Circuit Theory and Electronics Fundamentals

Department of Electrical and Computer Engineering, Técnico, University of Lisbon
Mestrado em Engenharia Aeroespacial

Laboratory 2 Report

Group 7

Diogo Matos Nº95778
Tiago Carneiro Nº95849
João Diz Nº95810

April 5th, 2021

Contents

1 Introduction

The objective of this laboratory assignment is to study a circuit containing a dependent voltage source (V_c) and independent (I_d, V_a) current and voltage sources, connected to resistors (R_1 to R_7) and to a capacitor (C). The circuit and its organization can be seen in Figure 1.

In Section 2, a theoretical analysis of the circuit is presented. In Section 3, the circuit is analysed by simulation, and the results are compared to the theoretical results obtained in Section 2. The conclusions of this study are outlined in Section 5.

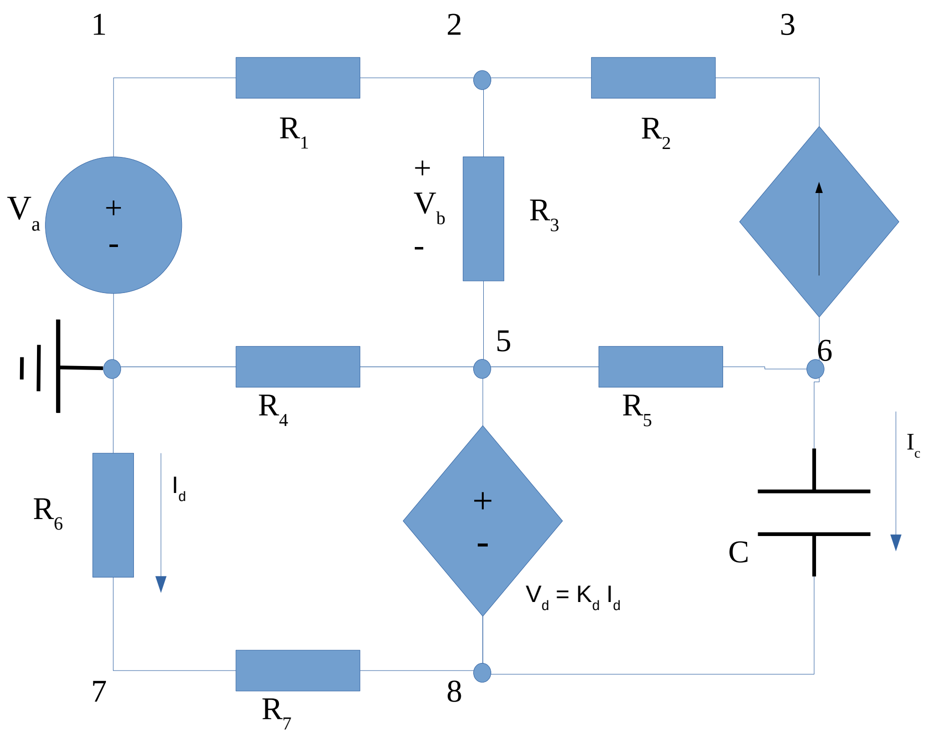

Figure 1: Circuit topography

2 Theoretical Analysis

In this section, the circuit shown in Figure 1 is analysed theoretically, through the method of Node Analysis.

2.1 Node analysis for t<0

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & G_1 & -G_1-G_3-G_2 & G_2 & G_3 & 0 & 0 & 0 \\ 0 & 0 & G_2+K_b & -G_2 & -K_b & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -G_6-G_7 & G_7 \\ 0 & 0 & 0 & 0 & 1 & 0 & K_d*G_6 & -1 \\ 0 & 0 & -K_b & 0 & G_5+K_b & -G_5 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & G_3 & 0 & -G_3-G_4-G_5 & G_5 & G_7 & -G_7 \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = \begin{bmatrix} V_s \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{1}$$

The results obtained after implementing the KNL in Octave can be seen in Tables 1 and 2:

Name	Value [V]
V_0	0.000000
V_1	5.103557
V_2	4.832610
V_3	4.274744
V_5	4.870590
V_6	5.723435
V_7	-1.944932
V_8	-2.897433

Table 1: Voltages obtained in the theoretical analysis in Octave.

