

# Competitive Programming Notebook

Programadores Roblox

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# 1 String

## 1.1 Trie

```

1 // Trie por array
2 int trie[MAXN][26];
3 int tot_nos = 0;
4 vector<bool> acaba(MAXN, false);
5 vector<int> contador(MAXN, 0);
6
7 void insere(string s) {
8     int no = 0;
9     for(auto &c : s) {
10         if(trie[no][c - 'a'] == 0) {
11             trie[no][c - 'a'] = ++tot_nos;
12         }
13         no = trie[no][c - 'a'];
14         contador[no]++;
15     }
16     acaba[no] = true;
17 }
18
19 bool busca(string s) {
20     int no = 0;
21     for(auto &c : s) {
22         if(trie[no][c - 'a'] == 0) {
23             return false;
24         }
25         no = trie[no][c - 'a'];
26     }
27     return acaba[no];
28 }
29
30 int isPref(string s) {
31     int no = 0;
32     for(auto &c : s) {
33         if(trie[no][c - 'a'] == 0) {
34             return -1;
35         }
36         no = trie[no][c - 'a'];
37     }
38     return contador[no];
39 }

```

## 1.2 Hashing

```

1 // String Hash template
2 // constructor(s) - 0(|s|)
3 // query(l, r) - returns the hash of the range [l,r]
4 // from left to right - 0(1)
5 // query_inv(l, r) from right to left - 0(1)
6 // patrocinado por tiagodfs
7
8 struct Hash {
9     const int X = 2147483647;
10    const int MOD = 1e9+7;
11    int n; string s;
12    vector<int> h, hi, p;
13    Hash() {}
14    Hash(string s): s(s), n(s.size()), h(n), hi(n), p
15    (n) {
16        for (int i=0;i<n;i++) p[i] = (i ? X*p[i-1]:1)
17        % MOD;
18        for (int i=0;i<n;i++)
19            h[i] = (s[i] + (i ? h[i-1]:0) * X) % MOD;
20        for (int i=n-1;i>=0;i--)
21            hi[i] = (s[i] + (i+1<n ? hi[i+1]:0) * X)
22            % MOD;
23    }
24    int query(int l, int r) {
25        int hash = (h[r] - (l ? h[l-1]*p[r-l+1]:MOD :
26        0));

```

```

22         return hash < 0 ? hash + MOD : hash;
23     }
24     int query_inv(int l, int r) {
25         int hash = (hi[l] - (r+1 < n ? hi[r+1]*p[r-1
26         +1] % MOD : 0));
27         return hash < 0 ? hash + MOD : hash;
28     }
29 };

```

## 1.3 Z Function

```

1 vector<int> z_function(string s) {
2     int n = s.size();
3     vector<int> z(n);
4     int l = 0, r = 0;
5     for(int i = 1; i < n; i++) {
6         if(i < r) {
7             z[i] = min(r - i, z[i - l]);
8         }
9         while(i + z[i] < n && s[z[i]] == s[i + z[i]])
10             z[i]++;
11     }
12     if(i + z[i] > r) {
13         l = i;
14         r = i + z[i];
15     }
16 }
17 return z;
18 }

```

## 1.4 Kmp

```

1 vector<int> kmp(string s) {
2     int n = (int)s.length();
3     vector<int> p(n+1);
4     p[0] = -1;
5     for (int i = 1; i < n; i++) {
6         int j = p[i-1];
7         while (j >= 0 && s[j] != s[i-1])
8             j = p[j-1];
9         p[i] = j+1;
10    }
11    return p;
12 }

```

# 2 DS

## 2.1 Segtree Iterativa

```

1 // Exemplo de uso:
2 // SegTree<int> st(vetor);
3 // range query e point update
4 template <typename T>
5 struct SegTree {
6     int n;
7     vector<T> tree;
8     T neutral_value = 0;
9     T combine(T a, T b) {
10         return a + b;
11     }
12
13     SegTree(const vector<T>& data) {
14         n = data.size();
15         tree.resize(2 * n, neutral_value);
16
17         for (int i = 0; i < n; i++)
18             tree[n + i] = data[i];
19
20         for (int i = n - 1; i > 0; --i)

```

```

21         tree[i] = combine(tree[i * 2], tree[i * 2
+ 1]);
22     }
23     T range_query(int l, int r) {
24         T res_l = neutral_value, res_r =
neutral_value;
25
26         for (l += n, r += n + 1; l < r; l >>= 1, r
>>= 1) {
27             if (l & 1) res_l = combine(res_l, tree[l
++]);
28             if (r & 1) res_r = combine(tree[--r],
res_r);
29         }
30
31         return combine(res_l, res_r);
32     }
33     void update(int pos, T new_val) {
34         tree[pos += n] = new_val;
35         for (pos >>= 1; pos > 0; pos >>= 1)
36             tree[pos] = combine(tree[2 * pos], tree[2
* pos + 1]);
37     }
38 };

```

## 2.2 Merge Sort Tree

```

1 struct SegTree {
2     int n;
3     vector<vector<int>> tree;
4
5     SegTree(vector<int> &a) {
6         n = a.size();
7         tree.resize(4 * n);
8         build(1, 0, n - 1, a);
9     }
10    void build(int x, int lx, int rx, vector<int> &a)
{
11        if (lx == rx) {
12            tree[x] = { a[lx] };
13            return;
14        }
15        int mid = lx + (rx - lx)/2;
16        build(2 * x, lx, mid, a);
17        build(2 * x + 1, mid + 1, rx, a);
18        auto &L = tree[2 * x], &R = tree[2 * x + 1];
19        tree[x].resize(L.size() + R.size());
20        merge(L.begin(), L.end(), R.begin(), R.end(),
tree[x].begin());
21    }
22    int query(int x, int lx, int rx, int l, int r) {
23        if (lx >= l && rx <= r) {
24            auto &v = tree[x];
25            return v.end() - upper_bound(v.begin(), v
.end(), r);
26        }
27        if (rx < l || lx > r) {
28            return 0;
29        }
30        int mid = lx + (rx - lx)/2;
31        return query(2 * x, lx, mid, l, r) + query(2
* x + 1, mid + 1, rx, l, r);
32    }
33    int query(int l, int r) {
34        return query(1, 0, n - 1, l, r);
35    }
36 }
37
38 // Checar se o range eh todo distinto
39 vector<int> nr(n);
40 map<int, int> mp;
41 for (int i = n - 1; i >= 0; i--) {
42     auto it = mp.find(a[i]);

```

```

nr[i] = it != mp.end() ? it->second : n;
44     mp[a[i]] = i;
45 }
46 SegTree seg(nr);

```

## 2.3 Ordered Set E Map

```

1
2 #include<ext/pb_ds/assoc_container.hpp>
3 #include<ext/pb_ds/tree_policy.hpp>
4 using namespace __gnu_pbds;
5 using namespace std;
6
7 template<typename T> using ordered_multiset = tree<T,
null_type, less_equal<T>, rb_tree_tag,
tree_order_statistics_node_update>;
8 template <typename T> using o_set = tree<T, null_type
, less<T>, rb_tree_tag,
tree_order_statistics_node_update>;
9 template <typename T, typename R> using o_map = tree<
T, R, less<T>, rb_tree_tag,
tree_order_statistics_node_update>;
10
11 int main() {
12     int i, j, k, n, m;
13     o_set<int> st;
14     st.insert(1);
15     st.insert(2);
16     cout << *st.find_by_order(0) << endl; /// k-esimo
elemento
17     cout << st.order_of_key(2) << endl; ///numero de
elementos menores que k
18     o_map<int, int> mp;
19     mp.insert({1, 10});
20     mp.insert({2, 20});
21     cout << mp.find_by_order(0)->second << endl; /// k-
esimo elemento
22     cout << mp.order_of_key(2) << endl; /// numero de
elementos (chave) menores que k
23     return 0;
24 }

```

## 2.4 Sparse Table

```

1 // 1-index, 0(1)
2 struct SparseTable {
3     vector<vector<int>> st;
4     int max_log;
5     SparseTable(vector<int> &arr) {
6         int n = arr.size();
7         max_log = floor(log2(n)) + 1;
8         st.resize(n, vector<int>(max_log));
9         for (int i = 0; i < n; i++) {
10             st[i][0] = arr[i];
11         }
12         for (int j = 1; j < max_log; j++) {
13             for (int i = 0; i + (1 << j) <= n; i++) {
14                 st[i][j] = max(st[i][j - 1], st[i +
(1 << (j - 1))][j - 1]);
15             }
16         }
17     }
18     int query(int L, int R) {
19         int tamanho = R - L + 1;
20         int k = floor(log2(tamanho));
21         return max(st[L][k], st[R - (1 << k) + 1][k])
;
22     }
23 };

