

# Competitive Programming Notebook

Programadores Roblox

## Contents

<b>1 String</b>	<b>2</b>	6.7 Acha Pontes . . . . .	10
1.1 Trie . . . . .	2	6.8 Edmonds-karp . . . . .	10
1.2 Hashing . . . . .	2	6.9 Kruskal . . . . .	10
1.3 Z Function . . . . .	2	6.10 Bellman Ford . . . . .	10
1.4 Kmp . . . . .	2	6.11 Lca Jc . . . . .	11
<b>2 DS</b>	<b>2</b>	6.12 Lca . . . . .	11
2.1 Seg Lazy Pa . . . . .	2	6.13 Kosaraju . . . . .	11
2.2 Segtree Iterativa . . . . .	3	6.14 Min Cost Max Flow . . . . .	12
2.3 Merge Sort Tree . . . . .	3	6.15 Diametro . . . . .	12
2.4 Ordered Set E Map . . . . .	3	<b>7 Primitives</b>	<b>12</b>
2.5 Sparse Table . . . . .	4	<b>8 DP</b>	<b>12</b>
2.6 Psum 2d . . . . .	4	8.1 Lis . . . . .	12
2.7 Segtree Lazy . . . . .	4	8.2 Edit Distance . . . . .	13
2.8 Min Queue . . . . .	4	8.3 Bitmask . . . . .	13
2.9 Dsu . . . . .	4	8.4 Lcs . . . . .	13
2.10 Bit . . . . .	5	8.5 Digit . . . . .	13
<b>3 Search and sort</b>	<b>5</b>	8.6 Knapsack . . . . .	13
3.1 Merge Sort . . . . .	5	8.7 Lis Seg . . . . .	13
3.2 Monotonic Stack . . . . .	5	8.8 Disjoint Blocks . . . . .	13
<b>4 Stress</b>	<b>5</b>	<b>9 General</b>	<b>14</b>
4.1 Gen . . . . .	5	9.1 Brute Choose . . . . .	14
<b>5 Math</b>	<b>5</b>	9.2 Struct . . . . .	14
5.1 Combinatorics . . . . .	5	9.3 Mex . . . . .	14
5.2 Equacao Diofantina . . . . .	6	9.4 Count Permutations . . . . .	14
5.3 Multiplicacao Matriz . . . . .	6	9.5 Bitwise . . . . .	14
5.4 Discrete Log . . . . .	6	<b>10 Geometry</b>	<b>15</b>
5.5 Segment Sieve . . . . .	6	10.1 Convex Hull . . . . .	15
5.6 Totient . . . . .	6	10.2 Inside Polygon . . . . .	15
5.7 Menor Fator Primo . . . . .	7	10.3 Point Location . . . . .	16
5.8 Exgcd . . . . .	7	10.4 Lattice Points . . . . .	16
5.9 Fexp . . . . .	7		
5.10 Divisores . . . . .	7		
5.11 Crivo . . . . .	7		
5.12 Mod Inverse . . . . .	7		
5.13 Base Calc . . . . .	7		
5.14 Fft . . . . .	8		
<b>6 Graph</b>	<b>8</b>		
6.1 Dijkstra . . . . .	8		
6.2 Floyd Warshall . . . . .	8		
6.3 Eulerian Path . . . . .	8		
6.4 Dinitz . . . . .	9		
6.5 Khan . . . . .	9		
6.6 Topological Sort . . . . .	10		

# 1 String

## 1.1 Trie

```

1 // Trie por array
2 int trie[MAXN][26];
3 int tot_nos = 0;
4 vector<bool> acaba(MAXN, false);
5 vector<int> contador(MAXN, 0);
6
7 void insere(string s) {
8     int no = 0;
9     for(auto &c : s) {
10         if(trie[no][c - 'a'] == 0) {
11             trie[no][c - 'a'] = ++tot_nos;
12         }
13         no = trie[no][c - 'a'];
14         contador[no]++;
15     }
16     acaba[no] = true;
17 }
18
19 bool busca(string s) {
20     int no = 0;
21     for(auto &c : s) {
22         if(trie[no][c - 'a'] == 0) {
23             return false;
24         }
25         no = trie[no][c - 'a'];
26     }
27     return acaba[no];
28 }
29
30 int isPref(string s) {
31     int no = 0;
32     for(auto &c : s) {
33         if(trie[no][c - 'a'] == 0) {
34             return -1;
35         }
36         no = trie[no][c - 'a'];
37     }
38     return contador[no];
39 }

```

## 1.2 Hashing

```

1 // String Hash template
2 // constructor(s) - 0(|s|)
3 // query(l, r) - returns the hash of the range [l,r] from
4 // left to right - 0(1)
5 // query_inv(l, r) from right to left - 0(1)
6 // patrocinado por tiagodfs
7
8 mt19937 rng(time(nullptr));
9
10 struct Hash {
11     const int X = rng();
12     const int MOD = 1e9+7;
13     int n; string s;
14     vector<int> h, hi, p;
15     Hash() {}
16     Hash(string s): s(s), n(s.size()), h(n), hi(n), p(n) {
17         for (int i=0;i<n;i++) p[i] = (i ? X*p[i-1]:1) % MOD;
18         for (int i=0;i<n;i++) {
19             h[i] = (s[i] + (i ? h[i-1]:0) * X) % MOD;
20             hi[i] = (s[i] + (i+1<n ? hi[i+1]:0) * X) % MOD;
21         }
22     }
23     int query(int l, int r) {
24         int hash = (h[r] - (l ? h[l-1]*p[r-l+1]:0)) % MOD;
25         return hash < 0 ? hash + MOD : hash;
26     }
27     int query_inv(int l, int r) {
28         int hash = (hi[l] - (r+1 < n ? hi[r+1]*p[r-l+1] % MOD : 0)) % MOD;
29         return hash < 0 ? hash + MOD : hash;
30     }
31 };

```

## 1.3 Z Function

```

1 vector<int> z_function(string s) {
2     int n = s.size();

```

```

3     vector<int> z(n);
4     int l = 0, r = 0;
5     for(int i = 1; i < n; i++) {
6         if(i < r) {
7             z[i] = min(r - i, z[i - l]);
8         }
9         while(i + z[i] < n && s[z[i]] == s[i + z[i]]) {
10             z[i]++;
11         }
12         if(i + z[i] > r) {
13             l = i;
14             r = i + z[i];
15         }
16     }
17     return z;
18 }

```

## 1.4 Kmp

```

1 vector<int> kmp(string &s) {
2     int m = s.size();
3     vector<int> lsp(m, 0);
4     for (int i = 1, j = 0; i < m; i++) {
5         while (j > 0 && s[j] != s[i]) j = lsp[j - 1];
6         if (s[i] == s[j]) j++;
7         lsp[i] = j;
8     }
9     return lsp;
10 }

```

# 2 DS

## 2.1 Seg Lazy Pa

```

1 /* Notas
2 PA eh da forma a0 (a) e razão (d)
3 na hora de propagar o lazy
4 aplica no no S = (a0 + (a0 + (n - 1) * d)) * n / 2 (
5 variante da soma total do no)
6 pros filhos, o da esquerda eh normal
7 lazy_a[esq] += lazy_a[x]
8 lazy_d[esq] += lazy_d[x]
9 pro da direita tem que mudar o a (que eh o elemento inicial
10 naquele no da direita)
11 lazy_a[dir] += a + len_esq * (d)
12 lazy_d[dir] += lazy_d[x]
13 */
14
15 int gauss(int n, int a, int d) {
16     // (a0 + an) * n / 2
17     if (n <= 0)
18         return 0;
19     return (a + (a + (n - 1) * d)) * n / 2;
20 }
21
22 struct SegTree {
23     int n;
24     vector<int> v, lazy_a, lazy_d, tree;
25     SegTree(vector<int> &a) : v(a), n(a.size()) {
26         tree.resize(4 * n);
27         lazy_a.resize(4 * n, 0ll);
28         lazy_d.resize(4 * n, 0ll);
29         build(1, 0, n - 1);
30     }
31     void build(int x, int lx, int rx) {
32         if (lx == rx) {
33             tree[x] = v[lx];
34             return;
35         }
36         int mid = (lx + rx) / 2;
37         build(2 * x, lx, mid);
38         build(2 * x + 1, mid + 1, rx);
39         tree[x] = tree[2 * x] + tree[2 * x + 1];
40     }
41     int query(int x, int lx, int rx, int l, int r) {
42         push(x, lx, rx);
43         if (lx >= l && rx <= r)
44             return tree[x];
45         if (lx > r || rx < l)
46             return 0;
47         int mid = (lx + rx) / 2;
48         return query(2 * x, lx, mid, l, r) + query(2 * x + 1, mid + 1, rx, l, r);

```

```

47 }
48 int query(int l, int r) {
49     return query(1, 0, n - 1, l, r);
50 }
51 void push(int x, int lx, int rx) {
52     if (lazy_a[x] && lazy_d[x]) {
53         int tam = rx - lx + 1;
54         tree[x] += gauss(tam, lazy_a[x], lazy_d[x]);
55         if (lx != rx) {
56             int mid = (lx + rx) / 2;
57             int tam_esq = mid - lx + 1;
58             lazy_a[2 * x] += lazy_a[x];
59             lazy_d[2 * x] += lazy_d[x];
60             lazy_a[2 * x + 1] += (lazy_a[x] + tam_esq * lazy_d[x]);
61             lazy_d[2 * x + 1] += lazy_d[x];
62         }
63         lazy_a[x] = lazy_d[x] = 0;
64     }
65 }
66 void update(int x, int lx, int rx, int l, int r, int a, int d) {
67     push(x, lx, rx);
68     if (lx >= l && rx <= r) {
69         lazy_a[x] += a + (lx - l) * d;
70         lazy_d[x] += d;
71         push(x, lx, rx);
72         return;
73     }
74     if (lx > r || rx < l)
75         return;
76     int mid = (lx + rx) / 2;
77     update(2 * x, lx, mid, l, r, a, d);
78     update(2 * x + 1, mid + 1, rx, l, r, a, d);
79     tree[x] = tree[2 * x] + tree[2 * x + 1];
80 }
81 void update(int l, int r, int a, int d) {
82     update(1, 0, n - 1, l, r, a, d);
83 }
84 };