The currents derived from the results of the node analysis are found in Table 2.

Name	Value [A]
$@I_1$	-0.000261
$@I_2$	-0.000273
$@I_3$	-0.000013
$@I_4$	0.001203
$@I_5$	0.000273
$@I_6$	-0.000942
$@I_7$	-0.000942

Table 2: Currents obtained in the theoretical analysis in Octave.

2.2 Node Analysis for t≥0 (Natural solution)

Here, we made use of Node Analysis to determine the current that passed through the capacitor. By short-circuting the independent voltage source V_s and by swapping the capacitor with a voltage source V_x = V_6 - V_8 we were able to calculate the current that flowed through the capacitor and the equivalent resistance. We short-circuted the independent voltage source because we are using the thevenin/norton theorems to calculate the resistence as seen by the capacitor and we swapped the capacitor with a voltage source as at t = 0 the capacitor begins discharging and it has a voltage of V_x (since it's voltage is at a peak). Node analysis matrix:

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & G_1 & -G_1-G_3-G_2 & G_2 & G_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & -G_2+K_b & G_2 & -K_b & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -G_6-G_7 & G_7 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & K_d*G_6 & -G_7 & 0 \\ 0 & 0 & K_b & 0 & G_5-K_b & -G_5 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & G_1 & 0 & G_4 & 0 & G_6 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ I_x \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ Vx \\ 0 \\ 0 \end{bmatrix} \quad (2)$$

$$R_eq = (V_x/I_x) \quad (3)$$

Name	Value [A], [V], [Ohm]
$@I_x$	0.002765
V_x	8.620868
R_{eq}	3118.377902

Table 3: Equivalent current, voltage and resistor obtained in the theoretical analysis in Octave.

$$V_n = V_x * exp(-t/(R_eq * C)) \quad (4)$$

With the results obtained before we ploted the natural solution in Figure 2.

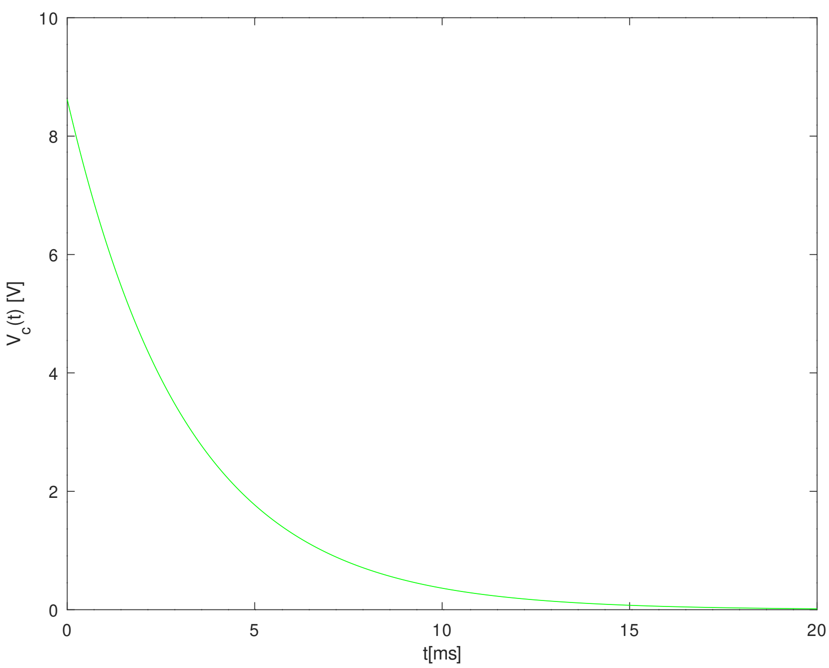

Figure 2: Natural solution for capacitor.

2.3 Node Analysis for t≥0 (Forced solution)

To determine the forced solution we used a phasor voltage source V_s = 1 V and then we applied the Node analysis (in which C was replaced by its impedance).

