

# Problem Module lv6

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## General info

- FIRE group number 75
- module number 6
- By
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- "We hereby declare that we have both actively participated in solving every exercise. All solutions are entirely our own work, without having taken part of other solutions.
- Number of hours spent for each one of you
  - Both 18h
- Number of hours spent in supervision for this module
  - 5h

## 1. Whales and Krill Problem

a)

Given the equations  $k' = (a - bw)k$  and  $w' = (-m + nk)w$  we can analyze what happens when we set some variables to different values. Let's say for example that there are no whales present  $w = 0$ , then  $k' = ak$  which means that rate of change for the krill population is a constant times current population. This tells us that  $a$  represents some kind of standard rate of change if there wouldn't be any whales present. Given this, we can assume that  $b$  stands for how much a given whale population affects rate of change for the krill population.

For  $w'$  we can see similarities to  $k'$ . When the krill population is zero, rate of change is going to be  $-mw$ . We can see that it is only dependent on current whale population and some constant. The constant  $m$  represents a decrease of whale population which is reasonable. Without any krill, the whales will not have any food and therefore decrease in population. Let's say that there are krill present, then rate of change is going to change in the positive direction. More food for whales results in a healthier population

where there are better conditions for reproduction. The effect a krill population has on the rate of change for whales are described with the constant  $n$ .

Now that we have interpreted the constants it is reasonable that the equation system tells us the affect krill and whale population have on the rate of change of the populations. When there are a lot of whales, the krill population decreases and when there are few whales the krill population increases. The same goes for the whale population but in reverse. When there are a lot of krill the population have a better chance to grow, otherwise it decreases.

b)

We know we have an equilibrium when rate of change is zero,  $k'$  and  $w' = 0$ . From the first equation  $w' = (-m + nk)w = 0$  Directly we can see that there is an equilibrium when  $w = 0$ . We can also simplify it.

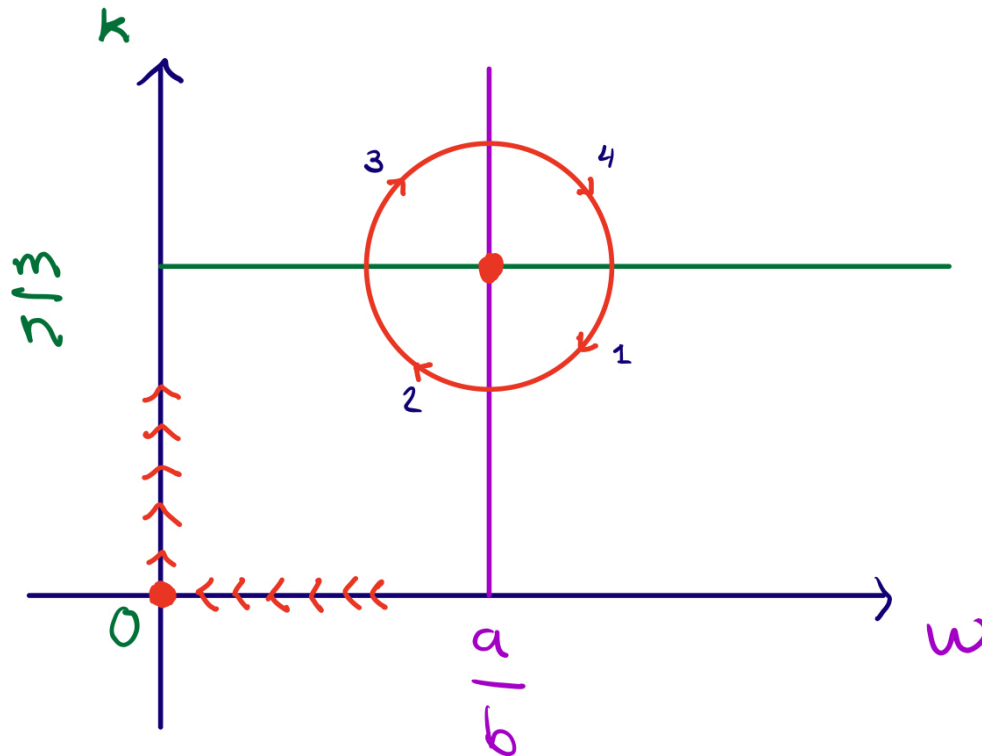
$$\begin{aligned}(-m + nk)w &= 0 \\ -mw + nkw &= 0 \\ -m + nk &= 0 \\ k &= \frac{m}{n}\end{aligned}$$

