Problem Module lv7

General info

- FIRE group number 75
- module number 7
- By

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- "We hereby declare that we have both actively participated in solving every exercise. All solutions are entirely our own work, without having taken part of other solutions.
- Number of hours spent for each one of you
  - Both 18h
- Number of hours spent in supervision for this module
  - 5h

0. SCHEDULING OF FINAL MEETING

- We are available wednesday October 30th. If possible, we would rather meet morning than afternoon.

1. STOCHASTIC TRAFFIC SIMULATION

We know that a determinstic model is a model where the material properties are deterministic and the applied load are determinstic aswell. This means that we can caluculate them exactly, they are not random. In a stochastic model the propoerties are random, i.e they are described with random variables that follow a distribution. The applied load is also random variable. What this means is that a stochastic model has the capacity to handle uncertainty in the inputs built into it. For a determinstic model the uncertainties are external to the model.

Knowing this we believe that diffrence in the prediction we can draw is that in a stochastic model we can make predictions that are broad and not very specific. We can study the effects of something to understand it better and find relations that might be lost in analytical or numerical treatment. We can make predictions and analyze the predictions performance with a loss function that can desscribe how bad it is to make errors of given size. An example of this is weather forecast, we can draw predictions about tomorrows weather and we can analyze the performance of the prediction, e.g that it has a 70% chance of raining at noon. With deterministic models we can calculate things exactly which means that predictions becomes more of a fact. We could for example draw a prediction about the position of the earth in a astronomical model of planetary motion, but since we could calculate it exactly with the model, the prediction would not vary.

2.CAR RENTAL PROBLEM

a)

We have two sites A and B. We know that the probability for a car rented at A also left at A is 0.6. The probability that a car rented at A left at B is 0.4. Summarized we have these probabilities:

- $A- > A = 0.6$
- $A- > B = 0.4$
- $B- > B = 0.7$
- $B- > A = 0.3$

From this we can see that we are looking for the probability that we are at one state give that we started at a state. The probability of going towards a state i after being in a state gets us thinking of a markov chain. With this knowledge we can model the transitionmatrix for the probabilties as:

$$P = \begin{bmatrix} 0.6 & 0.4 \\ 0.3 & 0.7 \end{bmatrix}$$

To calculate the probability of going to state A after n steps given that we started in state A we have the following function:

$$P(X_n = A | X_0 = A) = (P^n)_{AA}$$

This describes the probability that a single car rented in A is in A after n rentals.

b)

To find out what proportion of cars will be in A and B after a long time we need find an equlibrium. This means that after a long time the proportion of cars will be the same and we have found a steady state.

To do this we need to find the probability vector in stable state for our tranistion matrix:

$$P = \begin{bmatrix} 0.6 & 0.4 \\ 0.3 & 0.7 \end{bmatrix}$$

We do this by solving:

$$\begin{bmatrix} X & Y \end{bmatrix} * \begin{bmatrix} 0.6 & 0.4 \\ 0.3 & 0.7 \end{bmatrix} = \begin{bmatrix} X & Y \end{bmatrix}$$

We know that $X + Y = 1$ so the calculation for the probability vector in stable state will be:

$$[x \ y]\begin{bmatrix}0,6 & 0,4\\ 0,3 & 0,7\end{bmatrix} = [x \ y]$$

$$0,6x + 0,3y = x \longrightarrow 0,3y = 0,4x \longrightarrow x = \frac{3}{4}y$$

$$0,4x + 0,7y = y$$

$$x + y = 1 \longrightarrow \frac{3}{4}y + y = 1 \longrightarrow y = \frac{4}{7}$$

$$\downarrow$$

$$x + \frac{4}{7} = 1 \longrightarrow x = \frac{3}{7}$$

$$\left[\frac{3}{7} \ \frac{4}{7}\right]\begin{bmatrix}0,6 & 0,4\\ 0,3 & 0,7\end{bmatrix} = \left[\frac{3}{7} \ \frac{4}{7}\right] \approx [0,429 \quad 0,571]$$

This tells us that we have found the point where the proportion of cars starts to stabalize after a long time. The proportion of cars will be 43% in A and 57% in B.