Node analysis matrix:

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & G_1 & -G_1-G_3-G_2 & G_2 & G_3 & 0 & 0 & 0 \\ 0 & 0 & -G_2+K_b & G_2 & -K_b & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & G_6-G_7 & G_7 \\ 0 & 0 & 0 & 0 & 1 & 0 & K_d*G_6 & -1 \\ 0 & 0 & K_b & 0 & -G_5+K_b & G_5+1/Z_c & 0 & -1/Z_c \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -G_1 & 0 & 0 & G_4 & 0 & G_6 & 0 \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = \begin{bmatrix} 0 \\ exp(-j) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{5}$$

Name	Value [V]
V_0	0.000000+0.000000i
V_1	0.540302+-0.841471i
V_2	1.362896+-2.122584i
V_3	2.907933+-4.528837i
V_5	1.468083+-2.286404i
V_6	-0.111259+-0.457661i
V_7	0.325933+-0.507611i
V_8	0.166312+-0.259016i

Table 4: Complex voltages obtained in the theoretical analysis in Octave.

2.4 Final Solution

The final solution is:

$$V_t = V_6 - V_8, for\ t < 0 \tag{6}$$

$$V_t = V_n + abs(v_6) * cos(2 * pi * f * t + acos(Re(v_6)/abs(v_6))), for\ t >= 0 \tag{7}$$

The plot of V_t and V_s is in Figure 3.

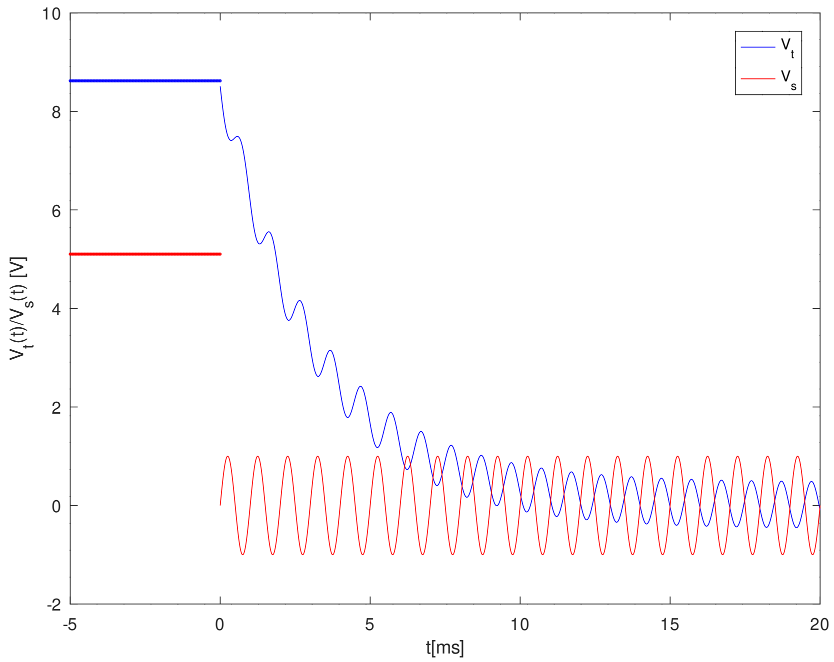

Figure 3: Final solution for V_s and V_t

Here, the model used is one in which, as can be seen in Figure 3, the natural solution of the capacitor tends to diminish into the forced solution given by the sinusoidal excitation of the voltage source V_s. Finally, at the end of the period studied (t=20ms),it can be observed that the phase of the voltage on the capacitor differs Pi from the phase of voltage source.

2.5 Frequency responses

In this subsection we present the frequency responses graphics. Since both V_6 and V_c have its amplitude, and therefore magnitude, dependent of frequency, we can observe that with the frequency increase comes a magnitude decrease of both signals. On the contrary, the amplitude of V_s does not depend on the frequency, and is constant and equal to 1, we can then observe it remains constant and equals to 0, as $log_{10}(1)$=0.

As can be seen in Figure 5, when the frequency increases the phase tends to negative values. It is therefore according to what was to be expected.

