

Competitive Programming Notebook

Programadores Roblox

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1 String

1.1 Trie Ponteiros

```

1 // Trie por ponteiros
2 // Inserir, busca e consulta de prefixo em O(N)
3
4 struct Node {
5     Node *filhos[26] = {};
6     bool acaba = false;
7     int contador = 0;
8 };
9
10 void insere(string s, Node *raiz) {
11     Node *cur = raiz;
12     for(auto &c : s) {
13         cur->contador++;
14         if(cur->filhos[c - 'a'] != NULL) {
15             cur = cur->filhos[c - 'a'];
16             continue;
17         }
18         cur->filhos[c - 'a'] = new Node();
19         cur = cur->filhos[c - 'a'];
20     }
21     cur->contador++;
22     cur->acaba = true;
23 }
24
25 bool busca(string s, Node *raiz) {
26     Node *cur = raiz;
27     for(auto &c : s) {
28         if (cur->filhos[c - 'a'] != NULL) {
29             cur = cur->filhos[c - 'a'];
30             continue;
31         }
32         return false;
33     }
34     return cur->acaba;
35 }
36
37 // Retorna se Ã prefixo e quantas strings tem s como
38 // prefixo
39 int isPref(string s, Node *raiz) {
40     Node *cur = raiz;
41     for(auto &c : s) {
42         if (cur->filhos[c - 'a'] != NULL) {
43             cur = cur->filhos[c - 'a'];
44             continue;
45         }
46         return -1;
47     }
48     return cur->contador;
49 }

```

1.2 Trie

```

1 // Trie por array
2 // Inserir, busca e consulta de prefixo em O(N)
3
4 int trie[MAXN][26];
5 int tot_nos = 0;
6 vector<bool> acaba(MAXN, false);
7 vector<int> contador(MAXN, 0);
8
9 void insere(string s) {
10     int no = 0;
11     for(auto &c : s) {
12         if(trie[no][c - 'a'] == 0) {
13             trie[no][c - 'a'] = ++tot_nos;
14         }
15         no = trie[no][c - 'a'];
16         contador[no]++;

```

```

17     }
18     acaba[no] = true;
19 }
20
21 bool busca(string s) {
22     int no = 0;
23     for(auto &c : s) {
24         if(trie[no][c - 'a'] == 0) {
25             return false;
26         }
27         no = trie[no][c - 'a'];
28     }
29     return acaba[no];
30 }
31
32 int isPref(string s) {
33     int no = 0;
34     for(auto &c : s) {
35         if(trie[no][c - 'a'] == 0){
36             return -1;
37         }
38         no = trie[no][c - 'a'];
39     }
40     return contador[no];
41 }

```

1.3 Hashing

```

1 // String Hash template
2 // constructor(s) - O(|s|)
3 // query(l, r) - returns the hash of the range [l,r]
4 // from left to right - O(1)
5 // query_inv(l, r) from right to left - O(1)
6 // patrocinado por tiagodfs
7
8 struct Hash {
9     const int X = 2147483647;
10    const int MOD = 1e9+7;
11    int n; string s;
12    vector<int> h, hi, p;
13    Hash() {}
14    Hash(string s): s(s), n(s.size()), h(n), hi(n), p
15    (n) {
16        for (int i=0;i<n;i++) p[i] = (i ? X*p[i-1]:1)
17        % MOD;
18        for (int i=0;i<n;i++)
19            h[i] = (s[i] + (i ? h[i-1]:0) * X) % MOD;
20        for (int i=n-1;i>=0;i--)
21            hi[i] = (s[i] + (i+1<n ? hi[i+1]:0) * X)
22            % MOD;
23    }
24    int query(int l, int r) {
25        int hash = (h[r] - (l ? h[l-1]*p[r-l+1]:0) %
26        MOD :
27        0));
28        return hash < 0 ? hash + MOD : hash;
29    }
30    int query_inv(int l, int r) {
31        int hash = (hi[l] - (r+1 < n ? hi[r+1]*p[r-
32        l+1] % MOD : 0));
33        return hash < 0 ? hash + MOD : hash;
34    }
35 };

```

1.4 Lcs

```

1 int lcs(string &s1, string &s2) {
2     int m = s1.size();
3     int n = s2.size();
4
5     vector<vector<int>> dp(m + 1, vector<int>(n + 1,
6     0));

```

```

7   for (int i = 1; i <= m; ++i) {
8       for (int j = 1; j <= n; ++j) {
9           if (s1[i - 1] == s2[j - 1])
10              dp[i][j] = dp[i - 1][j - 1] + 1;
11           else
12              dp[i][j] = max(dp[i - 1][j], dp[i][j
13 - 1]);
14       }
15   }
16   return dp[m][n];
17 }

```

1.5 Countpermutations

```

1 // Returns the number of distinct permutations
2 // that are lexicographically less than the string t
3 // using the provided frequency (freq) of the
4 // characters
5 // 0(n*freq.size())
6 int countPermLess(vector<int> freq, const string &t)
7 {
8     int n = t.size();
9     int ans = 0;
10
11     vector<int> fact(n + 1, 1), invfact(n + 1, 1);
12     for (int i = 1; i <= n; i++)
13         fact[i] = (fact[i - 1] * i) % MOD;
14     invfact[n] = fexp(fact[n], MOD - 2, MOD);
15     for (int i = n - 1; i >= 0; i--)
16         invfact[i] = (invfact[i + 1] * (i + 1)) % MOD;
17
18     // For each position in t, try placing a letter
19     // smaller than t[i] that is in freq
20     for (int i = 0; i < n; i++) {
21         for (char c = 'a'; c < t[i]; c++) {
22             if (freq[c - 'a'] > 0) {
23                 freq[c - 'a']--;
24                 int ways = fact[n - i - 1];
25                 for (int f : freq)
26                     ways = (ways * invfact[f]) % MOD;
27                 ans = (ans + ways) % MOD;
28                 freq[c - 'a']++;
29             }
30         }
31         if (freq[t[i] - 'a'] == 0) break;
32         freq[t[i] - 'a']--;
33     }
34     return ans;
35 }

```

1.6 Z Function

```

1 vector<int> z_function(string s) {
2     int n = s.size();
3     vector<int> z(n);
4     int l = 0, r = 0;
5     for (int i = 1; i < n; i++) {
6         if (i < r) {
7             z[i] = min(r - i, z[i - 1]);
8         }
9         while (i + z[i] < n && s[z[i]] == s[i + z[i]])
10             z[i]++;
11     }
12     if (i + z[i] > r) {
13         l = i;
14         r = i + z[i];
15     }
16 }
17 return z;

```

```

18 }

```

1.7 Kmp

```

1 vector<int> kmp(string s) {
2     int n = (int)s.length();
3     vector<int> p(n+1);
4     p[0] = -1;
5     for (int i = 1; i < n; i++) {
6         int j = p[i-1];
7         while (j >= 0 && s[j] != s[i-1])
8             j = p[j-1];
9         p[i] = j+1;
10    }
11    return p;
12 }

```

2 String copy

2.1 Trie Ponteiros

```

1 // Trie por ponteiros
2 // Inserir e buscar prefixo, busca e consulta de prefixo em O(N)
3
4 struct Node {
5     Node *filhos[26] = {};
6     bool acaba = false;
7     int contador = 0;
8 };
9
10 void insere(string s, Node *raiz) {
11     Node *cur = raiz;
12     for (auto &c : s) {
13         cur->contador++;
14         if (cur->filhos[c - 'a'] != NULL) {
15             cur = cur->filhos[c - 'a'];
16             continue;
17         }
18         cur->filhos[c - 'a'] = new Node();
19         cur = cur->filhos[c - 'a'];
20     }
21     cur->contador++;
22     cur->acaba = true;
23 }
24
25 bool busca(string s, Node *raiz) {
26     Node *cur = raiz;
27     for (auto &c : s) {
28         if (cur->filhos[c - 'a'] != NULL) {
29             cur = cur->filhos[c - 'a'];
30             continue;
31         }
32         return false;
33     }
34     return cur->acaba;
35 }
36
37 // Retorna se Ãl prefixo e quantas strings tem s como
38 // prefixo
39 int isPref(string s, Node *raiz) {
40     Node *cur = raiz;
41     for (auto &c : s) {
42         if (cur->filhos[c - 'a'] != NULL) {
43             cur = cur->filhos[c - 'a'];
44             continue;
45         }
46         return -1;
47     }
48     return cur->contador;
49 }

```

2.2 Hashing

```

1 // String Hash template
2 // constructor(s) - O(|s|)
3 // query(l, r) - returns the hash of the range [l,r]
  // from left to right - O(1)
4 // query_inv(l, r) from right to left - O(1)
5 // patrocinado por tiagodfs
6
7 mt19937 rng(time(nullptr));
8
9 struct Hash {
10     const int X = rng();
11     const int MOD = 1e9+7;
12     int n; string s;
13     vector<int> h, hi, p;
14     Hash() {}
15     Hash(string s): s(s), n(s.size()), h(n), hi(n), p
      (n) {
16         for (int i=0;i<n;i++) p[i] = (i ? X*p[i-1]:1)
          % MOD;
17         for (int i=0;i<n;i++)
18             h[i] = (s[i] + (i ? h[i-1]:0) * X) % MOD;
19         for (int i=n-1;i>=0;i--)
20             hi[i] = (s[i] + (i+1<n ? hi[i+1]:0) * X)
              % MOD;
21     }
22     int query(int l, int r) {
23         int hash = (h[r] - (l ? h[l-1]*p[r-l+1]%MOD :
          0));
24         return hash < 0 ? hash + MOD : hash;
25     }
26     int query_inv(int l, int r) {
27         int hash = (hi[l] - (r+1 < n ? hi[r+1]*p[r-l
          +1] % MOD : 0));
28         return hash < 0 ? hash + MOD : hash;
29     }
30 };

