费曼3作业8

郑子诺，物理41

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79.1

Because we have

$$f = \frac{4A}{h} = \frac{c}{\lambda}, \lambda \approx 21\text{cm}$$

then

$$A \sim 10^{-25}\text{J}$$

We also know that

$$\mu \approx \mu' \sim \frac{e\hbar}{m_e} \sim 10^{-23}\text{J}\,\text{T}^{-1}$$

(a)If $B = 10^{-5}\text{Gs} = 10^{-9}\text{T}$, we have $\mu B \ll A$, then

$$E_I = A + \mu B, E_{II} = A - \mu B, E_{III} = A$$

and we have

$$f_1 = \frac{\mu B}{h}, \lambda_1 = \frac{hc}{\mu B}$$
$$f_2 = \frac{\mu B}{h}, \lambda_2 = \frac{hc}{\mu B}$$
$$f_3 = \frac{2\mu B}{h}, \lambda_3 = \frac{hc}{2\mu B}$$

(b)If $B = 0.5\text{Gs} = 0.5 \times 10^{-4}\text{T}$, we also have $\mu B \ll A$, then we have the same result:

$$f_1 = \frac{\mu B}{h}, \lambda_1 = \frac{hc}{\mu B}$$
$$f_2 = \frac{\mu B}{h}, \lambda_2 = \frac{hc}{\mu B}$$
$$f_3 = \frac{2\mu B}{h}, \lambda_3 = \frac{hc}{2\mu B}$$

(c)If $B = 10^5\text{Gs} = 10\text{T}$, we have $\mu B \gg A$, thus

$$E_I = A + \mu B, E_{II} = A - \mu B, E_{III} = -A + \mu' B \approx -A + \mu B$$

and we have

$$f_1 = \frac{2A}{h}, \lambda_1 = \frac{hc}{2A}$$
$$f_2 = \frac{2\mu B}{h}, \lambda_2 = \frac{hc}{2\mu B}$$
$$f_3 = \frac{2\mu B - 2A}{h}, \lambda_3 = \frac{hc}{2\mu B - 2A}$$

80.1

(a)The Schrodinger equations are

$$i\hbar \frac{\mathrm{d}C_{n,i}}{\mathrm{d}t} = E_i C_{n,i} + AC_{n+1,j} + AC_{n-1,j} + BC_{n+1,i} + BC_{n-1,i}$$
$$i\hbar \frac{\mathrm{d}C_{n,j}}{\mathrm{d}t} = E_i C_{n,j} + AC_{n+1,i} + AC_{n-1,i} + BC_{n+1,j} + BC_{n-1,j}$$

Let

$$C_{n.i} = C_i e^{ikx_n - i\frac{E}{\hbar}t}, C_{n.j} = C_j e^{ikx_n - i\frac{E}{\hbar}t}$$

we have

$$\frac{E - E_i - 2B\cos kb}{2A\cos kb} = \frac{2A\cos kb}{E - E_j - 2B\cos kb}$$

Thus

$$E = \frac{E_i + E_j + 4B\cos kb \pm \sqrt{(E_i - E_j)^2 + 16A^2\cos kb}}{2}$$

(b)If $|E_i - E_j| \ll 2B$, we have

$$E \approx \frac{E_i + E_j + 4(B \pm A)\cos kb}{2}$$

Thus

$$E_1 = \frac{E_i + E_j}{2} + 2(B + A)\cos kb, E_2 = \frac{E_i + E_j}{2} + 2(B - A)\cos kb$$

Just like two energy bands which have the same center and have the width $4(B + A), 4(B - A)$ respectively.

If $|E_i - E_j| \gg 2B$, we have

$$E \approx \frac{E_i + E_j + 4B\cos kb \pm (E_i - E_j)}{2}$$

Thus

$$E_1 = E_i + 2B\cos kb, E_2 = E_j + 2B\cos kb$$

Just like two energy bands which have the same width $4B$ and have the center E_i, E_j respectively. Each is the same as energy band with just one state on an atom.

80.3

$$\begin{aligned}
i\hbar\frac{\mathrm{d}C_{-1}}{\mathrm{d}t} &= E_0 C_{-1} - AC_{-2} - BC_0 \\
i\hbar\frac{\mathrm{d}C_0}{\mathrm{d}t} &= E_0 C_0 - BC_{-1} - BC_1 \\
i\hbar\frac{\mathrm{d}C_1}{\mathrm{d}t} &= E_0 C_1 - AC_2 - BC_0
\end{aligned}$$

Let

$$\begin{aligned}
&C_n = e^{ikx_n - i\frac{E}{\hbar}t} + \beta e^{-ikx_n - i\frac{E}{\hbar}t}, n < 0 \\
&C_n = \gamma e^{ikx_n - i\frac{E}{\hbar}t}, n > 0 \\
&C_0 = ae^{-i\frac{E}{\hbar}t}
\end{aligned}$$

which satisfy the other equations when $E = E_0 - 2A\cos kb$. Use the three equations above, we have

$$\begin{aligned}
a &= \frac{iAB\sin kb}{B^2 e^{ikb} - A^2\cos kb} \\
\beta &= \frac{(A^2 - B^2)\cos kb}{B^2 e^{ikb} - A^2\cos kb} \\
\gamma &= \frac{iB^2\sin kb}{B^2 e^{ikb} - A^2\cos kb}
\end{aligned}$$

(b)

$$|\beta|^2 = \frac{(A^2 - B^2)^2\cos^2 kb}{(A^2 - B^2)^2\cos^2 kb + B^4\sin^2 kb}$$

$$|\gamma|^2 = \frac{B^4\sin^2 kb}{(A^2 - B^2)^2\cos^2 kb + B^4\sin^2 kb}$$

Obviously we have

$$|\beta|^2 + |\gamma|^2 = 1$$