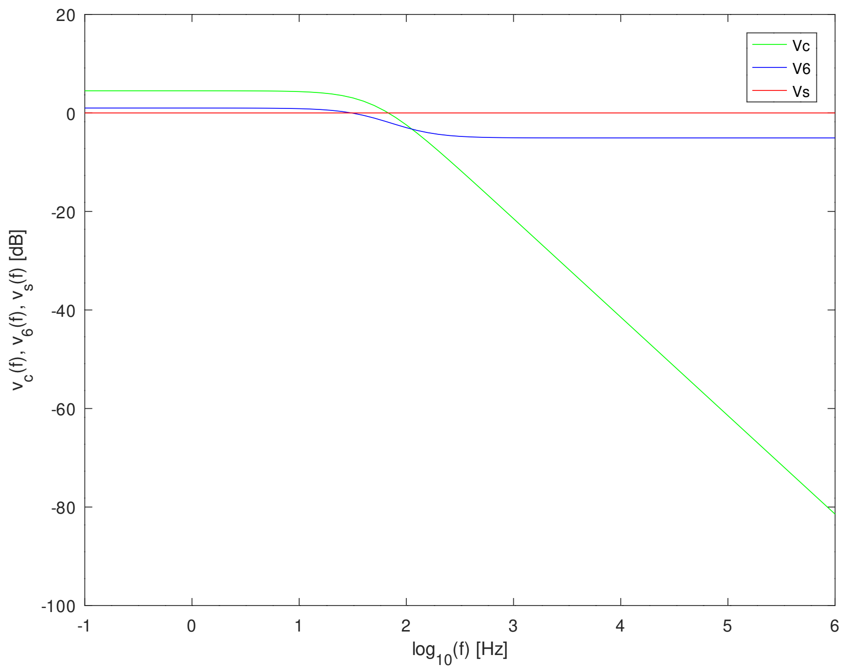

Figure 4: Theoretical analysis of magnitude in response to frequency changes.

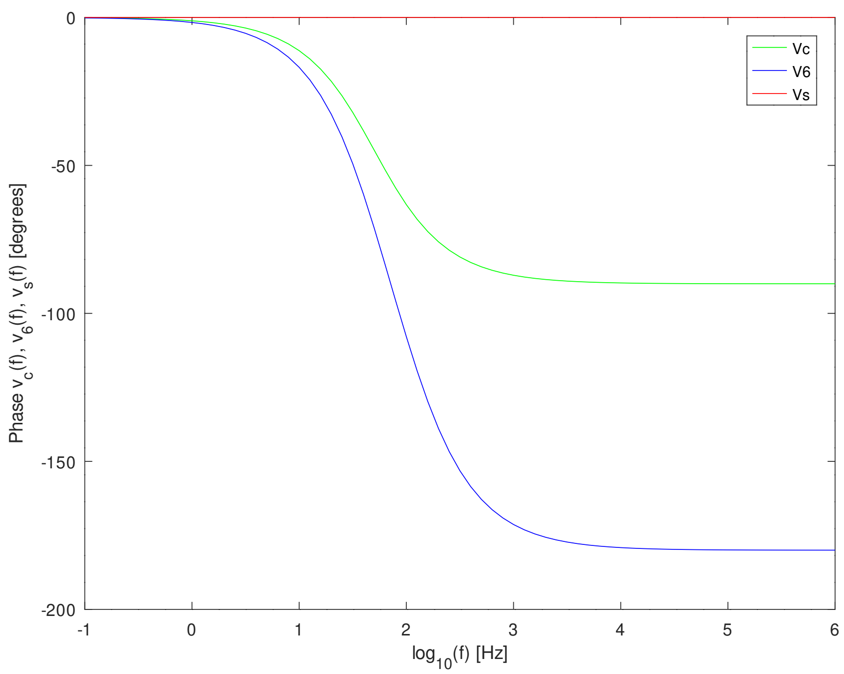

Figure 5: Theoretical analysis of phase in response to frequency changes.