```

## 2.5 Psum 2d

```

1 vector<vector<int>> psum(h+1, vector<int>(w+1, 0));
2 for (int i=1; i<=h; i++){
3     for (int j=1; j<=w; j++){
4         cin >> psum[i][j];
5         psum[i][j] += psum[i-1][j]+psum[i][j-1]-psum[
6             i-1][j-1];
7     }
8 // retorna a psum2d do intervalo inclusivo [(a, b), (
9     c, d)]
10 int retangulo(int a, int b, int c, int d){
11     c = min(c, h), d = min(d, w);
12     a = max(0LL, a-1), b = max(0LL, b-1);
13     return v[c][d]-v[a][d]-v[c][b]+v[a][b];
14 }

```

## 2.6 Segtree Sum

```

1 struct SegTree {
2     vector<int> tree;
3     vector<int> lazy;
4     int n;
5     SegTree(int N) {
6         n = N;
7         tree.resize(4 * n);
8         lazy.resize(4 * n, 0);
9         // build(a);
10    }
11    void push(int lx, int rx, int x) {
12        if (lazy[x] == 0) {
13            return;
14        }
15        // tree[x] += lazy[x];
16        tree[x] += (rx - lx + 1) * lazy[x];
17        if (lx != rx) {
18            lazy[2 * x] += lazy[x];
19            lazy[2 * x + 1] += lazy[x];
20        }
21        lazy[x] = 0;
22    }
23    void build(vector<int> &a) {
24        build(0, n - 1, 1, a);
25    }
26    void build(int lx, int rx, int x, vector<int> &a)
27    {
28        if (lx == rx) {
29            tree[x] = a[lx];
30            return;
31        }
32        int mid = lx + (rx - lx) / 2;
33        build(lx, mid, 2 * x, a);
34        build(mid + 1, rx, 2 * x + 1, a);
35        tree[x] = tree[2 * x] + tree[2 * x + 1];
36    }
37    void update(int l, int r, int val) {
38        update(0, n - 1, 1, l, r, val);
39    }
40    void update(int lx, int rx, int x, int l, int r,
41    int val) {
42        push(lx, rx, x);
43        if (rx < l || lx > r) {
44            return;
45        }
46        if (lx >= l && rx <= r) {
47            lazy[x] += val;
48            push(lx, rx, x);
49            return;
50        }
51        int mid = lx + (rx - lx) / 2;
52        update(lx, mid, 2 * x, l, r, val);
53        update(mid + 1, rx, 2 * x + 1, l, r, val);
54        tree[x] = tree[2 * x] + tree[2 * x + 1];
55    }

```

```

54 int query(int l, int r) {
55     return query(0, n - 1, 1, l, r);
56 }
57 int query(int lx, int rx, int x, int l, int r) {
58     push(lx, rx, x);
59     if (rx < l || lx > r) {
60         return 0;
61     }
62     if (lx >= l && rx <= r) {
63         return tree[x];
64     }
65     int mid = lx + (rx - lx) / 2;
66     int s1 = query(lx, mid, 2 * x, l, r);
67     int s2 = query(mid + 1, rx, 2 * x + 1, l, r);
68     return s1 + s2;
69 }
70 };

```

## 2.7 Dsu

```

1 struct DSU {
2     vector<int> par, rank, sz;
3     int c;
4     DSU(int n) : par(n + 1), rank(n + 1, 0), sz(n +
5     1, 1), c(n) {
6         for (int i = 1; i <= n; ++i) par[i] = i;
7     }
8     int find(int i) {
9         return (par[i] == i ? i : (par[i] = find(par[
10             i])));
11    }
12    bool same(int i, int j) {
13        return find(i) == find(j);
14    }
15    int get_size(int i) {
16        return sz[find(i)];
17    }
18    int count() {
19        return c; // quantos componentes conexos
20    }
21    int merge(int i, int j) {
22        if ((i = find(i)) == (j = find(j))) return
23        -1;
24        else --c;
25        if (rank[i] > rank[j]) swap(i, j);
26        par[i] = j;
27        sz[j] += sz[i];
28        if (rank[i] == rank[j]) rank[j]++;
29        return j;
30    }
31 }

```

## 2.8 Bit

```

1 struct BIT {
2     int n;
3     vector<int> bit;
4     BIT(int n = 0) : n(n), bit(n + 1, 0) {}
5     void add(int i, int delta) {
6         for (; i <= n; i += i & -i) bit[i] += delta;
7     }
8     int sum(int i) {
9         int r = 0;
10        for (; i > 0; i -= i & -i) r += bit[i];
11        return r;
12    }
13    int range_sum(int l, int r){
14        if (r < l) return 0;
15        return sum(r) - sum(l - 1);
16    }
17 };

```

## 3 Search and sort

### 3.1 Merge Sort

```

1 int mergeAndCount(vector<int>& v, int l, int m, int r) {
2     int x = m - l + 1;
3     int y = r - m;
4     vector<int> left(x), right(y);
5     for (int i = 0; i < x; i++) left[i] = v[l + i];
6     for (int j = 0; j < y; j++) right[j] = v[m + 1 + j];
7     int i = 0, j = 0, k = l;
8     int swaps = 0;
9     while (i < x && j < y) {
10         if (left[i] <= right[j]) {
11             v[k++] = left[i++];
12         } else {
13             v[k++] = right[j++];
14             swaps += (x - i);
15         }
16     }
17     while (i < x) v[k++] = left[i++];
18     while (j < y) v[k++] = right[j++];
19     return swaps;
20 }
21
22 int mergeSort(vector<int>& v, int l, int r) {
23     int swaps = 0;
24     if (l < r) {
25         int m = l + (r - l) / 2;
26         swaps += mergeSort(v, l, m);
27         swaps += mergeSort(v, m + 1, r);
28         swaps += mergeAndCount(v, l, m, r);
29     }
30     return swaps;
31 }

```

### 3.2 Monotonic Stack

```

1 vector<int> find_esq(vector<int> &v, bool maior) {
2     int n = v.size();
3     vector<int> result(n);
4     stack<int> s;
5
6     for (int i = 0; i < n; i++) {
7         while (!s.empty() && (maior ? v[s.top()] <= v[i] : v[s.top()] >= v[i])) {
8             s.pop();
9         }
10        if (s.empty()) {
11            result[i] = -1;
12        } else {
13            result[i] = v[s.top()];
14        }
15        s.push(i);
16    }
17    return result;
18 }
19
20 vector<int> find_dir(vector<int> &v, bool maior) {
21     int n = v.size();
22     vector<int> result(n);
23     stack<int> s;
24     for (int i = n - 1; i >= 0; i--) {
25         while (!s.empty() && (maior ? v[s.top()] <= v[i] : v[s.top()] >= v[i])) {
26             s.pop();
27         }
28         if (s.empty()) {
29             result[i] = -1;
30         } else {

```

```

31         result[i] = v[s.top()];
32     }
33     s.push(i);
34 }
35 return result;
36 }

```

## 4 Stress

### 4.1 Gen

```

1 #include <bits/stdc++.h>
2 #include <cstdlib>
3 #include <ctime>
4 using namespace std;
5
6 int randi(int L, int R) { return L + rand() % (R - L + 1); }
7 char randc(char L, char R) { return char(L + rand() % (R - L + 1)); }
8
9 int main(int argc, char** argv) {
10     if (argc > 1) srand(atoi(argv[1]));
11     else srand(time(0));
12
13     int n = randi(1, 100);
14     cout << n << '\n';
15     for (int i = 0; i < n; i++) {
16         cout << randi(-10, 20) << ' ';
17     }
18     cout << '\n';
19 }

```

## 5 Math

### 5.1 Combinatorics

```

1 const int MAXN_FATORIAL = 200005;
2 const int MOD = 1e9 + 7;
3 // DEFINE INT LONG LONG PLMDS
4 int fat[MAXN_FATORIAL], fati[MAXN_FATORIAL];
5
6 // (a^b) % m em O(log b)
7 // coloque o fexp
8
9 int inv(int n) { return fexp(n, MOD - 2); }
10
11 void precalc() {
12     fat[0] = 1;
13     fati[0] = 1;
14     for (int i = 1; i < MAXN_FATORIAL; i++) fat[i] = (fat[i - 1] * i) % MOD;
15     fati[MAXN_FATORIAL - 1] = inv(fat[MAXN_FATORIAL - 1]);
16     for (int i = MAXN_FATORIAL - 2; i >= 0; i--) fati[i] = (fati[i + 1] * (i + 1)) % MOD;
17 }
18
19 int choose(int n, int k) {
20     if (k < 0 || k > n) return 0;
21     return ((fat[n] * fati[k]) % MOD) * fati[n - k] % MOD;
22 }
23
24 // n! / (n-k)!
25 int perm(int n, int k) {
26     if (k < 0 || k > n) return 0;
27     return (fat[n] * fati[n - k]) % MOD;
28 }
29