We can control this by checking against our transition matrix after a large number of steps to see the probability of where cars will move depending on state:

$$P^{1000} = \begin{bmatrix}0.6 & 0.4\\ 0.3 & 0.7\end{bmatrix}^{1000} \approx \begin{bmatrix}0.43 & 0.57\\ 0.43 & 0.57\end{bmatrix}$$

This tells us we have found our equlibrium and that the proprtion of cars will be: 43% in A and 57% in B in the long run.

c)

We cannot find a way to determine the expected number of cars as a function of time with the given model in the first parts. This is because we need a correlation between time and rentals.

There are a number of ways to extend the model and get a solution. One way is to define time t as directly correlated to number of rentals n: ($t = n$). Let's say we have the variables:

- I_A: initial number of cars in A.
- I_B: initial number of cars in B.
- $A_C \text{ and } A_B$ number of cars in A and B

$$\begin{bmatrix}I_A & I_B\end{bmatrix}\begin{bmatrix}0.6 & 0.4\\ 0.3 & 0.7\end{bmatrix}^t = \begin{bmatrix}A_C & B_C\end{bmatrix}$$

Another extension could be to assume an average number of rentals per day avg_c. This would give us the model:

$$\begin{bmatrix} I_A & I_B \end{bmatrix} \begin{bmatrix} 0.6 & 0.4 \\ 0.3 & 0.7 \end{bmatrix}^{t*avg_c} = \begin{bmatrix} A_C & B_C \end{bmatrix}$$

Both these models do not consider the variation over time in rentals. Reasonably, number of rentals changes for day to day correlated to e.g. vacations etc. This could be described rentals as a function of time, which takes this into consideration $n(t)$.

$$\begin{bmatrix} I_A & I_B \end{bmatrix} \begin{bmatrix} 0.6 & 0.4 \\ 0.3 & 0.7 \end{bmatrix}^{n(t)} = \begin{bmatrix} A_C & B_C \end{bmatrix}$$

We could improve this model even further by taking the frequency of rentals from both sites A and B into consideration. This added to the model would look like this:

$$\begin{bmatrix} I_A * f_A(t) & I_B * f_B(t) \end{bmatrix} \begin{bmatrix} 0.6 & 0.4 \\ 0.3 & 0.7 \end{bmatrix}^{n(t)} = \begin{bmatrix} A_C & B_C \end{bmatrix}$$

This is important because the site's location could affect the number of rentals made in each of the sites. Imagine that one of the sites is located at the airport and the other at a random place outside of a city. Since the probability of where the car is returned varies between the sites, we need to take the frequency of rentals at a site at a time into consideration.

3. TEXT GENERATION

We approach the problem by selecting a text which will be used as our input. First text we try is a song lyric. We start by testing the text with order 1 and see that the text doesn't make any sense. Letters are all mixed up. One thing we notice is that most words are still possible to pronounce, this is because consonants often are followed by a vowel rather than another consonant.

When we change the order to 2, we can see that the output text becomes even more readable, even though few "words" are actual words. We have for instance the entire word "guessed" followed by "nobody" which both are medium sized correct words. Most words are still jibberish, but pronoucable.

Changing to order 3, the text has a majority of real words even though they do not build up complete logical sentences.

We we have order 4, we start seeing sentences that make sense. We can even see combination of words that are the same as in the input text.

With order 5, almost every word are actual words, and we see many similarities between the input and output texts.

We tried this with different types of input texts and different orders. Trying different languages also returns the same result.

What we learned from observing the different outputs with order 1, is that it didn't make a lot of sense since the algorithm only looks back one letter when determining the next one. However, we could se similarities between the outputs using diffrent inputs. The similarities occured with letters that are rarely used. Since they are rarely used the possible letters after this letter are few and this led to a recognizable pattern. When the program moved to next letter after the rare one, it got confusing right away. This is because the randomized letter with a common letter before has much more possibilities for outputs.