We see that there is an equilibrium when  $k = \frac{m}{n}$ . Let's do the same for the other equation  $k' = (a - bw)k = 0$ . Just like the first equation we can see that there is an equilibrium when  $k = 0$ . When we simplify we get:

$$\begin{aligned}(a - bw)k &= 0 \\ ak - bwk &= 0 \\ a - bw &= 0 \\ w &= \frac{a}{b}\end{aligned}$$

There's another equilibrium when  $w = \frac{a}{b}$ .

Now that we have found the equilibrium points we analyze the stability of each one. We do this by drawing a graph with the two populations in conjunction. Analyzing the first points when  $w$  and  $k$  are zero and moving along the axes we can see that it moves towards the equilibrium along the  $w$ -axis and away along the  $k$ -axis. This means that the equilibrium when  $(k = 0, w = 0)$  is unstable. This is reasonable because considering a non-present whale population, krill population can grow unhindered. It is also reasonable that whale population decreases when there is no krill population present.



Next, we analyze the equilibrium when  $w = \frac{a}{b}$  and  $k = \frac{m}{n}$ . Here we get that values circle around the equilibrium, we can describe this by giving a description of each numerated point in the graph.

1. This point represents when we have a large whale population and krill population is small. From this point it is going to move towards a point where whale population decreases until there is enough krill again.
2. This point represents when both whale and krill populations are small. Here krill is going to grow as there are few whales stopping the increase.
3. Here we have a large krill population but few whales. The large number of krill are going to stimulate whale population growth and it's going to move towards a larger whale population.
4. Here both whale and krill population is large. But more importantly the whale population has increased to a point where it's going to decelerate the krill population growth. We don't believe this equilibrium is stable as points nearby don't go directly towards the equilibrium point.

c)

With values for the constants we get the equations:

$$\begin{aligned} w' &= (-0.5 + 0.000001k)w \\ k' &= (0.2 - 0.0001w)k \end{aligned}$$

The equations for the current value of the population are then going to be

$$k_{n+1}(t) = k_n + tk'_n \text{ and } w_{n+1}(t) = k_n + tk'_n$$

Simulating this with Euler's method in a table of the values looks like this:

time	Approx $k$	Approx $w$
0	700000	3000
0.3	679000	3180
0.6	654963	3351

This means that with starting values 3'000 whales and 700'000 krill, results changes to 654'963 krill and 3'351 whales after 0.6 years.

d)

To start of this analysis we use want to see what it looks like along both axes.  $k = 20000, w = 0$ . We can see that the krill population is going to start off at 20'000 and then take off like crazy. The opposite goes for the whales. It's going to start start high and then drop to zero, where it will stay.

Using the same values as in **c)** we get a completely different result. Both the values from  $k$  and  $w$  are pending from high to low, depending on each other. We see that when the krill population is reaching its peak, the population of whales increases drastically and affects the krill population to decrease drastically. The same thing happens in reverse when whale population hits its peak.