```

2.3 Lcs

```

1 int lcs(string &s1, string &s2) {
2     int m = s1.size();
3     int n = s2.size();
4
5     vector<vector<int>> dp(m + 1, vector<int>(n + 1,
      0));
6
7     for (int i = 1; i <= m; ++i) {
8         for (int j = 1; j <= n; ++j) {
9             if (s1[i - 1] == s2[j - 1])
10                 dp[i][j] = dp[i - 1][j - 1] + 1;
11             else
12                 dp[i][j] = max(dp[i - 1][j], dp[i][j
          - 1]);
13         }
14     }
15
16     return dp[m][n];
17 }

```

2.4 Countpermutations

```

1 // Returns the number of distinct permutations
2 // that are lexicographically less than the string t
3 // using the provided frequency (freq) of the
  // characters
4 // O(n*freq.size())
5 int countPermLess(vector<int> freq, const string &t)
  {
6     int n = t.size();
7     int ans = 0;

```

```

8
9     vector<int> fact(n + 1, 1), invfact(n + 1, 1);
10    for (int i = 1; i <= n; i++)
11        fact[i] = (fact[i - 1] * i) % MOD;
12    invfact[n] = fexp(fact[n], MOD - 2, MOD);
13    for (int i = n - 1; i >= 0; i--)
14        invfact[i] = (invfact[i + 1] * (i + 1)) % MOD
      ;
15
16    // For each position in t, try placing a letter
  // smaller than t[i] that is in freq
17    for (int i = 0; i < n; i++) {
18        for (char c = 'a'; c < t[i]; c++) {
19            if (freq[c - 'a'] > 0) {
20                freq[c - 'a']--;
21                int ways = fact[n - i - 1];
22                for (int f : freq)
23                    ways = (ways * invfact[f]) % MOD;
24                ans = (ans + ways) % MOD;
25                freq[c - 'a']++;
26            }
27        }
28        if (freq[t[i] - 'a'] == 0) break;
29        freq[t[i] - 'a']--;
30    }
31    return ans;
32 }

```

2.5 Z Function

```

1 vector<int> z_function(string s) {
2     int n = s.size();
3     vector<int> z(n);
4     int l = 0, r = 0;
5     for(int i = 1; i < n; i++) {
6         if(i < r) {
7             z[i] = min(r - i, z[i - 1]);
8         }
9         while(i + z[i] < n && s[z[i]] == s[i + z[i]])
10             z[i]++;
11     }
12     if(i + z[i] > r) {
13         l = i;
14         r = i + z[i];
15     }
16 }
17 return z;
18 }

```

2.6 Kmp

```

1 vector<int> kmp(string s) {
2     int n = (int)s.length();
3     vector<int> p(n+1);
4     p[0] = -1;
5     for (int i = 1; i < n; i++) {
6         int j = p[i-1];
7         while (j >= 0 && s[j] != s[i-1])
8             j = p[j-1];
9         p[i] = j+1;
10    }
11    return p;
12 }

```

3 DS

3.1 Segtree Iterativa

```

1 // Exemplo de uso:
2 // SegTree<int> st(vetor);

```

```

3 // range query e point update
4
5 template <typename T>
6 struct SegTree {
7     int n;
8     vector<T> tree;
9     T neutral_value = 0;
10    T combine(T a, T b) {
11        return a + b;
12    }
13
14    SegTree(const vector<T>& data) {
15        n = data.size();
16        tree.resize(2 * n, neutral_value);
17
18        for (int i = 0; i < n; i++)
19            tree[n + i] = data[i];
20
21        for (int i = n - 1; i > 0; --i)
22            tree[i] = combine(tree[i * 2], tree[i * 2
23 + 1]);
24    }
25
26    T range_query(int l, int r) {
27        T res_l = neutral_value, res_r =
28        neutral_value;
29
30        for (l += n, r += n + 1; l < r; l >>= 1, r
31 >>= 1) {
32            if (l & 1) res_l = combine(res_l, tree[l
33 ++]);
34            if (r & 1) res_r = combine(tree[--r],
35 res_r);
36        }
37
38        return combine(res_l, res_r);
39    }
40
41    void update(int pos, T new_val) {
42        tree[pos += n] = new_val;
43
44        for (pos >>= 1; pos > 0; pos >>= 1)
45            tree[pos] = combine(tree[2 * pos], tree[2
46 * pos + 1]);
47    }
48 };

```

3.2 Segtree Gcd

```

1 int gcd(int a, int b) {
2     if (b == 0)
3         return a;
4     return gcd(b, a % b);
5 }
6
7 class SegmentTreeGCD {
8 private:
9     vector<int> tree;
10    int n;
11
12    void build(const vector<int>& arr, int node, int
13 start, int end) {
14        if (start == end) {
15            tree[node] = arr[start];
16        } else {
17            int mid = (start + end) / 2;
18            build(arr, 2 * node + 1, start, mid);
19            build(arr, 2 * node + 2, mid + 1, end);
20            tree[node] = gcd(tree[2 * node + 1], tree
21 [2 * node + 2]);
22        }
23    }
24 }

```

```

23 void update(int node, int start, int end, int idx
24 , int value) {
25     if (start == end) {
26         tree[node] = value;
27     } else {
28         int mid = (start + end) / 2;
29         if (idx <= mid) {
30             update(2 * node + 1, start, mid, idx,
31 value);
32         } else {
33             update(2 * node + 2, mid + 1, end,
34 idx, value);
35         }
36         tree[node] = gcd(tree[2 * node + 1], tree
37 [2 * node + 2]);
38     }
39 }
40
41 int query(int node, int start, int end, int l,
42 int r) {
43     if (r < start || l > end) {
44         return 0;
45     }
46     if (l <= start && end <= r) {
47         return tree[node];
48     }
49     int mid = (start + end) / 2;
50     int left_gcd = query(2 * node + 1, start, mid
51 , l, r);
52     int right_gcd = query(2 * node + 2, mid + 1,
53 end, l, r);
54     return gcd(left_gcd, right_gcd);
55 }
56
57 public:
58 SegmentTreeGCD(const vector<int>& arr) {
59     n = arr.size();
60     tree.resize(4 * n);
61     build(arr, 0, 0, n - 1);
62 }
63
64 void update(int idx, int value) {
65     update(0, 0, n - 1, idx, value);
66 }
67
68 int query(int l, int r) {
69     return query(0, 0, n - 1, l, r);
70 }
71 };

```

3.3 Merge Sort Tree

```

1 struct SegTree {
2     int n;
3     vector<vector<int>> tree;
4
5     SegTree(vector<int> &a) {
6         n = a.size();
7         tree.resize(4 * n);
8         build(1, 0, n - 1, a);
9     }
10
11    void build(int x, int lx, int rx, vector<int> &a)
12    {
13        if (lx == rx) {
14            tree[x] = { a[lx] };
15            return;
16        }
17        int mid = lx + (rx - lx)/2;
18        build(2 * x, lx, mid, a);
19        build(2 * x + 1, mid + 1, rx, a);
20        auto &L = tree[2 * x], &R = tree[2 * x + 1];
21        tree[x].resize(L.size() + R.size());
22        merge(L.begin(), L.end(), R.begin(), R.end(),
23 tree[x].begin());

```

```

22     }
23
24     int query(int x, int lx, int rx, int l, int r) {
25         if (lx >= l && rx <= r) {
26             auto &v = tree[x];
27             return v.end() - upper_bound(v.begin(), v
28 .end(), r);
29         }
30         if (rx < l || lx > r) {
31             return 0;
32         }
33         int mid = lx + (rx - lx)/2;
34         return query(2 * x, lx, mid, l, r) + query(2
35 * x + 1, mid + 1, rx, l, r);
36     }
37
38     int query(int l, int r) {
39         return query(1, 0, n - 1, l, r);
40     }
41 }
42
43 // Checar se o range Ã todo distinto
44 // Cada cara e sua prÃxima apariÃÃo a direita,
45 // conta quantos caras que a prÃxima apariÃÃo a
46 // direita ta dentro do range ainda
47
48 vector<int> nr(n);
49 map<int, int> mp;
50 for (int i = n - 1; i >= 0; i--) {
51     auto it = mp.find(a[i]);
52     nr[i] = it != mp.end() ? it->second : n;
53     mp[a[i]] = i;
54 }
55 SegTree seg(nr);

```

3.4 Ordered Set E Map

```

1
2 #include<ext/pb_ds/assoc_container.hpp>
3 #include<ext/pb_ds/tree_policy.hpp>
4 using namespace __gnu_pbds;
5 using namespace std;
6
7 template<typename T> using ordered_multiset = tree<T,
8     null_type, less_equal<T>, rb_tree_tag,
9     tree_order_statistics_node_update>;
10
11 template<typename T> using o_set = tree<T, null_type
12     , less<T>, rb_tree_tag,
13     tree_order_statistics_node_update>;
14
15 template<typename T, typename R> using o_map = tree<
16     T, R, less<T>, rb_tree_tag,
17     tree_order_statistics_node_update>;
18
19 int main() {
20     int i, j, k, n, m;
21     o_set<int> st;
22     st.insert(1);
23     st.insert(2);
24     cout << *st.find_by_order(0) << endl; /// k-esimo
25     elemento
26     cout << st.order_of_key(2) << endl; /// numero de
27     elementos menores que k
28     o_map<int, int> mp;
29     mp.insert({1, 10});
30     mp.insert({2, 20});
31     cout << mp.find_by_order(0)->second << endl; /// k-
32     esimo elemento
33     cout << mp.order_of_key(2) << endl; /// numero de
34     elementos (chave) menores que k
35     return 0;
36 }

