费曼3作业11

郑子诺，物理41

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84.1

According to the conservation of angular momentum, $A(\theta)$ is just proportional to the possibility amplitude of $|-\hat{n}\rangle$. Because the symmetry, we can only consider the rotation along y-axis. We know that

$$|1,0\rangle = \frac{|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle}{\sqrt{2}}$$

and

$$R_y(\theta) = \begin{bmatrix} \cos\frac{\theta}{2} & \sin\frac{\theta}{2} \\ -\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix}$$

hence

$$|1,0\rangle = \frac{1}{\sqrt{2}}\sin\theta\,|1,1\rangle' + \cos\theta\,|1,0\rangle' - \frac{1}{\sqrt{2}}\sin\theta\,|1,-1\rangle'$$

Then we have

$$A(\theta) \propto \sin\theta$$

84.3

(a)Because of the conservation of angular momentum, the angular momentum along $+z$ direction of p^* need be the same as the sum of the angular momentum of the right hand circularly photon and the initial proton, which is $1+\frac{1}{2}$ or $1-\frac{1}{2}$. Thus only $m = \frac{3}{2}, \frac{1}{2}$ are allowed. The total angular momentum of the final state is $j = \frac{1}{2}$, thus only $m' = \frac{1}{2}, -\frac{1}{2}$ are allowed.

(b)Because the parity is conserved, we have $c = d$.

For the $|\frac{3}{2}, \frac{1}{2}\rangle$, we have

$$|\frac{3}{2}, \frac{1}{2}\rangle = \frac{|\uparrow,\uparrow,\downarrow\rangle + |\uparrow,\downarrow,\uparrow\rangle + |\downarrow,\uparrow,\uparrow\rangle}{\sqrt{3}}$$

and then

$$|\frac{3}{2},\frac{1}{2}\rangle = -\sqrt{3}\sin\frac{\theta}{2}\cos^2\frac{\theta}{2}\,|\frac{3}{2},\frac{3}{2}\rangle' + (\cos^3\frac{\theta}{2} - 2\cos\frac{\theta}{2}\sin^2\frac{\theta}{2})\,|\frac{3}{2},\frac{1}{2}\rangle'$$
$$+ (-\sin^3\frac{\theta}{2} + 2\cos^2\frac{\theta}{2}\sin\frac{\theta}{2})\,|\frac{3}{2},-\frac{1}{2}\rangle' + \sqrt{3}\sin^2\frac{\theta}{2}\cos\frac{\theta}{2}\,|\frac{3}{2},-\frac{3}{2}\rangle'$$

also we have

$$|\frac{3}{2},\frac{3}{2}\rangle = \cos^3\frac{\theta}{2}\,|\frac{3}{2},\frac{3}{2}\rangle' + \sqrt{3}\cos^2\frac{\theta}{2}\sin\frac{\theta}{2}\,|\frac{3}{2},\frac{1}{2}\rangle'$$
$$+ \sqrt{3}\cos\frac{\theta}{2}\sin^2\frac{\theta}{2}\,|\frac{3}{2},-\frac{1}{2}\rangle' + \sin^3\frac{\theta}{2}\,|\frac{3}{2},\frac{3}{2}\rangle'$$

Thus the $f(\theta)$ is

$$f(\theta) = (|\cos^3\frac{\theta}{2} - 2\cos\frac{\theta}{2}\sin^2\frac{\theta}{2}|^2 + |-\sin^3\frac{\theta}{2} + 2\cos^2\frac{\theta}{2}\sin\frac{\theta}{2}|^2)|a|^2|c|^2$$
$$+ (|\sqrt{3}\cos^2\frac{\theta}{2}\sin\frac{\theta}{2}|^2 + |\sqrt{3}\cos\frac{\theta}{2}\sin^2\frac{\theta}{2}|^2)|b|^2|c|^2$$
$$= |c|^2((1-\frac{3}{4}\sin^2\theta)|a|^2 + \frac{3}{4}\sin^2\theta|b|^2)$$

84.5

We know

$$|1,1\rangle = \frac{1}{\sqrt{2}}(1+\cos\theta)\,|1,1\rangle' - \frac{1}{\sqrt{2}}\sin\theta\,|1,0\rangle' + \frac{1}{\sqrt{2}}\,|1,-1\rangle'$$

If the excited state has even parity, which means the state of photons has even parity, then the possibility amplitude of $+z$-axis photon is the same one of $-z$-axis photon. Conversely, it will differ a signal.

(a)If photons are detected regardless of polarization, then the possibility distribution is always proportion to

$$1+\cos^2\theta$$

(b)Detect the x'-polarized photon, if even, we have

$$f(\theta) \propto |\frac{1}{\sqrt{2}}(1+\cos\theta) + \frac{1}{\sqrt{2}}(1-\cos\theta)|^2 = 2$$

and then is uniform. Conversely, we have

$$f(\theta) \propto |\frac{1}{\sqrt{2}}(1+\cos\theta) - \frac{1}{\sqrt{2}}(1-\cos\theta)|^2 = 2\cos^2\theta$$

hence we can distinguish the two case form the angular distribution of x'-polarized photons.