```

```

30 // C_n = (1 / (n+1)) * C(2n, n)
31 int catalan(int n) {
32     if (n < 0 || 2 * n >= MAXN_FATORIAL) return 0;
33     int c2n_n = choose(2 * n, n);
34     return (c2n_n * inv(n + 1)) % MOD;
35 }

```

## 5.2 Equacao Diofantina

```

1 int extended_gcd(int a, int b, int& x, int& y) {
2     if (a == 0) {
3         x = 0;
4         y = 1;
5         return b;
6     }
7     int x1, y1;
8     int gcd = extended_gcd(b % a, a, x1, y1);
9     x = y1 - (b / a) * x1;
10    y = x1;
11    return gcd;
12 }
13
14 bool solve(int a, int b, int c, int& x0, int& y0) {
15     int x, y;
16     int g = extended_gcd(abs(a), abs(b), x, y);
17     if (c % g != 0) {
18         return false;
19     }
20     x0 = x * (c / g);
21     y0 = y * (c / g);
22     if (a < 0) x0 = -x0;
23     if (b < 0) y0 = -y0;
24     return true;
25 }

```

## 5.3 Discrete Log

```

1 // returns minimum x for which a^x = b (mod m), a and
  m are coprime.
2 // if the answer dont need to be greater than some
  value, the vector<int> can be removed
3 int discrete_log(int a, int b, int m) {
4     a %= m, b %= m;
5     int n = sqrt(m) + 1;
6
7     int an = 1;
8     for (int i = 0; i < n; ++i)
9         an = (an * 1ll * a) % m;
10
11    unordered_map<int, vector<int>> vals;
12    for (int q = 0, cur = b; q <= n; ++q) {
13        vals[cur].push_back(q);
14        cur = (cur * 1ll * a) % m;
15    }
16
17    int res = LLONG_MAX;
18
19    for (int p = 1, cur = 1; p <= n; ++p) {
20        cur = (cur * 1ll * an) % m;
21        if (vals[cur].count()) {
22            for (int q: vals[cur]) {
23                int ans = n * p - q;
24                res = min(res, ans);
25            }
26        }
27    }
28    return res;
29 }

```

## 5.4 Segment Sieve

```

1 // Retorna quantos primos tem entre [l, r] (inclusivo
  )
2 // precisa de um vetor com os primos ate sqrt(r)
3 int seg_sieve(int l, int r){
4     if (l > r) return 0;
5     vector<bool> is_prime(r - l + 1, true);
6     if (l == 1) is_prime[0] = false;
7
8     for (int p : primos){
9         if (p * p > r) break;
10        int start = max(p * p, (l + p - 1) / p * p);
11        for (int j = start; j <= r; j += p){
12            if (j >= 1) {
13                is_prime[j - l] = false;
14            }
15        }
16    }
17
18    return accumulate(all(is_prime), 0ll);
19 }

```

## 5.5 Totient

```

1 // phi(n) = n * (1 - 1/p1) * (1 - 1/p2) * ...
2 int phi(int n) {
3     int result = n;
4     for (int i = 2; i * i <= n; i++) {
5         if (n % i == 0) {
6             while (n % i == 0)
7                 n /= i;
8             result -= result / i;
9         }
10    }
11    if (n > 1) // SE n sobrou, ele Ã um fator primo
12        result -= result / n;
13    return result;
14 }
15
16 // crivo phi
17 const int MAXN_PHI = 1000001;
18 int phiv[MAXN_PHI];
19 void phi_sieve() {
20     for (int i = 0; i < MAXN_PHI; i++) phiv[i] = i;
21     for (int i = 2; i < MAXN_PHI; i++) {
22         if (phiv[i] == i) {
23             for (int j = i; j < MAXN_PHI; j += i)
24                 phiv[j] -= phiv[j] / i;
25         }
26     }

```

## 5.6 Menor Fator Primo

```

1 const int MAXN = 1000001;
2 int spf[MAXN];
3 vector<int> primos;
4
5 void crivo() {
6     for (int i = 0; i < MAXN; i++) spf[i] = [i];
7     for (int i = 2; i * i < MAXN; i++) {
8         if (spf[i] == i) {
9             for (int j = i * i; j < MAXN; j += i) {
10                 if (spf[j] == j) {
11                     spf[j] = i;
12                 }
13             }
14         }
15     }
16     for (int i = 2; i < MAXN; i++) {
17         if (spf[i] == i) {
18             primos.push_back(i);
19         }

```

```

20     }
21 }
22
23 map<int, int> fatorar(int n) {
24     map<int, int> fatores;
25     while (n > 1) {
26         fatores[spf[n]]++;
27         n /= spf[n];
28     }
29     return fatores;
30 }
31
32 int numero_de_divisores(int n) {
33     if (n == 1) return 1;
34     map<int, int> fatores = fatorar(n);
35     int nod = 1;
36     for (auto &[primo, expoente] : fatores) nod *= (
37         expoente + 1);
38     return nod;
39 }
40
41 // DEFINE INT LONG LONG
42 int soma_dos_divisores(int n) {
43     if (n == 1) return 1;
44     map<int, int> fatores = fatorar(n);
45     int sod = 1;
46     for (auto &[primo, expoente] : fatores) {
47         int termo_soma = 1;
48         int potencia_primo = 1;
49         for (int i = 0; i < expoente; i++) {
50             potencia_primo *= primo;
51             termo_soma += potencia_primo;
52         }
53         sod *= termo_soma;
54     }
55     return sod;
56 }

```

## 5.7 Exgcd

```

1 // 0 retorno da funcao eh {n, m, g}
2 // e significa que gcd(a, b) = g e
3 // n e m sao inteiros tais que an + bm = g
4 array<ll, 3> exgcd(int a, int b) {
5     if (b == 0) return {1, 0, a};
6     auto [m, n, g] = exgcd(b, a % b);
7     return {n, m - a / b * n, g};
8 }

```

## 5.8 Fexp

```

1 // a^e mod m
2 // 0(log n)
3
4 int fexp(int a, int e, int m) {
5     a %= m;
6     int ans = 1;
7     while (e > 0) {
8         if (e & 1) ans = ans * a % m;
9         a = a * a % m;
10        e /= 2;
11    }
12    return ans % m;
13 }

```

## 5.9 Divisores

```

1 // Retorna um vetor com os divisores de x
2 // eh preciso ter o crivo implementado
3 // 0(divisores)
4

```

```

5 vector<int> divs(int x) {
6     vector<int> ans = {1};
7     vector<array<int, 2>> primos; // {primo, expoente}
8
9     while (x > 1) {
10        int p = crivo[x], cnt = 0;
11        while (x % p == 0) cnt++, x /= p;
12        primos.push_back({p, cnt});
13    }
14
15    for (int i=0; i<primos.size(); i++){
16        int cur = 1, len = ans.size();
17
18        for (int j=0; j<primos[i][1]; j++){
19            cur *= primos[i][0];
20            for (int k=0; k<len; k++)
21                ans.push_back(cur*ans[k]);
22        }
23    }
24
25    return ans;
26 }

```

## 5.10 Crivo

```

1 // 0(n*log(log(n)))
2 bool composto[MAX]
3 for(int i = 1; i <= n; i++) {
4     if(composto[i]) continue;
5     for(int j = 2*i; j <= n; j += i)
6         composto[j] = 1;
7 }

```

## 5.11 Mod Inverse

```

1 array<int, 2> extended_gcd(int a, int b) {
2     if (b == 0) return {1, 0};
3     auto [x, y] = extended_gcd(b, a % b);
4     return {y, x - (a / b) * y};
5 }
6
7 int mod_inverse(int a, int m) {
8     auto [x, y] = extended_gcd(a, m);
9     return (x % m + m) % m;
10 }