```

3.5 Sparse Table

```

1 // 1-index, 0(1)
2 struct SparseTable {
3     vector<vector<int>> st;
4     int max_log;
5     SparseTable(vector<int>& arr) {
6         int n = arr.size();
7         max_log = floor(log2(n)) + 1;
8         st.resize(n, vector<int>(max_log));
9         for (int i = 0; i < n; i++) {
10             st[i][0] = arr[i];
11         }
12         for (int j = 1; j < max_log; j++) {
13             for (int i = 0; i + (1 << j) <= n; i++) {
14                 st[i][j] = max(st[i][j - 1], st[i +
15 (1 << (j - 1))][j - 1]);
16             }
17         }
18     }
19     int query(int L, int R) {
20         int tamanho = R - L + 1;
21         int k = floor(log2(tamanho));
22         return max(st[L][k], st[R - (1 << k) + 1][k]);
23     };
24 };

```

3.6 Psum 2d

```

1 vector<vector<int>> psum(h+1, vector<int>(w+1, 0));
2
3 for (int i=1; i<=h; i++){
4     for (int j=1; j<=w; j++){
5         cin >> psum[i][j];
6         psum[i][j] += psum[i-1][j]+psum[i][j-1]-psum[
7 i-1][j-1];
8     }
9 }
10 // retorna a psum2d do intervalo inclusivo [(a, b), (
11 c, d)]
12 int retangulo(int a, int b, int c, int d){
13     c = min(c, h), d = min(d, w);
14     a = max(0LL, a-1), b = max(0LL, b-1);
15     return v[c][d]-v[a][d]-v[c][b]+v[a][b];
16 }

```

3.7 Segtree Sum

```

1 struct SegTree {
2     ll merge(ll a, ll b) { return a + b; }
3     const ll neutral = 0;
4     int n;
5     vector<ll> t, lazy;
6     vector<bool> replace;
7     inline int lc(int p) { return p * 2; }
8     inline int rc(int p) { return p * 2 + 1; }
9     void push(int p, int l, int r) {
10         if (replace[p]) {
11             t[p] = lazy[p] * (r - l + 1);
12             if (l != r) {
13                 lazy[lc(p)] = lazy[p];
14                 lazy[rc(p)] = lazy[p];
15                 replace[lc(p)] = true;
16                 replace[rc(p)] = true;
17             }
18         } else if (lazy[p] != 0) {
19             t[p] += lazy[p] * (r - l + 1);
20             if (l != r) {
21                 lazy[lc(p)] += lazy[p];
22                 lazy[rc(p)] += lazy[p];
23             }
24         }
25     }
26 };

```

```

24     }
25     replace[p] = false;
26     lazy[p] = 0;
27 }
28 void build(int p, int l, int r, const vector<ll>
&v) {
29     if (l == r) {
30         t[p] = v[l];
31     } else {
32         int mid = (l + r) / 2;
33         build(lc(p), l, mid, v);
34         build(rc(p), mid + 1, r, v);
35         t[p] = merge(t[lc(p)], t[rc(p)]);
36     }
37 }
38 void build(int _n) {
39     n = _n;
40     t.assign(n * 4, neutral);
41     lazy.assign(n * 4, 0);
42     replace.assign(n * 4, false);
43 }
44 void build(const vector<ll> &v) {
45     n = (int)v.size();
46     t.assign(n * 4, neutral);
47     lazy.assign(n * 4, 0);
48     replace.assign(n * 4, false);
49     build(1, 0, n - 1, v);
50 }
51 void build(ll *bg, ll *en) {
52     build(vector<ll>(bg, en));
53 }
54 ll query(int p, int l, int r, int L, int R) {
55     push(p, l, r);
56     if (l > R || r < L) return neutral;
57     if (l >= L && r <= R) return t[p];
58     int mid = (l + r) / 2;
59     auto ql = query(lc(p), l, mid, L, R);
60     auto qr = query(rc(p), mid + 1, r, L, R);
61     return merge(ql, qr);
62 }
63 ll query(int l, int r) { return query(1, 0, n -
1, l, r); }
64 void update(int p, int l, int r, int L, int R, ll
val, bool repl = 0) {
65     push(p, l, r);
66     if (l > R || r < L) return;
67     if (l >= L && r <= R) {
68         lazy[p] = val;
69         replace[p] = repl;
70         push(p, l, r);
71     } else {
72         int mid = (l + r) / 2;
73         update(lc(p), l, mid, L, R, val, repl);
74         update(rc(p), mid + 1, r, L, R, val, repl);
75     }
76     t[p] = merge(t[lc(p)], t[rc(p)]);
77 }
78 void sumUpdate(int l, int r, ll val) { update(1,
0, n - 1, l, r, val, 0); }
79 void assignUpdate(int l, int r, ll val) { update
(1, 0, n - 1, l, r, val, 1); }
80 } segsum;

```

3.8 Dsu

```

1 struct DSU {
2     vector<int> par, rank, sz;
3     int c;
4     DSU(int n) : par(n + 1), rank(n + 1, 0), sz(n +
1, 1), c(n) {
5         for (int i = 1; i <= n; ++i) par[i] = i;
6     }

```

```

7     int find(int i) {
8         return (par[i] == i ? i : (par[i] = find(par[
i])));
9     }
10    bool same(int i, int j) {
11        return find(i) == find(j);
12    }
13    int get_size(int i) {
14        return sz[find(i)];
15    }
16    int count() {
17        return c; // quantos componentes conexos
18    }
19    int merge(int i, int j) {
20        if ((i = find(i)) == (j = find(j))) return
-1;
21        else --c;
22        if (rank[i] > rank[j]) swap(i, j);
23        par[i] = j;
24        sz[j] += sz[i];
25        if (rank[i] == rank[j]) rank[j]++;
26        return j;
27    }
28 };

```

3.9 Bit

```

1 class BIT {
2     vector<int> bit;
3     int n;
4     int sum(int idx) {
5         int result = 0;
6         while (idx > 0) {
7             result += bit[idx];
8             idx -= idx & -idx;
9         }
10        return result;
11    }
12
13 public:
14     BIT(int size) {
15         n = size;
16         bit.assign(n + 1, 0); // BIT indexada em 1
17     }
18     void update(int idx, int delta) {
19         while (idx <= n) {
20             bit[idx] += delta;
21             idx += idx & -idx;
22         }
23     }
24     int query(int idx) {
25         return sum(idx);
26     }
27     int range_query(int l, int r) {
28         return sum(r) - sum(l - 1);
29     }
30 };
31
32 BIT fenwick(n);
33 for(int i = 1; i <= n; i++) {
34     fenwick.update(i, arr[i]);
35 }

```

4 Search and sort

4.1 Pilha Monotonic

```

1 vector<int> find_esq(vector<int> &v, bool maior) {
2     int n = v.size();
3     vector<int> result(n);
4     stack<int> s;

```

```

5
6 for (int i = 0; i < n; i++) {
7     while (!s.empty() && (maior ? v[s.top()] <= v
8 [i] : v[s.top()] >= v[i])) {
9         s.pop();
10    }
11    if (s.empty()) {
12        result[i] = -1;
13    } else {
14        result[i] = v[s.top()];
15    }
16    s.push(i);
17 }
18 return result;
19 }
20 // maior = true -> encontra o primeiro maior à
21 // direita
22 vector<int> find_dir(vector<int> &v, bool maior) {
23     int n = v.size();
24     vector<int> result(n);
25     stack<int> s;
26     for (int i = n - 1; i >= 0; i--) {
27         while (!s.empty() && (maior ? v[s.top()] <= v
28 [i] : v[s.top()] >= v[i])) {
29             s.pop();
30         }
31         if (s.empty()) {
32             result[i] = -1;
33         } else {
34             result[i] = v[s.top()];
35         }
36         s.push(i);
37     }
38     return result;
39 }

```

4.2 Mergeandcount

```

1
2 // Realiza a mesclagem de dois subarrays e conta o
3 // número de trocas necessárias.
4 int mergeAndCount(vector<int> &v, int l, int m, int r
5 ) {
6     int x = m - 1 + 1; // Tamanho do subarray
7     // esquerdo.
8     int y = r - m; // Tamanho do subarray direito.
9
10    // Vetores temporários para os subarray esquerdo
11    // e direito.
12    vector<int> left(x), right(y);
13
14    for (int i = 0; i < x; i++) left[i] = v[l + i];
15    for (int j = 0; j < y; j++) right[j] = v[m + 1 +
16 j];
17
18    int i = 0, j = 0, k = 1;
19    int swaps = 0;
20
21    while (i < x && j < y) {
22        if (left[i] <= right[j]) {
23            // Se o elemento da esquerda for menor ou
24            // igual, coloca no vetor original.
25            v[k++] = left[i++];
26        } else {
27            // Caso contrário, coloca o elemento da
28            // direita e conta as trocas.
29            v[k++] = right[j++];
30            swaps += (x - i);
31        }
32    }
33 }

```

```

27 // Adiciona os elementos restantes do subarray
28 // esquerdo (se houver).
29 while (i < x) v[k++] = left[i++];
30
31 // Adiciona os elementos restantes do subarray
32 // direito (se houver).
33 while (j < y) v[k++] = right[j++];
34
35 return swaps; // Retorna o numero total de
36 // trocas realizadas.
37 }
38
39 int mergeSort(vector<int> &v, int l, int r) {
40     int swaps = 0;
41
42     if (l < r) {
43         // Encontra o ponto medio para dividir o
44         // vetor.
45         int m = l + (r - l) / 2;
46
47         // Chama merge sort para a metade esquerda.
48         swaps += mergeSort(v, l, m);
49         // Chama merge sort para a metade direita.
50         swaps += mergeSort(v, m + 1, r);
51
52         // Mescla as duas metades e conta as trocas.
53         swaps += mergeAndCount(v, l, m, r);
54     }
55
56     return swaps; // Retorna o numero total de
57     // trocas no vetor.
58 }