Another conclusion we can draw by picking different starting points is that the values start to circle around the equilibrium. If the krill is low and whale is high the values are changing, meaning that krill starts increasing and whales decreasing. This justifies the equilibrium that we found in b.

e)

Adding the term  $-rk$  gives us the equation  $k' = (a - bw)k - rk$ . Our first interpretation of this is that  $r$  has to be a coefficient representing the magnitude of the fishing relative to the current population. Logically the rate of change for  $k$  has to do with krill reproduction, the term  $-rk$  decelerates the reproduction. To visualize this we try different values for  $r$  in mathematica. We start off with the same value for  $r$  as for  $a$  just to get a better understanding of the effect. We can see that the magnitude of the fishing is greater than the reproduction of krill which results in a heavy deceleration of the population until extinct. This is reasonable because we fish out more krill than is

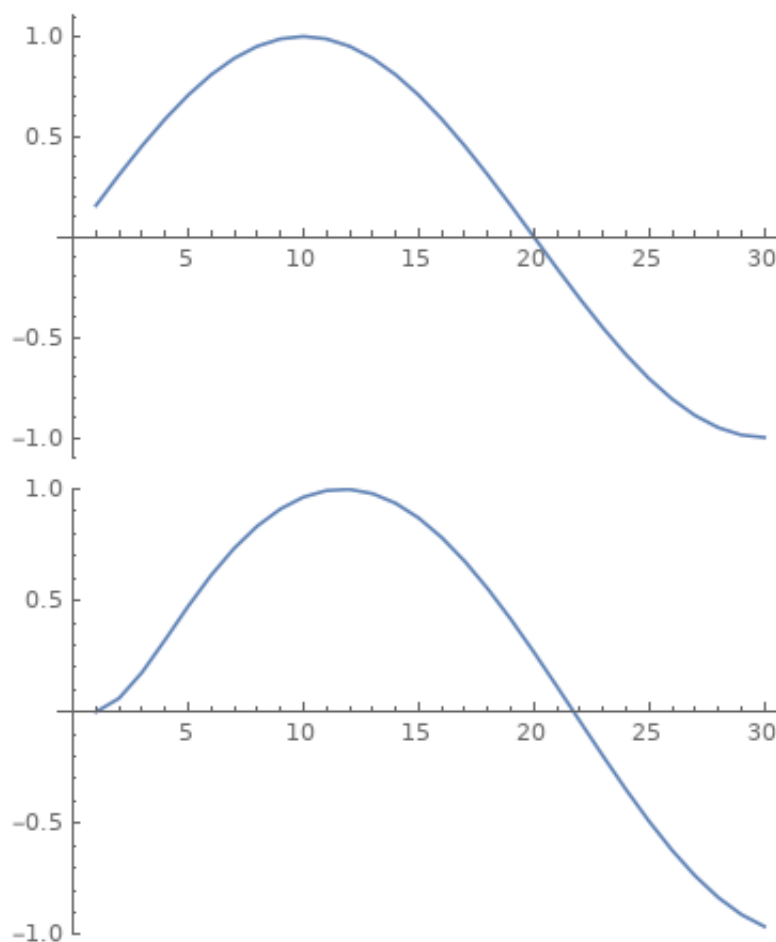
reproduced over the same time period. This also helps justify our model extended with  $-rk$ . Trying with different values for  $r$  we can see that extended fishing results in a longer cycle for both whales and krill. This means that more fishing slows down the positive rate of change for krill and prolongs the time to its peak, which also delays the time for the start of positive increase of whales.

f)