3 Simulation Analysis

3.1 Operating Point Analysis for t$<$0

While using Ngspice we encountered some problems with the use of current-controlled voltage sources. To overcome them, we introduced a dummy 0V voltage source between the resistors 6 and 7. The voltage source V_c depends on the current I_c, which is the current on R_6. However, since Ngspice could not take the current in R_6, we introduced the null voltage source, since the current there will be I_c, which is the current that V_c depends on.

Table 5 shows the simulated operating point results for the circuit under analysis. Compared to the theoretical analysis results, one notices the following differences: describe and explain the differences.

Comparing the Tables 1 and 2 with Table 5, we see that the difference between the different voltages and currents is apromixametly null.

Name	Value [A or V]
@cd[i]	0.000000e+00
@gib[i]	-2.73490e-04
@rr1[i]	2.609385e-04
@rr2[i]	-2.73490e-04
@rr3[i]	-1.25516e-05
@rr4[i]	1.202716e-03
@rr5[i]	-2.73490e-04
@rr6[i]	9.417776e-04
@rr7[i]	9.417776e-04
v(1)	5.103557e+00
v(2)	4.832610e+00
v(3)	4.274744e+00
v(5)	4.870590e+00
v(6)	5.723435e+00
v(7)	-1.94493e+00
v(8)	-2.89743e+00
v(9)	0.000000e+00

Table 5: Operating point. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

3.2 Transient Analysis for t$>$0 (natural solution)

By short-circuting the independent voltage source V_s and by swapping the capacitor with a voltage source V_x = V_6 - V_8 we were able to calculate the current that flowed through the capacitor and the equivalent resistance. We short-circuted the independent voltage source because we are using the thevenin/norton theorems to calculate the resistence as seen by the capacitor and we swapped the capacitor with a voltage source as at t = 0 the capacitor begins discharging and it has a voltage of V_x (since it's voltage is at a peak).

The current source I_x and voltage V_6 can be found in table 6.

Name	Value [A or V]
@rr5[i]	-2.76454e-03
v(6)	8.620868e+00

Table 6: Operating point of point 2 of the simulation. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

The graphic for the natural solution, V_6(t), (using transient analysis) can be seen in Figure 6.

Comparing Table 3 with 6, we can see that the difference between current and voltage obtained is nearly null. Moreover, looking at Figures 2 and 6 we can easily observe the similarities between graphics. This points us to the fact that the theorical model used aproximates very well the simulations.

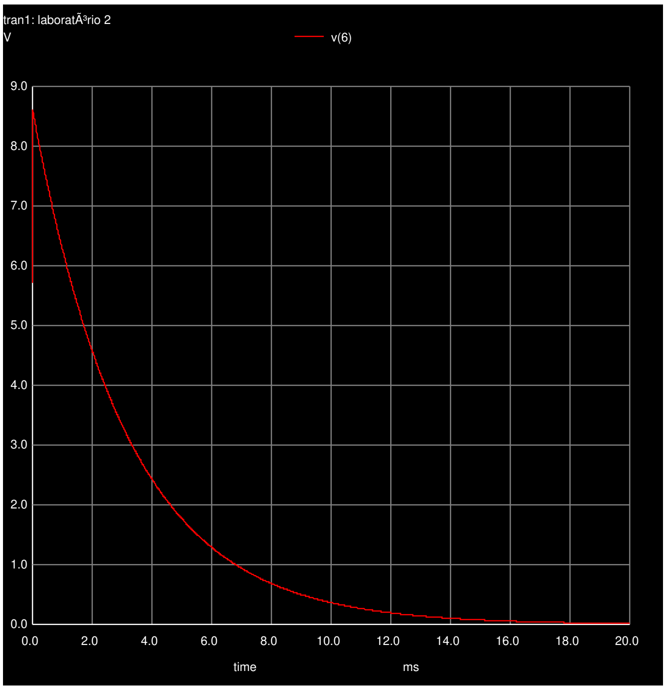

Figure 6: Natural solution for capacitor.