Increasing the order, we follow a pattern of the output text becoming more similar to the input text. This is because increasing the number of letters the alorithm is looking back on, we narrow the probability of what letters can appear next. Meaning, when increasing the order to near the maximum, we can see many sequences that also is found in the input. That is because it not only looks back on the previous letters, it utilized the entire matrix for the input text that it creates to see that some sequences are repetetive in the input. This increases the probability of getting the right letter, many times in a row.

To conclude we can say that to make the output recognizable so that it is similar to the input. It is good the maximize the steps we are looking back so that the probability of getting the right letter next increases. An extension to this could be to add pattern rocognition. This would make it possible the see sequenses of words appearing several times in a text compared to words appearing for the first time.

4

Let's start by defining what we know: We denote a person having the disease with A and a positive test result with B. With these variables we set know:

- Probability that a person has the disease: $P[A] = 0.0033$
- Probability that the test shows positive given that a person has the disease $P[A|B] = 0.99$
- Probability that a test shows positive given that a person does not have the disease: $P[B|A] = 0.03$

Given these probabilities we want to find the probability that a person is sick, given that the test shows positive $P[A|B]$. To find this we can use Bayes' theorem:

$$P[A|B] = \frac{P[A] * P[B|A]}{P[B]}$$

First we need to find $P[B]$ which is the probability that a test shows positive. We know this to be the sum of the people having the disease and getting a correct diagnose and all healthy people getting the wrong diagnose. We know that 0.33% of the population have the disease. Out of these, 99% will get the correct diagnose, meaning they will get a positive test diagnose. With this we know that 99.67% of the population is healthy and out of these, 3% will get a positive test result. Given this, we can calculate the probability of getting a positive test result.

$$P[B] = 0.0033 * 0.99 + 0.9967 * 0.03 = 0.033168$$

This gives us the equation:

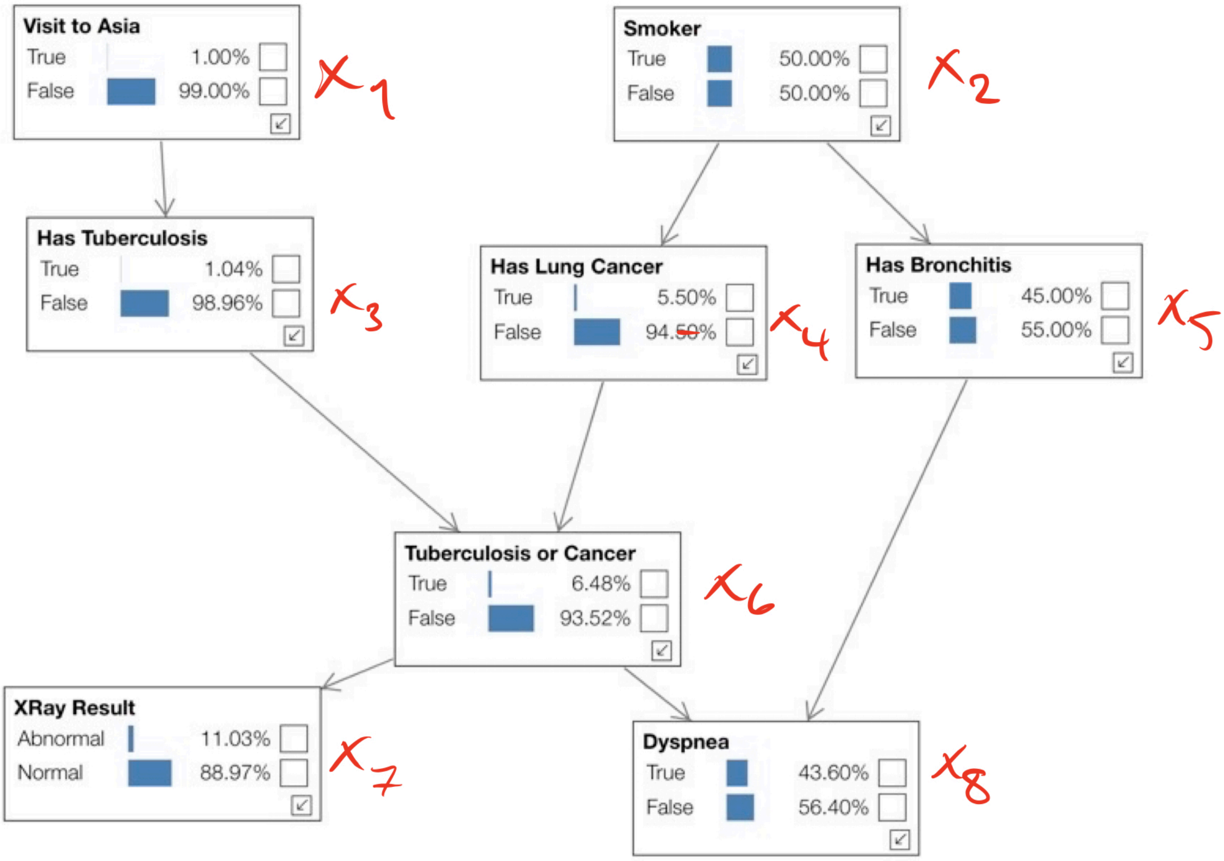
$$P[A|B] = \frac{P[A] * P[B|A]}{P[B]} = \frac{0.0033 * 0.99}{0.033168} = 0.0985$$