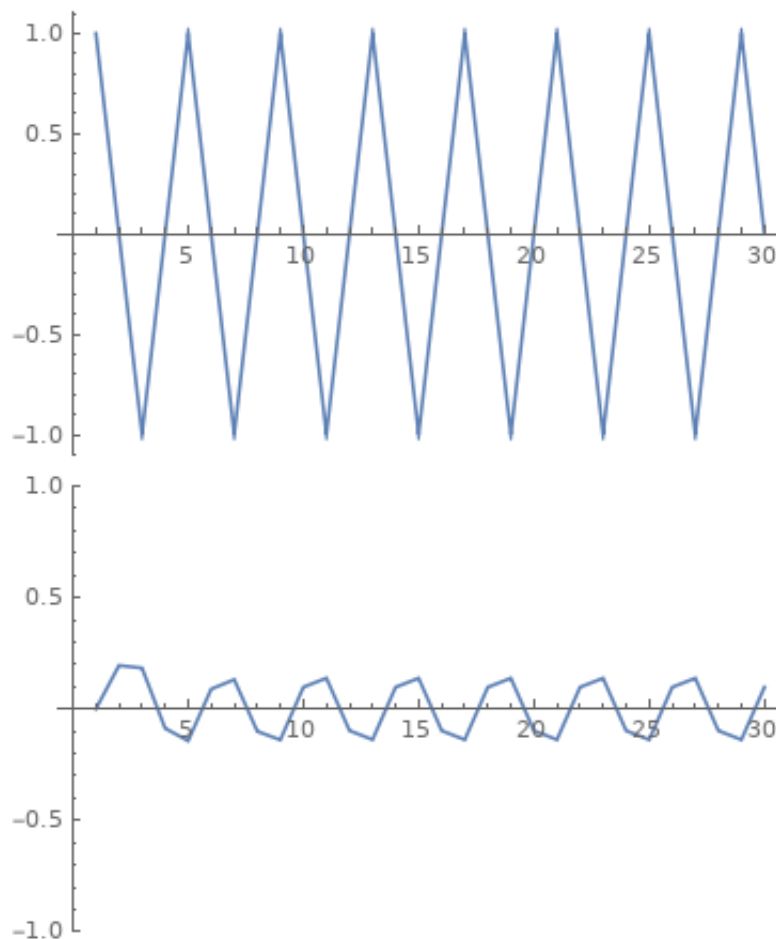
We can find a few different ways to improve the model. One thing we noticed is that with the code and starting values given, the whale population can become negative. An improvement to the model is to not allow this. Considering that we analyze the fishing of krill, it is reasonable to also include hunting of whales, as this also affects the populations. Another thing that should be taken into consideration is that if whales go extinct, krill population moves towards infinity. This is unreasonable as the krill also feeds on something. The population of krill should have a limit.

## 2. Signal Processing Problem

We started by analyzing the different plots with different frequencies. The first thing we noticed was when using low frequencies for both the input function and the output function. For  $frequency = 200$ . We get the two plots below.



If we analyze these figures we see that they are almost identical except that they are phase shifted a bit. This does not tell us much, it's likely that this is just the delay between input and output of sound. What's actually interesting is that they are so alike. When inputting low frequencies we get a big similarity between output and input. If we now instead look at the plots when the frequency for both of them are high, we get another interesting result.



These figures above represents the two plots at  $frequency = 2000$ . From this frequency we can see a much more distinct difference. The amplitude of the output sound is much smaller than for the input sound. From the input sound we can also see that it has spikes instead of rounded tops. From the lower frequencies we noticed that both the input and the output looks a lot like sinus waves, but when we increase the frequency, the input turning points becomes sharp and the whole output plot becomes truncated. These spikes are a result of having too few sampling points for such a high frequency. When this happens the amplitude of the output decreases.

Since the component would reduce the high frequencies in sound, we think it's reasonable that such a component could be used to remove distortion in sound. That the component is used in variety of ways with the purpose of reducing/removing high frequency buzz and interference from audio.

If we spoke into the equation we believe that the output would not be as clear as our regular speaking voice. That it would be more flat and harder to interpret. We believe this because voice frequency is usually said to be between 300 and 3000 Hz, and with this component we start to reduce the amplitude of sounds much earlier than 3000 Hz.

### 3. Drug Dosage Problem

There are a number of things we would have to consider determining dose and interval for a sample patient. Reasonably different people absorb the drug differently. As a drug company most likely don't know who uses their drugs, we have to find a model working for a generic case.