```

5 Math

5.1 Combinatorics

```

1 const int MAXN_FATORIAL = 200005;
2 const int MOD = 1e9 + 7;
3 // DEFINE INT LONG LONG PLMDS
4 int fat[MAXN_FATORIAL], fati[MAXN_FATORIAL];
5
6 // (a^b) % m em O(log b)
7 // coloque o fexp
8
9 int inv(int n) { return fexp(n, MOD - 2); }
10
11 void precalc() {
12     fat[0] = 1;
13     fati[0] = 1;
14     for (int i = 1; i < MAXN_FATORIAL; i++) fat[i] =
15     (fat[i - 1] * i) % MOD;
16     fati[MAXN_FATORIAL - 1] = inv(fat[MAXN_FATORIAL -
17 1]);
18     for (int i = MAXN_FATORIAL - 2; i >= 0; i--) fati
19     [i] = (fati[i + 1] * (i + 1)) % MOD;
20 }
21
22 int choose(int n, int k) {
23     if (k < 0 || k > n) return 0;
24     return ((fat[n] * fati[k]) % MOD) * fati[n - k]
25     % MOD;
26 }
27
28 // n! / (n-k)!
29 int perm(int n, int k) {
30     if (k < 0 || k > n) return 0;
31     return (fat[n] * fati[n - k]) % MOD;
32 }
33
34 // C_n = (1 / (n+1)) * C(2n, n)
35 int catalan(int n) {

```



```

32     if (n < 0 || 2 * n >= MAXN_FATORIAL) return 0;
33     int c2n_n = choose(2 * n, n);
34     return (c2n_n * inv(n + 1)) % MOD;
35 }

```

5.2 Equacao Diofantina

```

1  int extended_gcd(int a, int b, int& x, int& y) {
2      if (a == 0) {
3          x = 0;
4          y = 1;
5          return b;
6      }
7      int x1, y1;
8      int gcd = extended_gcd(b % a, a, x1, y1);
9      x = y1 - (b / a) * x1;
10     y = x1;
11     return gcd;
12 }
13
14 bool solve(int a, int b, int c, int& x0, int& y0) {
15     int x, y;
16     int g = extended_gcd(abs(a), abs(b), x, y);
17     if (c % g != 0) {
18         return false;
19     }
20     x0 = x * (c / g);
21     y0 = y * (c / g);
22     if (a < 0) x0 = -x0;
23     if (b < 0) y0 = -y0;
24     return true;
25 }

```

5.3 Discrete Log

```

1  // Returns minimum x for which a^x = b (mod m), a and
2  // m are coprime.
3  // if the answer dont need to be greater than some
4  // value, the vector<int> can be removed
5  int discrete_log(int a, int b, int m) {
6      a %= m, b %= m;
7      int n = sqrt(m) + 1;
8
9      int an = 1;
10     for (int i = 0; i < n; ++i)
11         an = (an * 111 * a) % m;
12
13     unordered_map<int, vector<int>> vals;
14     for (int q = 0, cur = b; q <= n; ++q) {
15         vals[cur].push_back(q);
16         cur = (cur * 111 * a) % m;
17     }
18
19     int res = LLONG_MAX;
20
21     for (int p = 1, cur = 1; p <= n; ++p) {
22         cur = (cur * 111 * an) % m;
23         if (vals.count(cur)) {
24             for (int q: vals[cur]) {
25                 int ans = n * p - q;
26                 res = min(res, ans);
27             }
28         }
29     }
30     return res;
31 }

```

5.4 Segment Sieve

```

1  // Retorna quantos primos tem entre [l, r] (inclusivo
2  // precisa de um vetor com os primos atÃ sqrt(r)

```

```

3  int seg_sieve(int l, int r){
4      if (l > r) return 0;
5      vector<bool> is_prime(r - l + 1, true);
6      if (l == 1) is_prime[0] = false;
7
8      for (int p : primos){
9          if (p * p > r) break;
10         int start = max(p * p, (l + p - 1) / p * p);
11         for (int j = start; j <= r; j += p){
12             if (j >= 1) {
13                 is_prime[j - l] = false;
14             }
15         }
16     }
17
18     return accumulate(all(is_prime), 0ll);
19 }

```

5.5 Totient

```

1  // phi(n) = n * (1 - 1/p1) * (1 - 1/p2) * ...
2  int phi(int n) {
3      int result = n;
4      for (int i = 2; i * i <= n; i++) {
5          if (n % i == 0) {
6              while (n % i == 0)
7                  n /= i;
8              result -= result / i;
9          }
10     }
11     if (n > 1) // SE n sobrou, ele Ã um fator primo
12         result -= result / n;
13     return result;
14 }
15
16 // crivo phi
17 const int MAXN_PHI = 1000001;
18 int phiv[MAXN_PHI];
19 void phi_sieve() {
20     for (int i = 0; i < MAXN_PHI; i++) phiv[i] = i;
21     for (int i = 2; i < MAXN_PHI; i++) {
22         if (phiv[i] == i) {
23             for (int j = i; j < MAXN_PHI; j += i)
24                 phiv[j] -= phiv[j] / i;
25         }
26     }
27 }

```

5.6 Menor Fator Primo

```

1  const int MAXN = 1000001; // Limite para o Crivo.
2  int spf[MAXN];
3  vector<int> primos;
4
5  void crivo() {
6      for (int i = 2; i * i < MAXN; i++) {
7          if (spf[i] == i) {
8              for (int j = i * i; j < MAXN; j += i) {
9                  if (spf[j] == j) {
10                     spf[j] = i;
11                 }
12             }
13         }
14     }
15     for (int i = 2; i < MAXN; i++) {
16         if (spf[i] == i) {
17             primos.push_back(i);
18         }
19     }
20 }
21
22 map<int, int> fatora(int n) {

```

```

23     map<int, int> fatores;
24     while (n > 1) {
25         fatores[spf[n]]++;
26         n /= spf[n];
27     }
28     return fatores;
29 }
30
31 int numero_de_divisores(int n) {
32     if (n == 1) return 1;
33     map<int, int> fatores = fatorar(n);
34     int nod = 1;
35     for (auto &[primo, expoente] : fatores) nod *= (
36         expoente + 1);
37     return nod;
38 }
39 // DEFINE INT LONG LONG
40 int soma_dos_divisores(int n) {
41     if (n == 1) return 1;
42     map<int, int> fatores = fatorar(n);
43     int sod = 1;
44     for (auto &[primo, expoente] : fatores) {
45         int termo_soma = 1;
46         int potencia_primo = 1;
47         for (int i = 0; i < expoente; i++) {
48             potencia_primo *= primo;
49             termo_soma += potencia_primo;
50         }
51         sod *= termo_soma;
52     }
53     return sod;
54 }

```

5.7 Exgcd

```

1 // 0 retorno da funcao eh {n, m, g}
2 // e significa que gcd(a, b) = g e
3 // n e m sao inteiros tais que an + bm = g
4 array<ll, 3> exgcd(int a, int b) {
5     if (b == 0) return {1, 0, a};
6     auto [m, n, g] = exgcd(b, a % b);
7     return {n, m - a / b * n, g};
8 }

```

5.8 Fexp

```

1 // a^e mod m
2 // 0(log n)
3
4 int fexp(int a, int e, int m) {
5     a %= m;
6     int ans = 1;
7     while (e > 0) {
8         if (e & 1) ans = ans * a % m;
9         a = a * a % m;
10        e /= 2;
11    }
12    return ans % m;
13 }

```

5.9 Divisores

```

1 // Retorna um vetor com os divisores de x
2 // eh preciso ter o crivo implementado
3 // 0(divisores)
4
5 vector<int> divs(int x) {
6     vector<int> ans = {1};
7     vector<array<int, 2>> primos; // {primo, expoente}
8 }

```

```

9     while (x > 1) {
10        int p = crivo[x], cnt = 0;
11        while (x % p == 0) cnt++, x /= p;
12        primos.push_back({p, cnt});
13    }
14
15    for (int i=0; i<primos.size(); i++){
16        int cur = 1, len = ans.size();
17
18        for (int j=0; j<primos[i][1]; j++){
19            cur *= primos[i][0];
20            for (int k=0; k<len; k++){
21                ans.push_back(cur*ans[k]);
22            }
23        }
24    }
25    return ans;
26 }

```

5.10 Crivo

```

1 // 0(n*log(log(n)))
2 bool composto[MAX]
3 for(int i = 1; i <= n; i++) {
4     if(composto[i]) continue;
5     for(int j = 2*i; j <= n; j += i)
6         composto[j] = 1;
7 }

```

5.11 Mod Inverse

```

1 array<int, 2> extended_gcd(int a, int b) {
2     if (b == 0) return {1, 0};
3     auto [x, y] = extended_gcd(b, a % b);
4     return {y, x - (a / b) * y};
5 }
6
7 int mod_inverse(int a, int m) {
8     auto [x, y] = extended_gcd(a, m);
9     return (x % m + m) % m;
10 }

```

5.12 Base Calc

```

1 int char_to_val(char c) {
2     if (c >= '0' && c <= '9') return c - '0';
3     else return c - 'A' + 10;
4 }
5
6 char val_to_char(int val) {
7     if (val >= 0 && val <= 9) return val + '0';
8     else return val - 10 + 'A';
9 }
10
11 int to_base_10(string &num, int bfrom) {
12     int result = 0;
13     int pot = 1;
14     for (int i = num.size() - 1; i >= 0; i--) {
15         if (char_to_val(num[i]) >= bfrom) return -1;
16         result += char_to_val(num[i]) * pot;
17         pot *= bfrom;
18     }
19     return result;
20 }
21
22 string from_base_10(int n, int bto) {
23     if (n == 0) return "0";
24     string result = "";
25     while (n > 0) {
26         result += val_to_char(n % bto);
27         n /= bto;
28     }

```

```

29     reverse(result.begin(), result.end());
30     return result;
31 }
32
33 string convert_base(string &num, int bfrom, int bto)
34 {
35     int n_base_10 = to_base_10(num, bfrom);
36     return from_base_10(n_base_10, bto);
37 }