The probability of a person having the disease given a positive test result is 9.95%.

5

Studyind the Asia online application with different input that objects not directly connected to eachother can have an effect on their outcome probability. Analyzing how the probability of having bronchitis changes as we change the input for XRay result, we observe that an abnormal result yields a 50.63% risk of having bronchitis and normal result yields a 44.3% risk.

Let's structure the variables with a drawing:



Given this, the joint probability distribution $p(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8)$ can be described with cause/effect relationship:

$$p(x1)p(x_2)p(x_3|x_1)p(x_4|x_2)p(x_5|x_2)p(x_6|x_3, x_4)p(x_7|x_6)p(x_8|x_5, x_6)$$

This describes each variable's connection to eachother, for example: $p(x_3|x_1)$: x_3 is connected to x_1. We also have $p(x_8|x_5, x_6)$ which describes that x_8 is connected to both x_5 and x_6.

When analyzing the online Asia app and seeing that these values change, we are observing how Bayes' theorem can tell us to update current values expressed as a probability distribution. This is expressed as

$$p(x|\text{info}) = \frac{1}{p(\text{info})} p(x)p(\text{info}|x)$$

When we update a value and tell our model that we have got new information, the info becomes a constant.

$$p(x|\text{info} = \text{value}) \propto p(x)p(\text{info} = \text{value}|x)$$

This notation describes that the posterior is proportional to a prior and a likelyhood function. In this example, $p(x|\text{info} = \text{value})$ is posterior, $p(x)$ is prior and $p(\text{info} = \text{value}|x)$ is the likelyhood function. This can be applied to the previous problem when describing the probability that someone is sick, given a positive test result. In this case, x would represent that someone is sick, and info is the positive test result. The probability that someone is sick given a positive test result is $p(x|\ \text{info} = \text{value})$, the posterior probability (when we have taken everything else into account).

When we are updating the value for the XRay, we are giving values to the likelyhood function. The XRay x_7 is connected to *Tubercolosis or Cancer* x_6 with $p(x_7|x_6)$ as defined above. As we are giving a value to XRay we are changing the value for x_7, $p(x_7 = \text{positive}|x_6)$ With this likelyhood function and a prior probability for $p(x_6)$ We can calculate $p(x_6|x_7 = \text{ positive })$. With this new probability value for x_6 we can calculate the posterior probabilities for all other variables connected to it. This is why something not directly connected to x_7 can change as it is given an updated value. How all of these are connected with updates values can also be visualized in a table

To better be able to explain what happens when new information is updates, we calculate $p(x_1, \ldots, x_8)$ as a single big table as it is extended above. From the given values in *Probability, Uncertainty and Logic*, we can easily compute the full value.

$$p(x1)p(x_2)p(x_3|x_1)p(x_4|x_2)p(x_5|x_2)p(x_6|x_3, x_4)p(x_7|x_6)p(x_8|x_5, x_6) = \\ = 0.01 * 0.5 * 0.05 * 0.1 * 0.6 * 1 * 0.98 * 0.9 = 0.00001323$$

Now that we have the standard values for the table, we can see what happens when we apply prior, posterior probability. We know that the posterior is going to be proportional to the prior times the likelyhood funciton.