```

## 2.2 Segtree Iterativa

```

1 // Exemplo de uso:
2 // SegTree<int> st(vetor);
3 // range query e point update
4 template <typename T>
5 struct SegTree {
6     int n;
7     vector<T> tree;
8     T neutral_value = 0;
9     T combine(T a, T b) {
10         return a + b;
11     }
12
13     SegTree(const vector<T>& data) {
14         n = data.size();
15         tree.resize(2 * n, neutral_value);
16
17         for (int i = 0; i < n; i++)
18             tree[n + i] = data[i];
19
20         for (int i = n - 1; i > 0; --i)
21             tree[i] = combine(tree[i * 2], tree[i * 2 + 1]);
22     }
23     T range_query(int l, int r) {
24         T res_l = neutral_value, res_r = neutral_value;
25
26         for (l += n, r += n + 1; l < r; l >>= 1, r >>= 1) {
27             if (l & 1) res_l = combine(res_l, tree[l++]);
28             if (r & 1) res_r = combine(tree[--r], res_r);
29         }
30
31         return combine(res_l, res_r);
32     }
33     void update(int pos, T new_val) {
34         tree[pos += n] = new_val;
35         for (pos >>= 1; pos > 0; pos >>= 1)
36             tree[pos] = combine(tree[2 * pos], tree[2 * pos + 1]);
37     }
38 };

```

## 2.3 Merge Sort Tree

```

1 #define all(x) x.begin(), x.end()
2
3 struct MST {
4     int n;
5     vector<vector<int>> tree;
6     MST(vector<int> &a) {
7         n = a.size();
8         tree.resize(4 * n);
9         build(1, 0, n - 1, a);
10    }
11    void build(int x, int lx, int rx, vector<int> &a) {
12        if (lx == rx) {
13            tree[x] = { a[lx] };
14            return;
15        }
16        int mid = lx + (rx - lx) / 2;
17        build(2 * x, lx, mid, a);
18        build(2 * x + 1, mid + 1, rx, a);
19        auto &L = tree[2 * x], &R = tree[2 * x + 1];
20        tree[x].resize(L.size() + R.size());
21        merge(all(L), all(R), tree[x].begin());
22    }
23    int query(int x, int lx, int rx, int l, int r, int val) {
24        if (lx > r || rx < l) return 0;
25        if (lx >= l && rx <= r) {
26            auto &v = tree[x];
27            return lower_bound(all(v), val) - v.begin();
28        }
29        int mid = lx + (rx - lx) / 2;
30        return query(2 * x, lx, mid, l, r, val) + query(2 * x + 1, mid + 1, rx, l, r, val);
31    }
32    int query(int l, int r, int val) {
33        if (l > r) return 0;
34        return query(1, 0, n - 1, l, r, val);
35    }
36 };
37
38 /* mst.query(l, r, l) retorna quantos caras distintos no
39    range
40    map<int, int> last;
41    for (int i = 0; i < n; i++) {
42        if (last.count(a[i])) {
43            esq[i] = last[a[i]];
44        } else {
45            esq[i] = -1;
46        }
47        last[a[i]] = i;
48    }
49    MST mst(esq);
50    jogar vetor de ultima aparicao na seg
51 */

```

## 2.4 Ordered Set E Map

```

1
2 #include <ext/pb_ds/assoc_container.hpp>
3 #include <ext/pb_ds/tree_policy.hpp>
4 using namespace __gnu_pbds;
5 using namespace std;
6
7 template <typename T> using ordered_multiset = tree<T,
8     null_type, less_equal<T>, rb_tree_tag,
9     tree_order_statistics_node_update>;
10 template <typename T> using o_set = tree<T, null_type, less<T>,
11     rb_tree_tag, tree_order_statistics_node_update>;
12 template <typename T, typename R> using o_map = tree<T, R,
13     less<T>, rb_tree_tag, tree_order_statistics_node_update>;
14
15 int main() {
16     int i, j, k, n, m;
17     o_set<int> st;
18     st.insert(1);
19     st.insert(2);
20     cout << *st.find_by_order(0) << endl; /// k-esimo elemento
21     cout << st.order_of_key(2) << endl; /// numero de elementos
22     /// menores que k
23     o_map<int, int> mp;
24     mp.insert({1, 10});
25     mp.insert({2, 20});
26     cout << mp.find_by_order(0) ->second << endl; /// k-esimo
27     /// elemento
28     cout << mp.order_of_key(2) << endl; /// numero de elementos
29     /// (chave) menores que k
30     return 0;
31 }

```

24 }

## 2.5 Sparse Table

```

1 // 0-index, 0(1)
2 struct SparseTable {
3     vector<vector<int>> st;
4     int max_log;
5     SparseTable(vector<int>& arr) {
6         int n = arr.size();
7         max_log = floor(log2(n)) + 1;
8         st.resize(n, vector<int>(max_log));
9         for (int i = 0; i < n; i++) {
10             st[i][0] = arr[i];
11         }
12         for (int j = 1; j < max_log; j++) {
13             for (int i = 0; i + (1 << j) <= n; i++) {
14                 st[i][j] = max(st[i][j - 1], st[i + (1 << (j
15 - 1))][j - 1]);
16             }
17         }
18     }
19     int query(int L, int R) {
20         int tamanho = R - L + 1;
21         int k = floor(log2(tamanho));
22         return max(st[L][k], st[R - (1 << k) + 1][k]);
23     };

```

## 2.6 Psum 2d

```

1 vector<vector<int>> psum(h+1, vector<int>(w+1, 0));
2 for (int i=1; i<=h; i++){
3     for (int j=1; j<=w; j++){
4         cin >> psum[i][j];
5         psum[i][j] += psum[i-1][j]+psum[i][j-1]-psum[i-1][j
6 -1];
7     }
8 }
9 // retorna a psum2d do intervalo inclusivo [(a, b), (c, d)]
10 int retangulo(int a, int b, int c, int d){
11     c = min(c, h), d = min(d, w);
12     a = max(0LL, a-1), b = max(0LL, b-1);
13     return v[c][d]-v[a][d]-v[c][b]+v[a][b];

```

## 2.7 Segtree Lazy

```

1 /*
2 Seg com double Lazy de soma e de set
3 tipo 1 - soma em range
4 tipo 2 - set em range
5 */
6 struct SegTree {
7     int n;
8     vector<int> v, tree;
9     vector<int> lazy_soma, lazy_set;
10     SegTree(vector<int> &a) : v(a), n(a.size()) {
11         tree.resize(4 * n);
12         lazy_soma.resize(4 * n, 0);
13         lazy_set.resize(4 * n, -1);
14         build(1, 0, n - 1);
15     };
16     void build(int x, int lx, int rx) {
17         if (lx == rx) {
18             tree[x] = v[lx];
19             return;
20         }
21         int mid = (lx + rx) / 2;
22         build(2 * x, lx, mid);
23         build(2 * x + 1, mid + 1, rx);
24         tree[x] = tree[2 * x] + tree[2 * x + 1];
25     }
26     void seta(int x, int lx, int rx, int val) {
27         tree[x] = val * (rx - lx + 1);
28         lazy_set[x] = val;
29         lazy_soma[x] = 0;
30     }
31     void soma(int x, int lx, int rx, int val) {
32         tree[x] += val * (rx - lx + 1);
33         if (lazy_set[x] != -1) {
34             lazy_set[x] += val;
35         } else {

```

```

36         lazy_soma[x] += val;
37     }
38 }
39 void push(int x, int lx, int rx) {
40     int mid = (lx + rx) / 2;
41     if (lazy_set[x] != -1) {
42         if (lx != rx) {
43             seta(2 * x, lx, mid, lazy_set[x]);
44             seta(2 * x + 1, mid + 1, rx, lazy_set[x]);
45         }
46         lazy_set[x] = -1;
47     }
48     if (lazy_soma[x] != 0) {
49         if (lx != rx) {
50             soma(2 * x, lx, mid, lazy_soma[x]);
51             soma(2 * x + 1, mid + 1, rx, lazy_soma[x]);
52         }
53         lazy_soma[x] = 0;
54     }
55 }
56 void update(int x, int lx, int rx, int l, int r, int val,
57 int type) {
58     push(x, lx, rx);
59     if (lx >= l && rx <= r) {
60         if (type == 1) soma(x, lx, rx, val);
61         else seta(x, lx, rx, val);
62         push(x, lx, rx);
63         return;
64     }
65     int mid = (lx + rx) / 2;
66     update(2 * x, lx, mid, l, r, val, type);
67     update(2 * x + 1, mid + 1, rx, l, r, val, type);
68     tree[x] = tree[2 * x] + tree[2 * x + 1];
69 }
70 void update(int l, int r, int val, int type) {
71     update(1, 0, n - 1, l, r, val, type);
72 }
73 int query(int x, int lx, int rx, int l, int r) {
74     push(x, lx, rx);
75     if (lx >= l && rx <= r) return tree[x];
76     if (lx > r || rx < l) return 0;
77     int mid = (lx + rx) / 2;
78     return query(2 * x, lx, mid, l, r) + query(2 * x + 1, mid
79 + 1, rx, l, r);
80 }
81 int query(int l, int r) {
82     return query(1, 0, n - 1, l, r);
83 };

