费曼3作业9

郑子诺，物理41

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82.1

(a)Neglect the interaction between the electrons, then we just need solve the one-electron Schrodinger equations, where the matrix of Hamiltonian is

$$\begin{bmatrix} E_0 & -A & 0 & 0 \\ -A & E_0 & -A & 0 \\ 0 & -A & E_0 & -A \\ 0 & 0 & -A & E_0 \end{bmatrix}$$

Solve the characteristic equation we have

$$E = E_0 \pm \sqrt{\frac{3+\sqrt{5}}{2}}A, E_0 \pm \sqrt{\frac{3-\sqrt{5}}{2}}A$$

The energy gap between the first excited state and the ground state is just $2\sqrt{\frac{3-\sqrt{5}}{2}}A \approx 1.236\text{eV}$, and therefore the wavelength λ is

$$\lambda \approx 1003\text{nm}$$

(b)Solve the equations, we know that the distribution is just $C \sin kx$. The two low energies correspond to $k = \frac{\pi}{5}, \frac{2\pi}{5}$. Thus Two electrons will have the same distribution which spreads like a wave packet, and the rest electron will spread symmetrically about the center of the molecule.

83.1

(a)If not, the left side of the Schrodinger equation

$$-\frac{\hbar^2}{2m}\frac{\mathrm{d}^2 u_0(x)}{\mathrm{d}x^2} + V_0 u_0(x) = E u_0(x)$$

will diverge. Hence we need $u_0(x) = 0$ for $x < 0, x > a$.

(b)We have

$$u(x) = C_1 \cos kx + C_2 \sin kx, k = \sqrt{\frac{2mE}{\hbar^2}}$$

according to the condition above and the normalization condition, we have

$$u(x) = \sqrt{\frac{2}{a}} \sin kx, k = \frac{n\pi}{a}, n = 1, 2, \ldots$$

(c)

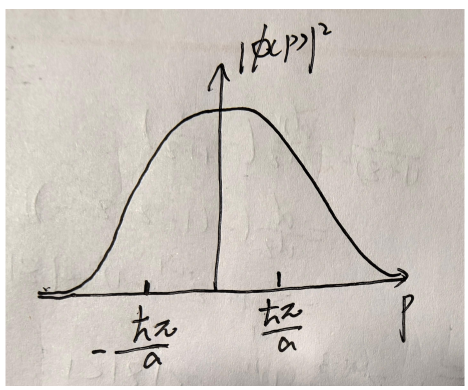
$$E_0 = \frac{\pi^2\hbar^2}{2ma^2}, u_0 = \sqrt{\frac{2}{a}} \sin \frac{\pi}{a} x$$

$u_0(x)$ is just like a wave packet.

(d)

$$\Delta E = E_1 - E_0 = \frac{3\pi^2\hbar^2}{2ma^2}$$

(e)



83.2

(a)Solve the Schrodinger equation, we have

$$\psi(x) = Ae^{\beta x} + B^{-\beta x}, x > a$$
$$\psi(x) = C_1 \cos \alpha x + C_2 \sin \alpha x, -a < x < a$$
$$\psi(x) = Ce^{\beta x} + De^{-\beta x}$$

According to the boundary conditions, we have

$$A = D = 0$$

$$C_1 \cos \alpha a + C_2 \sin \alpha a = Be^{-\beta a}, -\alpha C_1 \sin \alpha a + \alpha C_2 \cos \alpha a = -\beta B e^{-\beta a}$$

$$C_1 \cos \alpha a - C_2 \sin \alpha a = Ce^{-\beta a}, \alpha C_1 \sin \alpha a + \alpha C_2 \cos \alpha a = \beta C e^{-\beta a}$$

Then we have

$$\alpha \tan \alpha a = \beta$$

or

$$\alpha \cot \alpha a = -\beta$$

(b) The equation now is

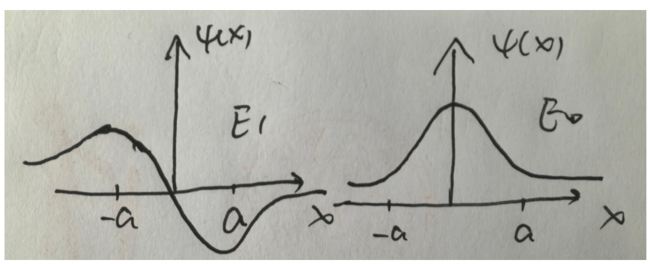
$$y \tan y = \sqrt{4 - y^2}, y = \alpha a$$

or

$$y \cot y = -\sqrt{4 - y^2}$$

Therefore we have

$$E_0 \approx 1.03 \frac{\hbar^2}{2ma}, E_1 \approx 1.90 \frac{\hbar^2}{2ma}$$



(c)Only one. Because $\sqrt{\frac{1}{4} - y^2}$ will be zero before $\frac{\pi}{2}$.