费曼3作业5

郑子诺，物理41

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75.2

$$C' = R_y(\frac{\pi}{4})C$$
$$\begin{bmatrix} \cos\frac{\pi}{8} & \sin\frac{\pi}{8} \\ -\sin\frac{\pi}{8} & \cos\frac{\pi}{8} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\frac{\pi}{8} \\ -\sin\frac{\pi}{8} \end{bmatrix}$$
$$\therefore |\phi(t)\rangle = \cos\frac{\pi}{8} e^{i\frac{\mu B}{\hbar}t} |+'\rangle - \sin\frac{\pi}{8} e^{-i\frac{\mu B}{\hbar}t} |-'\rangle$$

$$\begin{aligned} P_{+x} &= |\langle +x|+'\rangle + \langle +x|-'\rangle|^2 \\ &= |\cos^2\frac{\pi}{8} e^{i\frac{\mu B}{\hbar}t} - \sin^2\frac{\pi}{8} e^{-i\frac{\mu B}{\hbar}t}|^2 \\ &= |\cos\frac{\pi}{4}\cos(\frac{\mu B}{\hbar}t) + i\sin(\frac{\mu B}{\hbar}t)|^2 \\ &= \frac{1}{2}(1+\sin^2(\frac{\mu B}{\hbar}t)) \end{aligned}$$

$$\because C_y = R_z(\frac{\pi}{2})C_x \to C_{+y} = e^{i\frac{\pi}{4}} C_{+x}$$
$$\therefore P_{+y} = P_{+x} = \frac{1}{2}(1+\sin^2(\frac{\mu B}{\hbar}t))$$

75.3

(a)According to the condition, we have

$$\frac{dC_+}{dt} = i\frac{A}{\hbar}C_-$$
$$\frac{dC_-}{dt} = i\frac{A}{\hbar}C_+$$

Thus we have

$$\frac{d^2C_+}{dt^2} + \frac{A^2}{\hbar^2}C_+ = 0$$

and the initial condition

$$C_+(0) = 1, \frac{dC_+}{dt}(0) = 0$$

$$\therefore P_+ = |C_+(t)|^2 = \cos^2(\frac{A}{\hbar}t)$$

(b)We can find the eigenvectors of H

$$\begin{vmatrix} -\lambda & -A \\ -A & -\lambda \end{vmatrix} = 0 \rightarrow \lambda = A, -A$$

$$\alpha_1 = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}, \alpha_2 = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix}$$

Thus we have the stationary states

$$|\phi_1\rangle = \frac{1}{\sqrt{2}} |+\rangle + \frac{1}{\sqrt{2}} |-\rangle$$

$$|\phi_2\rangle = \frac{1}{\sqrt{2}} |+\rangle - \frac{1}{\sqrt{2}} |-\rangle$$

and the new Hamiltonian matrix

$$H' = \begin{bmatrix} A & 0 \\ 0 & -A \end{bmatrix}$$

Hence the energies of the two states are

$$E_1 = A, E_2 = -A$$

(c)For an arbitrary direction,we have

$$|+\rangle = \cos\frac{\theta}{2} e^{i\frac{\phi}{2}} |+'\rangle - i\sin\frac{\theta}{2} e^{i\frac{\phi}{2}} |-'\rangle$$

$$|-\rangle = \sin\frac{\theta}{2} e^{-i\frac{\phi}{2}} |+'\rangle + i\cos\frac{\theta}{2} e^{-i\frac{\phi}{2}} |-'\rangle$$

which can be acquired by multiple $R_z(\phi - \frac{\pi}{2})$ and $R_x(\theta)$.
Thus we have

$$C_{+'} = \cos(\frac{A}{\hbar}t)\cos\frac{\theta}{2} e^{i\frac{\phi}{2}} + i\sin(\frac{A}{\hbar}t)\sin\frac{\theta}{2} e^{-i\frac{\phi}{2}}$$

$$P_{+'} = |C_{+'}|^2 = \sin^2\frac{\theta}{2} + \cos^2(\frac{A}{\hbar}t)\cos\theta + \sin(\frac{A}{\hbar}t)\cos(\frac{A}{\hbar}t)\sin\theta\sin\phi = 1$$

$$\cos 2(\frac{A}{\hbar}t)\cos\theta + \sin 2(\frac{A}{\hbar}t)\sin\theta\sin\phi = 1$$

$$\sqrt{\cos^2\theta + \sin^2\theta\sin^2\phi}\cos(2\frac{A}{\hbar}t - \delta) = 1, \tan\delta = \frac{\sin\theta\sin\phi}{\cos\theta}$$

$$\therefore \phi = \frac{\pi}{2}, \theta = 2\frac{A}{\hbar}t + 2k\pi$$

Another method:

$$\cos\frac{\theta}{2}e^{-i\frac{\phi}{2}} = c\cos\frac{A}{\hbar}t$$

$$\sin\frac{\theta}{2}e^{i\frac{\phi}{2}} = ci\sin\frac{A}{\hbar}t$$

$$\therefore \phi = \frac{\pi}{2}, \theta = 2\frac{A}{\hbar}t + 2k\pi$$

(d)An equipment with x-axis uniform magnetic field.

76.1

(a)The probability of transmission is

$$P'(T) = 4\pi^2(\frac{\mu^2}{4\pi\epsilon_0\hbar^2 c})\mathcal{J}(\omega_0)T$$

Thus the probability per unit time is

$$P(I \to II) = 4\pi^2(\frac{\mu^2}{4\pi\epsilon_0\hbar^2 c})\mathcal{J}(\omega_0)$$

(b)Because the equations are symmetric, we immediately have

$$P(II \to I) = 4\pi^2(\frac{\mu^2}{4\pi\epsilon_0\hbar^2 c})\mathcal{J}(\omega_0)$$

(c)We have

$$B_{I\to II} = B_{II\to I} = 4\pi^2(\frac{\mu^2}{4\pi\epsilon_0\hbar^2 c})$$

(d)

$$A_{I\to II} = 4\pi^2(\frac{\mu^2}{4\pi\epsilon_0\hbar^2 c})\frac{\hbar\omega_0^3}{\pi^2 c^2}$$