费曼3作业7

郑子诺，物理41

2024 年 11 月 20 日

78.1

(a)

$$\boldsymbol{\sigma} \times \boldsymbol{\sigma} = \begin{bmatrix} \sigma_y\sigma_z - \sigma_z\sigma_y \\ \sigma_z\sigma_x - \sigma_x\sigma_z \\ \sigma_x\sigma_y - \sigma_y\sigma_x \end{bmatrix}$$

We have

$$[\sigma_i, \sigma_j] = 2i\epsilon_{ijk}\sigma_k$$

$$\sigma_i^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Thus

$$\boldsymbol{\sigma} \times \boldsymbol{\sigma} = 2i\boldsymbol{\sigma}$$

$$\boldsymbol{\sigma} \cdot \boldsymbol{\sigma} = 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

(b) First, form $\boldsymbol{\sigma} \times \boldsymbol{\sigma} = 2i\boldsymbol{\sigma}$ we have

$$Tr(\sigma_i) = 0$$

According to the symmetry, we have

$$Tr(\sigma_i^2) = 2$$

We know that we can always choose a basis to make σ_z be like

$$\begin{bmatrix} c & 1 \\ 0 & c \end{bmatrix}$$

or

$$\begin{bmatrix} c_1 & 0 \\ 0 & c_2 \end{bmatrix}$$

and form the conditions above we have $c_1 = -c_2 = \pm 1$. Choose $c_1 = 1$, then we have

$$\sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Let

$$\sigma_y = \begin{bmatrix} c_1 & c_2 \\ c_3 & c_4 \end{bmatrix}$$

Use the conditions we have $c_1 = -c_4, c_1^2 + c_2c_3 = 1$. We can then calculate σ_x, which is

$$\begin{bmatrix} 0 & ic_2 \\ -ic_3 & 0 \end{bmatrix}$$

Use $Tr(\sigma_x^2) = 1$ we have $c_2c_3 = 1, c_1 = 0$. We can always choose a basis to make $c_2 = -i, c_3 = i$, because multiplying a complex number on a base vector makes $c_2 \to zc_2, c_3 \to \frac{c_3}{z}$. Then we have

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

78.2

(a)

$$\begin{aligned} i\hbar\frac{dC_1}{dt} &= E_O C_1 - AC_2 \\ i\hbar\frac{dC_2}{dt} &= -AC_1 + E_C C_2 - AC_3 \\ i\hbar\frac{dC_3}{dt} &= -AC_2 + E_O C_3 \end{aligned}$$

We have the characteristic polynomial

$$\begin{vmatrix} \lambda - E_O & A & 0 \\ A & \lambda - E_C & A \\ 0 & A & \lambda - E_O \end{vmatrix} = (\lambda - E_O)(\lambda^2 - (E_O + E_C)\lambda + E_O E_C - 2A^2)$$

And thus the energy levels are

$$E = E_O, \frac{E_O + E_C \pm \sqrt{(E_O - E_C)^2 + 8A^2}}{2}$$

(b)If $E_O = E_C$, we have

$$E = E_O, E_O + \sqrt{2}A, E_O - \sqrt{2}A$$

The first one caused by the stationary state that is symmetrical about carbon atom and has the opposite symbol on each oxygen atom, and thus make the probability amplitude on the carbon atom be 0.

The second and third one are caused by the exchange force because of the chance for the electron to jump between an oxygen atom and the carbon atom.

78.4

(a)(b)

$$\begin{aligned}
i\hbar\frac{dC_1}{dt} &= E_0C_1 + AC_2 + AC_6 \\
i\hbar\frac{dC_2}{dt} &= E_0C_2 + AC_1 + AC_3 \\
i\hbar\frac{dC_3}{dt} &= E_0C_3 + AC_2 + AC_4 \\
i\hbar\frac{dC_4}{dt} &= E_0C_4 + AC_3 + AC_5 \\
i\hbar\frac{dC_5}{dt} &= E_0C_5 + AC_4 + AC_6 \\
i\hbar\frac{dC_6}{dt} &= E_0C_6 + AC_5 + AC_1
\end{aligned}$$

If $C_k = \frac{1}{\sqrt{6}}e^{-\frac{i}{\hbar}E_I t}$, then the equations is simply satisfied if

$$E_I = E_0 + 2A$$

(c)(d) The second to fourth equations give that

$$i\hbar\frac{C_1}{dt} = (E_0 + 2A\cos\delta)C_1$$

which means

$$C_1 = Ce^{-\frac{i}{\hbar}(E_0+2A\cos\delta)t}$$

Use the first and last equation, we have

$$e^{i6\delta} = 1, \rightarrow \delta = \frac{i\pi}{3}, i = 0, 1, \ldots, 5$$

Normalizing the result, we have

$$C_k = \frac{1}{\sqrt{6}} e^{-\frac{i}{\hbar} Et + i(k-1)\delta}, E = E_0 + 2A\cos\delta, \delta = \frac{i\pi}{3}, i = 0, 1, \ldots, 5$$

The energy levels are

$$E_0 + 2A, E_0 + A, E_0 - A, E_0 - 2A, E_0 - A, E_0 + A$$

Thus the spacings are $A, 2A, A, A, 2A, A$.