```

6 Graph

6.1 Dijkstra

```

1 // SSP com pesos positivos.
2 // O((V + E) log V).
3
4 vector<int> dijkstra(int S) {
5     vector<bool> vis(MAXN, 0);
6     vector<ll> dist(MAXN, LLONG_MAX);
7     dist[S] = 0;
8     priority_queue<pii, vector<pii>, greater<pii>> pq
9     ;
10    pq.push({0, S});
11    while(pq.size()) {
12        ll v = pq.top().second;
13        pq.pop();
14        if(vis[v]) continue;
15        vis[v] = 1;
16        for(auto &[peso, vizinho] : adj[v]) {
17            if(dist[vizinho] > dist[v] + peso) {
18                dist[vizinho] = dist[v] + peso;
19                pq.push({dist[vizinho], vizinho});
20            }
21        }
22    }
23    return dist;
24 }

```

6.2 Floyd Warshall

```

1 // SSP e acha ciclos.
2 // Bom com constraints menores.
3 // O(n^3)
4
5 int dist[501][501];
6
7 void floydWarshall() {
8     for(int k = 0; k < n; k++) {
9         for(int i = 0; i < n; i++) {
10             for(int j = 0; j < n; j++) {
11                 dist[i][j] = min(dist[i][j], dist[i][k]
12                 + dist[k][j]);
13             }
14         }
15     }
16 }
17
18 void solve() {
19     int m, q;
20     cin >> n >> m >> q;
21     for(int i = 0; i < n; i++) {
22         for(int j = i; j < n; j++) {
23             if(i == j) {
24                 dist[i][j] = dist[j][i] = 0;
25             } else {
26                 dist[i][j] = dist[j][i] = linf;
27             }
28         }
29     }
30     for(int i = 0; i < m; i++) {
31         int u, v, w;

```

```

30         cin >> u >> v >> w; u--; v--;
31         dist[u][v] = min(dist[u][v], w);
32         dist[v][u] = min(dist[v][u], w);
33     }
34     floydWarshall();
35     while(q--) {
36         int u, v;
37         cin >> u >> v; u--; v--;
38         if(dist[u][v] == linf) cout << -1 << '\n';
39         else cout << dist[u][v] << '\n';
40     }
41 }

```

6.3 Eulerian Path

```

1 /**
2  * Versão que assume: #define int long long
3  *
4  * Retorna um caminho/ciclo euleriano em um grafo (se
5  * existir).
6  * - g: lista de adjacência (vector<vector<int>>).
7  * - directed: true se o grafo for dirigido.
8  * - s: vértice inicial.
9  * - e: vértice final (opcional). Se informado,
10   tenta caminho de s até e.
11   * - O(Nlog(N))
12   * Retorna vetor com a sequência de vértices, ou
13   vazio se impossível.
14   */
15 vector<int> eulerian_path(const vector<vector<int>>&
16 g, bool directed, int s, int e = -1) {
17     int n = (int)g.size();
18     // cãpia das adjacências em multiset para
19     permitir remoção específica
20     vector<multiset<int>> h(n);
21     vector<int> in_degree(n, 0);
22     vector<int> result;
23     stack<int> st;
24     // preencher h e indegrees
25     for (int u = 0; u < n; ++u) {
26         for (auto v : g[u]) {
27             ++in_degree[v];
28             h[u].emplace(v);
29         }
30     }
31     st.emplace(s);
32     if (e != -1) {
33         int out_s = (int)h[s].size();
34         int out_e = (int)h[e].size();
35         int diff_s = in_degree[s] - out_s;
36         int diff_e = in_degree[e] - out_e;
37         if (diff_s * diff_e != -1) return {}; //
38         impossível
39     }
40     for (int u = 0; u < n; ++u) {
41         if (e != -1 && (u == s || u == e)) continue;
42         int out_u = (int)h[u].size();
43         if (in_degree[u] != out_u || (!directed && (
44         in_degree[u] & 1))) {
45             return {};
46         }
47     }
48     while (!st.empty()) {
49         int u = st.top();
50         if (h[u].empty()) {
51             result.emplace_back(u);
52             st.pop();
53         } else {
54             int v = *h[u].begin();
55             auto it = h[u].find(v);
56             if (it != h[u].end()) h[u].erase(it);
57             --in_degree[v];
58             if (!directed) {

```

```

52         auto it2 = h[v].find(u);
53         if (it2 != h[v].end()) h[v].erase(it2);
54     };
55     --in_degree[u];
56     }
57     st.emplace(v);
58 }
59 for (int u = 0; u < n; ++u) {
60     if (in_degree[u] != 0) return {};
61 }
62 reverse(result.begin(), result.end());
63 return result;
64 }

```

6.4 Dinitz

```

1 // Complexidade: O(V^2E)
2
3 struct FlowEdge {
4     int v, u;
5     long long cap, flow = 0;
6     FlowEdge(int v, int u, long long cap) : v(v), u(u), cap(cap) {}
7 };
8
9 struct Dinic {
10     const long long flow_inf = 1e18;
11     vector<FlowEdge> edges;
12     vector<vector<int>> adj;
13     int n, m = 0;
14     int s, t;
15     vector<int> level, ptr;
16     queue<int> q;
17
18     Dinic(int n, int s, int t) : n(n), s(s), t(t) {
19         adj.resize(n);
20         level.resize(n);
21         ptr.resize(n);
22     }
23
24     void add_edge(int v, int u, long long cap) {
25         edges.emplace_back(v, u, cap);
26         edges.emplace_back(u, v, 0);
27         adj[v].push_back(m);
28         adj[u].push_back(m + 1);
29         m += 2;
30     }
31
32     bool bfs() {
33         while (!q.empty()) {
34             int v = q.front();
35             q.pop();
36             for (int id : adj[v]) {
37                 if (edges[id].cap == edges[id].flow)
38                     continue;
39                 if (level[edges[id].u] != -1)
40                     continue;
41                 level[edges[id].u] = level[v] + 1;
42                 q.push(edges[id].u);
43             }
44         }
45         return level[t] != -1;
46     }
47
48     long long dfs(int v, long long pushed) {
49         if (pushed == 0)
50             return 0;
51         if (v == t)
52             return pushed;
53         for (int& cid = ptr[v]; cid < (int)adj[v].size(); cid++) {
54             int id = adj[v][cid];

```

```

55         int u = edges[id].u;
56         if (level[v] + 1 != level[u])
57             continue;
58         long long tr = dfs(u, min(pushed, edges[id].cap - edges[id].flow));
59         if (tr == 0)
60             continue;
61         edges[id].flow += tr;
62         edges[id ^ 1].flow -= tr;
63         return tr;
64     }
65     return 0;
66 }
67
68 long long flow() {
69     long long f = 0;
70     while (true) {
71         fill(level.begin(), level.end(), -1);
72         level[s] = 0;
73         q.push(s);
74         if (!bfs())
75             break;
76         fill(ptr.begin(), ptr.end(), 0);
77         while (long long pushed = dfs(s, flow_inf)) {
78             f += pushed;
79         }
80     }
81     return f;
82 }
83 };

```

6.5 Khan

```

1 // topo-sort DAG
2 // lexicograficamente menor.
3 // N: número de vértices (1-indexado)
4 // adj: lista de adjacência do grafo
5
6 const int MAXN = 5 * 1e5 + 2;
7 vector<int> adj[MAXN];
8 int N;
9
10 vector<int> kahn() {
11     vector<int> indegree(N + 1, 0);
12     for (int u = 1; u <= N; u++) {
13         for (int v : adj[u]) {
14             indegree[v]++;
15         }
16     }
17     priority_queue<int, vector<int>, greater<int>> pq;
18     for (int i = 1; i <= N; i++) {
19         if (indegree[i] == 0) {
20             pq.push(i);
21         }
22     }
23     vector<int> result;
24     while (!pq.empty()) {
25         int u = pq.top();
26         pq.pop();
27         result.push_back(u);
28         for (int v : adj[u]) {
29             indegree[v]--;
30             if (indegree[v] == 0) {
31                 pq.push(v);
32             }
33         }
34     }
35     if (result.size() != N) {
36         return {};
37     }
38     return result;

```

39 }

6.6 Topological Sort

```
1 vector<int> adj[MAXN];
2 vector<int> estado(MAXN); // 0: nao visitado 1:
   processamento 2: processado
3 vector<int> ordem;
4 bool temCiclo = false;
5
6 void dfs(int v) {
7     if(estado[v] == 1) {
8         temCiclo = true;
9         return;
10    }
11    if(estado[v] == 2) return;
12    estado[v] = 1;
13    for(auto &nei : adj[v]) {
14        if(estado[nei] != 2) dfs(nei);
15    }
16    estado[v] = 2;
17    ordem.push_back(v);
18    return;
19 }
```

6.7 Acha Pontes

```
1 vector<int> d, low, pai; // d[v] Tempo de
   descoberta (discovery time)
2 vector<bool> vis;
3 vector<int> pontos_articulacao;
4 vector<pair<int, int>> pontes;
5 int tempo;
6
7 vector<vector<int>> adj;
8
9 void dfs(int u) {
10    vis[u] = true;
11    tempo++;
12    d[u] = low[u] = tempo;
13    int filhos_dfs = 0;
14    for (int v : adj[u]) {
15        if (v == pai[u]) continue;
16        if (vis[v]) { // back edge
17            low[u] = min(low[u], d[v]);
18        } else {
19            pai[v] = u;
20            filhos_dfs++;
21            dfs(v);
22            low[u] = min(low[u], low[v]);
23            if (pai[u] == -1 && filhos_dfs > 1) {
24                pontos_articulacao.push_back(u);
25            }
26            if (pai[u] != -1 && low[v] >= d[u]) {
27                pontos_articulacao.push_back(u);
28            }
29            if (low[v] > d[u]) {
30                pontes.push_back({min(u, v), max(u, v)});
31            }
32        }
33    }
34 }
```

