Homework One

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Problem 1

$$\text{Find the sets } A, B, C \ : A - (B \cup C) \neq (A - B) \cup C$$

<u>ANSWER</u>

Suppose that $A \subseteq C \subseteq B$
To start let us define our set operations as

$$x \in A - B \iff x \in A \wedge x \notin B$$

$$x \in A \cup B \iff x \in A \vee x \in B$$

$$x \in A \cap B \iff x \in A \wedge \ x \in B$$

One can say that the left side of the equation is

$x \in A - (B \cup C) \Leftrightarrow$
$x \in A \wedge x \notin (B \cup C) \Leftrightarrow$
$x \in A \wedge (x \notin B \vee x \notin C) \Leftrightarrow$
$(x \in A \wedge x \notin B) \vee (x \in A \wedge x \notin C)$

One can then take the left side of the equation and write it as
$x \in (A - B) \cup C \Leftrightarrow$
$x \in (A - B) \vee x \in C \Leftrightarrow$
$(x \in A \wedge x \notin B) \vee x \in C \Leftrightarrow$
$(x \in C \vee x \in A) \vee (x \in C \wedge x \notin B)$
At this point there is a contradiction because on the left hand one get that $x \in C \vee x \in A$. Where as on the right side one get that $x \in A \wedge x \notin C$.

$\square$

From this we can find that there are some sets that exemplify that $A - (B \cup C) \neq (A - B) \cup C$
$A = \{1, 2, 3, 4, 5\}$
$B = \{-1, 0, 1, 2\}$
$C = \{3, 4, 5, 6, 7\}$ When working these out we get that the left side is equal to the empty set. While the Right side equals the set $\{3, 4, 5, 6, 7\}$

Problem 2

Consider the set $D = 7x + 3y : x \in \mathbb{Z}, y \in \mathbb{Z}$.
(a) Show that $1 \in D$ holds.
(b) Use (a) to show that $D = \mathbb{Z}$

ANSWER (a)

Let one take the equation $7x + 3y$ and write it as $7x + 3Y = 1$. One knows that x and y can be any number in $\mathbb{Z}$. One can now make $x = 10$ and $y = -23$ as both 10 and -23 are integers this shows that $1 \in D$

ANSWER (B)

One can define a function $f(w) = 7x + 3(-2w)$ where $w \in \mathbb{Z}$. The range of this function is $\mathbb{Z}$. From this it can be said that $D = \mathbb{Z}$

$\square$