```

## 2.8 Min Queue

```

1 struct MinQueue {
2     deque<pair<int, int>> dq;
3     void push(int val, int idx) {
4         while (!dq.empty() && dq.back().first >= val) dq.
5             pop_back();
6         dq.emplace_back(val, idx);
7     }
8     void pop(int idx) {
9         if (!dq.empty() && dq.front().second == idx) {
10             dq.pop_front();
11         }
12     }
13     int get() {
14         return dq.front().first;
15     }
16     bool empty() {
17         return dq.empty();
18     }
19 };
20 /*
21 considerando janela de tamanho K
22 mq.push(v[i], i);
23 if (i >= k) {
24     mq.pop(i - k);
25 }
26 if (i >= k - 1) {
27     result.push_back(mq.get());
28 }
29 */

```

## 2.9 Dsu

```

1 struct DSU {
2     vector<int> par, rank, sz;
3     int c;
4     DSU(int n) : par(n + 1), rank(n + 1, 0), sz(n + 1, 1), c(
5         n) {
6         for (int i = 1; i <= n; ++i) par[i] = i;
7     }
8     int find(int i) {
9         return (par[i] == i ? i : (par[i] = find(par[i])));
10    }
11    bool same(int i, int j) {
12        return find(i) == find(j);
13    }
14    int get_size(int i) {
15        return sz[find(i)];
16    }
17    int count() {
18        return c; // quantos componentes conexos
19    }
20    int merge(int i, int j) {
21        if ((i = find(i)) == (j = find(j))) return -1;
22        else --c;
23        if (rank[i] > rank[j]) swap(i, j);
24        par[i] = j;
25        sz[j] += sz[i];
26        if (rank[i] == rank[j]) rank[j]++;
27        return j;
28    };

```

## 2.10 Bit

```

1 struct BIT {
2     int n;
3     vector<int> bit;
4     BIT(int n = 0) : n(n), bit(n + 1, 0) {}
5     void add(int i, int delta) {
6         for (; i <= n; i += i & -i) bit[i] += delta;
7     }
8     int sum(int i) {
9         int r = 0;
10        for (; i > 0; i -= i & -i) r += bit[i];
11        return r;
12    }
13    int range_sum(int l, int r) {
14        if (r < l) return 0;
15        return sum(r) - sum(l - 1);
16    }
17 };

```

# 3 Search and sort

## 3.1 Merge Sort

```

1 int mergeAndCount(vector<int>& v, int l, int m, int r) {
2     int x = m - l + 1;
3     int y = r - m;
4     vector<int> left(x), right(y);
5     for (int i = 0; i < x; i++) left[i] = v[l + i];
6     for (int j = 0; j < y; j++) right[j] = v[m + 1 + j];
7     int i = 0, j = 0, k = l;
8     int swaps = 0;
9     while (i < x && j < y) {
10        if (left[i] <= right[j]) {
11            v[k++] = left[i++];
12        } else {
13            v[k++] = right[j++];
14            swaps += (x - i);
15        }
16    }
17    while (i < x) v[k++] = left[i++];
18    while (j < y) v[k++] = right[j++];
19    return swaps;
20 }
21
22 int mergeSort(vector<int>& v, int l, int r) {
23     int swaps = 0;
24     if (l < r) {
25         int m = l + (r - l) / 2;
26         swaps += mergeSort(v, l, m);
27         swaps += mergeSort(v, m + 1, r);
28         swaps += mergeAndCount(v, l, m, r);
29     }

```

```

30     return swaps;
31 }

```

## 3.2 Monotonic Stack

```

1 vector<int> find_esq(vector<int> &v, bool maior) {
2     int n = v.size();
3     vector<int> result(n);
4     stack<int> s;
5
6     for (int i = 0; i < n; i++) {
7         while (!s.empty() && (maior ? v[s.top()] <= v[i] : v[
8             s.top()] >= v[i])) {
9             s.pop();
10        }
11        if (s.empty()) {
12            result[i] = -1;
13        } else {
14            result[i] = v[s.top()];
15        }
16        s.push(i);
17    }
18    return result;
19 }
20
21 vector<int> find_dir(vector<int> &v, bool maior) {
22     int n = v.size();
23     vector<int> result(n);
24     stack<int> s;
25     for (int i = n - 1; i >= 0; i--) {
26         while (!s.empty() && (maior ? v[s.top()] <= v[i] : v[
27             s.top()] >= v[i])) {
28             s.pop();
29        }
30        if (s.empty()) {
31            result[i] = -1;
32        } else {
33            result[i] = v[s.top()];
34        }
35        s.push(i);
36    }
37    return result;
38 }

```

## 4 Stress

### 4.1 Gen

```

1 // pre-compilar os headers:
2 // compilar template com g++ -H e procurar onde esta stdc++.h
3 // criar a pasta bits e incluir com "" ou invés de <>
4 // compilar stress com: g++ -pipe -O3 -flto -march=native -
5 // mtune=native gen.cpp
6 // faz bastante diferença no runtime
7
8 #include <bits/stdc++.h>
9 #include <cstdlib>
10 #include <ctime>
11 using namespace std;
12
13 int randi(int L, int R) { return L + rand() % (R - L + 1); }
14 char randc(char L, char R) { return char(L + rand() % (R - L
15     + 1)); }
16
17 int main(int argc, char** argv) {
18     if (argc > 1) srand(atoi(argv[1]));
19     else srand(time(0));
20
21     int n = randi(1, 100);
22     cout << n << '\n';
23     for (int i = 0; i < n; i++) {
24         cout << randi(-10, 20) << ' ';
25     }
26     cout << '\n';
27 }

```

## 5 Math

### 5.1 Combinatorics

```

1 const int MAXN_FATORIAL = 200005;
2 const int MOD = 1e9 + 7;
3 // DEFINE INT LONG LONG PLMDS
4 int fat[MAXN_FATORIAL], fati[MAXN_FATORIAL];
5
6 // (a^b) % m em O(log b)
7 // coloque o fexp
8
9 int inv(int n) { return fexp(n, MOD - 2); }
10
11 void precalc() {
12     fat[0] = 1;
13     fati[0] = 1;
14     for (int i = 1; i < MAXN_FATORIAL; i++) fat[i] = (fat[i - 1] * i) % MOD;
15     fati[MAXN_FATORIAL - 1] = inv(fat[MAXN_FATORIAL - 1]);
16     for (int i = MAXN_FATORIAL - 2; i >= 0; i--) fati[i] = (fati[i + 1] * (i + 1)) % MOD;
17 }
18
19 int choose(int n, int k) {
20     if (k < 0 || k > n) return 0;
21     return ((fat[n] * fati[k]) % MOD * fati[n - k]) % MOD;
22 }
23
24 // n! / (n-k)!
25 int perm(int n, int k) {
26     if (k < 0 || k > n) return 0;
27     return (fat[n] * fati[n - k]) % MOD;
28 }
29
30 // C_n = (1 / (n+1)) * C(2n, n)
31 int catalan(int n) {
32     if (n < 0 || 2 * n >= MAXN_FATORIAL) return 0;
33     int c2n_n = choose(2 * n, n);
34     return (c2n_n * inv(n + 1)) % MOD;
35 }

```

## 5.2 Equacao Diofantina

```

1 int extended_gcd(int a, int b, int& x, int& y) {
2     if (a == 0) {
3         x = 0;
4         y = 1;
5         return b;
6     }
7     int x1, y1;
8     int gcd = extended_gcd(b % a, a, x1, y1);
9     x = y1 - (b / a) * x1;
10    y = x1;
11    return gcd;
12 }
13
14 bool solve(int a, int b, int c, int& x0, int& y0) {
15     int x, y;
16     int g = extended_gcd(abs(a), abs(b), x, y);
17     if (c % g != 0) {
18         return false;
19     }
20     x0 = x * (c / g);
21     y0 = y * (c / g);
22     if (a < 0) x0 = -x0;
23     if (b < 0) y0 = -y0;
24     return true;
25 }

```

## 5.3 Multiplicacao Matriz

```

1 // multiplica matrizes de tamanhos variados, resultando em
   uma matrix N*M
2 vector<vector<int>> mm(vector<vector<int>> A, vector<vector<
   int>> B) {
3     int N = A.size(), M = B[0].size(), K = B.size();
4     vector<vector<int>> C(N, vector<int>(M));
5
6     for (int i = 0; i < N; ++i)
7         for (int j = 0; j < M; ++j)
8             for (int k = 0; k < K; ++k)
9                 C[i][j] = (C[i][j] + A[i][k] * B[k][j] % mod) %
10                mod;
11
12     return C;

```

## 5.4 Discrete Log

```

1 // returns minimum x for which a^x = b (mod m), a and m are
   coprime.
2 // if the answer dont need to be greater than some value, the
   vector<int> can be removed
3 int discrete_log(int a, int b, int m) {
4     a %= m, b %= m;
5     int n = sqrt(m) + 1;
6
7     int an = 1;
8     for (int i = 0; i < n; ++i)
9         an = (an * 11l * a) % m;
10
11    unordered_map<int, vector<int>> vals;
12    for (int q = 0, cur = b; q <= n; ++q) {
13        vals[cur].push_back(q);
14        cur = (cur * 11l * a) % m;
15    }
16
17    int res = LLONG_MAX;
18
19    for (int p = 1, cur = 1; p <= n; ++p) {
20        cur = (cur * 11l * an) % m;
21        if (vals.count(cur)) {
22            for (int q: vals[cur]) {
23                int ans = n * p - q;
24                res = min(res, ans);
25            }
26        }
27    }
28    return res;
29 }

