$i := 0..6$ $\quad d_i := 5 \cdot i$ These are indices and positions d(cm) of the measured beam profile.

I contains the measured beam profile in arbitrary units (i.e. normalized to isocenter)

$$I_{central} := \begin{pmatrix} 1 \\ 0.842105263 \\ 0.555263158 \\ 0.248684211 \\ 0.15 \\ 0.109210526 \\ 0.082894737 \end{pmatrix} \qquad d = \begin{pmatrix} 0 \\ 5 \\ 10 \\ 15 \\ 20 \\ 25 \\ 30 \end{pmatrix}$$

theta contains the angular positions of the measurement points in radians. The tube is directly overhead and the measurements were along the x axis. The distance from focal spot to isocenter is 57 cm for the Siemens 16 slice.

$$theta_i := \operatorname{atan}\left(\frac{d_i}{57}\right) \qquad theta = \begin{pmatrix} 0 \\ 0.087 \\ 0.174 \\ 0.257 \\ 0.337 \\ 0.413 \\ 0.484 \end{pmatrix}$$

The r values are the distance from the focal spot to the measurement locations. The goal is to inverse square correct the measurements to get values that are at a constant 57 cm from the focal spot at the same angular positions as the x-axis profile measurements. In other words our profile will be a function of angle, not position along the x-axis.

$$r_i := \sqrt{57^2 + \left(d_i\right)^2} \qquad r = \begin{pmatrix} 57 \\ 57.219 \\ 57.871 \\ 58.941 \\ 60.407 \\ 62.241 \\ 64.413 \end{pmatrix}$$

The P values are the inverse square corrected intensities at 57 cm from the FS.

$$P_i := I_{central_i} \cdot \frac{(r_i)^2}{57^2} \qquad P = \begin{pmatrix} 1 \\ 0.849 \\ 0.572 \\ 0.266 \\ 0.168 \\ 0.13 \\ 0.106 \end{pmatrix}$$

Beam Profile inverse square corrected to constant 57 cm from focal spot.

Now we can do the integration over the tube angular position to get the contribution to the ctdi at each of the original measurement positions.

$p := 0..359$ 360 tube positions for fine resolution. Will do the conversion to radians in the calculations since the trig functions want radians.

$$T_{x_p} := 57 \cdot \cos\left(p \cdot \frac{\pi}{180}\right) \qquad T_{y_p} := 57 \cdot \sin\left(p \cdot \frac{\pi}{180}\right)$$

Focal Spot coordinates in cm. 0 degrees is tube overhead.

$$\Phi_i := \operatorname{atan}\left(\frac{d_i}{57}\right)$$

These are the angles between the focal spot to isocenter ray and the focal spot to beam profile measurement points, these will be used for interpolating the beam profile values along the 57 cm arc.

$vs := \text{regress}(\Phi, P, 4)$ Curve fit for interpolation purposes, uses 4th order poly.

$I(\phi) := \text{interp}(vs, \Phi, P, \phi)$ This function returns the beam intensity at an angle ϕ from the central ray at a distance of 57 cm from the spot.

Use the vector dot product to calculate the angle between the FS-isocenter and FS-measurement point. This is just the function definition at this point, it is used in next calculation. I had to take the real component, doesn't seem like imaginary would happen though?

$$\theta(a, b, c) := \operatorname{Re}\left[\operatorname{acos}\left[\frac{\begin{pmatrix} -b \\ -c \end{pmatrix} \cdot \begin{pmatrix} a - b \\ -c \end{pmatrix}}{\sqrt{(a-b)^2 + c^2} \cdot \sqrt{b^2 + c^2}}\right]\right]$$

This calculates the angle between the ray to the point of interest and the ray to the isocenter
a = x value of point at which to calculate the CTDI
b = x coordinate of focal spot
c = y coordinate of focal spot

Now we just need to do the sum at a bunch of points to get the ctdi profile

We'll do the same x positions as used for the beam profile measurement

The final calculation just sums over all tube positions, adding the interpolated beam intensity that is inverse square corrected to the distance from FS to measurement point. Note that this is essentially a double loop, for each position d_i we sum over all angular tube positions p.

$$CTDI_{air_i} := \sum_p \left[I\left(\theta\left(d_i, T_{x_p}, T_{y_p}\right)\right) \cdot \frac{57^2}{\sqrt{\left(d_i - T_{x_p}\right)^2 + \left(T_{y_p}\right)^2}^{\,2}} \right]$$

$$\mathrm{maxval} := \max\left(CTDI_{air}\right)$$ Used to normalize the CTDI values.

$$CTDI_{air} := \frac{CTDI_{air}}{\mathrm{maxval}} \qquad CTDI_{air} = \begin{pmatrix} 1 \\ 0.936 \\ 0.76 \\ 0.576 \\ 0.444 \\ 0.372 \\ 0.292 \end{pmatrix}$$