```

## 5.12 Base Calc

```

1 int char_to_val(char c) {
2     if (c >= '0' && c <= '9') return c - '0';
3     else return c - 'A' + 10;
4 }
5
6 char val_to_char(int val) {
7     if (val >= 0 && val <= 9) return val + '0';
8     else return val - 10 + 'A';
9 }
10
11 int to_base_10(string &num, int bfrom) {
12     int result = 0;
13     int pot = 1;
14     for (int i = num.size() - 1; i >= 0; i--) {
15         if (char_to_val(num[i]) >= bfrom) return -1;
16         result += char_to_val(num[i]) * pot;
17         pot *= bfrom;
18     }
19     return result;
20 }
21
22 string from_base_10(int n, int bto) {
23     if (n == 0) return "0";
24

```

```

24     string result = "";
25     while (n > 0) {
26         result += val_to_char(n % bto);
27         n /= bto;
28     }
29     reverse(result.begin(), result.end());
30     return result;
31 }
32
33 string convert_base(string &num, int bfrom, int bto)
34 {
35     int n_base_10 = to_base_10(num, bfrom);
36     return from_base_10(n_base_10, bto);
37 }

```

## 6 Graph

### 6.1 Dijkstra

```

1 // SSP com pesos positivos.
2 // O((V + E) log V).
3
4 vector<int> dijkstra(int S) {
5     vector<bool> vis(MAXN, 0);
6     vector<ll> dist(MAXN, LLONG_MAX);
7     dist[S] = 0;
8     priority_queue<pii, vector<pii>, greater<pii>> pq;
9     pq.push({0, S});
10    while(pq.size()) {
11        ll v = pq.top().second;
12        pq.pop();
13        if(vis[v]) continue;
14        vis[v] = 1;
15        for(auto &[peso, vizinho] : adj[v]) {
16            if(dist[vizinho] > dist[v] + peso) {
17                dist[vizinho] = dist[v] + peso;
18                pq.push({dist[vizinho], vizinho});
19            }
20        }
21    }
22    return dist;
23 }

```

### 6.2 Floyd Warshall

```

1 // SSP e acha ciclos.
2 // Bom com constraints menores.
3 // O(n^3)
4
5 int dist[501][501];
6
7 void floydWarshall() {
8     for(int k = 0; k < n; k++) {
9         for(int i = 0; i < n; i++) {
10            for(int j = 0; j < n; j++) {
11                dist[i][j] = min(dist[i][j], dist[i][k]
12                + dist[k][j]);
13            }
14        }
15    }
16    void solve() {
17        int m, q;
18        cin >> n >> m >> q;
19        for(int i = 0; i < n; i++) {
20            for(int j = i; j < n; j++) {
21                if(i == j) {
22                    dist[i][j] = dist[j][i] = 0;
23                } else {
24                    dist[i][j] = dist[j][i] = linf;

```

```

25        }
26    }
27    }
28    for(int i = 0; i < m; i++) {
29        int u, v, w;
30        cin >> u >> v >> w; u--; v--;
31        dist[u][v] = min(dist[u][v], w);
32        dist[v][u] = min(dist[v][u], w);
33    }
34    floydWarshall();
35    while(q--) {
36        int u, v;
37        cin >> u >> v; u--; v--;
38        if(dist[u][v] == linf) cout << -1 << '\n';
39        else cout << dist[u][v] << '\n';
40    }
41 }

```

### 6.3 Eulerian Path

```

1 /**
2  * versao que assume: #define int long long
3  *
4  * Retorna um caminho/ciclo euleriano em um grafo (se
5  * existir).
6  * - g: lista de adjacencia (vector<vector<int>>).
7  * - directed: true se o grafo for dirigido.
8  * - s: vertice inicial.
9  * - e: vertice final (opcional). Se informado, tenta
10   * caminho de s ate e.
11   * - O(Nlog(N))
12   * Retorna vetor com a sequencia de vertices, ou
13   * vazio se impossivel.
14   */
15 vector<int> eulerian_path(const vector<vector<int>>&
16 g, bool directed, int s, int e = -1) {
17     int n = (int)g.size();
18     // copia das adjacencias em multiset para
19     // permitir remoção especifica
20     vector<multiset<int>> h(n);
21     vector<int> in_degree(n, 0);
22     vector<int> result;
23     stack<int> st;
24     // preencher h e indegrees
25     for (int u = 0; u < n; ++u) {
26         for (auto v : g[u]) {
27             ++in_degree[v];
28             h[u].emplace(v);
29         }
30     }
31     st.emplace(s);
32     if (e != -1) {
33         int out_s = (int)h[s].size();
34         int out_e = (int)h[e].size();
35         int diff_s = in_degree[s] - out_s;
36         int diff_e = in_degree[e] - out_e;
37         if (diff_s * diff_e != -1) return {}; //
38         impossivel
39     }
40     for (int u = 0; u < n; ++u) {
41         if (e != -1 && (u == s || u == e)) continue;
42         int out_u = (int)h[u].size();
43         if (in_degree[u] != out_u || (!directed && (
44         in_degree[u] & 1))) {
45             return {};
46         }
47     }
48     while (!st.empty()) {
49         int u = st.top();
50         if (h[u].empty()) {
51             result.emplace_back(u);
52             st.pop();
53         } else {

```



```

47         int v = *h[u].begin();
48         auto it = h[u].find(v);
49         if (it != h[u].end()) h[u].erase(it);
50         --in_degree[v];
51         if (!directed) {
52             auto it2 = h[v].find(u);
53             if (it2 != h[v].end()) h[v].erase(it2);
54         }
55         --in_degree[u];
56         st.emplace(v);
57     }
58 }
59 for (int u = 0; u < n; ++u) {
60     if (in_degree[u] != 0) return {};
61 }
62 reverse(result.begin(), result.end());
63 return result;
64 }

```

## 6.4 Dinitz

```

1 // Complexidade: O(V^2E)
2
3 struct FlowEdge {
4     int from, to;
5     long long cap, flow = 0;
6     FlowEdge(int from, int to, long long cap) : from(
7         from), to(to), cap(cap) {}
8 };
9
10 struct Dinic {
11     const long long flow_inf = 1e18;
12     vector<FlowEdge> edges;
13     vector<vector<int>> adj;
14     int n, m = 0;
15     int s, t;
16     vector<int> level, ptr;
17     queue<int> q;
18
19     Dinic(int n, int s, int t) : n(n), s(s), t(t) {
20         adj.resize(n);
21         level.resize(n);
22         ptr.resize(n);
23     }
24
25     void add_edge(int from, int to, long long cap) {
26         edges.emplace_back(from, to, cap);
27         edges.emplace_back(to, from, 0);
28         adj[from].push_back(m);
29         adj[to].push_back(m + 1);
30         m += 2;
31     }
32
33     bool bfs() {
34         while (!q.empty()) {
35             int from = q.front();
36             q.pop();
37             for (int id : adj[from]) {
38                 if (edges[id].cap == edges[id].flow)
39                     continue;
40                 if (level[edges[id].to] != -1)
41                     continue;
42                 level[edges[id].to] = level[from] +
43                     1;
44                 q.push(edges[id].to);
45             }
46         }
47         return level[t] != -1;
48     }
49
50     long long dfs(int from, long long pushed) {
51         if (pushed == 0)

```

```

52             return 0;
53             if (from == t)
54                 return pushed;
55             for (int& cid = ptr[from]; cid < (int)adj[
56                 from].size(); cid++) {
57                 int id = adj[from][cid];
58                 int to = edges[id].to;
59                 if (level[from] + 1 != level[to])
60                     continue;
61                 long long tr = dfs(to, min(pushed, edges[
62                     id].cap - edges[id].flow));
63                 if (tr == 0)
64                     continue;
65                 edges[id].flow += tr;
66                 edges[id ^ 1].flow -= tr;
67                 return tr;
68             }
69             return 0;
70         }
71     }
72
73     long long flow() {
74         long long f = 0;
75         while (true) {
76             fill(level.begin(), level.end(), -1);
77             level[s] = 0;
78             q.push(s);
79             if (!bfs())
80                 break;
81             fill(ptr.begin(), ptr.end(), 0);
82             while (long long pushed = dfs(s, flow_inf
83                 )) {
84                 f += pushed;
85             }
86         }
87         return f;
88     }
89 };

