symmetric by definition, and the relation R is symmetric.

3. Transitivity: For a relation to be transitive, it must satisfy aRb and $bRc \implies aRc$ for all circles $a, b, c \in C$. Consider three circles a, b, and c such that:

- Circle a intersects circle b. - Circle b intersects circle c.

However, it is not guaranteed that circle a intersects circle c. For example, imagine circles a and c are disjoint but both intersect circle b, which lies in between them. In this case, the transitivity property does not hold. Therefore, the relation R is not transitive.

In summary, the relation R on the set C of circles is symmetric but not reflexive or transitive. □

Definition 2. *Let D be the set of all lines in a plane, and let S be a relation on D defined as: for any two lines h and k, $h\ S\ k$ if lines h and k are either parallel or perpendicular. We will prove that relation S is an equivalence relation.*

Theorem 2. *The relation S on set D is an equivalence relation.*

Proof. To show that S is an equivalence relation, we must prove that it is reflexive, symmetric, and transitive.

1. *Reflexivity*: For a relation to be reflexive, it must satisfy $h\ S\ h$ for all lines $h \in D$. By definition, a line is parallel to itself. Therefore, $h\ S\ h$ holds for all lines $h \in D$, and the relation S is reflexive.

2. *Symmetry*: For a relation to be symmetric, it must satisfy $h\ S\ k \implies k\ S\ h$ for all lines $h, k \in D$. If line h is parallel or perpendicular to line k, then line k is also parallel or perpendicular to line h. Thus, the relation S is symmetric.

3. *Transitivity*: For a relation to be transitive, it must satisfy $h\ S\ k$ and $k\ S\ l \implies h\ S\ l$ for all lines $h, k, l \in D$. Consider the following cases:

   - If h is parallel to k and k is parallel to l, then by the transitive property of parallelism, h is parallel to l.
   - If h is perpendicular to k and k is perpendicular to l, then h and l are parallel, since the angle between them is $90° + 90° = 180°$.
   - If h is parallel to k and k is perpendicular to l, then h is perpendicular to l.
   - If h is perpendicular to k and k is parallel to l, then h is perpendicular to l.

   In all cases, $h\ S\ l$ holds, so the relation S is transitive.

In summary, the relation S on the set D of lines is an equivalence relation, as it is reflexive, symmetric, and transitive. □