group
$(G, \cdot)$
G
G
Associativity:
$a, b, c \in G$
$(a \cdot b) \cdot c = a \cdot (b \cdot c)$
Identity Element:
$e \in G$
$a \in G$
$a \cdot e = e \cdot a = a$
Inverse Element:
$a \in G$
$a^{-1} \in G$
$a \cdot a^{-1} = a^{-1} \cdot a = e$
$(G, \cdot)$
H
G
subgroup
G
$H \leq G$
$(H, \cdot)$
H
$a, b \in H$
$a \cdot b \in H$
$a \in H$
$a^{-1} \in H$
$(G, \cdot_G)$
$(H, \cdot_H)$
$\phi : G \to H$
group homomorphism
$a, b \in G$
$\phi(a \cdot_G b) = \phi(a) \cdot_H \phi(b)$

$\phi : G \to H$
kernel
ϕ
$\ker(\phi)$
G
H

$$\ker(\phi) = \{g \in G \mid \phi(g) = e_H\}$$

$\phi : G \to H$
image
ϕ
$im(\phi)$
H
G

$$im(\phi) = \{h \in H \mid \exists g \in G, \phi(g) = h\}$$

N
G
normal subgroup

$a, b \in$
G
$\phi :$
$G \to$
H
$\phi(e_G) =$
e_H
$\phi(g^{-1}) =$
$(\phi(g))^{-1}$
$g \in$
G
$\phi(e_G) =$
$\phi(e_G \cdot$
$e_G) =$
$\phi(e_G) \cdot$
$\phi(e_G)$
$(\phi(e_G))^{-1}$
H
$\phi(e_G)(\phi(e_G))^{-1} =$
$(\phi(e_G)\phi(e_G))(\phi(e_G))^{-1}$
$e_H =$
$\phi(e_G)$
$g \in$
G
$e_H =$
$\phi(e_G) =$
$\phi(g \cdot$
$g^{-1}) =$
$\phi(g) \cdot$
$\phi(g^{-1})$
$(\phi(g))^{-1}$
$(\phi(g))^{-1} \cdot$
$e_H =$
$(\phi(g))^{-1} \cdot$
$(\phi(g) \cdot$
$\phi(g^{-1}))$
$(\phi(g))^{-1} =$
$\phi(g^{-1})$
$\phi :$
$G \to$
H
$\ker(\phi)$
G
??
??
$\phi(e_G) =$
e_H
$e_G \in$
$\ker(\phi)$
$\ker(\phi)$
$a, b \in$
$\ker(\phi)$
$\phi(a) =$
e_H
$\phi(b) =$
e_H
??
$\phi(ab) =$
$\phi(a)\phi(b) =$
$e_H e_H =$
e_H
$ab \in$
$\ker(\phi)$
$a \in$
$\ker(\phi)$
$\phi(a) =$
e_H
??
$\phi(a^{-1}) =$
$(\phi(a))^{-1} =$
$(e_H)^{-1} =$
e_H
$a^{-1} \in$
$\ker(\phi)$
$\ker(\phi)$
G
$\phi :$
$G \to$
H
$\ker(\phi)$
G
??
$K =$
$\ker(\phi)$
G
$k \in$
K
$g \in$
G
$gkg^{-1} \in$
K