```

## 6.5 Khan

```

1 // topo-sort DAG
2 // lexicograficamente menor.
3 // N: numero de vÃrtices (1-indexado)
4 // adj: lista de adjacencia do grafo
5
6 const int MAXN = 5 * 1e5 + 2;
7 vector<int> adj[MAXN];
8 int N;
9
10 vector<int> kahn() {
11     vector<int> indegree(N + 1, 0);
12     for (int u = 1; u <= N; u++) {
13         for (int v : adj[u]) {
14             indegree[v]++;
15         }
16     }
17     priority_queue<int, vector<int>, greater<int>> pq;
18     ;
19     for (int i = 1; i <= N; i++) {
20         if (indegree[i] == 0) {
21             pq.push(i);
22         }
23     }
24     vector<int> result;
25     while (!pq.empty()) {
26         int u = pq.top();
27         pq.pop();
28         result.push_back(u);
29         for (int v : adj[u]) {
30             indegree[v]--;
31             if (indegree[v] == 0) {
32                 pq.push(v);
33             }
34         }
35     }
36     return result;
37 }

```

```

33     }
34 }
35 if (result.size() != N) {
36     return {};
37 }
38 return result;
39 }

```

## 6.6 Topological Sort

```

1 vector<int> adj[MAXN];
2 vector<int> estado(MAXN); // 0: nao visitado 1:
   processamento 2: processado
3 vector<int> ordem;
4 bool temCiclo = false;
5
6 void dfs(int v) {
7     if(estado[v] == 1) {
8         temCiclo = true;
9         return;
10    }
11    if(estado[v] == 2) return;
12    estado[v] = 1;
13    for(auto &nei : adj[v]) {
14        if(estado[v] != 2) dfs(nei);
15    }
16    estado[v] = 2;
17    ordem.push_back(v);
18    return;
19 }

```

## 6.7 Acha Pontes

```

1 vector<int> d, low, pai; // d[v] Tempo de
   descoberta (discovery time)
2 vector<bool> vis;
3 vector<int> pontos_articulacao;
4 vector<pair<int, int>> pontes;
5 int tempo;
6
7 vector<vector<int>> adj;
8
9 void dfs(int u) {
10     vis[u] = true;
11     tempo++;
12     d[u] = low[u] = tempo;
13     int filhos_dfs = 0;
14     for (int v : adj[u]) {
15         if (v == pai[u]) continue;
16         if (vis[v]) { // back edge
17             low[u] = min(low[u], d[v]);
18         } else {
19             pai[v] = u;
20             filhos_dfs++;
21             dfs(v);
22             low[u] = min(low[u], low[v]);
23             if (pai[u] == -1 && filhos_dfs > 1) {
24                 pontos_articulacao.push_back(u);
25             }
26             if (pai[u] != -1 && low[v] >= d[u]) {
27                 pontos_articulacao.push_back(u);
28             }
29             if (low[v] > d[u]) {
30                 pontes.push_back({min(u, v), max(u, v)});
31             }
32         }
33     }
34 }

```

## 6.8 Edmonds-karp

```

1 // Edmonds-Karp com scalling  $O(E \log(F))$ 
2
3 int n, m;
4 const int MAXN = 510;
5 vector<vector<int>> capacity(MAXN, vector<int>(MAXN,
   0));
6 vector<vector<int>> adj(MAXN);
7
8 int bfs(int s, int t, int scale, vector<int>& parent)
   {
9     fill(parent.begin(), parent.end(), -1);
10    parent[s] = -2;
11    queue<pair<int, int>> q;
12    q.push({s, LLONG_MAX});
13
14    while (!q.empty()) {
15        int cur = q.front().first;
16        int flow = q.front().second;
17        q.pop();
18
19        for (int next : adj[cur]) {
20            if (parent[next] == -1 && capacity[cur][
   next] >= scale) {
21                parent[next] = cur;
22                int new_flow = min(flow, capacity[cur
   ][next]);
23                if (next == t)
24                    return new_flow;
25                q.push({next, new_flow});
26            }
27        }
28    }
29
30    return 0;
31 }
32
33 int maxflow(int s, int t) {
34     int flow = 0;
35     vector<int> parent(MAXN);
36     int new_flow;
37     int scalling = 1ll << 62;
38
39     while (scalling > 0) {
40         while (new_flow = bfs(s, t, scalling, parent)
   ){
41             if (new_flow == 0) continue;
42             flow += new_flow;
43             int cur = t;
44             while (cur != s) {
45                 int prev = parent[cur];
46                 capacity[prev][cur] -= new_flow;
47                 capacity[cur][prev] += new_flow;
48                 cur = prev;
49             }
50             scalling /= 2;
51         }
52     }
53
54     return flow;
55 }

```

## 6.9 Kruskal

```

1 // Ordena as arestas por peso, insere se ja nao
   estiver no mesmo componente
2 //  $O(E \log E)$ 
3
4 struct Edge {
5     int u, v, w;
6     bool operator <(Edge const & other) {
7         return weight < other.weight;
8     }
9 }

```

```

10
11 vector<Edge> kruskal(int n, vector<Edge> edges) {
12     vector<Edge> mst;
13     DSU dsu = DSU(n + 1);
14     sort(edges.begin(), edges.end());
15     for (Edge e : edges) {
16         if (dsu.find(e.u) != dsu.find(e.v)) {
17             mst.push_back(e);
18             dsu.join(e.u, e.v);
19         }
20     }
21     return mst;
22 }

```

## 6.10 Bellman Ford

```

1 struct Edge {
2     int u, v, w;
3 };
4
5 // se x = -1, nao tem ciclo
6 // se x != -1, pegar pais de x pra formar o ciclo
7
8 int n, m;
9 vector<Edge> edges;
10 vector<int> dist(n);
11 vector<int> pai(n, -1);
12
13 for (int i = 0; i < n; i++) {
14     x = -1;
15     for (Edge &e : edges) {
16         if (dist[e.u] + e.w < dist[e.v]) {
17             dist[e.v] = max(-INF, dist[e.u] + e.w);
18             pai[e.v] = e.u;
19             x = e.v;
20         }
21     }
22 }
23
24 // achando caminho (se precisar)
25 for (int i = 0; i < n; i++) x = pai[x];
26
27 vector<int> ciclo;
28 for (int v = x;; v = pai[v]) {
29     cycle.push_back(v);
30     if (v == x && ciclo.size() > 1) break;
31 }
32 reverse(ciclo.begin(), ciclo.end());

```

## 6.11 Lca Jc

```

1 const int MAXN = 200005;
2 int N;
3 int LOG;
4
5 vector<vector<int>> adj;
6 vector<int> profundidade;
7 vector<vector<int>> cima; // cima[v][j] eh o 2^j-
8 // esimo ancestral de v
9
10 void dfs(int v, int p, int d) {
11     profundidade[v] = d;
12     cima[v][0] = p; // o pai direto eh o 2^0-ésimo
13     // ancestral
14     for (int j = 1; j < LOG; j++) {
15         // se o ancestral 2^(j-1) existir, calculamos
16         // o 2^j
17         if (cima[v][j - 1] != -1) {
18             cima[v][j] = cima[cima[v][j - 1]][j - 1];
19         } else {
20             cima[v][j] = -1; // nao tem ancestral
21             // superior
22         }
23     }
24 }

```

```

18     }
19 }
20 for (int nei : adj[v]) {
21     if (nei != p) {
22         dfs(nei, v, d + 1);
23     }
24 }
25 }
26
27 void build(int root) {
28     LOG = ceil(log2(N));
29     profundidade.assign(N + 1, 0);
30     cima.assign(N + 1, vector<int>(LOG, -1));
31     dfs(root, -1, 0);
32 }
33
34 int get_lca(int a, int b) {
35     if (profundidade[a] < profundidade[b]) {
36         swap(a, b);
37     }
38     // sobe 'a' ate a mesma profundidade de 'b'
39     for (int j = LOG - 1; j >= 0; j--) {
40         if (profundidade[a] - (1 << j) >=
41             profundidade[b]) {
42             a = cima[a][j];
43         }
44     }
45     // se 'b' era um ancestral de 'a', então 'a'
46     // agora é igual a 'b'
47     if (a == b) {
48         return a;
49     }
50     // sobe os dois juntos ate encontrar os filhos do
51     // LCA
52     for (int j = LOG - 1; j >= 0; j--) {
53         if (cima[a][j] != -1 && cima[a][j] != cima[b]
54             [j]) {
55             a = cima[a][j];
56             b = cima[b][j];
57         }
58     }
59     return cima[a][0];
60 }

```

## 6.12 Lca

```

1 // LCA - CP algorithm
2 // preprocessing O(NlogN)
3 // lca O(logN)
4 // Uso: criar LCA com a quantidade de vertices (n) e
5 // lista de adjacencias (adj)
6 // chamar a funcao preprocess com a raiz da arvore
7
8 struct LCA {
9     int n, l, timer;
10     vector<vector<int>> adj;
11     vector<int> tin, tout;
12     vector<vector<int>> up;
13
14     LCA(int n, const vector<vector<int>>& adj) : n(n)
15     , adj(adj) {}
16
17     void dfs(int v, int p) {
18         tin[v] = ++timer;
19         up[v][0] = p;
20         for (int i = 1; i <= LOG; ++i)
21             up[v][i] = up[up[v][i-1]][i-1];
22
23         for (int u : adj[v]) {
24             if (u != p)
25                 dfs(u, v);
26         }
27     }
28 }