6.8 Edmonds-karp

```
1 // Edmonds-Karp com scalling  $O(E \log(F))$ 
2
3 int n, m;
4 const int MAXN = 510;
5 vector<vector<int>> capacity(MAXN, vector<int>(MAXN,
   0));
```

```
6 vector<vector<int>> adj(MAXN);
7
8 int bfs(int s, int t, int scale, vector<int>& parent)
9 {
10    fill(parent.begin(), parent.end(), -1);
11    parent[s] = -2;
12    queue<pair<int, int>> q;
13    q.push({s, LLONG_MAX});
14
15    while (!q.empty()) {
16        int cur = q.front().first;
17        int flow = q.front().second;
18        q.pop();
19
20        for (int next : adj[cur]) {
21            if (parent[next] == -1 && capacity[cur][
   next] >= scale) {
22                parent[next] = cur;
23                int new_flow = min(flow, capacity[cur]
   [next]);
24                if (next == t)
25                    return new_flow;
26                q.push({next, new_flow});
27            }
28        }
29    }
30    return 0;
31 }
32
33 int maxflow(int s, int t) {
34    int flow = 0;
35    vector<int> parent(MAXN);
36    int new_flow;
37    int scalling = 1ll << 62;
38
39    while (scalling > 0) {
40        while (new_flow = bfs(s, t, scalling, parent)
   ){
41            if (new_flow == 0) continue;
42            flow += new_flow;
43            int cur = t;
44            while (cur != s) {
45                int prev = parent[cur];
46                capacity[prev][cur] -= new_flow;
47                capacity[cur][prev] += new_flow;
48                cur = prev;
49            }
50            scalling /= 2;
51        }
52    }
53    return flow;
54 }
55 }
```

6.9 Kruskal

```
1 // Ordena as arestas por peso, insere se ja nao
   estiver no mesmo componente
2 //  $O(E \log E)$ 
3
4 struct DSU {
5     vector<int> par, rank, sz;
6     int c;
7     DSU(int n) : par(n + 1), rank(n + 1, 0), sz(n +
   1, 1), c(n) {
8         for (int i = 1; i <= n; ++i) par[i] = i;
9     }
10    int find(int i) {
11        return (par[i] == i ? i : (par[i] = find(par[
   i])));
12    }
13    bool same(int i, int j) {
```

```

14     return find(i) == find(j);
15 }
16 int get_size(int i) {
17     return sz[find(i)];
18 }
19 int count() {
20     return c; // quantos componentes conexos
21 }
22 int merge(int i, int j) {
23     if ((i = find(i)) == (j = find(j))) return
-1;
24     else --c;
25     if (rank[i] > rank[j]) swap(i, j);
26     par[i] = j;
27     sz[j] += sz[i];
28     if (rank[i] == rank[j]) rank[j]++;
29     return j;
30 }
31 };
32
33 struct Edge {
34     int u, v, w;
35     bool operator <(Edge const & other) {
36         return weight < other.weight;
37     }
38 }
39
40 vector<Edge> kruskal(int n, vector<Edge> edges) {
41     vector<Edge> mst;
42     DSU dsu = DSU(n + 1);
43     sort(edges.begin(), edges.end());
44     for (Edge e : edges) {
45         if (dsu.find(e.u) != dsu.find(e.v)) {
46             mst.push_back(e);
47             dsu.join(e.u, e.v);
48         }
49     }
50     return mst;
51 }

```

6.10 Bellman Ford

```

1 struct Edge {
2     int u, v, w;
3 };
4
5 // se x = -1, não tem ciclo
6 // se x != -1, pegar pais de x pra formar o ciclo
7
8 int n, m;
9 vector<Edge> edges;
10 vector<int> dist(n);
11 vector<int> pai(n, -1);
12
13 for (int i = 0; i < n; i++) {
14     x = -1;
15     for (Edge &e : edges) {
16         if (dist[e.u] + e.w < dist[e.v]) {
17             dist[e.v] = max(-INF, dist[e.u] + e.w
18         );
19         pai[e.v] = e.u;
20         x = e.v;
21     }
22 }
23
24 // achando caminho (se precisar)
25 for (int i = 0; i < n; i++) x = pai[x];
26
27 vector<int> ciclo;
28 for (int v = x; v = pai[v]) {
29     cycle.push_back(v);
30     if (v == x && ciclo.size() > 1) break;

```

```

31 }
32 reverse(ciclo.begin(), ciclo.end());

```

6.11 Lca Jc

```

1 const int MAXN = 200005;
2 int N;
3 int LOG;
4
5 vector<vector<int>> adj;
6 vector<int> profundidade;
7 vector<vector<int>> cima; // cima[v][j] Ã o 2^j-
    Ãsimo ancestral de v
8
9 void dfs(int v, int p, int d) {
10     profundidade[v] = d;
11     cima[v][0] = p; // o pai direto Ã o 2^0-Ãsimo
    ancestral
12     for (int j = 1; j < LOG; j++) {
13         // se o ancestral 2^(j-1) existir, calculamos
    o 2^j
14         if (cima[v][j - 1] != -1) {
15             cima[v][j] = cima[cima[v][j - 1]][j - 1];
16         } else {
17             cima[v][j] = -1; // nÃo tem ancestral
    superior
18         }
19     }
20     for (int nei : adj[v]) {
21         if (nei != p) {
22             dfs(nei, v, d + 1);
23         }
24     }
25 }
26
27 void build(int root) {
28     LOG = ceil(log2(N));
29     profundidade.assign(N + 1, 0);
30     cima.assign(N + 1, vector<int>(LOG, -1));
31     dfs(root, -1, 0);
32 }
33
34 int get_lca(int a, int b) {
35     if (profundidade[a] < profundidade[b]) {
36         swap(a, b);
37     }
38     // sobe 'a' atÃ a mesma profundidade de 'b'
39     for (int j = LOG - 1; j >= 0; j--) {
40         if (profundidade[a] - (1 << j) >=
    profundidade[b]) {
41             a = cima[a][j];
42         }
43     }
44     // se 'b' era um ancestral de 'a', entÃo 'a'
    agora Ã igual a 'b'
45     if (a == b) {
46         return a;
47     }
48
49     // sobe os dois nÃs juntos atÃ encontrar os
    filhos do LCA
50     for (int j = LOG - 1; j >= 0; j--) {
51         if (cima[a][j] != -1 && cima[a][j] != cima[b
    ][j]) {
52             a = cima[a][j];
53             b = cima[b][j];
54         }
55     }
56     return cima[a][0];
57 }

```

6.12 Lca

```

1 // LCA - CP algorithm
2 // preprocessing O(NlogN)
3 // lca O(logN)
4 // Uso: criar LCA com a quantidade de vÃrtices (n) e
   lista de adjacÃncia (adj)
5 // chamar a funÃÃo preprocess com a raiz da Ãrvore
6
7 struct LCA {
8     int n, l, timer;
9     vector<vector<int>> adj;
10    vector<int> tin, tout;
11    vector<vector<int>> up;
12
13    LCA(int n, const vector<vector<int>>& adj) : n(n)
14    , adj(adj) {}
15
16    void dfs(int v, int p) {
17        tin[v] = ++timer;
18        up[v][0] = p;
19        for (int i = 1; i <= l; ++i)
20            up[v][i] = up[up[v][i-1]][i-1];
21
22        for (int u : adj[v]) {
23            if (u != p)
24                dfs(u, v);
25        }
26
27        tout[v] = ++timer;
28    }
29
30    bool is_ancestor(int u, int v) {
31        return tin[u] <= tin[v] && tout[u] >= tout[v];
32    };
33
34    int lca(int u, int v) {
35        if (is_ancestor(u, v))
36            return u;
37        if (is_ancestor(v, u))
38            return v;
39        for (int i = l; i >= 0; --i) {
40            if (!is_ancestor(up[u][i], v))
41                u = up[u][i];
42        }
43        return up[u][0];
44    }
45
46    void preprocess(int root) {
47        tin.resize(n);
48        tout.resize(n);
49        timer = 0;
50        l = ceil(log2(n));
51        up.assign(n, vector<int>(l + 1));
52        dfs(root, root);
53    };

```

6.13 Kosaraju

```

1 bool vis[MAXN];
2 vector<int> order;
3 int component[MAXN];
4 int N, m;
5 vector<int> adj[MAXN], adj_rev[MAXN];
6
7 // dfs no grafo original para obter a ordem (pÃs-
   order)
8 void dfs1(int u) {
9     vis[u] = true;
10    for (int v : adj[u]) {
11        if (!vis[v]) {
12            dfs1(v);
13        }

```

```

14    }
15    order.push_back(u);
16 }
17
18 // dfs o grafo reverso para encontrar os SCCs
19 void dfs2(int u, int c) {
20     component[u] = c;
21     for (int v : adj_rev[u]) {
22         if (component[v] == -1) {
23             dfs2(v, c);
24         }
25     }
26 }
27
28 int kosaraju() {
29     order.clear();
30     fill(vis + 1, vis + N + 1, false);
31     for (int i = 1; i <= N; i++) {
32         if (!vis[i]) {
33             dfs1(i);
34         }
35     }
36     fill(component + 1, component + N + 1, -1);
37     int c = 0;
38     reverse(order.begin(), order.end());
39     for (int u : order) {
40         if (component[u] == -1) {
41             dfs2(u, c++);
42         }
43     }
44     return c;
45 }

```

6.14 Pega Ciclo

```

1 // encontra um ciclo em g (direcionado ou nÃo)
2 // g[u] = vector<pair<id_aresta, vizinho>>
3 // rec_arestas: true -> retorna ids das arestas do
   ciclo; false -> retorna vÃrtices do ciclo
4 // directed: grafo direcionado?
5
6 const int MAXN = 5 * 1e5 + 2;
7 vector<pair<int, int>> g[MAXN];
8 int N;
9 bool DIRECTED = false;
10 vector<int> color(MAXN), parent(MAXN, -1), edgein(
   MAXN, -1); // color: 0,1,2 ; edgein[v] = id da
   aresta que entra em v
11 int ini_ciclo = -1, fim_ciclo = -1, back_edge_id =
   -1;
12
13 bool dfs(int u, int pai_edge){
14     color[u] = 1; // cinza
15     for (auto [id, v] : g[u]) {
16         if (!DIRECTED && id == pai_edge) continue; //
   ignorar aresta de volta ao pai em nÃo-dir
17         if (color[v] == 0) {
18             parent[v] = u;
19             edgein[v] = id;
20             if (dfs(v, id)) return true;
21         } else if (color[v] == 1) {
22             // back-edge u -> v detectado
23             ini_ciclo = u;
24             fim_ciclo = v;
25             back_edge_id = id;
26             return true;
27         }
28     }
29     // se color[v] == 2, ignora
30 }
31 color[u] = 2; // preto
32 return false;
33 }

