费曼3作业4

郑子诺，物理41

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74.1

(a)

$$V = +\mu B, 0, -\mu B$$

$$\Delta(\frac{p_0^2}{2M}) \approx \frac{p_0 \Delta p_0}{M} = -\Delta V$$

$$\Delta\phi = \Delta p_0 L = -\frac{ML}{p_0}\Delta V = p_y \Delta z$$

$$\therefore \delta\theta \approx \frac{p_y}{p_0} = -\frac{ML}{p_0^2}\frac{\Delta V}{\Delta z} \approx -\frac{ML}{p_0^2}\frac{\partial V}{\partial z}$$

For different states,we have

$$\delta\theta = -\frac{ML}{p_0^2}\frac{\mu\partial B}{\partial z}, 0, +\frac{ML}{p_0^2}\frac{\mu\partial B}{\partial z}$$

(b)

$$|+x(t)\rangle = \langle +z| + x\rangle \, |+z(t)\rangle + \langle 0z| + x\rangle \, |0z(t)\rangle + \langle -z| + x\rangle \, |-z(t)\rangle$$

$$= \frac{1}{2}e^{-i\frac{\mu B}{\hbar}t} |+z\rangle - \frac{1}{\sqrt{2}} |0z\rangle + \frac{1}{2}e^{i\frac{\mu B}{\hbar}t} |-z\rangle$$

$$|0x(t)\rangle = \frac{1}{\sqrt{2}}e^{-i\frac{\mu B}{\hbar}t} |+z\rangle - \frac{1}{\sqrt{2}}e^{i\frac{\mu B}{\hbar}t} |-z\rangle$$

$$|-x(t)\rangle = \frac{1}{2}e^{-i\frac{\mu B}{\hbar}t} |+z\rangle + \frac{1}{\sqrt{2}} |0z\rangle + \frac{1}{2}e^{i\frac{\mu B}{\hbar}t} |-z\rangle$$

$$\langle +x| + x(t)\rangle = \langle +x| + z(t)\rangle \langle +z| + x\rangle + \langle +x|0z(t)\rangle \langle 0z| + x\rangle + \langle +x| - z(t)\rangle \langle -z| + x\rangle$$

$$= \frac{1}{2}(\cos(\frac{\mu B}{\hbar}t) + 1)$$

$$\langle 0x|0x(t)\rangle = \cos(\frac{\mu B}{\hbar}t)$$

$$\langle -x| - x(t)\rangle = \frac{1}{2}(\cos(\frac{\mu B}{\hbar}t) + 1)$$

(c) Method 1:Measure the deflection of a beam of particles that passed through an inhomogeneous magnetic field,then we have

$$\mu = \frac{p_0^2}{ML\frac{\partial B}{\partial z}}\delta\theta$$

Method 2:Measure the oscillation angular frequency of the probability of detecting a particle that has a spin on $+x$ direction,which is put in a uniform magnetic field with $+z$ direction.Then we have

$$\mu = \frac{\hbar}{B}\omega$$

73.3

(a)

$$\begin{aligned}
N &= N_0|\langle +U| + T\rangle\langle +T| - S\rangle + \langle +U| - T\rangle\langle -T| - S\rangle|^2 \\
&= N_0|\langle +U| - S\rangle|^2
\end{aligned}$$

(b)

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\therefore \langle +T| - S\rangle = 1, \langle -T| - S\rangle = 0$$

(c)

$$\begin{bmatrix} C_1' \\ C_2' \end{bmatrix} = R_y(-\frac{\pi}{2})R_z(\theta)R_y(\frac{\pi}{2})\begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} \cos\frac{\theta}{2} & i\sin\frac{\theta}{2} \\ i\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix}\begin{bmatrix} C_1 \\ C_2 \end{bmatrix}$$

$$\therefore \langle +U| - S\rangle = i\sin\frac{\theta}{2}$$

(d)

$$\theta = 0, \langle +U| - S\rangle = 0$$

$$\theta = \pi, \langle +U| - S\rangle = i$$

(e) Because U need a rotation along z-axis to become T.

$$\begin{bmatrix} -i & 0 \\ 0 & i \end{bmatrix}\begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

73.6

(a)

$$P_{+z} = |\sqrt{1 + \frac{v}{c}} \sin \frac{\theta}{2}|^2 = (1 + \frac{v}{c}) \sin^2 \frac{\theta}{2}$$

$$P_{-z} = (1 - \frac{v}{c}) \cos^2 \frac{\theta}{2}$$

(b)

$$P_{+x} = |\frac{1}{\sqrt{2}} \sqrt{1 + \frac{v}{c}} \sin \frac{\theta}{2} + \frac{1}{\sqrt{2}} \sqrt{1 - \frac{v}{c}} \cos \frac{\theta}{2}|^2$$

$$= \frac{1}{2}(1 - \frac{v}{c} \cos \theta + \sqrt{1 - \frac{v^2}{c^2}} \sin \theta)$$

$$P_{-x} = \frac{1}{2}(1 - \frac{v}{c} \cos \theta - \sqrt{1 - \frac{v^2}{c^2}} \sin \theta)$$

(c)

$$P_{+y} = |\frac{1}{\sqrt{2}} \sqrt{1 + \frac{v}{c}} \sin \frac{\theta}{2} - i\frac{1}{\sqrt{2}} \sqrt{1 - \frac{v}{c}} \cos \frac{\theta}{2}|^2$$

$$= \frac{1}{2}(1 - \frac{v}{c} \cos \theta)$$

$$P_{-y} = \frac{1}{2}(1 - \frac{v}{c} \cos \theta)$$

(d)

$$P_{+z} = \int_0^\pi \frac{1}{2} \sin \theta (1 + \frac{v}{c}) \sin^2 \frac{\theta}{2} d\theta = \frac{1}{2}(1 + \frac{v}{c})$$

$$P_{-z} = \frac{1}{2}(1 - \frac{v}{c})$$