```

```

26     tout[v] = ++timer;
27 }
28
29 bool is_ancestor(int u, int v) {
30     return tin[u] <= tin[v] && tout[u] >= tout[v];
31 };
32
33 int lca(int u, int v) {
34     if (is_ancestor(u, v))
35         return u;
36     if (is_ancestor(v, u))
37         return v;
38     for (int i = 1; i >= 0; --i) {
39         if (!is_ancestor(up[u][i], v))
40             u = up[u][i];
41     }
42     return up[u][0];
43 }
44
45 void preprocess(int root) {
46     tin.resize(n);
47     tout.resize(n);
48     timer = 0;
49     l = ceil(log2(n));
50     up.assign(n, vector<int>(l + 1));
51     dfs(root, root);
52 }
53 };

```

### 6.13 Kosaraju

```

1 bool vis[MAXN];
2 vector<int> order;
3 int component[MAXN];
4 int N, m;
5 vector<int> adj[MAXN], adj_rev[MAXN];
6
7 // dfs no grafo original para obter a ordem (pos-
8 // order)
9 void dfs1(int u) {
10     vis[u] = true;
11     for (int v : adj[u]) {
12         if (!vis[v]) {
13             dfs1(v);
14         }
15     }
16     order.push_back(u);
17 }
18
19 // dfs o grafo reverso para encontrar os SCCs
20 void dfs2(int u, int c) {
21     component[u] = c;
22     for (int v : adj_rev[u]) {
23         if (component[v] == -1) {
24             dfs2(v, c);
25         }
26     }
27 }
28
29 int kosaraju() {
30     order.clear();
31     fill(vis + 1, vis + N + 1, false);
32     for (int i = 1; i <= N; i++) {
33         if (!vis[i]) {
34             dfs1(i);
35         }
36     }
37     fill(component + 1, component + N + 1, -1);
38     int c = 0;
39     reverse(order.begin(), order.end());
40     for (int u : order) {
41         if (component[u] == -1) {

```

```

41         dfs2(u, c++);
42     }
43 }
44     return c;
45 }

```

### 6.14 Pega Ciclo

```

1 // encontra um ciclo em g
2 // g[u] = vector<pair<id_aresta, vizinho>>
3 // rec_arestas: true -> retorna ids das arestas do
4 // ciclo; false -> retorna vertices do ciclo
5 // directed: grafo direcionado?
6
7 const int MAXN = 5 * 1e5 + 2;
8 vector<pair<int, int>> g[MAXN];
9 int N;
10 bool DIRECTED = false;
11 vector<int> color(MAXN), parent(MAXN, -1), edgein(
12     MAXN, -1); // color: 0,1,2 ; edgein[v] = id da
13 // aresta que entra em v
14 int ini_ciclo = -1, fim_ciclo = -1, back_edge_id =
15     -1;
16
17 bool dfs(int u, int pai_edge) {
18     color[u] = 1; // cinza
19     for (auto [id, v] : g[u]) {
20         if (!DIRECTED && id == pai_edge) continue; //
21         // ignorar aresta de volta ao pai em n-dir
22         if (color[v] == 0) {
23             parent[v] = u;
24             edgein[v] = id;
25             if (dfs(v, id)) return true;
26         } else if (color[v] == 1) {
27             // back-edge u -> v detectado
28             ini_ciclo = u;
29             fim_ciclo = v;
30             back_edge_id = id;
31             return true;
32         }
33         // se color[v] == 2, ignora
34     }
35     color[u] = 2; // preto
36     return false;
37 }
38
39 // retorna ids das arestas do ciclo
40 vector<int> pega_ciclo(bool rec_arestas) {
41     for (int u = 1; u <= N; u++) {
42         if (color[u] != 0) continue;
43         if (dfs(u, -1)) {
44             // caminho u -> ... -> v via parent
45             vector<int> path;
46             int cur = ini_ciclo;
47             path.push_back(cur);
48             while (cur != fim_ciclo) {
49                 cur = parent[cur];
50                 path.push_back(cur);
51             }
52             // path = [u, ..., v] -> inverter para [v,
53             // ..., u]
54             reverse(path.begin(), path.end());
55             if (!rec_arestas) return path;
56             // converte para ids das arestas: edgein[
57             // node] eh a aresta que entra em node
58             vector<int> edges;
59             for (int i = 1; i < path.size(); i++)
60                 edges.push_back(edgein[path[i]]);
61             // adiciona a aresta de retorno u -> v
62             edges.push_back(back_edge_id);
63             return edges;
64         }
65     }
66 }

```

```

58     }
59     return {};
60 }

```

## 6.15 Min Cost Max Flow

```

1 // Encontra o menor custo para passar K de fluxo em
  // um grafo com N vertices
2 // Funciona com multiplas arestas para o mesmo par de
  // vertices
3 // Para encontrar o min cost max flow eh so fazer K =
  // infinito
4
5 struct Edge {
6     int from, to, capacity, cost, id;
7 };
8
9 vector<vector<array<int, 2>>> adj;
10 vector<Edge> edges; // arestas pares sao as normais e
  // suas reversas sao as impares
11
12 const int INF = LLONG_MAX;
13
14 void shortest_paths(int n, int v0, vector<int>& dist,
  // vector<int>& edge_to) {
15     dist.assign(n, INF);
16     dist[v0] = 0;
17     vector<bool> in_queue(n, false);
18     queue<int> q;
19     q.push(v0);
20     edge_to.assign(n, -1);
21
22     while (!q.empty()) {
23         int u = q.front();
24         q.pop();
25         in_queue[u] = false;
26         for (auto [v, id] : adj[u]) {
27             if (edges[id].capacity > 0 && dist[v] >
28                 dist[u] + edges[id].cost) {
29                 dist[v] = dist[u] + edges[id].cost;
30                 edge_to[v] = id;
31                 if (!in_queue[v]) {
32                     in_queue[v] = true;
33                     q.push(v);
34                 }
35             }
36         }
37     }
38
39 void add_edge(int from, int to, int capacity, int
  // cost){
40     edges.push_back({from, to, capacity, cost, (int)
41         edges.size()});
42     edges.push_back({to, from, 0, -cost, (int)edges.
43         size()}); // reversa
44 }
45
46 int min_cost_flow(int N, int K, int s, int t) {
47     adj.assign(N, vector<array<int, 2>>());
48
49     for (Edge e : edges) {
50         adj[e.from].push_back({e.to, e.id});
51     }
52
53     int flow = 0;
54     int cost = 0;
55     vector<int> dist, edge_to;
56     while (flow < K) {
57         shortest_paths(N, s, dist, edge_to);
58         if (dist[t] == INF)
59             break;

```

```

59 // find max flow on that path
60 int f = K - flow;
61 int cur = t;
62 while (cur != s) {
63     f = min(f, edges[edge_to[cur]].capacity);
64     cur = edges[edge_to[cur]].from;
65 }
66
67 // apply flow
68 flow += f;
69 cost += f * dist[t];
70 cur = t;
71 while (cur != s) {
72     int edge = edge_to[cur];
73     int rev_edge = edge^1;
74
75     edges[edge].capacity -= f;
76     edges[rev_edge].capacity += f;
77     cur = edges[edge].from;
78 }
79
80 }
81
82 if (flow < K)
83     return -1;
84 else
85     return cost;
86 }

```

## 7 Primitives

## 8 DP

### 8.1 Lis

```

1 int lis_nlogn(vector<int> &v) {
2     vector<int> lis;
3     lis.push_back(v[0]);
4     for (int i = 1; i < v.size(); i++) {
5         if (v[i] > lis.back()) {
6             // estende a lis
7             lis.push_back(v[i]);
8         } else {
9             // encontra o primeiro elemento em lis
10             // que >= v[i].
11             auto it = lower_bound(lis.begin(), lis.
12                 end(), v[i]);
13             *it = v[i];
14         }
15     }
16     return lis.size();
17 }
18
19 // lis na tree do problema da sub
20 const int MAXN_TREE = 100001;
21 vector<int> adj[MAXN_TREE];
22 int values[MAXN_TREE];
23 int ans = 0;
24
25 void dfs(int u, int p, vector<int>& tails) {
26     auto it = lower_bound(tails.begin(), tails.end(),
27         values[u]);
28     int prev = -1;
29     bool coloquei = false;
30     if (it == tails.end()) {
31         tails.push_back(values[u]);
32         coloquei = true;
33     } else {
34         prev = *it;
35         *it = values[u];
36     }
37     ans = max(ans, (int)tails.size());
38 }

