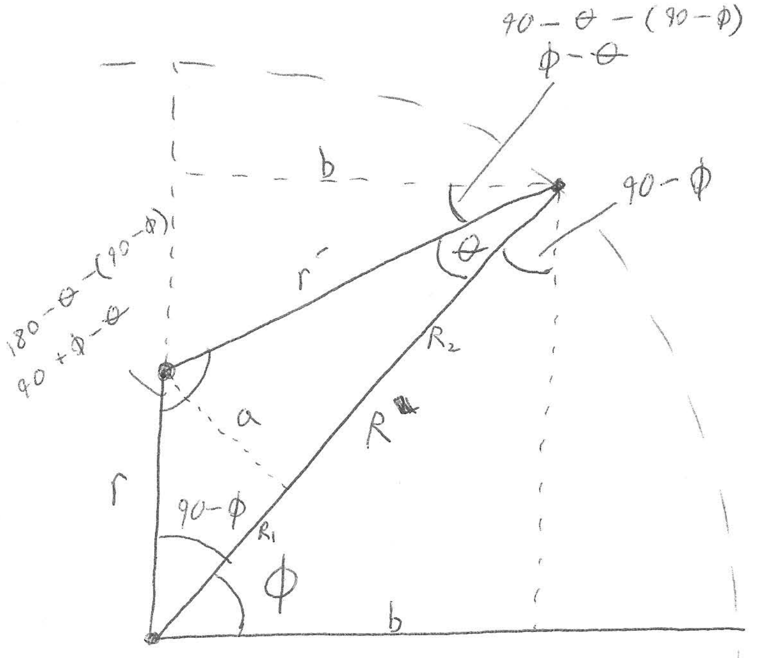


$\phi = \omega t$

$r' = f(t)$

$\theta = g(t)$

$$CTDI(r) = \int_0^t \frac{R^2 H(\theta)}{r'^2}\, dt$$

① Law of sin

$\sin(90-\phi) = \frac{a}{r}$ $\quad \sin(\theta) = \frac{a}{r'}$

$r'\sin(\theta) = r\sin(90-\phi)$

$\sin(90)\cos(\phi) - \cancel{\cos(90)}^{0}\sin(\phi)$

$\boxed{r'\sin(\theta) = r\cos(\phi)}$

$\frac{\partial}{\partial t}$ $\quad \frac{\partial}{\partial t}$

$\dot{r}'\sin(\theta) + r'\cos(\theta)\dot{\theta} = -r\sin(\phi)\dot{\phi}$

$\boxed{\dot{r}'\sin(\theta) + r'\cos(\theta)\dot{\theta} = -r\omega\sin(\phi)}$

② Law of cosine

$R^2 = r^2 + r'^2 - 2rr'\cos(90+\phi-\theta)$

$\cancel{\cos(90)}^{0}\cos(\phi-\theta) - \sin(90)^{1}\sin(\phi-\theta)$

$\boxed{R^2 = r^2 + r'^2 + 2rr'\sin(\phi-\theta)}$

$\frac{\partial}{\partial t}$ $\quad \frac{\partial}{\partial t}$

$\varnothing = \varnothing + 2r'\dot{r}' + 2r\dot{r}'\sin(\phi-\theta)$
$+ 2rr'\cos(\phi-\theta)[\underbrace{\dot{\phi}}_{\omega} - \dot{\theta}]$

$\boxed{0 = 2r'\dot{r}' + 2r\dot{r}'\sin(\phi-\theta) + 2rr'\cos(\phi-\theta)[\omega-\dot{\theta}]}$

③ Length of b

$\cos(\phi) = \frac{b}{R}$ $\quad \cos(\phi-\theta) = \frac{b}{r'}$

$\boxed{R\cos(\phi) = r'\cos(\phi-\theta)}$

$-R\sin(\phi)\dot{\phi} = \dot{r}'\cos(\phi-\theta) - r'\sin(\phi-\theta)[\dot{\phi}-\dot{\theta}]$

$\boxed{-R\omega\sin(\phi) = \dot{r}'\cos(\phi-\theta) - r'\sin(\phi-\theta)[\omega-\dot{\theta}]}$

