A Very Simple $\LaTeX\,2_\varepsilon$ Template

John E. Palenchar
Department of Mathamatics
Student—University of Florida

February 22, 2023

This starts the second part of the course.
Induction
First group of induction arguments.
Second group Strong induction
Third group Proof by considering the least counterexample.

Simple Induction Proving statements of the form $\forall n \in \mathbb{N} P(n)$
basis steps: $P(0)$
induction step: $\forall n \in \mathbb{N}(P(n) \to P(n+1))$
conclude that $\forall n P(n)$ holds
Strong induction: Proving statements $\forall n \in \mathbb{N} P(n)$
Prove $\forall n(\forall m < n P(m)) \to P(n)$
conclude $\forall n P(n)$ holds

Proof by least counterexample: proving $\forall n \in \mathbb{N} P(n)$
argue by contradiction. Assume that $P(n)$ fails for some n. Let n be smallest such that $P(n)$ fails. get a contradiction.

Simple induction proof.
Summation Formulas:
Th. For every Number $n \in \mathbb{N}$
$0 + 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$
please rewrite as $\sum_{k=0}^{n} k = \frac{n(n+1)}{2}$
Proof.
basis step $\sum_{k=0}^{0} k = 0 = \frac{0(0+1)}{2}$

induction step. Opponent hands you $n \in \mathbb{N}$ say $P(n)$ holds you need to conclude that $P(n+1)$ holds.

The induction hypothesi in this case that $\sum_{k=0}^{n} k = \frac{n(n+1)}{2}$ $\sum_{k=0}^{n+1} k = \sum_{k=0}^{n} x + n + 1 = \frac{n(n+1)}{2} + n + 1 = \frac{(n+1)(n+2)}{2}$

$\square$

<u>Th.</u> For every number $n \in \mathbb{N}$ $1 + 3 + 5 + \cdots + (2n+1) = (n+1)^2$
rewrite $\sum_{k=0}^{n}(2k+1) = (n+1)^2$

<u>Proof.</u> This is the proof
basis step: $\sum_{k=0}^{0}(2k+1) = 1 = (0+1)^2$
induction step: Opponent hands you $n \in \mathbb{N}$ say $P(n)$ holds you need to conclude that $P(n+1)$ holds.

$\sum_{k=0}^{n}(2k+1) = (n+1)^2$

$\sum_{k=0}^{n+1}(2k+1) = \sum_{k=0}^{n}(2k+1) + 2n + 1 + 1 = (n+1)^2 + 2n + 1 = (n+2)^2$

$\square$

<u>Strong Induction:</u> Fibonacci Numbers.
Defined by recursion:
$F_0 = 0, F_1 = 1, F_{n+2} = F_n + F_{n+1}$
Example $0, 1, 1, 2, 3, 5, 8$

<u>Th.</u> for every $n \in \mathbb{N} F_n < 2^n$
Proof: assume that this has been checked for all values up to $n >= 2$
We need to conclude that it is true at n as well
$F_n = F_{n-1} + F_{n-2} < 2^{n-1} + 2^{n-2} < 2(2^{n-1}) = 2^n$
$F_{n-1} < 2^{n-1}$ $f_{n-2} < 2^{n-2}$

$\square$