```

## 5.5 Segment Sieve

```

1 // Retorna quantos primos tem entre [l, r] (inclusivo)
2 // precisa de um vetor com os primos ate sqrt(r)
3 int seg_sieve(int l, int r) {
4     if (l > r) return 0;
5     vector<bool> is_prime(r - l + 1, true);
6     if (l == 1) is_prime[0] = false;
7
8     for (int p : primos) {
9         if (p * p > r) break;
10        int start = max(p * p, (l + p - 1) / p * p);
11        for (int j = start; j <= r; j += p) {
12            if (j >= 1) {
13                is_prime[j - l] = false;
14            }
15        }
16    }
17
18    return accumulate(all(is_prime), 0ll);
19 }

```

## 5.6 Totient

```

1 // phi(n) = n * (1 - 1/p1) * (1 - 1/p2) * ...
2 int phi(int n) {
3     int result = n;
4     for (int i = 2; i * i <= n; i++) {
5         if (n % i == 0) {
6             while (n % i == 0)
7                 n /= i;
8             result -= result / i;
9         }
10    }
11
12    if (n > 1) // SE n sobrou, ele Ã um fator primo
13        result -= result / n;
14    return result;
15 }
16
17 // crivo phi
18 const int MAXN_PHI = 1000001;
19 int phiv[MAXN_PHI];
20 void phi_sieve() {
21     for (int i = 0; i < MAXN_PHI; i++) phiv[i] = i;
22     for (int i = 2; i < MAXN_PHI; i++) {
23         if (phiv[i] == i) {
24             for (int j = i; j < MAXN_PHI; j += i) phiv[j] -=
25                 phiv[j] / i;
26         }
27     }
28 }

```

## 5.7 Menor Fator Primo

```

1  const int MAXN = 1000001;
2  int spf[MAXN];
3  vector<int> primos;
4
5  void crivo() {
6      for (int i = 0; i < MAXN; i++) spf[i] = [i];
7      for (int i = 2; i * i < MAXN; i++) {
8          if (spf[i] == i) {
9              for (int j = i * i; j < MAXN; j += i) {
10                 if (spf[j] == j) {
11                     spf[j] = i;
12                 }
13             }
14         }
15     }
16     for (int i = 2; i < MAXN; i++) {
17         if (spf[i] == i) {
18             primos.push_back(i);
19         }
20     }
21 }
22
23 map<int, int> fatorar(int n) {
24     map<int, int> fatores;
25     while (n > 1) {
26         fatores[spf[n]]++;
27         n /= spf[n];
28     }
29     return fatores;
30 }
31
32 int numero_de_divisores(int n) {
33     if (n == 1) return 1;
34     map<int, int> fatores = fatorar(n);
35     int nod = 1;
36     for (auto &[primo, expoente] : fatores) nod *= (expoente
37         + 1);
38     return nod;
39 }
40
41 // DEFINE INT LONG LONG
42 int soma_dos_divisores(int n) {
43     if (n == 1) return 1;
44     map<int, int> fatores = fatorar(n);
45     int sod = 1;
46     for (auto &[primo, expoente] : fatores) {
47         int termo_soma = 1;
48         int potencia_primo = 1;
49         for (int i = 0; i < expoente; i++) {
50             potencia_primo *= primo;
51             termo_soma += potencia_primo;
52         }
53         sod *= termo_soma;
54     }
55     return sod;
56 }

```

## 5.8 Exgcd

```

1  // 0 retorno da funcao eh {n, m, g}
2  // e significa que gcd(a, b) = g e
3  // n e m sao inteiros tais que an + bm = g
4  array<ll, 3> exgcd(int a, int b) {
5      if (b == 0) return {1, 0, a};
6      auto [m, n, g] = exgcd(b, a % b);
7      return {n, m - a / b * n, g};
8  }

```

## 5.9 Fexp

```

1  // a^e mod m
2  // 0(log n)
3
4  int fexp(int a, int e, int m) {
5      a %= m;
6      int ans = 1;
7      while (e > 0) {
8          if (e & 1) ans = ans * a % m;
9          a = a * a % m;
10         e /= 2;
11     }

```

```

12     return ans % m;
13 }

```

## 5.10 Divisores

```

1  // Retorna um vetor com os divisores de x
2  // eh preciso ter o crivo implementado
3  // 0(divisores)
4
5  vector<int> divs(int x) {
6      vector<int> ans = {1};
7      vector<array<int, 2>> primos; // {primo, expoente}
8
9      while (x > 1) {
10         int p = crivo[x], cnt = 0;
11         while (x % p == 0) cnt++, x /= p;
12         primos.push_back({p, cnt});
13     }
14
15     for (int i=0; i<primos.size(); i++){
16         int cur = 1, len = ans.size();
17
18         for (int j=0; j<primos[i][1]; j++){
19             cur *= primos[i][0];
20             for (int k=0; k<len; k++){
21                 ans.push_back(cur*ans[k]);
22             }
23         }
24     }
25     return ans;
26 }

```

## 5.11 Crivo

```

1  // 0(n*log(log(n)))
2  bool composto[MAX]
3  for (int i = 1; i <= n; i++) {
4      if (composto[i]) continue;
5      for (int j = 2*i; j <= n; j += i)
6          composto[j] = 1;
7  }

```

## 5.12 Mod Inverse

```

1  array<int, 2> extended_gcd(int a, int b) {
2      if (b == 0) return {1, 0};
3      auto [x, y] = extended_gcd(b, a % b);
4      return {y, x - (a / b) * y};
5  }
6
7  int mod_inverse(int a, int m) {
8      auto [x, y] = extended_gcd(a, m);
9      return (x % m + m) % m;
10 }

```

## 5.13 Base Calc

```

1  int char_to_val(char c) {
2      if (c >= '0' && c <= '9') return c - '0';
3      else return c - 'A' + 10;
4  }
5
6  char val_to_char(int val) {
7      if (val >= 0 && val <= 9) return val + '0';
8      else return val - 10 + 'A';
9  }
10
11 int to_base_10(string &num, int bfrom) {
12     int result = 0;
13     int pot = 1;
14     for (int i = num.size() - 1; i >= 0; i--) {
15         if (char_to_val(num[i]) >= bfrom) return -1;
16         result += char_to_val(num[i]) * pot;
17         pot *= bfrom;
18     }
19     return result;
20 }
21
22 string from_base_10(int n, int bto) {
23     if (n == 0) return "0";
24     string result = "";
25     while (n > 0) {

```

```

26         result += val_to_char(n % bto);
27         n /= bto;
28     }
29     reverse(result.begin(), result.end());
30     return result;
31 }
32
33 string convert_base(string &num, int bfrom, int bto) {
34     int n_base_10 = to_base_10(num, bfrom);
35     return from_base_10(n_base_10, bto);
36 }

```

## 5.14 Fft

```

1 // multiplica dois polinomios em O(NlogN)
2
3 using cd = complex<double>;
4 const double PI = acos(-1);
5
6 void fft(vector<cd> &A, bool invert) {
7     int N = size(A);
8
9     for (int i = 1, j = 0; i < N; i++) {
10         int bit = N >> 1;
11         for (; j & bit; bit >>= 1)
12             j ^= bit;
13         j ^= bit;
14
15         if (i < j)
16             swap(A[i], A[j]);
17     }
18
19     for (int len = 2; len <= N; len <<= 1) {
20         double ang = 2 * PI / len * (invert ? -1 : 1);
21         cd wlen(cos(ang), sin(ang));
22         for (int i = 0; i < N; i += len) {
23             cd w(1);
24             for (int j = 0; j < len/2; j++) {
25                 cd u = A[i+j], v = A[i+j+len/2] * w;
26                 A[i+j] = u + v;
27                 A[i+j+len/2] = u - v;
28                 w *= wlen;
29             }
30         }
31     }
32
33     if (invert) {
34         for (auto &x : A)
35             x /= N;
36     }
37 }
38
39 vector<int> multiply(vector<int> const& A, vector<int> const& B) {
40     vector<cd> fa(begin(A), end(A)), fb(begin(B), end(B));
41     int N = 1;
42     while (N < size(A) + size(B))
43         N <<= 1;
44     fa.resize(N);
45     fb.resize(N);
46
47     fft(fa, false);
48     fft(fb, false);
49     for (int i = 0; i < N; i++)
50         fa[i] *= fb[i];
51     fft(fa, true);
52
53     vector<int> result(N);
54     for (int i = 0; i < N; i++)
55         result[i] = round(fa[i].real());
56     return result;
57 }

```

# 6 Graph

## 6.1 Dijkstra

```

1 // SSP com pesos positivos.
2 // O((V + E) log V).
3
4 vector<int> dijkstra(int S) {
5     vector<bool> vis(MAXN, 0);
6     vector<ll> dist(MAXN, LLONG_MAX);

```

```

7     dist[S] = 0;
8     priority_queue<pii, vector<pii>, greater<pii>> pq;
9     pq.push({0, S});
10    while(pq.size()) {
11        ll v = pq.top().second;
12        pq.pop();
13        if(vis[v]) continue;
14        vis[v] = 1;
15        for(auto &[peso, vizinho] : adj[v]) {
16            if(dist[vizinho] > dist[v] + peso) {
17                dist[vizinho] = dist[v] + peso;
18                pq.push({dist[vizinho], vizinho});
19            }
20        }
21    }
22    return dist;
23 }