To find the optimal drug dosage considering time interval and some kind of upper and lower limit of what is a "good" interval of concentration. We know that the dose value is going to be constant and same for each dose. As we want the concentration to always be below max limit and above minimum limit. If we assume that we want to take the dose as few times as possible, reasonably the dose is then going to be as large as possible  $maxC - minC$  without damaging the patient. This is visualized in the figure below:



Dose  $\rightarrow$  Fixed  $\rightarrow$  constant

$$\text{Dose} = \text{Max } C - \text{Min } C$$

Let's say that we have a rate of decay  $k$  for the dose, meaning how much of the dose the body absorbs per time unit  $t$ . We will have the equation  $maxC - kt = minC$ . Let's say that  $maxC = 10$  and  $minC = 2$ , we have rate of decay  $k = 2$ . Then we will get time interval by:

$$maxC - kt = minC =$$

$$\begin{aligned}10 - 2t &= 2 = \\2t &= 8 \Rightarrow \\t &= 4\end{aligned}$$

If the time unit was in hours then the time interval between doses should be 4 hours. One issue we encountered directly with this model is that we are using time as a discrete variable. This does not represent reality perfectly as time is continuous.

An improvement of this model could be found with a simulation of a patient. Where we have a max concentration and minimum concentration for the drug given. We assume we know how long it takes for the drug to half its concentration after intake and we have the absorption rate per hour. With this information we could simulate the patient to get the dose size and time interval between intakes.

The algorithm used to solve this is explained below.

```
public static void main(String[] args) {  
    int maxConcentration = 150;  
    int minConcentration = 100;  
  
    double concentration = minConcentration;  
    int offset = 10; //For safety, do not want to be on absolute max everytime we take a dose;  
    double dose = maxConcentration - minConcentration + offset;  
  
    double drugHalfLifeRate = 10;  
    double intakePerTime = 40;  
    double bodyIntake = 0;  
  
    ArrayList<Integer> doseAtTimeList = new ArrayList<>();  
    int intervalListSize = 20;
```

Here we have the basic parameters that we say that we get for this problem, how we defined the problem. As you can see, the concentration has the similar equation used above, except for an added safety offset. We also create a list to save the times that a patient takes another dose.



```

int loopTimeValue = 0;

while (loopTimeValue <= 2000) {

    boolean pill = false;

    if (concentration < minConcentration + offset) {
        bodyIntake += dose;
        System.out.print("Takes pill at hour; " + loopTimeValue + " -----> ");
        System.out.print("concentration before pill: " + concentration + " -----> ");
        doseAtTimeList.add(loopTimeValue);

        if (doseAtTimeList.size() > intervalListSize)
            doseAtTimeList.remove(index: 0);

        pill = true;
    }

    if (bodyIntake >= intakePerTime) {
        concentration += intakePerTime;
        bodyIntake -= intakePerTime;
    } else if (bodyIntake < intakePerTime){
        concentration += bodyIntake;
        bodyIntake = 0;
    }

    concentration *= Math.pow(0.5, (1.0 / drugHalfLifeRate));

    if (pill)
        System.out.println("concentration after pill: " + concentration);
}

```

Here the simulation starts. The loop describes the simulation in hours, where every index in the loop is a time unit. We start by checking the concentration level and if the concentration level is below the minimum + offset, we give the patient a dose. We also keep the list of times at a size limit for simplicity reasons. Then we check the amount in the body compared to the intake per hour and adding the concentration to the blood and removing it from the digestion accordingly. Then we adjust the concentration in line with the drug's halvation time.

```

if (doseAtTimeList.size() == intervalListSize) {
    ArrayList<Integer> clone = (ArrayList) doseAtTimeList.clone();

    for (int i = intervalListSize - 1; i > 0; i--) {
        clone.set(i, clone.get(i) - clone.get(i - 1));
    }

    clone.remove(index: 0);

    double sum = 0;
    for (Integer s : clone) {
        sum += s;
    }

    double average = sum / clone.size();

    int timeInterval = 0;
    for (int i = 0; i < clone.size(); i++) {
        if (clone.get(i) < average + 1 && clone.get(i) > average - 1)
            timeInterval++;
    }

    if (timeInterval == clone.size()) {
        System.out.println("Average time between doses: " + average);
        System.out.println("Dose size: " + dose);
        break;
    }
}
loopTimeValue ++;
}

