费曼3作业10

郑子诺，物理41

2024 年 12 月 4 日

80.4

(a)Suppose $\beta = a + ib$, use $|\beta|^2 + |1+\beta|^2 = 1$ we have

$$b^2 = -a(1+a)$$

and we also have

$$\frac{\beta}{1+\beta} = \frac{a+ib}{1+a+ib} = \frac{a(1+a)+b^2+ib}{(1+a)^2+b^2}$$

and therefore

$$\mathrm{Re}\{\frac{\beta}{1+\beta}\} = \frac{a(1+a)+b^2}{(1+a)^2+b^2} = 0$$

(b)Let $\frac{\beta}{1+\beta} = i\tan\eta$, we have

$$\beta = \frac{i\tan\eta}{1-i\tan\eta} = \frac{i\sin\eta}{\cos\eta - i\sin\eta} = i(\cos\eta + \sin\eta)\sin\eta = ie^{i\eta}\sin\eta$$

83.4

(a)Use the Schrodinger equation, we have

$$i\hbar(-\frac{\mathrm{d}a}{\mathrm{d}t}x^2 - \frac{\mathrm{d}c}{\mathrm{d}t}) = -\frac{\hbar^2}{2m}(4a^2x^2 - 2a)$$

and then

$$(i\hbar\frac{\mathrm{d}a}{\mathrm{d}t} - \frac{2\hbar^2}{m}a^2)x^2 + i\hbar\frac{\mathrm{d}c}{\mathrm{d}t} + \frac{\hbar^2}{m}a = 0$$

Because this equation should be correct for all x, we have

$$\frac{\mathrm{d}a}{\mathrm{d}t} = -\frac{2i\hbar}{m}a^2$$

and

$$\frac{1}{a(t)} = \frac{1}{a_0} + \frac{2i\hbar}{m}t$$

(b)Use the equation above, we also have

$$c(t) = \frac{1}{2}\ln(1 + \frac{2i\hbar a_0}{m}t) + c_0$$

(c)We have

$$\mathrm{Re}\{a(t)\} = \frac{a_0}{1 + \frac{4\hbar^2 a_0^2}{m^2}t^2}$$

Let $\dfrac{1}{\sqrt{a_0}} = 1\text{Å}, t = 1\text{s}$, we have

$$d = \frac{1}{\sqrt{\mathrm{Re}\{a(t)\}}} = 2.3 \times 10^6\mathrm{m}$$

(d)We have

$$\phi(p) = \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi\hbar}} e^{-i\frac{p}{\hbar}x}\psi(x)\mathrm{d}x = \frac{K}{\sqrt{2a\hbar}} e^{-(\frac{p^2}{4a\hbar^2}+c)}$$

(e)We have

$$\Delta p = \sqrt{4\mathrm{Re}\{a(t)\}\hbar^2} = \sqrt{\frac{4a_0\hbar^2}{1 + \frac{4\hbar^2 a_0^2}{m^2}t^2}}$$

(f)Obviously we have

$$\frac{\mathrm{d}\frac{1}{\sqrt{a(t)}}}{\mathrm{d}t} = i\hbar\sqrt{a(t)}$$

Then

$$2|i\hbar\sqrt{a(t)}| = 2\sqrt{|a(t)|}\hbar = \Delta p$$