(4) $R = R_1 + R_2$

$\cos(90-\phi) = \frac{R_1}{r}$ $\quad \cos(\theta) = \frac{R_2}{r'}$

$\cancel{\cos(90)}\cos(\phi) + \sin(90)\sin(\phi)$

$\sin\phi = \frac{R_1}{r}$

$$\boxed{R = r\sin(\phi) + r'\cos(\theta)}$$

$\frac{\partial}{\partial t} \quad \frac{\partial}{\partial t}$

$0 = r\sin(\phi)\dot{\phi} + \dot{r}'\cos(\theta) - r'\sin(\theta)\dot{\theta}$

$$\boxed{0 = \omega r\sin(\phi) + \dot{r}'\cos(\theta) - r'\sin(\theta)\dot{\theta}}$$

to solve for $r'(t)$ subst. take out $\sin(\theta)$ + $\cos(\theta)$

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$\cancel{(1)\ r'\sin(\theta) + r'\cos(\theta)}$

(1)' $r'\sin(\theta) = r\cos(\phi)$ $\Rightarrow$ $\boxed{\sin(\theta) = \frac{r\cos(\phi)}{r'}}$

(2)' $R^2 = r^2 + r'^2 + 2rr'\sin(\phi-\theta) = r^2 + r'^2 + 2rr'[\sin(\phi)\cos(\theta) - \sin(\theta)\cos(\phi)]$

(3)' $\frac{R\cos(\phi)}{r'} = \cos(\phi-\theta) = \cos(\phi)\cos(\theta) + \sin(\phi)\sin(\theta)$

(4)'' $R = r\sin(\phi) + r'\cos(\theta)$ $\Rightarrow$ $\boxed{\cos(\theta) = \frac{R - r\sin(\phi)}{r'}}$

Take ②′ + sub in ①′ ~~+④′~~

$$R^2 = r^2 + r'^2 + 2rr'\left[\sin(\phi)\cos(\theta) - \sin(\theta)\cos(\phi)\right]$$

$$R^2 = r^2 + r'^2 + 2r\cancel{r'}\left[\frac{\sin(\phi)\left[R - r\sin(\phi)\right]}{\cancel{r'}} - \frac{r\cos(\phi)}{\cancel{r'}}\cos(\phi)\right]$$

$$R^2 = r^2 + r'^2 + 2r\left[R\sin(\phi) - r\sin^2(\phi) - r\cos^2(\phi)\right]$$

$$R^2 = r^2 + r'^2 + 2r\left[R\sin\phi - r\right]$$

$$R^2 = r^2 + r'^2 + 2rR\sin\phi - 2r^2$$

$$R^2 = r'^2 - r^2 + 2rR\sin\phi$$

$$\boxed{r'^2 = R^2 + r^2 - 2Rr\sin\phi}$$

Can also reach this using

$$1 = \sin^2\theta + \cos^2\theta$$

$$1 = \left(\frac{r\cos(\phi)}{r'}\right)^2 + \left(\frac{R - r\sin(\phi)}{r'}\right)^2$$

@ $\phi = 0$ $r'$ should be $\sqrt{R^2 + r^2}$

$$r'^2 = R^2 + r^2 - \cancel{2Rr\sin(0)}^{\,0}$$

$$r' = \sqrt{R^2 + r^2} \checkmark$$

@ $\phi = 90$ $r'$ should be $R - r$

$$r'^2 = R^2 + r^2 - 2Rr\sin(90)$$

$$r'^2 = R^2 + r^2 - 2Rr$$

$$r'^2 = (R - r)^2$$

$$r' = R - r \checkmark$$

Aside if take

③′ + sub in ①′ + ④′

$$\frac{R\cos(\phi)}{r'} = \cos(\phi)\cos(\theta) + \sin(\phi)\sin(\theta)$$

$$\frac{R\cos(\phi)}{\cancel{r'}} = \cos(\phi)\left[\frac{R - r\sin(\phi)}{\cancel{r'}}\right] + \sin(\phi)\left[\frac{r\cos\phi}{\cancel{r'}}\right]$$

$$R\cos\phi = R\cos\phi - r\cos\phi\sin\phi + r\cos\phi\sin\phi$$

$$0 = 0$$

$$CTDI(r) = \int_0^t \frac{R^2 H(\theta)}{r'^2}\, dt$$

$$= \int_0^t \frac{R^2 H(\theta)}{\left(R^2 + r^2 - 2Rr\sin(\phi)\right)}\, dt$$

If $H(\theta)$ represented as first few FFT frequencies can substitute $\sin\theta$ + $\cos\theta$ in for known functions of $t$

$$\sin(\theta) = \frac{r\cos(\phi)}{r'} = \frac{r\cos(wt)}{\sqrt{R^2+r^2-2Rr\sin(wt)}}$$

$$\cos(\theta) = \frac{R - r\sin(\phi)}{r'} = \frac{R - r\sin(wt)}{\sqrt{R^2+r^2-2Rr\sin(wt)}}$$

with unknown constants for each frequency that can be solved for if a CTDI(r) known.