```

```

34 // retorna ids das arestas do ciclo (vazio se não
35   há)
36 vector<int> pega_ciclo(bool rec_arestas) {
37     for (int u = 1; u <= N; u++) {
38         if (color[u] != 0) continue;
39         if (dfs(u, -1)) {
40             // reconstrói caminho u -> ... -> v via
41             parent
42             vector<int> path;
43             int cur = ini_ciclo;
44             path.push_back(cur);
45             while (cur != fim_ciclo) {
46                 cur = parent[cur];
47                 path.push_back(cur);
48             }
49             // path = [u, ..., v] -> inverter para [v
50             , ..., u]
51             reverse(path.begin(), path.end());
52             if (!rec_arestas) return path;
53             // converte para ids das arestas: edgein[
54             node] é a aresta que entra em node
55             vector<int> edges;
56             for (int i = 1; i < path.size(); i++)
57                 edges.push_back(edgein[path[i]]);
58             // adiciona a aresta de retorno u -> v
59             edges.push_back(back_edge_id);
60             return edges;
61         }
62     }
63     return {};
64 }

```

6.15 Min Cost Max Flow

```

1 // Encontra o menor custo para passar K de fluxo em
2   um grafo com N vertices
3 // Funciona com multiplas arestas para o mesmo par de
4   vertices
5 // Para encontrar o min cost max flow é só fazer K
6   = infinito
7
8 struct Edge {
9     int from, to, capacity, cost, id;
10 };
11
12 vector<vector<array<int, 2>>> adj;
13 vector<Edge> edges; // arestas pares são as normais
14   e suas reversas são as impares
15
16 const int INF = LLONG_MAX;
17
18 void shortest_paths(int n, int v0, vector<int>& dist,
19   vector<int>& edge_to) {
20     dist.assign(n, INF);
21     dist[v0] = 0;
22     vector<bool> in_queue(n, false);
23     queue<int> q;
24     q.push(v0);
25     edge_to.assign(n, -1);
26
27     while (!q.empty()) {
28         int u = q.front();
29         q.pop();
30         in_queue[u] = false;
31         for (auto [v, id] : adj[u]) {
32             if (edges[id].capacity > 0 && dist[v] >
33                 dist[u] + edges[id].cost) {
34                 dist[v] = dist[u] + edges[id].cost;
35                 edge_to[v] = id;
36                 if (!in_queue[v]) {
37                     in_queue[v] = true;
38                     q.push(v);
39                 }
40             }
41         }
42     }
43 }

```

```

33     }
34 }
35
36 }
37
38 int min_cost_flow(int N, int K, int s, int t) {
39     adj.assign(N, vector<array<int, 2>>());
40
41     for (Edge e : edges) {
42         adj[e.from].push_back({e.to, e.id});
43     }
44
45     int flow = 0;
46     int cost = 0;
47     vector<int> dist, edge_to;
48     while (flow < K) {
49         shortest_paths(N, s, dist, edge_to);
50         if (dist[t] == INF)
51             break;
52
53         // find max flow on that path
54         int f = K - flow;
55         int cur = t;
56         while (cur != s) {
57             f = min(f, edges[edge_to[cur]].capacity);
58             cur = edges[edge_to[cur]].from;
59         }
60
61         // apply flow
62         flow += f;
63         cost += f * dist[t];
64         cur = t;
65         while (cur != s) {
66             int edge = edge_to[cur];
67             int rev_edge = edge^1;
68
69             edges[edge].capacity -= f;
70             edges[rev_edge].capacity += f;
71             cur = edges[edge].from;
72         }
73     }
74
75     if (flow < K)
76         return -1;
77     else
78         return cost;
79 }
80

```

7 Primitives

8 DP

8.1 Lis

```

1 int lis_nlogn(vector<int> &v) {
2     vector<int> lis;
3     lis.push_back(v[0]);
4     for (int i = 1; i < v.size(); i++) {
5         if (v[i] > lis.back()) {
6             // estende a LIS.
7             lis.push_back(v[i]);
8         } else {
9             // encontra o primeiro elemento em lis
10            que é >= v[i].
11            // subsequência de mesmo comprimento,
12            mas com um final menor.
13            auto it = lower_bound(lis.begin(), lis.
14            end(), v[i]);
15            *it = v[i];
16        }
17    }
18 }

```



```

14     }
15     return lis.size();
16 }
17
18 // LIS NA ARVORE
19 const int MAXN_TREE = 100001;
20 vector<int> adj[MAXN_TREE];
21 int values[MAXN_TREE];
22 int ans = 0;
23
24
25 void dfs(int u, int p, vector<int>& tails) {
26     auto it = lower_bound(tails.begin(), tails.end(),
27         values[u]);
28     int prev = -1;
29     bool coloquei = false;
30     if (it == tails.end()) {
31         tails.push_back(values[u]);
32         coloquei = true;
33     } else {
34         prev = *it;
35         *it = values[u];
36     }
37     ans = max(ans, (int)tails.size());
38     for (int v : adj[u]) {
39         if (v != p) {
40             dfs(v, u, tails);
41         }
42     }
43     if (coloquei) {
44         tails.pop_back();
45     } else {
46         *it = prev;
47     }
48 }

```

8.2 Edit Distance

```

1 vector<vector<int>> dp(n+1, vector<int>(m+1, LINF));
2
3 for(int j = 0; j <= m; j++) {
4     dp[0][j] = j;
5 }
6
7 for(int i = 0; i <= n; i++) {
8     dp[i][0] = i;
9 }
10
11 for(int i = 1; i <= n; i++) {
12     for(int j = 1; j <= m; j++) {
13         if(a[i-1] == b[j-1]) {
14             dp[i][j] = dp[i-1][j-1];
15         } else {
16             dp[i][j] = min({dp[i-1][j] + 1, dp[i][j-1] + 1, dp[i-1][j-1] + 1});
17         }
18     }
19 }
20
21 cout << dp[n][m];

```

8.3 Bitmask

```

1 // dp de intervalos com bitmask
2 int prox(int idx) {
3     return lower_bound(S.begin(), S.end(), array<int, 4>{S[idx][1], 011, 011, 011}) - S.begin();
4 }
5
6 int dp[1002][(int)(111 << 10)];
7

```

```

8 int rec(int i, int vis) {
9     if (i == (int)S.size()) {
10         if (__builtin_popcountll(vis) == N) return 0;
11         return LLONG_MIN;
12     }
13     if (dp[i][vis] != -1) return dp[i][vis];
14     int ans = rec(i + 1, vis);
15     ans = max(ans, rec(prox(i), vis | (111 << S[i][3])) + S[i][2]);
16     return dp[i][vis] = ans;
17 }

```

8.4 Lcs

```

1 string s1, s2;
2 int dp[1001][1001];
3
4 int lcs(int i, int j) {
5     if (i < 0 || j < 0) return 0;
6     if (dp[i][j] != -1) return dp[i][j];
7     if (s1[i] == s2[j]) {
8         return dp[i][j] = 1 + lcs(i - 1, j - 1);
9     } else {
10         return dp[i][j] = max(lcs(i - 1, j), lcs(i, j - 1));
11     }
12 }

```

8.5 Digit

```

1 vector<int> digits;
2
3 int dp[20][10][2][2];
4
5 int rec(int i, int last, int flag, int started) {
6     if (i == (int)digits.size()) return 1;
7     if (dp[i][last][flag][started] != -1) return dp[i][last][flag][started];
8     int lim;
9     if (flag) lim = 9;
10    else lim = digits[i];
11    int ans = 0;
12    for (int d = 0; d <= lim; d++) {
13        if (started && d == last) continue;
14        int new_flag = flag;
15        int new_started = started;
16        if (d > 0) new_started = 1;
17        if (!flag && d < lim) new_flag = 1;
18        ans += rec(i + 1, d, new_flag, new_started);
19    }
20    return dp[i][last][flag][started] = ans;
21 }

```

8.6 Knapsack

```

1 // dp[i][j] => i-esimo item com j-carga sobrando na mochila
2 // O(N * W)
3
4 for(int j = 0; j < MAXN; j++) {
5     dp[0][j] = 0;
6 }
7
8 for(int i = 1; i <= N; i++) {
9     for(int j = 0; j <= W; j++) {
10        if (items[i].first > j) {
11            dp[i][j] = dp[i-1][j];
12        } else {
13            dp[i][j] = max(dp[i-1][j], dp[i-1][j - items[i].first] + items[i].second);
14        }
15    }
16 }

```

8.7 Lis Seg

```

1  vector<int> a(n);
2  for (int i = 0; i < n; i++) cin >> a[i];
3  vector<int> sorted_a = a;
4  sort(sorted_a.begin(), sorted_a.end());
5  for (int i = 0; i < n; i++) {
6      a[i] = lower_bound(sorted_a.begin(), sorted_a
7      .end(), a[i]) - sorted_a.begin();
8  }
9  SegTreeMx segmx;
10 segmx.build(n);
11 vector<int> dp(n, 1);
12 for (int k = 0; k < n; k++) {
13     if (a[k] > 0) {
14         dp[k] = segmx.query(0, a[k] - 1) + 1;
15     }
16     segmx.update(a[k], dp[k]);
17 }
18 cout << *max_element(dp.begin(), dp.end()) << '\n';