```

## 6.2 Floyd Warshall

```

1 // SSP e acha ciclos.
2 // Bom com constraints menores.
3 // O(n^3)
4
5 int dist[501][501];
6
7 void floydWarshall() {
8     for(int k = 0; k < n; k++) {
9         for(int i = 0; i < n; i++) {
10             for(int j = 0; j < n; j++) {
11                 dist[i][j] = min(dist[i][j], dist[i][k] +
12                                 dist[k][j]);
13             }
14         }
15     }
16
17     void solve() {
18         int m, q;
19         cin >> n >> m >> q;
20         for(int i = 0; i < n; i++) {
21             for(int j = i; j < n; j++) {
22                 if(i == j) {
23                     dist[i][j] = dist[j][i] = 0;
24                 } else {
25                     dist[i][j] = dist[j][i] = linf;
26                 }
27             }
28         }
29         for(int i = 0; i < m; i++) {
30             int u, v, w;
31             cin >> u >> v >> w; u--; v--;
32             dist[u][v] = min(dist[u][v], w);
33             dist[v][u] = min(dist[v][u], w);
34         }
35         floydWarshall();
36         while(q--) {
37             int u, v;
38             cin >> u >> v; u--; v--;
39             if(dist[u][v] == linf) cout << -1 << '\n';
40             else cout << dist[u][v] << '\n';
41         }
42     }
43 }

```

## 6.3 Eulerian Path

```

1 /**
2  * versao que assume: #define int long long
3  *
4  * Retorna um caminho/ciclo euleriano em um grafo (se existir).
5  * - g: lista de adjacencia (vector<vector<int>>).
6  * - directed: true se o grafo for dirigido.
7  * - s: vertice inicial.
8  * - e: vertice final (opcional). Se informado, tenta caminho
9  *   de s ate e.
10  * - O(Nlog(N))
11  * Retorna vetor com a sequencia de vertices, ou vazio se
12  *   impossivel.
13  */
14 vector<int> eulerian_path(const vector<vector<int>>& g, bool
15     directed, int s, int e = -1) {
16     int n = (int)g.size();
17     // copia das adjacencias em multiset para permitir
18     // remoção especifica

```



```

15 vector<multiset<int>> h(n);
16 vector<int> in_degree(n, 0);
17 vector<int> result;
18 stack<int> st;
19 // preencher h e indegrees
20 for (int u = 0; u < n; ++u) {
21     for (auto v : g[u]) {
22         ++in_degree[v];
23         h[u].emplace(v);
24     }
25 }
26 st.emplace(s);
27 if (e != -1) {
28     int out_s = (int)h[s].size();
29     int out_e = (int)h[e].size();
30     int diff_s = in_degree[s] - out_s;
31     int diff_e = in_degree[e] - out_e;
32     if (diff_s * diff_e != -1) return {}; // impossible
33 }
34 for (int u = 0; u < n; ++u) {
35     if (e != -1 && (u == s || u == e)) continue;
36     int out_u = (int)h[u].size();
37     if (in_degree[u] != out_u || (!directed && (in_degree
38         [u] & 1))) {
39         return {};
40     }
41 }
42 while (!st.empty()) {
43     int u = st.top();
44     if (h[u].empty()) {
45         result.emplace_back(u);
46         st.pop();
47     } else {
48         int v = *h[u].begin();
49         auto it = h[u].find(v);
50         if (it != h[u].end()) h[u].erase(it);
51         --in_degree[v];
52         if (!directed) {
53             auto it2 = h[v].find(u);
54             if (it2 != h[v].end()) h[v].erase(it2);
55             --in_degree[u];
56         }
57         st.emplace(v);
58     }
59 }
60 for (int u = 0; u < n; ++u) {
61     if (in_degree[u] != 0) return {};
62 }
63 reverse(result.begin(), result.end());
64 return result;

```

## 6.4 Dinitz

```

1 // Complexidade: O(V^2E)
2
3 struct FlowEdge {
4     int from, to;
5     long long cap, flow = 0;
6     FlowEdge(int from, int to, long long cap) : from(from),
7         to(to), cap(cap) {}
8 };
9
10 struct Dinic {
11     const long long flow_inf = 1e18;
12     vector<FlowEdge> edges;
13     vector<vector<int>> adj;
14     int n, m = 0;
15     int s, t;
16     vector<int> level, ptr;
17     queue<int> q;
18
19     Dinic(int n, int s, int t) : n(n), s(s), t(t) {
20         adj.resize(n);
21         level.resize(n);
22         ptr.resize(n);
23     }
24
25     void add_edge(int from, int to, long long cap) {
26         edges.emplace_back(from, to, cap);
27         edges.emplace_back(to, from, 0);
28         adj[from].push_back(m);
29         adj[to].push_back(m + 1);
30         m += 2;
31     }

```

```

32 bool bfs() {
33     while (!q.empty()) {
34         int from = q.front();
35         q.pop();
36         for (int id : adj[from]) {
37             if (edges[id].cap == edges[id].flow)
38                 continue;
39             if (level[edges[id].to] != -1)
40                 continue;
41             level[edges[id].to] = level[from] + 1;
42             q.push(edges[id].to);
43         }
44     }
45     return level[t] != -1;
46 }
47
48 long long dfs(int from, long long pushed) {
49     if (pushed == 0)
50         return 0;
51     if (from == t)
52         return pushed;
53     for (int& cid = ptr[from]; cid < (int)adj[from].size
54         (); cid++) {
55         int id = adj[from][cid];
56         int to = edges[id].to;
57         if (level[from] + 1 != level[to])
58             continue;
59         long long tr = dfs(to, min(pushed, edges[id].cap
60             - edges[id].flow));
61         if (tr == 0)
62             continue;
63         edges[id].flow += tr;
64         edges[id ^ 1].flow -= tr;
65         return tr;
66     }
67     return 0;
68 }
69
70 long long flow() {
71     long long f = 0;
72     while (true) {
73         fill(level.begin(), level.end(), -1);
74         level[s] = 0;
75         q.push(s);
76         if (!bfs())
77             break;
78         fill(ptr.begin(), ptr.end(), 0);
79         while (long long pushed = dfs(s, flow_inf)) {
80             f += pushed;
81         }
82     }
83     return f;
84 }

```

## 6.5 Khan

```

1 // topo-sort DAG
2 // lexicograficamente menor.
3 // N: numero de vertices (1-indexado)
4 // adj: lista de adjacencia do grafo
5
6 const int MAXN = 5 * 1e5 + 2;
7 vector<int> adj[MAXN];
8 int N;
9
10 vector<int> kahn() {
11     vector<int> indegree(N + 1, 0);
12     for (int u = 1; u <= N; u++) {
13         for (int v : adj[u]) {
14             indegree[v]++;
15         }
16     }
17     priority_queue<int, vector<int>, greater<int>> pq;
18     for (int i = 1; i <= N; i++) {
19         if (indegree[i] == 0) {
20             pq.push(i);
21         }
22     }
23     vector<int> result;
24     while (!pq.empty()) {
25         int u = pq.top();
26         pq.pop();
27         result.push_back(u);
28         for (int v : adj[u]) {

```

```

29         indegree[v]--;
30         if (indegree[v] == 0) {
31             pq.push(v);
32         }
33     }
34 }
35 if (result.size() != N) {
36     return {};
37 }
38 return result;
39 }

```

## 6.6 Topological Sort

```

1 vector<int> adj[MAXN];
2 vector<int> estado(MAXN); // 0: nao visitado 1: processamento
3     2: processado
4 vector<int> ordem;
5 bool temCiclo = false;
6 void dfs(int v) {
7     if(estado[v] == 1) {
8         temCiclo = true;
9         return;
10    }
11    if(estado[v] == 2) return;
12    estado[v] = 1;
13    for(auto &nei : adj[v]) {
14        if(estado[nei] != 2) dfs(nei);
15    }
16    estado[v] = 2;
17    ordem.push_back(v);
18    return;
19 }

```

## 6.7 Acha Pontes

```

1 vector<int> d, low, pai; // d[v] Tempo de descoberta (
2     discovery time)
3 vector<bool> vis;
4 vector<int> pontos_articulacao;
5 vector<pair<int, int>> pontes;
6 int tempo;
7 vector<vector<int>> adj;
8
9 void dfs(int u) {
10     vis[u] = true;
11     tempo++;
12     d[u] = low[u] = tempo;
13     int filhos_dfs = 0;
14     for (int v : adj[u]) {
15         if (v == pai[u]) continue;
16         if (vis[v]) { // back edge
17             low[u] = min(low[u], d[v]);
18         } else {
19             pai[v] = u;
20             filhos_dfs++;
21             dfs(v);
22             low[u] = min(low[u], low[v]);
23             if (pai[u] == -1 && filhos_dfs > 1) {
24                 pontos_articulacao.push_back(u);
25             }
26             if (pai[u] != -1 && low[v] >= d[u]) {
27                 pontos_articulacao.push_back(u);
28             }
29             if (low[v] > d[u]) {
30                 pontes.push_back({min(u, v), max(u, v)});
31             }
32         }
33     }
34 }

```

## 6.8 Edmonds-karp

```

1 // Edmonds-Karp com scalling  $O(E^2 \log(F))$ 
2
3 int n, m;
4 const int MAXN = 510;
5 vector<vector<int>> capacity(MAXN, vector<int>(MAXN, 0));
6 vector<vector<int>> adj(MAXN);
7
8 int bfs(int s, int t, int scale, vector<int>& parent) {