```

This is still in the simulation loop. Here we create a list with the offset of the times as elements. We calculate the average of the offsets in times, then we check if the interval stabilizes over time. When we have found a stabilization, we output the dose value and interval in which the drugs should be taken. See output below.

```

Takes pill at hour; 52 -----> concentration before pill: 108.25280586094229 -----> conecetration after pill: 138.3247589561605
Takes pill at hour; 59 -----> concentration before pill: 104.45538595693272 -----> conecetration after pill: 134.78164090300106
Takes pill at hour; 65 -----> concentration before pill: 109.44714788569316 -----> conecetration after pill: 139.43911946843193
Takes pill at hour; 72 -----> concentration before pill: 105.19058971263023 -----> conecetration after pill: 135.4676102625686
Takes pill at hour; 78 -----> concentration before pill: 109.93220147152957 -----> conecetration after pill: 139.89169046668056
Takes pill at hour; 85 -----> concentration before pill: 105.48917521881796 -----> conecetration after pill: 135.74620039063646
Takes pill at hour; 92 -----> concentration before pill: 102.75417174413262 -----> conecetration after pill: 133.19435191678727
Takes pill at hour; 98 -----> concentration before pill: 108.32476507983864 -----> conecetration after pill: 138.39189928143603
Takes pill at hour; 105 -----> concentration before pill: 104.49968205209919 -----> conecetration after pill: 134.82297062118764
Takes pill at hour; 111 -----> concentration before pill: 109.47637240968741 -----> conecetration after pill: 139.46638691348056
Takes pill at hour; 118 -----> concentration before pill: 105.2085795173544 -----> conecetration after pill: 135.48439534388754
Takes pill at hour; 124 -----> concentration before pill: 109.94407031635293 -----> conecetration after pill: 139.90276449047218
Average time between doses: 6.526315789473684
Dose size: 60.0

```

This algorithm could be used to get the dose size and average interval between doses, meaning it calculates the optimal time interval between doses for a given drug with these given parameters.

## 4. Socratic Questioning

a)

- *Question:* Have you done a plot of the data points in a graph?

- *Question:* What can we say about the data points? Are they linear or not?
- *Question:* Can you guess the form of the function? *Discussion erupts.*
- *Question:* Based on the form you guessed, can you use the values in the table to define the function more precisely to fit the data? Can it be justified?

b)

- *Question:* Have you plotted the function in combination with points? How well does your function fit with all the points?
- *Question:* What are your functions edge cases? Like when  $x$  goes to zero or infinity. Is it reasonable that the data should behave similarly for the problem?
- *Question:* What other form of functions follows a similar pattern, try plotting!

c)

- *Question:* What do we know? We start listing the values we know from the question.
- *Question:* Is there a relation between these values? Have you tried plotting them?
- *Question:* How do we know that the machine is worth buying? How do we know it's a good investment?
- *Question:* Is there anything else they can do to increase sales? What does the data say?

d)

- *Question:* What do you think the question is asking for? (This starts a discussion where the group starts talking about the problem, which can lead to them understanding it more without a supervisor telling them anything. A classic *rubber duck* situation).
- *Question:* What part of the question are you having trouble with? (We can try to formulate it in another way)
- *Question:* Can we break it down to smaller parts?
- *Situation:* If they give suggestions that are not relevant, try to ask questions that disproves it. If they are on the right track, just let them continue and ask if they feel like they can move on.