费曼3作业4

郑子诺，物理41

2024 年 10 月 26 日

74.1

(a)

$$V = +\mu B, 0, -\mu B$$

$$\Delta(\frac{p_0^2}{2M}) \approx \frac{p_0 \Delta p_0}{M} = -\Delta V$$

$$\Delta\phi = \Delta p_0 L = -\frac{ML}{p_0}\Delta V = p_y \Delta z$$

$$\therefore \delta\theta \approx \frac{p_y}{p_0} = -\frac{ML}{p_0^2}\frac{\Delta V}{\Delta z} \approx -\frac{ML}{p_0^2}\frac{\partial V}{\partial z}$$

For different states,we have

$$\delta\theta = -\frac{ML}{p_0^2}\frac{\mu\partial B}{\partial z}, 0, +\frac{ML}{p_0^2}\frac{\mu\partial B}{\partial z}$$

(b)

$$\begin{aligned}
|{+x(t)}\rangle &= \langle +z|+x\rangle\,|{+z(t)}\rangle + \langle 0z|+x\rangle\,|0z(t)\rangle + \langle -z|+x\rangle\,|{-z(t)}\rangle \\
&= \frac{1}{2}e^{-i\frac{\mu B}{\hbar}t}\,|{+z}\rangle - \frac{1}{\sqrt{2}}\,|0z\rangle + \frac{1}{2}e^{i\frac{\mu B}{\hbar}t}\,|{-z}\rangle
\end{aligned}$$

$$|0x(t)\rangle = \frac{1}{\sqrt{2}}e^{-i\frac{\mu B}{\hbar}t}\,|{+z}\rangle - \frac{1}{\sqrt{2}}e^{i\frac{\mu B}{\hbar}t}\,|{-z}\rangle$$

$$|{-x(t)}\rangle = \frac{1}{2}e^{-i\frac{\mu B}{\hbar}t}\,|{+z}\rangle + \frac{1}{\sqrt{2}}\,|0z\rangle + \frac{1}{2}e^{i\frac{\mu B}{\hbar}t}\,|{-z}\rangle$$

$$\begin{aligned}
\langle +x|+x(t)\rangle &= \langle +x|+z(t)\rangle\,\langle +z|+x\rangle + \langle +x|0z(t)\rangle\,\langle 0z|+x\rangle + \langle +x|-z(t)\rangle\,\langle -z|+x\rangle \\
&= \frac{1}{2}(\cos(\frac{\mu B}{\hbar}t) + 1)
\end{aligned}$$

$$\langle 0x|0x(t)\rangle = \cos(\frac{\mu B}{\hbar}t)$$
$$\langle -x|-x(t)\rangle = \frac{1}{2}(\cos(\frac{\mu B}{\hbar}t)+1)$$

(c) Method 1:Measure the deflection of a beam of particles that passed through an inhomogeneous magnetic field,then we have

$$\mu = \frac{p_0^2}{ML\frac{\partial B}{\partial z}}\delta\theta$$

Method 2:Measure the oscillation angular frequency of the probability of detecting a particle that has a spin on $+x$ direction,which is put in a uniform magnetic field with $+z$ direction.Then we have

$$\mu = \frac{\hbar}{B}\omega$$

73.3

(a)

$$\begin{aligned} N &= N_0|\langle +U|+T\rangle\langle +T|-S\rangle + \langle +U|-T\rangle\langle -T|-S\rangle|^2 \\ &= N_0|\langle +U|-S\rangle|^2 \end{aligned}$$

(b)

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$
$$\therefore \langle +T|-S\rangle = 1, \langle -T|-S\rangle = 0$$

(c)

$$\begin{bmatrix} C_1' \\ C_2' \end{bmatrix} = R_y(-\frac{\pi}{2})R_z(\theta)R_y(\frac{\pi}{2})\begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} \cos\frac{\theta}{2} & i\sin\frac{\theta}{2} \\ i\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix}\begin{bmatrix} C_1 \\ C_2 \end{bmatrix}$$
$$\therefore \langle +U|-S\rangle = i\sin\frac{\theta}{2}$$

(d)

$$\theta = 0, \langle +U|-S\rangle = 0$$
$$\theta = \pi, \langle +U|-S\rangle = i$$

(e) Because U need a rotation along z-axis to become T.

$$\begin{bmatrix} -i & 0 \\ 0 & i \end{bmatrix}\begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

73.6

(a)

$$P_{+z} = |\sqrt{1-\frac{v}{c}}\sin\frac{\theta}{2}|^2 = (1-\frac{v}{c})\sin^2\frac{\theta}{2}$$

$$P_{-z} = (1+\frac{v}{c})\cos^2\frac{\theta}{2}$$

(b)

$$\begin{aligned} P_{+x} &= |\frac{1}{\sqrt{2}}\sqrt{1-\frac{v}{c}}\sin\frac{\theta}{2} + \frac{1}{\sqrt{2}}\sqrt{1+\frac{v}{c}}\cos\frac{\theta}{2}|^2 \\ &= \frac{1}{2}(1+\frac{v}{c}\cos\theta + \sqrt{1-\frac{v^2}{c^2}}\sin\theta) \end{aligned}$$

$$P_{-x} = \frac{1}{2}(1+\frac{v}{c}\cos\theta - \sqrt{1-\frac{v^2}{c^2}}\sin\theta)$$

(c)

$$\begin{aligned} P_{+y} &= |\frac{1}{\sqrt{2}}\sqrt{1-\frac{v}{c}}\sin\frac{\theta}{2} - i\frac{1}{\sqrt{2}}\sqrt{1+\frac{v}{c}}\cos\frac{\theta}{2}|^2 \\ &= \frac{1}{2}(1+\frac{v}{c}\cos\theta) \end{aligned}$$

$$P_{-y} = \frac{1}{2}(1+\frac{v}{c}\cos\theta)$$

(d)

$$P_{+z} = \int_0^\pi \frac{1}{2}\sin\theta(1-\frac{v}{c})\sin^2\frac{\theta}{2}d\theta = \frac{1}{2}(1-\frac{v}{c})$$

$$P_{-z} = \frac{1}{2}(1+\frac{v}{c})$$