```

```

35     for (int v : adj[u]) {
36         if (v != p) {
37             dfs(v, u, tails);
38         }
39     }
40     if (coloquei) {
41         tails.pop_back();
42     } else {
43         *it = prev;
44     }
45 }

```

## 8.2 Edit Distance

```

1     vector<vector<int>> dp(n+1, vector<int>(m+1,
      LLONG_MIN));
2     for(int j = 0; j <= m; j++) dp[0][j] = j;
3     for(int i = 0; i <= n; i++) dp[i][0] = i;
4     for(int i = 1; i <= n; i++) {
5         for(int j = 1; j <= m; j++) {
6             if(a[i-1] == b[j-1]) {
7                 dp[i][j] = dp[i-1][j-1];
8             } else {
9                 dp[i][j] = min({dp[i-1][j] + 1, dp[i-1][j-1] + 1, dp[i][j-1] + 1});
10            }
11        }
12    }
13    cout << dp[n][m];

```

## 8.3 Bitmask

```

1 // dp de intervalos com bitmask
2 int prox(int idx) {
3     return lower_bound(S.begin(), S.end(), array<int, 4>{S[idx][1], 011, 011, 011}) - S.begin();
4 }
5
6 int dp[1002][(int)(111 << 10)];
7
8 int rec(int i, int vis) {
9     if (i == (int)S.size()) {
10         if (__builtin_popcountll(vis) == N) return 0;
11         return LLONG_MIN;
12     }
13     if (dp[i][vis] != -1) return dp[i][vis];
14     int ans = rec(i + 1, vis);
15     ans = max(ans, rec(prox(i), vis | (111 << S[i][3])) + S[i][2]);
16     return dp[i][vis] = ans;
17 }

```

## 8.4 Lcs

```

1 string s1, s2;
2 int dp[1001][1001];
3 int lcs(int i, int j) {
4     if (i < 0 || j < 0) return 0;
5     if (dp[i][j] != -1) return dp[i][j];
6     if (s1[i] == s2[j]) return dp[i][j] = 1 + lcs(i - 1, j - 1);
7     else return dp[i][j] = max(lcs(i - 1, j), lcs(i, j - 1));
8 }

```

## 8.5 Digit

```

1 vector<int> digits;
2
3 int dp[20][10][2][2];
4

```

```

5 int rec(int i, int last, int flag, int started) {
6     if (i == (int)digits.size()) return 1;
7     if (dp[i][last][flag][started] != -1) return dp[i][last][flag][started];
8     int lim;
9     if (flag) lim = 9;
10    else lim = digits[i];
11    int ans = 0;
12    for (int d = 0; d <= lim; d++) {
13        if (started && d == last) continue;
14        int new_flag = flag;
15        int new_started = started;
16        if (d > 0) new_started = 1;
17        if (!flag && d < lim) new_flag = 1;
18        ans += rec(i + 1, d, new_flag, new_started);
19    }
20    return dp[i][last][flag][started] = ans;
21 }

```

## 8.6 Knapsack

```

1 // dp[i][j] => i-esimo item com j-carga sobrando na mochila
2 // O(N * W)
3 vector<vector<int>> dp(N + 1, vector<int>(W + 1, 0));
4 for (int i = 1; i <= N; i++) dp[i][0] = 0;
5 for (int j = 0; j <= W; j++) dp[0][j] = 0;
6 for (int i = 1; i <= N; i++) {
7     for (int j = 0; j <= W; j++) {
8         dp[i][j] = dp[i - 1][j];
9         if (j >= items[i-1].first) {
10             dp[i][j] = max(dp[i][j], dp[i - 1][j - items[i-1].first] + items[i-1].second);
11         }
12     }
13 }
14 cout << dp[N][W] << '\n';

```

## 8.7 Lis Seg

```

1 // comprime coordenadas pra quando ai <= 1e9
2 vector<int> vals(a.begin(), a.end());
3 sort(vals.begin(), vals.end());
4 vals.erase(unique(vals.begin(), vals.end()), vals.end());
5 auto id = [&](int x) -> int {
6     return lower_bound(vals.begin(), vals.end(), x) - vals.begin();
7 };
8 for (int i = 0; i < n; i++) a[i] = id(a[i]);
9 // fim da parte de compr
10 SegTree seg(n);
11 // dp[i]: lis que termina em i
12 vector<int> dp(n, 1); // dp[i] = 1 base case
13 for (int i = 0; i < n; i++) {
14     if (a[i] > 0) dp[i] = seg.query(0, max(011, a[i] - 1)) + 1;
15     seg.update(a[i], dp[i]);
16 }
17 cout << *max_element(dp.begin(), dp.end()) << '\n';

```

## 8.8 Disjoint Blocks

```

1 // num max de subarrays disjuntos com soma x usando apenas
2 // prefixo i (ou seja, considerando prefixo a[1..i]).
3 int disjointSumX(vector<int> &a, int x) {
4     int n = a.size();
5     map<int, int> best; // best[pref] = melhor dp visto para esse pref
6     best[0] = 0;
7     int pref = 0;

```

```

8     vector<int> dp(n + 1, 0); // dp[0] = 0
9     for (int i = 1; i <= n; i++) {
10         pref += a[i - 1];
11         dp[i] = dp[i-1];
12         auto it = best.find(pref - x);
13         if (it != best.end()) {
14             dp[i] = max(dp[i], it->second + 1);
15         }
16         best[pref] = max(best[pref], dp[i]);
17     }
18     return dp[n];
19 }

```

## 9 General

### 9.1 Brute Choose

```

1 vector<int> elements;
2 int N, K;
3 vector<int> comb;
4
5 void brute_choose(int i) {
6     if (comb.size() == K) {
7         for (int j = 0; j < comb.size(); j++) {
8             cout << comb[j] << ' ';
9         }
10        cout << '\n';
11        return;
12    }
13    if (i == N) return;
14    int r = N - i;
15    int preciso = K - comb.size();
16    if (r < preciso) return;
17    comb.push_back(elements[i]);
18    brute_choose(i + 1);
19    comb.pop_back();
20    brute_choose(i + 1);
21 }

```

### 9.2 Struct

```

1 struct Pessoa{
2     // Atributos
3     string nome;
4     int idade;
5
6     // Comparador
7     bool operator<(const Pessoa& other) const{
8         if(idade != other.idade) return idade > other
9         .idade;
10        else return nome > other.nome;
11    }
12 }

```

### 9.3 Mex

```

1 struct MEX {
2     map<int, int> f;
3     set<int> falta;
4     int tam;
5     MEX(int n) : tam(n) {
6         for (int i = 0; i <= n; i++) falta.insert(i);
7     }
8     void add(int x) {
9         f[x]++;
10        if (f[x] == 1 && x >= 0 && x <= tam) {
11            falta.erase(x);
12        }
13    }
14    void rem(int x) {
15        if (f.count(x) && f[x] > 0) {

```

```

16            f[x]--;
17            if (f[x] == 0 && x >= 0 && x <= tam) {
18                falta.insert(x);
19            }
20        }
21    }
22    int get() {
23        if (falta.empty()) return tam + 1;
24        return *falta.begin();
25    }
26 };

```

### 9.4 Count Permutations

```

1 // Returns the number of distinct permutations
2 // that are lexicographically less than the string t
3 // using the provided frequency (freq) of the
4 // characters
5 // 0(n*freq.size())
6 int countPermLess(vector<int> freq, const string &t)
7 {
8     int n = t.size();
9     int ans = 0;
10
11    vector<int> fact(n + 1, 1), invfact(n + 1, 1);
12    for (int i = 1; i <= n; i++)
13        fact[i] = (fact[i - 1] * i) % MOD;
14    invfact[n] = fexp(fact[n], MOD - 2, MOD);
15    for (int i = n - 1; i >= 0; i--)
16        invfact[i] = (invfact[i + 1] * (i + 1)) % MOD;
17
18    // For each position in t, try placing a letter
19    // smaller than t[i] that is in freq
20    for (int i = 0; i < n; i++) {
21        for (char c = 'a'; c < t[i]; c++) {
22            if (freq[c - 'a'] > 0) {
23                freq[c - 'a']--;
24                int ways = fact[n - i - 1];
25                for (int f : freq)
26                    ways = (ways * invfact[f]) % MOD;
27                ans = (ans + ways) % MOD;
28                freq[c - 'a']++;
29            }
30        }
31        if (freq[t[i] - 'a'] == 0) break;
32        freq[t[i] - 'a']--;
33    }
34    return ans;
35 }