```

8.8 Disjoint Blocks

```

1 // Número máximo de subarrays disjuntos com soma x
2 // usando apenas
3 // prefixo até i (ou seja, considerando prefixo a
4 // [1..i]).
5 int disjointSumX(vector<int> &a, int x) {
6     int n = a.size();
7     map<int, int> best; // best[pref] = melhor dp
8     visto para esse pref
9     best[0] = 0;
10    int pref = 0;
11    vector<int> dp(n + 1, 0); // dp[0] = 0
12    for (int i = 1; i <= n; i++) {
13        pref += a[i - 1];
14        // não pegar subarray terminando em i
15        dp[i] = dp[i-1];
16        // pega se existir prefixo anterior e
17        atualiza best
18        auto it = best.find(pref - x);
19        if (it != best.end()) {
20            dp[i] = max(dp[i], it->second + 1);
21        }
22        best[pref] = max(best[pref], dp[i]);
23    }
24    return dp[n];
25 }

```

9 General

9.1 Brute Choose

```

1 vector<int> elements;
2 int N, K;
3 vector<int> comb;
4
5 void brute_choose(int i) {
6     if (comb.size() == K) {
7         for (int j = 0; j < comb.size(); j++) {
8             cout << comb[j] << ' ';
9         }
10        cout << '\n';
11        return;
12    }
13    if (i == N) return;
14    int r = N - i;
15    int preciso = K - comb.size();
16    if (r < preciso) return;

```

```

18    comb.push_back(elements[i]);
19    brute_choose(i + 1);
20    comb.pop_back();
21    brute_choose(i + 1);
22 }

```

9.2 Struct

```

1 struct Pessoa{
2     // Atributos
3     string nome;
4     int idade;
5
6     // Comparador
7     bool operator<(const Pessoa& other) const{
8         if(idade != other.idade) return idade > other
9         .idade;
10        else return nome > other.nome;
11    }
12 }

```

9.3 Bitwise

```

1 int check_kth_bit(int x, int k) {
2     return (x >> k) & 1;
3 }
4
5 void print_on_bits(int x) {
6     for (int k = 0; k < 32; k++) {
7         if (check_kth_bit(x, k)) {
8             cout << k << ' ';
9         }
10    }
11    cout << '\n';
12 }
13
14 int count_on_bits(int x) {
15     int ans = 0;
16     for (int k = 0; k < 32; k++) {
17         if (check_kth_bit(x, k)) {
18             ans++;
19         }
20    }
21    return ans;
22 }
23
24 bool is_even(int x) {
25     return ((x & 1) == 0);
26 }
27
28 int set_kth_bit(int x, int k) {
29     return x | (1 << k);
30 }
31
32 int unset_kth_bit(int x, int k) {
33     return x & ~(1 << k);
34 }
35
36 int toggle_kth_bit(int x, int k) {
37     return x ^ (1 << k);
38 }
39
40 bool check_power_of_2(int x) {
41     return count_on_bits(x) == 1;
42 }

```

10 Geometry

10.1 Convex Hull

```

1 #include <bits/stdc++.h>

```

```

2
3 using namespace std;
4 #define int long long
5 typedef int cod;
6
7 struct point
8 {
9     cod x,y;
10     point(cod x = 0, cod y = 0): x(x), y(y)
11     {}
12
13     double modulo()
14     {
15         return sqrt(x*x + y*y);
16     }
17
18     point operator+(point o)
19     {
20         return point(x+o.x, y+o.y);
21     }
22     point operator-(point o)
23     {
24         return point(x - o.x , y - o.y);
25     }
26     point operator*(cod t)
27     {
28         return point(x*t, y*t);
29     }
30     point operator/(cod t)
31     {
32         return point(x/t, y/t);
33     }
34
35     cod operator*(point o)
36     {
37         return x*o.x + y*o.y;
38     }
39     cod operator^(point o)
40     {
41         return x*o.y - y * o.x;
42     }
43     bool operator<(point o)
44     {
45         if( x != o.x) return x < o.x;
46         return y < o.y;
47     }
48 };
49
50 int ccw(point p1, point p2, point p3)
51 {
52     cod cross = (p2-p1) ^ (p3-p1);
53     if(cross == 0) return 0;
54     else if(cross < 0) return -1;
55     else return 1;
56 }
57
58 vector <point> convex_hull(vector<point> p)
59 {
60     sort(p.begin(), p.end());
61     vector<point> L,U;
62
63     //Lower
64     for(auto pp : p)
65     {
66         while(L.size() >= 2 and ccw(L[L.size() - 2],
67         L.back(), pp) == -1)
68         {
69             // Ãl -1 pq eu nÃo quero excluir os
70             // colineares
71             L.pop_back();
72         }
73         L.push_back(pp);
74     }

```

```

73     }
74
75     reverse(p.begin(), p.end());
76
77     //Upper
78     for(auto pp : p)
79     {
80         while(U.size() >= 2 and ccw(U[U.size()-2], U
81         .back(), pp) == -1)
82         {
83             U.pop_back();
84         }
85         U.push_back(pp);
86     }
87
88     L.pop_back();
89     L.insert(L.end(), U.begin(), U.end()-1);
90     return L;
91
92 cod area(vector<point> v)
93 {
94     int ans = 0;
95     int aux = (int)v.size();
96     for(int i = 2; i < aux; i++)
97     {
98         ans += ((v[i] - v[0])^(v[i-1] - v[0]))/2;
99     }
100     ans = abs(ans);
101     return ans;
102 }
103
104 int bound(point p1 , point p2)
105 {
106     return __gcd(abs(p1.x-p2.x), abs(p1.y-p2.y));
107 }
108 //teorema de pick [pontos = A - (bound+points)/2 + 1]
109
110 int32_t main()
111 {
112
113     int n;
114     cin >> n;
115
116     vector<point> v(n);
117     for(int i = 0; i < n; i++)
118     {
119         cin >> v[i].x >> v[i].y;
120     }
121
122     vector <point> ch = convex_hull(v);
123
124     cout << ch.size() << '\n';
125     for(auto p : ch) cout << p.x << " " << p.y << "\n
126     ";
127
128     return 0;
129 }

```

10.2 Inside Polygon

```

1 // Convex O(logn)
2
3 bool insideT(point a, point b, point c, point e){
4     int x = ccw(a, b, e);
5     int y = ccw(b, c, e);
6     int z = ccw(c, a, e);
7     return !((x==1 or y==1 or z==1) and (x==-1 or y
8     ==-1 or z==-1));
9 }
10 bool inside(vp &p, point e){ // ccw
11     int l=2, r=(int)p.size()-1;

```

```

12 while(l<r){
13     int mid = (l+r)/2;
14     if(ccw(p[0], p[mid], e) == 1)
15         l=mid+1;
16     else{
17         r=mid;
18     }
19 }
20 // bordo
21 // if(r==(int)p.size()-1 and ccw(p[0], p[r], e)
22 ==0) return false;
23 // if(r==2 and ccw(p[0], p[1], e)==0) return
24 false;
25 // if(ccw(p[r], p[r-1], e)==0) return false;
26 return insideT(p[0], p[r-1], p[r], e);
27 }
28 // Any O(n)
29
30 int inside(vp &p, point pp){
31     // 1 - inside / 0 - boundary / -1 - outside
32     int n = p.size();
33     for(int i=0; i<n; i++){
34         int j = (i+1)%n;
35         if(line({p[i], p[j]}).inside_seg(pp))
36             return 0;
37     }
38     int inter = 0;
39     for(int i=0; i<n; i++){
40         int j = (i+1)%n;
41         if(p[i].x <= pp.x and pp.x < p[j].x and ccw(p
42 [i], p[j], pp)==1)
43             inter++; // up
44         else if(p[j].x <= pp.x and pp.x < p[i].x and
45 ccw(p[i], p[j], pp)==-1)
46             inter++; // down
47     }
48     if(inter%2==0) return -1; // outside
49     else return 1; // inside
50 }

```

10.3 Point Location

```

1 int32_t main(){
2     sws;
3
4     int t; cin >> t;

```

```

6 while(t--){
7
8     int x1, y1, x2, y2, x3, y3; cin >> x1 >> y1
9     >> x2 >> y2 >> x3 >> y3;
10
11     int deltax1 = (x1-x2), deltax2 = (x1-x3), deltax3 = (x1-x2);
12     int deltax2 = (x2-x3), deltax3 = (x2-x1);
13     int compx = (x1-x3), compy = (y1-y3);
14     int ans = (deltax1*compy) - (compx*deltax2);
15
16     if(ans == 0){cout << "TOUCH\n"; continue;}
17     if(ans < 0){cout << "RIGHT\n"; continue;}
18     if(ans > 0){cout << "LEFT\n"; continue;}
19 }
20 return 0;
21 }
22 }

```

10.4 Lattice Points

```

1 ll gcd(ll a, ll b) {
2     return b == 0 ? a : gcd(b, a % b);
3 }
4 ll area_triangulo(ll x1, ll y1, ll x2, ll y2, ll x3,
5 ll y3) {
6     return abs(x1 * (y2 - y3) + x2 * (y3 - y1) + x3 *
7 (y1 - y2));
8 }
9 ll pontos_borda(ll x1, ll y1, ll x2, ll y2) {
10     return gcd(abs(x2 - x1), abs(y2 - y1));
11 }
12 int32_t main() {
13     ll x1, y1, x2, y2, x3, y3;
14     cin >> x1 >> y1;
15     cin >> x2 >> y2;
16     cin >> x3 >> y3;
17     ll area = area_triangulo(x1, y1, x2, y2, x3, y3);
18     ll tot_borda = pontos_borda(x1, y1, x2, y2) +
19 pontos_borda(x2, y2, x3, y3) + pontos_borda(x3,
20 y3, x1, y1);
21
22     ll ans = (area - tot_borda) / 2 + 1;
23     cout << ans << endl;
24
25     return 0;
26 }

```