```

```

9     fill(parent.begin(), parent.end(), -1);
10    parent[s] = -2;
11    queue<pair<int, int>> q;
12    q.push({s, LLONG_MAX});
13
14    while (!q.empty()) {
15        int cur = q.front().first;
16        int flow = q.front().second;
17        q.pop();
18
19        for (int next : adj[cur]) {
20            if (parent[next] == -1 && capacity[cur][next] >=
21                scale) {
22                parent[next] = cur;
23                int new_flow = min(flow, capacity[cur][next]);
24            }
25            if (next == t)
26                return new_flow;
27            q.push({next, new_flow});
28        }
29    }
30    return 0;
31 }
32
33 int maxflow(int s, int t) {
34     int flow = 0;
35     vector<int> parent(MAXN);
36     int new_flow;
37     int scalling = 1ll << 62;
38
39     while (scalling > 0) {
40         while (new_flow = bfs(s, t, scalling, parent)){
41             if (new_flow == 0) continue;
42             flow += new_flow;
43             int cur = t;
44             while (cur != s) {
45                 int prev = parent[cur];
46                 capacity[prev][cur] -= new_flow;
47                 capacity[cur][prev] += new_flow;
48                 cur = prev;
49             }
50             scalling /= 2;
51         }
52     }
53     return flow;
54 }
55 }

```

## 6.9 Kruskal

```

1 // Ordena as arestas por peso, insere se ja nao estiver no
2     mesmo componente
3 //  $O(E \log E)$ 
4 struct Edge {
5     int u, v, w;
6     bool operator <(Edge const & other) {
7         return weight < other.weight;
8     }
9 }
10
11 vector<Edge> kruskal(int n, vector<Edge> edges) {
12     vector<Edge> mst;
13     DSU dsu = DSU(n + 1);
14     sort(edges.begin(), edges.end());
15     for (Edge e : edges) {
16         if (dsu.find(e.u) != dsu.find(e.v)) {
17             mst.push_back(e);
18             dsu.join(e.u, e.v);
19         }
20     }
21     return mst;
22 }

```

## 6.10 Bellman Ford

```

1 struct Edge {
2     int u, v, w;
3 };
4
5 // se x = -1, nao tem ciclo
6 // se x != -1, pegar pais de x pra formar o ciclo

```

```

7
8 int n, m;
9 vector<Edge> edges;
10 vector<int> dist(n);
11 vector<int> pai(n, -1);
12
13 for (int i = 0; i < n; i++) {
14     x = -1;
15     for (Edge &e : edges) {
16         if (dist[e.u] + e.w < dist[e.v]) {
17             dist[e.v] = max(-INF, dist[e.u] + e.w);
18             pai[e.v] = e.u;
19             x = e.v;
20         }
21     }
22 }
23
24 // achando caminho (se precisar)
25 for (int i = 0; i < n; i++) x = pai[x];
26
27 vector<int> ciclo;
28 for (int v = x; v = pai[v]) {
29     cycle.push_back(v);
30     if (v == x && ciclo.size() > 1) break;
31 }
32 reverse(ciclo.begin(), ciclo.end());

```

## 6.11 Lca Jc

```

1 const int MAXN = 200005;
2 int N;
3 int LOG;
4
5 vector<vector<int>> adj;
6 vector<int> profundidade;
7 vector<vector<int>> cima; // cima[v][j] eh o 2^j-esimo
8                          // ancestral de v
9
10 void dfs(int v, int p, int d) {
11     profundidade[v] = d;
12     cima[v][0] = p; // o pai direto eh o 2^0-ésimo ancestral
13     for (int j = 1; j < LOG; j++) {
14         // se o ancestral 2^(j-1) existir, calculamos o 2^j
15         if (cima[v][j-1] != -1) {
16             cima[v][j] = cima[cima[v][j-1]][j-1];
17         } else {
18             cima[v][j] = -1; // nao tem ancestral superior
19         }
20     }
21     for (int nei : adj[v]) {
22         if (nei != p) {
23             dfs(nei, v, d + 1);
24         }
25     }
26
27 void build(int root) {
28     LOG = ceil(log2(N));
29     profundidade.assign(N + 1, 0);
30     cima.assign(N + 1, vector<int>(LOG, -1));
31     dfs(root, -1, 0);
32 }
33
34 int get_lca(int a, int b) {
35     if (profundidade[a] < profundidade[b]) {
36         swap(a, b);
37     }
38     // sobe 'a' ate a mesma profundidade de 'b'
39     for (int j = LOG - 1; j >= 0; j--) {
40         if (profundidade[a] - (1 << j) >= profundidade[b]) {
41             a = cima[a][j];
42         }
43     }
44     // se 'b' era um ancestral de 'a', então 'a' agora é
45     // igual a 'b'
46     if (a == b) {
47         return a;
48     }
49     // sobe os dois juntos ate encontrar os filhos do LCA
50     for (int j = LOG - 1; j >= 0; j--) {
51         if (cima[a][j] != -1 && cima[a][j] != cima[b][j]) {
52             a = cima[a][j];
53             b = cima[b][j];
54         }
55     }

```

```

55     return cima[a][0];
56 }

```

## 6.12 Lca

```

1 // LCA - CP algorithm
2 // preprocessing O(NlogN)
3 // lca O(logN)
4 // Uso: criar LCA com a quantidade de vertices (n) e lista de
5 // adjacencias (adj)
6 // chamar a funcao preprocess com a raiz da arvore
7
8 struct LCA {
9     int n, l, timer;
10     vector<vector<int>> adj;
11     vector<int> tin, tout;
12     vector<vector<int>> up;
13
14     LCA(int n, const vector<vector<int>>& adj) : n(n), adj(
15         adj) {}
16
17     void dfs(int v, int p) {
18         tin[v] = ++timer;
19         up[v][0] = p;
20         for (int i = 1; i <= l; ++i)
21             up[v][i] = up[up[v][i-1]][i-1];
22
23         for (int u : adj[v]) {
24             if (u != p)
25                 dfs(u, v);
26         }
27         tout[v] = ++timer;
28     }
29
30     bool is_ancestor(int u, int v) {
31         return tin[u] <= tin[v] && tout[u] >= tout[v];
32     }
33
34     int lca(int u, int v) {
35         if (is_ancestor(u, v))
36             return u;
37         if (is_ancestor(v, u))
38             return v;
39         for (int i = l; i >= 0; --i) {
40             if (!is_ancestor(up[u][i], v))
41                 u = up[u][i];
42         }
43         return up[u][0];
44     }
45
46     void preprocess(int root) {
47         tin.resize(n);
48         tout.resize(n);
49         timer = 0;
50         l = ceil(log2(n));
51         up.assign(n, vector<int>(l + 1));
52         dfs(root, root);
53     }

```

## 6.13 Kosaraju

```

1 bool vis[MAXN];
2 vector<int> order;
3 int component[MAXN];
4 int N, m;
5 vector<int> adj[MAXN], adj_rev[MAXN];
6
7 // dfs no grafo original para obter a ordem (pos-order)
8 void dfs1(int u) {
9     vis[u] = true;
10     for (int v : adj[u]) {
11         if (!vis[v]) {
12             dfs1(v);
13         }
14     }
15     order.push_back(u);
16 }
17
18 // dfs o grafo reverso para encontrar os SCCs
19 void dfs2(int u, int c) {
20     component[u] = c;
21     for (int v : adj_rev[u]) {

```

```

22         if (component[v] == -1) {
23             dfs2(v, c);
24         }
25     }
26 }
27
28 int kosaraju() {
29     order.clear();
30     fill(vis + 1, vis + N + 1, false);
31     for (int i = 1; i <= N; i++) {
32         if (!vis[i]) {
33             dfs1(i);
34         }
35     }
36     fill(component + 1, component + N + 1, -1);
37     int c = 0;
38     reverse(order.begin(), order.end());
39     for (int u : order) {
40         if (component[u] == -1) {
41             dfs2(u, c++);
42         }
43     }
44     return c;
45 }

```

## 6.14 Min Cost Max Flow

```

1 // Encontra o menor custo para passar K de fluxo em um grafo
  // com N vertices
2 // Funciona com multiplas arestas para o mesmo par de
  // vertices
3 // Para encontrar o min cost max flow eh so fazer K =
  // infinito
4
5 struct Edge {
6     int from, to, capacity, cost, id;
7 };
8
9 vector<vector<array<int, 2>>> adj;
10 vector<Edge> edges; // arestas pares sao as normais e suas
    // reversas sao as impares
11
12 const int INF = LLONG_MAX;
13
14 void shortest_paths(int n, int v0, vector<int>& dist, vector<
    int>& edge_to) {
15     dist.assign(n, INF);
16     dist[v0] = 0;
17     vector<bool> in_queue(n, false);
18     queue<int> q;
19     q.push(v0);
20     edge_to.assign(n, -1);
21
22     while (!q.empty()) {
23         int u = q.front();
24         q.pop();
25         in_queue[u] = false;
26         for (auto [v, id] : adj[u]) {
27             if (edges[id].capacity > 0 && dist[v] > dist[u] +
                edges[id].cost) {
28                 dist[v] = dist[u] + edges[id].cost;
29                 edge_to[v] = id;
30                 if (!in_queue[v]) {
31                     in_queue[v] = true;
32                     q.push(v);
33                 }
34             }
35         }
36     }
37 }
38
39 void add_edge(int from, int to, int capacity, int cost) {
40     edges.push_back({from, to, capacity, cost, (int)edges.
        size()});
41     edges.push_back({to, from, 0, -cost, (int)edges.size()});
    // reversa
42 }
43
44 int min_cost_flow(int N, int K, int s, int t) {
45     adj.assign(N, vector<array<int, 2>>());
46
47     for (Edge e : edges) {
48         adj[e.from].push_back({e.to, e.id});
49     }
50
51     int flow = 0;

