Home Work 2

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Problem 1

Show that the cube root of two is irrational.

<u>Proof:</u> *BWOC*

Let one assume that $\sqrt[3]{2} = \frac{a}{b}$ where $a, b \in \mathbb{Z}$ and a, b are co-prime. One starts by $2 = \frac{a^3}{b^3} \to 2b^3 = a^3$. At this point one can now say that $a = 2k$ when rewriting $2b^3 = (2k)^3$ as $2b^3 = 8k^3$ one can simplify to $b^3 = 4k^3$. $\perp$ This is a $\perp$ because b is even and a, b are co-prime.

$\square$

Problem 2

Use proof by contrapositive to show that for all integers a, b if both ab and $a + b$ are even then both a and b are even.

<u>Proof:</u> by contrapositive

Suppose that a is an odd number $a = 2k + 1$ where $k \in \mathbb{Z}$.
$ab = (2k + 1)b \Rightarrow 2kb + b$ this expression is odd if b is odd and even if b even. As for $a + b = 2k + 1 + b$ this expression odd if b is even and even

if b is odd. It follows that if b is even or odd one of the expression will be odd.

This argument holds if b is supposed to be is odd.

$\square$