```

### 9.5 Bitwise

```

1 int check_kth_bit(int x, int k) {
2     return (x >> k) & 1;
3 }
4
5 void print_on_bits(int x) {
6     for (int k = 0; k < 32; k++) {
7         if (check_kth_bit(x, k)) {
8             cout << k << ' ';
9         }
10    }
11    cout << '\n';
12 }
13
14 int count_on_bits(int x) {
15     int ans = 0;
16     for (int k = 0; k < 32; k++) {
17         if (check_kth_bit(x, k)) {
18             ans++;
19         }
20    }
21 }

```

```

20 }
21 return ans;
22 }
23
24 bool is_even(int x) {
25     return ((x & 1) == 0);
26 }
27
28 int set_kth_bit(int x, int k) {
29     return x | (1 << k);
30 }
31
32 int unset_kth_bit(int x, int k) {
33     return x & ~(1 << k);
34 }
35
36 int toggle_kth_bit(int x, int k) {
37     return x ^ (1 << k);
38 }
39
40 bool check_power_of_2(int x) {
41     return count_on_bits(x) == 1;
42 }

```

## 10 Geometry

### 10.1 Convex Hull

```

1 #include <bits/stdc++.h>
2
3 using namespace std;
4 #define int long long
5 typedef int cod;
6
7 struct point
8 {
9     cod x,y;
10     point(cod x = 0, cod y = 0): x(x), y(y)
11     {}
12
13     double modulo()
14     {
15         return sqrt(x*x + y*y);
16     }
17
18     point operator+(point o)
19     {
20         return point(x+o.x, y+o.y);
21     }
22     point operator-(point o)
23     {
24         return point(x - o.x , y - o.y);
25     }
26     point operator*(cod t)
27     {
28         return point(x*t, y*t);
29     }
30     point operator/(cod t)
31     {
32         return point(x/t, y/t);
33     }
34
35     cod operator*(point o)
36     {
37         return x*o.x + y*o.y;
38     }
39     cod operator^(point o)
40     {
41         return x*o.y - y * o.x;
42     }
43     bool operator<(point o)
44     {

```

```

45         if( x != o.x) return x < o.x;
46         return y < o.y;
47     }
48
49 };
50
51 int ccw(point p1, point p2, point p3)
52 {
53     cod cross = (p2-p1) ^ (p3-p1);
54     if(cross == 0) return 0;
55     else if(cross < 0) return -1;
56     else return 1;
57 }
58
59 vector <point> convex_hull(vector<point> p)
60 {
61     sort(p.begin(), p.end());
62     vector<point> L,U;
63
64     //Lower
65     for(auto pp : p)
66     {
67         while(L.size() >= 2 and ccw(L[L.size() - 2],
68             L.back(), pp) == -1)
69             // Ãr -1 pq eu nÃo quero excluir os
70             // colineares
71             L.pop_back();
72         L.push_back(pp);
73     }
74
75     reverse(p.begin(), p.end());
76
77     //Upper
78     for(auto pp : p)
79     {
80         while(U.size() >= 2 and ccw(U[U.size()-2], U
81             .back(), pp) == -1)
82             {
83                 U.pop_back();
84             }
85         U.push_back(pp);
86     }
87
88     L.pop_back();
89     L.insert(L.end(), U.begin(), U.end()-1);
90     return L;
91
92 cod area(vector<point> v)
93 {
94     int ans = 0;
95     int aux = (int)v.size();
96     for(int i = 2; i < aux; i++)
97     {
98         ans += ((v[i] - v[0])^(v[i-1] - v[0]))/2;
99     }
100     ans = abs(ans);
101     return ans;
102 }
103
104 int bound(point p1 , point p2)
105 {
106     return __gcd(abs(p1.x-p2.x), abs(p1.y-p2.y));
107 }
108 //teorema de pick [pontos = A - (bound+points)/2 + 1]
109
110 int32_t main()
111 {
112
113     int n;
114     cin >> n;

```



```

115 vector<point> v(n);
116 for(int i = 0; i < n; i++)
117 {
118     cin >> v[i].x >> v[i].y;
119 }
120
121 vector <point> ch = convex_hull(v);
122
123 cout << ch.size() << '\n';
124 for(auto p : ch) cout << p.x << " " << p.y << "\n";
125
126 return 0;
127 }
128

```

## 10.2 Inside Polygon

```

1 // Convex O(logn)
2
3 bool insideT(point a, point b, point c, point e){
4     int x = ccw(a, b, e);
5     int y = ccw(b, c, e);
6     int z = ccw(c, a, e);
7     return !((x==1 or y==1 or z==1) and (x==-1 or y
8 ==-1 or z==-1));
9 }
10
11 bool inside(vp &p, point e){ // ccw
12     int l=2, r=(int)p.size()-1;
13     while(l<r){
14         int mid = (l+r)/2;
15         if(ccw(p[0], p[mid], e) == 1)
16             l=mid+1;
17         else{
18             r=mid;
19         }
20     }
21     // bordo
22     // if(r==(int)p.size()-1 and ccw(p[0], p[r], e)
23 ==0) return false;
24     // if(r==2 and ccw(p[0], p[1], e)==0) return
25 false;
26     // if(ccw(p[r], p[r-1], e)==0) return false;
27     return insideT(p[0], p[r-1], p[r], e);
28 }
29
30 // Any O(n)
31
32 int inside(vp &p, point pp){
33     // 1 - inside / 0 - boundary / -1 - outside
34     int n = p.size();
35     for(int i=0; i<n; i++){
36         int j = (i+1)%n;
37         if(line({p[i], p[j]}).inside_seg(pp))
38             return 0;
39     }
40     int inter = 0;
41     for(int i=0; i<n; i++){
42         int j = (i+1)%n;
43         if(p[i].x <= pp.x and pp.x < p[j].x and ccw(p
44 [i], p[j], pp)==1)
45             inter++; // up
46         else if(p[j].x <= pp.x and pp.x < p[i].x and

```

```

44         ccw(p[i], p[j], pp)==-1)
45             inter++; // down
46     }
47     if(inter%2==0) return -1; // outside
48     else return 1; // inside
49 }

```

## 10.3 Point Location

```

1
2 int32_t main(){
3     sws;
4
5     int t; cin >> t;
6
7     while(t--){
8
9         int x1, y1, x2, y2, x3, y3; cin >> x1 >> y1
10 >> x2 >> y2 >> x3 >> y3;
11
12         int deltax1 = (x1-x2), deltax2 = (x2-x3), deltax3 = (x3-x1);
13         int deltay1 = (y1-y2), deltay2 = (y2-y3), deltay3 = (y3-y1);
14
15         int compx = (deltax1*deltay2 - deltax2*deltay1);
16         int compy = (deltax2*deltay3 - deltax3*deltay2);
17         int ans = (compx + compy) / 2;
18
19         if(ans == 0){cout << "TOUCH\n"; continue;}
20         if(ans < 0){cout << "RIGHT\n"; continue;}
21         if(ans > 0){cout << "LEFT\n"; continue;}
22     }
23     return 0;
24 }

```

## 10.4 Lattice Points

```

1 ll gcd(ll a, ll b) {
2     return b == 0 ? a : gcd(b, a % b);
3 }
4 ll area_triangulo(ll x1, ll y1, ll x2, ll y2, ll x3,
5 ll y3) {
6     return abs(x1 * (y2 - y3) + x2 * (y3 - y1) + x3 *
7 (y1 - y2));
8 }
9 ll pontos_borda(ll x1, ll y1, ll x2, ll y2) {
10     return gcd(abs(x2 - x1), abs(y2 - y1));
11 }
12
13 int32_t main() {
14     ll x1, y1, x2, y2, x3, y3;
15     cin >> x1 >> y1;
16     cin >> x2 >> y2;
17     cin >> x3 >> y3;
18     ll area = area_triangulo(x1, y1, x2, y2, x3, y3);
19     ll tot_borda = pontos_borda(x1, y1, x2, y2) +
20 pontos_borda(x2, y2, x3, y3) + pontos_borda(x3,
21 y3, x1, y1);
22     ll ans = (area - tot_borda) / 2 + 1;
23     cout << ans << endl;
24
25     return 0;
26 }

```