费曼3作业6

郑子诺，物理41

2024 年 11 月 8 日

77.1

$$i\hbar\dot{a} = -\mu B\cos\theta a - \mu B\sin\theta e^{i\omega t} b$$
$$i\hbar\dot{b} = -\mu B\sin\theta e^{-i\omega t} a + \mu B\cos\theta b$$

The zero order:

$$i\hbar\dot{a_0} = -\mu B a_0$$
$$i\hbar\dot{b_0} = \mu B b_0$$

we have

$$a_0 = e^{i\frac{\mu B}{\hbar}t}, b_0 = 0$$

Then we consider the first order. We notice that the first equation has no first order.

$$i\hbar\dot{b} = -\mu B\sin\theta e^{-i\omega t} e^{i\frac{\mu B}{\hbar}t} + \mu B b$$

we have

$$b = \frac{\frac{\mu B}{\hbar}\sin\theta}{\frac{2\mu B}{\hbar} - \omega}(e^{i(\frac{\mu B}{\hbar}-\omega)t} - e^{-i\frac{\mu B}{\hbar}t})$$

Obviously, ω must be $\dfrac{2\mu B}{\hbar}$ for resonance. Then we have

$$b = i\frac{\mu B}{\hbar} t e^{-i\frac{\mu B}{\hbar}t}\sin\theta$$

$$P_{-z} = |b|^2 = \frac{\mu^2 B^2}{\hbar^2} t^2 \sin^2\theta$$

77.2

Like 77.1, let z-axis along B_0, we have

$$a_0 = 0, b_0 = e^{-i\frac{\mu B_0}{\hbar}t}$$

$$i\hbar\dot{a} = -\mu B_n(e^{i\omega t} + e^{-i\omega t})e^{i\frac{\mu B_0}{\hbar}t} - \mu B_0 a$$

If $\omega \neq \frac{2\mu B_0}{\hbar}, -\frac{2\mu B_0}{\hbar}$, we have

$$a = \frac{\frac{\mu B_n}{\hbar}}{\omega - \frac{2\mu B_0}{\hbar}}e^{i(\omega-\frac{\mu B_0}{\hbar})t} - \frac{\frac{\mu B_n}{\hbar}}{\omega + \frac{2\mu B_0}{\hbar}}e^{-i(\omega+\frac{\mu B_0}{\hbar})t} - \frac{\frac{4\mu^2 B_0 B_n}{\hbar^2}}{\omega^2 - \frac{4\mu^2 B_0^2}{\hbar^2}}e^{i\frac{\mu B_0}{\hbar}t}$$

$$\begin{aligned}
P_{||} &= |a|^2 \\
&= (\frac{\frac{\mu B_n}{\hbar}}{\omega - \frac{2\mu B_0}{\hbar}})^2 + (\frac{\frac{\mu B_n}{\hbar}}{\omega + \frac{2\mu B_0}{\hbar}})^2 + (\frac{\frac{4\mu^2 B_0 B_n}{\hbar^2}}{\omega^2 - \frac{4\mu^2 B_0^2}{\hbar^2}})^2 \\
&- \frac{\frac{2\mu^2 B_n^2}{\hbar^2}}{\omega^2 - \frac{4\mu^2 B_0^2}{\hbar^2}}\cos 2\omega t - 2\frac{\frac{\mu B_n}{\hbar}}{\omega - \frac{2\mu B_0}{\hbar}}\frac{\frac{4\mu^2 B_0 B_n}{\hbar^2}}{\omega^2 - \frac{4\mu^2 B_0^2}{\hbar^2}}\cos(\omega - \frac{2\mu B_0}{\hbar})t \\
&+ 2\frac{\frac{\mu B_n}{\hbar}}{\omega + \frac{2\mu B_0}{\hbar}}\frac{\frac{4\mu^2 B_0 B_n}{\hbar^2}}{\omega^2 - \frac{4\mu^2 B_0^2}{\hbar^2}}\cos(\omega + \frac{2\mu B_0}{\hbar})t
\end{aligned}$$

If $\omega = \frac{2\mu B_0}{\hbar}$ or $-\frac{2\mu B_0}{\hbar}$, we have

$$a \approx i\frac{\mu B_n}{\hbar}te^{i\frac{\mu B_0}{\hbar}t}$$

$$P_{||} = |a|^2 = \frac{\mu^2 B_n^2}{\hbar^2}t^2$$