The updated node probability $p(x_i|observation)$.. can be calculated as

$$p(x_i|observation) = \prod_{x_n} \sum_{x_n \in \text{variables}-\text{observation}-x_i} p(x_1 \rightarrow x_n)$$

What is meant by this is that we can calculate the probability of each variable given the new observed value for one of the variables. That is the product of the sum of all marginal distributions for every variable except for the value we want to calculate and the observed value.

If we would want to do this for the example $p(x_5|x_7 = value)$, we would take

$$p(x_5|x_7 = value) = \sum_{x_1}\sum_{x_1}\sum_{x_2}\sum_{x_3}\sum_{x_4}\sum_{x_5}\sum_{x_6}\sum_{x_8} p(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8)$$

Where $\sum_{x_1}$ is the marginal distribution for x_1 and

$$p(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) = p(x1)p(x_2)p(x_3|x_1)p(x_4|x_2)p(x_5|x_2)p(x_6|x_3, x_4)p(x_7|x_6)p(x_8|x_5, x_6)$$

Comparing a Bayesian network with a CLIPS expert system we can clearly see their differences. While a CLIPS expert system is fairly simple and works well when having clearly defined outcomes like, it rains $\rightarrow$ bring an umbrella. A bayesian network is better adapted for when we cannot make definite assertion. A classic example for this is in health care when making a diagnosis one cannot be entierly sure the diagnosis is correct. It is also favorable in making diagnosises as a test outcome can affect the probabilities for other diseases.

6

Approaching the problem of predicting the weather for May 19th, we first have to evaluate the given options:

- **The relative frequency of precipitation on May 19 for the last five years**

The positives of basing the prediction on the relative frequency of precipitatin on May 19 for the last five years is that reasonable, the earth's planetary position and distance to the sun should be the same for all data points. This is viewed positive as the sun has a large impact on weather. Even though basing our prediction on the same date is generally good, one outlier could have a large impact on our prediction given these few data points.

- **The relative frequency of precipitation on all days in May during these years**

Basing our prediction on a sample of relative preception frequency for all days in May includes more data points. This decreases the risk of few ouliers having a large negative influence on our prediction. One could also argue that since weather is reasonably similar throughout the month, even data points days from the given date is relevant.

- **The relative frequency of precipitation on all days during these years?**

We don't want to use a sample from all days the past five years as this data would not give us correct information for the specific month of May. We would have much data wich is usually good for giving a good probability, the majority of the data would be misleading since its not telling us much about the weather in May. Its reasonable that ther percipitation frequency in december will not tell us much about May 19.

Considering all of these pros and cons about the models we think it reasonable to choose: **The relative frequency of precipitation on all days in May during these years**.

Analyzing the same models but with data gathered from the past hundred years we arrive at the same conclusion, to consider all dates in May. We believe that if we were to choose the model that only takes May 19 into consideration, we still only have 100 data points. This is still a small sample and it would not produce a reliable result. If we were to choose all days within a hundred years as the last model suggest, we have the same issue as before but worse. We have a huge set of data but many of these data points would be misleading.

For all three models there is something important we have to consider. When choosing the model, we have to make critical assumptions. Examples of this is that the weather climate has remained the same throughout the 100 years. We know that this is not true due to pollution factours, industrial growth and large increases in human population. Considering that these assumptions has to be made and that they are dependant on so many variables we believe that choosing a model does require human judgement. However, this must not always be the case. If we were model a problem that we human could not give valueble input to, e.q quantum problems that we can't visualize or determine based on history. This could be a situation where a computer would do a better job of choosing the model based on the set of rules we can define for the problem.