```

```

52     int cost = 0;
53     vector<int> dist, edge_to;
54     while (flow < K) {
55         shortest_paths(N, s, dist, edge_to);
56         if (dist[t] == INF)
57             break;
58
59         // find max flow on that path
60         int f = K - flow;
61         int cur = t;
62         while (cur != s) {
63             f = min(f, edges[edge_to[cur]].capacity);
64             cur = edges[edge_to[cur]].from;
65         }
66
67         // apply flow
68         flow += f;
69         cost += f * dist[t];
70         cur = t;
71         while (cur != s) {
72             int edge = edge_to[cur];
73             int rev_edge = edge^1;
74
75             edges[edge].capacity -= f;
76             edges[rev_edge].capacity += f;
77             cur = edges[edge].from;
78         }
79     }
80
81     if (flow < K)
82         return -1;
83     else
84         return cost;
85 }

```

## 6.15 Diametro

```

1 /*
2 0(N + M), retorna cara mais distante, len
3 do caminho em arestas e os caras do caminho em um vetor
4 Para pegar o diametro da arvore, chamar duas vezes
5 auto [a, DA, PA] = calc(1);
6 auto [b, len_diametro, caminho_diametro] = calc(a);
7 */
8 auto calc = [&](int S) -> tuple<int, int, vector<int>> {
9     queue<pair<int, int>> q;
10     q.push({S, 0});
11     vector<int> dp(n + 1, -1), pai(n + 1, 0);
12     dp[S] = 0;
13     pai[S] = -1;
14     int longe = S;
15     int dist = 0;
16     while (!q.empty()) {
17         auto [u, d] = q.front();
18         q.pop();
19         if (d > dist) {
20             dist = d;
21             longe = u;
22         }
23         for (int v : adj[u]) {
24             if (dp[v] == -1) {
25                 dp[v] = d + 1;
26                 pai[v] = u;
27                 q.push({v, d + 1});
28             }
29         }
30     }
31     int cur = longe;
32     vector<int> caminho;
33     while (cur != -1) {
34         caminho.push_back(cur);
35         cur = pai[cur];
36     }
37     return { longe, dist, caminho };
38 };

```

## 7 Primitives

## 8 DP

### 8.1 Lis

```

1 int lis_nlogn(vector<int> &v) {
2     vector<int> lis;
3     lis.push_back(v[0]);
4     for (int i = 1; i < v.size(); i++) {
5         if (v[i] > lis.back()) {
6             // estende a lis
7             lis.push_back(v[i]);
8         } else {
9             // encontra o primeiro elemento em lis que >= v[i]
10            auto it = lower_bound(lis.begin(), lis.end(), v[i]);
11            *it = v[i];
12        }
13    }
14    return lis.size();
15 }
16
17 // lis na tree do problema da sub
18 const int MAXN_TREE = 100001;
19 vector<int> adj[MAXN_TREE];
20 int values[MAXN_TREE];
21 int ans = 0;
22
23 void dfs(int u, int p, vector<int> &tails) {
24     auto it = lower_bound(tails.begin(), tails.end(), values[u]);
25     int prev = -1;
26     bool coloquei = false;
27     if (it == tails.end()) {
28         tails.push_back(values[u]);
29         coloquei = true;
30     } else {
31         prev = *it;
32         *it = values[u];
33     }
34     ans = max(ans, (int)tails.size());
35     for (int v : adj[u]) {
36         if (v != p) {
37             dfs(v, u, tails);
38         }
39     }
40     if (coloquei) {
41         tails.pop_back();
42     } else {
43         *it = prev;
44     }
45 }

```

## 8.2 Edit Distance

```

1 vector<vector<int>> dp(n+1, vector<int>(m+1, LLONG_MIN));
2 for(int j = 0; j <= m; j++) dp[0][j] = j;
3 for(int i = 0; i <= n; i++) dp[i][0] = i;
4 for(int i = 1; i <= n; i++) {
5     for(int j = 1; j <= m; j++) {
6         if(a[i-1] == b[j-1]) {
7             dp[i][j] = dp[i-1][j-1];
8         } else {
9             dp[i][j] = min({dp[i-1][j] + 1, dp[i][j-1] + 1, dp[i-1][j-1] + 1});
10        }
11    }
12 }
13 cout << dp[n][m];

```

## 8.3 Bitmask

```

1 // dp de intervalos com bitmask
2 int prox(int idx) {
3     return lower_bound(S.begin(), S.end(), array<int, 4>{S[idx][1], 011, 011, 011}) - S.begin();
4 }
5
6 int dp[1002][(int)(111 << 10)];
7
8 int rec(int i, int vis) {
9     if (i == (int)S.size()) {
10        if (__builtin_popcountll(vis) == N) return 0;
11        return LLONG_MIN;
12    }
13    if (dp[i][vis] != -1) return dp[i][vis];
14    int ans = rec(i + 1, vis);
15    ans = max(ans, rec(prox(i), vis | (111 << S[i][3])) + S[i][2]);

```

```

16     return dp[i][vis] = ans;
17 }

```

## 8.4 Lcs

```

1 string s1, s2;
2 int dp[1001][1001];
3 int lcs(int i, int j) {
4     if (i < 0 || j < 0) return 0;
5     if (dp[i][j] != -1) return dp[i][j];
6     if (s1[i] == s2[j]) return dp[i][j] = 1 + lcs(i - 1, j - 1);
7     else return dp[i][j] = max(lcs(i - 1, j), lcs(i, j - 1));
8 }

```

## 8.5 Digit

```

1 vector<int> digits;
2
3 int dp[20][10][2][2];
4
5 int rec(int i, int last, int flag, int started) {
6     if (i == (int)digits.size()) return 1;
7     if (dp[i][last][flag][started] != -1) return dp[i][last][flag][started];
8     int lim;
9     if (flag) lim = 9;
10    else lim = digits[i];
11    int ans = 0;
12    for (int d = 0; d <= lim; d++) {
13        if (started && d == last) continue;
14        int new_flag = flag;
15        int new_started = started;
16        if (d > 0) new_started = 1;
17        if (!flag && d < lim) new_flag = 1;
18        ans += rec(i + 1, d, new_flag, new_started);
19    }
20    return dp[i][last][flag][started] = ans;
21 }

```

## 8.6 Knapsack

```

1 // dp[i][j] => i-esimo item com j-carga sobrando na mochila
2 // 0(N * W)
3 vector<vector<int>> dp(N + 1, vector<int>(W + 1, 0));
4 for (int i = 1; i <= N; i++) dp[i][0] = 0;
5 for (int j = 0; j <= W; j++) dp[0][j] = 0;
6 for (int i = 1; i <= N; i++) {
7     for (int j = 0; j <= W; j++) {
8         dp[i][j] = dp[i-1][j];
9         if (j >= items[i-1].first) {
10            dp[i][j] = max(dp[i][j], dp[i-1][j - items[i-1].first] + items[i-1].second);
11        }
12    }
13 }
14 cout << dp[N][W] << '\n';

```

## 8.7 Lis Seg

```

1 // comprime coordenadas pra quando ai <= 1e9
2 vector<int> vals(a.begin(), a.end());
3 sort(vals.begin(), vals.end());
4 vals.erase(unique(vals.begin(), vals.end(), vals.end()));
5 auto id = [&](int x) -> int {
6     return lower_bound(vals.begin(), vals.end(), x) - vals.begin();
7 };
8 for (int i = 0; i < n; i++) a[i] = id(a[i]);
9 // fim da parte de compr
10 SegTree seg(n);
11 // dp[i]: lis que termina em i
12 vector<int> dp(n, 1); // dp[i] = 1 base case
13 for (int i = 0; i < n; i++) {
14     if (a[i] > 0) dp[i] = seg.query(0, max(011, a[i] - 1)) + 1;
15     seg.update(a[i], dp[i]);
16 }
17 cout << *max_element(dp.begin(), dp.end()) << '\n';

```

## 8.8 Disjoint Blocks

```

1 // num max de subarrays disjuntos com soma x usando apenas
2 // prefixo i (ou seja, considerando prefixo a[1..i]).
3 int disjointSumX(vector<int> &a, int x) {
4     int n = a.size();
5     map<int, int> best; // best[pref] = melhor dp visto para
6     // esse pref
7     best[0] = 0;
8     int pref = 0;
9     vector<int> dp(n + 1, 0); // dp[0] = 0
10    for (int i = 1; i <= n; i++) {
11        pref += a[i - 1];
12        dp[i] = dp[i - 1];
13        auto it = best.find(pref - x);
14        if (it != best.end()) {
15            dp[i] = max(dp[i], it->second + 1);
16        }
17        best[pref] = max(best[pref], dp[i]);
18    }
19    return dp[n];
20 }

```

## 9 General

### 9.1 Brute Choose

```

1 vector<int> elements;
2 int N, K;
3 vector<int> comb;
4
5 void brute_choose(int i) {
6     if (comb.size() == K) {
7         for (int j = 0; j < comb.size(); j++) {
8             cout << comb[j] << ' ';
9         }
10        cout << '\n';
11        return;
12    }
13    if (i == N) return;
14    int r = N - i;
15    int preciso = K - comb.size();
16    if (r < preciso) return;
17    comb.push_back(elements[i]);
18    brute_choose(i + 1);
19    comb.pop_back();
20    brute_choose(i + 1);
21 }

```

### 9.2 Struct

```

1 struct Pessoa{
2     // Atributos
3     string nome;
4     int idade;
5
6     // Comparador
7     bool operator< (const Pessoa& other) const{
8         if(idade != other.idade) return idade > other.idade;
9         else return nome > other.nome;
10    }
11 }

```

### 9.3 Mex

```

1 struct MEX {
2     map<int, int> f;
3     set<int> falta;
4     int tam;
5     MEX(int n) : tam(n) {
6         for (int i = 0; i <= n; i++) falta.insert(i);
7     }
8     void add(int x) {
9         f[x]++;
10        if (f[x] == 1 && x >= 0 && x <= tam) {
11            falta.erase(x);
12        }
13    }
14    void rem(int x) {
15        if (f.count(x) && f[x] > 0) {
16            f[x]--;
17            if (f[x] == 0 && x >= 0 && x <= tam) {
18                falta.insert(x);
19            }
20        }
21    }
22 }

