

# Homework One

John E. Palenchar  
Department of Mathematics  
Student—University of Florida

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## Problem 1

Find the sets  $A, B, C$  :  $A - (B \cup C) \neq (A - B) \cup C$

ANSWER

Suppose that  $A \subseteq C \subseteq B$   
To start let us define our set operations as

$$x \in A - B \iff x \in A \wedge x \notin B$$

$$x \in A \cup B \iff x \in A \vee x \in B$$

$$x \in A \cap B \iff x \in A \wedge x \in B$$

One can say that the left side of the equation is

$$\begin{aligned} x \in A - (B \cup C) &\iff \\ x \in A \wedge x \notin (B \cup C) &\iff \\ x \in A \wedge (x \notin B \vee x \notin C) &\iff \\ (x \in A \wedge x \notin B) \vee (x \in A \wedge x \notin C) &\end{aligned}$$

One can then take the left side of the equation and write it as

$$x \in (A - B) \cup C \Leftrightarrow$$

$$x \in (A - B) \vee x \in C \Leftrightarrow$$

$$(x \in A \wedge x \notin B) \vee x \in C \Leftrightarrow$$

$$(x \in C \vee x \in A) \vee (x \in C \wedge x \notin B)$$

At this point there is a contradiction because on the left hand one get that  $x \in C \vee x \in A$ . Where as on the right side one get that  $x \in A \wedge x \notin C$ .

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From this we can find that there are some sets that exemplify that  $A - (B \cup C) \neq (A - B) \cup C$

$$A = \{1, 2, 3, 4, 5\}$$

$$B = \{-1, 0, 1, 2\}$$

$C = \{3, 4, 5, 6, 7\}$  When working these out we get that the left side is equal to the empty set. While the Right side equals the set  $\{3, 4, 5, 6, 7\}$

## Problem 2

Consider the set  $D = 7x + 3y : x \in \mathbb{Z}, y \in \mathbb{Z}$ .

- (a) Show that  $1 \in D$  holds.
- (b) Use (a) to show that  $D = \mathbb{Z}$

ANSWER (a)

Let one take the equation  $7x + 3y$  and write it as  $7x + 3Y = 1$ . One knows that  $x$  and  $y$  can be any number in  $\mathbb{Z}$ . One can now make  $x = 10$  and  $y = -23$  as both 10 and  $-23$  are integers this shows that  $1 \in D$

ANSWER (B)

Let one take  $7x + 3y : x, y \in \mathbb{Z}$ . Suppose that  $7x + 3y = a$  where  $a \in \mathbb{Z}$ . from this one can define a function  $f(x) = 7x + 3(-2x)$  where  $x \in \mathbb{Z}$ . The range of this function is  $\mathbb{Z}$  when the domain is  $\mathbb{Z}$ . From this it can be said that  $D = \mathbb{Z}$

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