```

```

20     }
21 }
22 int get() {
23     if (falta.empty()) return tam + 1;
24     return *falta.begin();
25 }
26 };

```

### 9.4 Count Permutations

```

1 // Returns the number of distinct permutations
2 // that are lexicographically less than the string t
3 // using the provided frequency (freq) of the characters
4 // 0(n*freq.size())
5 int countPermLess(vector<int> freq, const string &t) {
6     int n = t.size();
7     int ans = 0;
8
9     vector<int> fact(n + 1, 1), invfact(n + 1, 1);
10    for (int i = 1; i <= n; i++)
11        fact[i] = (fact[i - 1] * i) % MOD;
12    invfact[n] = fexp(fact[n], MOD - 2, MOD);
13    for (int i = n - 1; i >= 0; i--)
14        invfact[i] = (invfact[i + 1] * (i + 1)) % MOD;
15
16    // For each position in t, try placing a letter smaller
17    // than t[i] that is in freq
18    for (int i = 0; i < n; i++) {
19        for (char c = 'a'; c < t[i]; c++) {
20            if (freq[c - 'a'] > 0) {
21                freq[c - 'a']--;
22                int ways = fact[n - i - 1];
23                for (int f : freq)
24                    ways = (ways * invfact[f]) % MOD;
25                ans = (ans + ways) % MOD;
26                freq[c - 'a']++;
27            }
28            if (freq[t[i] - 'a'] == 0) break;
29            freq[t[i] - 'a']--;
30        }
31    }
32    return ans;
33 }

```

### 9.5 Bitwise

```

1 int check_kth_bit(int x, int k) {
2     return (x >> k) & 1;
3 }
4
5 void print_on_bits(int x) {
6     for (int k = 0; k < 32; k++) {
7         if (check_kth_bit(x, k)) {
8             cout << k << ' ';
9         }
10    }
11    cout << '\n';
12 }
13
14 int count_on_bits(int x) {
15     int ans = 0;
16     for (int k = 0; k < 32; k++) {
17         if (check_kth_bit(x, k)) {
18             ans++;
19         }
20    }
21    return ans;
22 }
23
24 bool is_even(int x) {
25     return ((x & 1) == 0);
26 }
27
28 int set_kth_bit(int x, int k) {
29     return x | (1 << k);
30 }
31
32 int unset_kth_bit(int x, int k) {
33     return x & ~(1 << k);
34 }
35
36 int toggle_kth_bit(int x, int k) {
37     return x ^ (1 << k);
38 }

```

```

39
40 bool check_power_of_2(int x) {
41     return count_on_bits(x) == 1;
42 }

```

## 10 Geometry

### 10.1 Convex Hull

```

1 #include <bits/stdc++.h>
2
3 using namespace std;
4 #define int long long
5 typedef int cod;
6
7 struct point
8 {
9     cod x,y;
10    point(cod x = 0, cod y = 0): x(x), y(y)
11    {}
12
13    double modulo()
14    {
15        return sqrt(x*x + y*y);
16    }
17
18    point operator+(point o)
19    {
20        return point(x+o.x, y+o.y);
21    }
22    point operator-(point o)
23    {
24        return point(x - o.x , y - o.y);
25    }
26    point operator*(cod t)
27    {
28        return point(x*t, y*t);
29    }
30    point operator/(cod t)
31    {
32        return point(x/t, y/t);
33    }
34
35    cod operator*(point o)
36    {
37        return x*o.x + y*o.y;
38    }
39    cod operator^(point o)
40    {
41        return x*o.y - y * o.x;
42    }
43    bool operator<(point o)
44    {
45        if( x != o.x) return x < o.x;
46        return y < o.y;
47    }
48 };
49
50 int ccw(point p1, point p2, point p3)
51 {
52     cod cross = (p2-p1) ^ (p3-p1);
53     if(cross == 0) return 0;
54     else if(cross < 0) return -1;
55     else return 1;
56 }
57
58 vector <point> convex_hull(vector<point> p)
59 {
60     sort(p.begin(), p.end());
61     vector<point> L,U;
62
63     //Lower
64     for(auto pp : p)
65     {
66         while(L.size() >= 2 and ccw(L[L.size()-2], L.back(),
67             , pp) == -1)
68         {
69             // Não quero excluir os colineares
70             L.pop_back();
71         }
72         L.push_back(pp);
73     }
74

```

```

75     reverse(p.begin(), p.end());
76
77     //Upper
78     for(auto pp : p)
79     {
80         while(U.size() >= 2 and ccw(U[U.size()-2], U.back(),
81             pp) == -1)
82         {
83             U.pop_back();
84         }
85         U.push_back(pp);
86     }
87
88     L.pop_back();
89     L.insert(L.end(), U.begin(), U.end()-1);
90     return L;
91
92 cod area(vector<point> v)
93 {
94     int ans = 0;
95     int aux = (int)v.size();
96     for(int i = 2; i < aux; i++)
97     {
98         ans += ((v[i] - v[0])^(v[i-1] - v[0]))/2;
99     }
100    ans = abs(ans);
101    return ans;
102 }
103
104 int bound(point p1 , point p2)
105 {
106     return __gcd(abs(p1.x-p2.x), abs(p1.y-p2.y));
107 }
108 //teorema de pick [pontos = A - (bound+points)/2 + 1]
109
110 int32_t main()
111 {
112
113     int n;
114     cin >> n;
115
116     vector<point> v(n);
117     for(int i = 0; i < n; i++)
118     {
119         cin >> v[i].x >> v[i].y;
120     }
121
122     vector <point> ch = convex_hull(v);
123
124     cout << ch.size() << '\n';
125     for(auto p : ch) cout << p.x << " " << p.y << "\n";
126
127     return 0;
128 }

```

### 10.2 Inside Polygon

```

1 // Convex O(logn)
2
3 bool insideT(point a, point b, point c, point e){
4     int x = ccw(a, b, e);
5     int y = ccw(b, c, e);
6     int z = ccw(c, a, e);
7     return !((x==1 or y==1 or z==1) and (x==-1 or y==-1 or z
8         ==-1));
9 }
10 bool inside(vp &p, point e){ // ccw
11     int l=2, r=(int)p.size()-1;
12     while(l<r){
13         int mid = (l+r)/2;
14         if(ccw(p[0], p[mid], e) == 1)
15             l=mid+1;
16         else{
17             r=mid;
18         }
19     }
20     // bordo
21     // if(r==(int)p.size()-1 and ccw(p[0], p[r], e)==0)
22     // return false;
23     // if(r==2 and ccw(p[0], p[1], e)==0) return false;
24     // if(ccw(p[r], p[r-1], e)==0) return false;
25     return insideT(p[0], p[r-1], p[r], e);
26 }

```

```

27 // Any 0(n)
28
29
30 int inside(vp &p, point pp){
31     // 1 - inside / 0 - boundary / -1 - outside
32     int n = p.size();
33     for(int i=0;i<n;i++){
34         int j = (i+1)%n;
35         if(line({p[i], p[j]}).inside_seg(pp))
36             return 0;
37     }
38     int inter = 0;
39     for(int i=0;i<n;i++){
40         int j = (i+1)%n;
41         if(p[i].x <= pp.x and pp.x < p[j].x and ccw(p[i], p[j], pp)==1)
42             inter++; // up
43         else if(p[j].x <= pp.x and pp.x < p[i].x and ccw(p[i], p[j], pp)==-1)
44             inter++; // down
45     }
46
47     if(inter%2==0) return -1; // outside
48     else return 1; // inside
49 }

```

### 10.3 Point Location

```

1
2 int32_t main(){
3     sws;
4
5     int t; cin >> t;
6
7     while(t--){
8
9         int x1, y1, x2, y2, x3, y3; cin >> x1 >> y1 >> x2 >> y2 >> x3 >> y3;
10
11         int deltax1 = (x1-x2), deltax1 = (y1-y2);
12

```

```

13         int compx = (x1-x3), compy = (y1-y3);
14
15         int ans = (deltax1*compy) - (compx*deltay1);
16
17         if(ans == 0){cout << "TOUCH\n"; continue;}
18         if(ans < 0){cout << "RIGHT\n"; continue;}
19         if(ans > 0){cout << "LEFT\n"; continue;}
20     }
21     return 0;
22 }

```

### 10.4 Lattice Points

```

1 ll gcd(ll a, ll b) {
2     return b == 0 ? a : gcd(b, a % b);
3 }
4 ll area_triangulo(ll x1, ll y1, ll x2, ll y2, ll x3, ll y3) {
5     return abs(x1 * (y2 - y3) + x2 * (y3 - y1) + x3 * (y1 - y2));
6 }
7 ll pontos_borda(ll x1, ll y1, ll x2, ll y2) {
8     return gcd(abs(x2 - x1), abs(y2 - y1));
9 }
10
11 int32_t main() {
12     ll x1, y1, x2, y2, x3, y3;
13     cin >> x1 >> y1;
14     cin >> x2 >> y2;
15     cin >> x3 >> y3;
16     ll area = area_triangulo(x1, y1, x2, y2, x3, y3);
17     ll tot_borda = pontos_borda(x1, y1, x2, y2) + pontos_borda(x2, y2, x3, y3) + pontos_borda(x3, y3, x1, y1);
18
19     ll ans = (area - tot_borda) / 2 + 1;
20     cout << ans << endl;
21
22     return 0;
23 }

```