3.3 Transient analysis for t>0 (natural and forced solution)

The graphic for the natural and forced responses on V_6(t) and on V_1 (voltage source) can be observed in Figure 9. Once again, we can conclude that the natural response of the capacitor tends to fade, while the forced response starts to prevail. At the end of the period in study, we can observe that capacitor has a difference of Pi in phase with the voltage source.

Comparing Figure 3 and Figure 7, from Octave and Ngspice respectively, we can observe that the results are identical, which is to be expected, since the circuit is composed by linear components only.

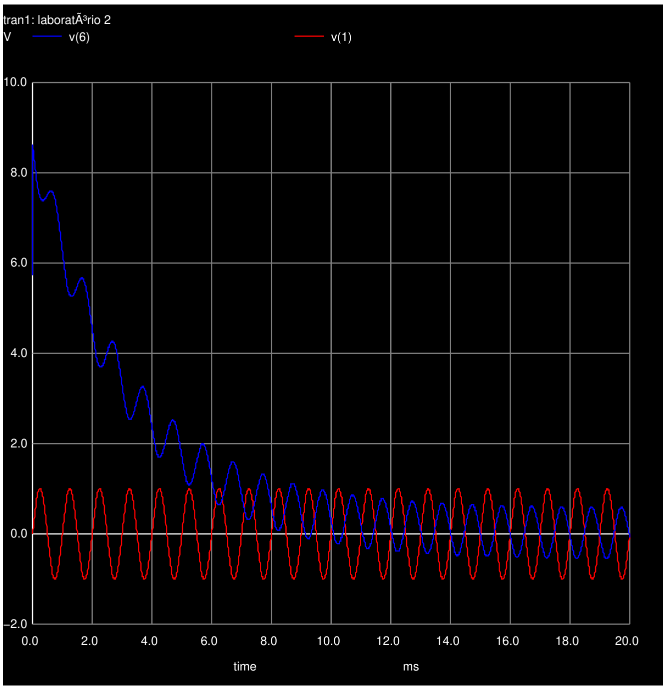

Figure 7: Forced solution of voltage source and capacitor.

3.4 Frequency response

The graphics for the frequency response are seen in Figure 8 and 9. As can be seen in the graph, V_c's magnitude is constant, since it is independent from the frequency. V_c and V_6 are dependent of the frequency and this leads V_c to 0, since V_c is inversely proporcional to the frequency. Octave's (Figure 4) and ngspice's (Figure 8) graphs are similiar. Since the circuit is linear, this was expected, so it can be concluded that the model used is a very good aproximation. Once again, we can observe that the phase drops when the frequency increases, pointing out the very similar results in Figure 5 and Figure 9.

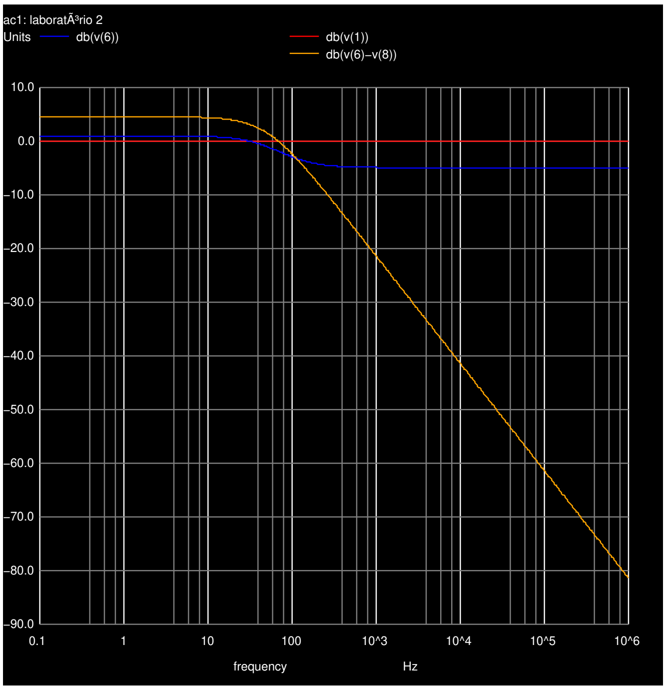

Figure 8: Magnitude in response to frequency changes.

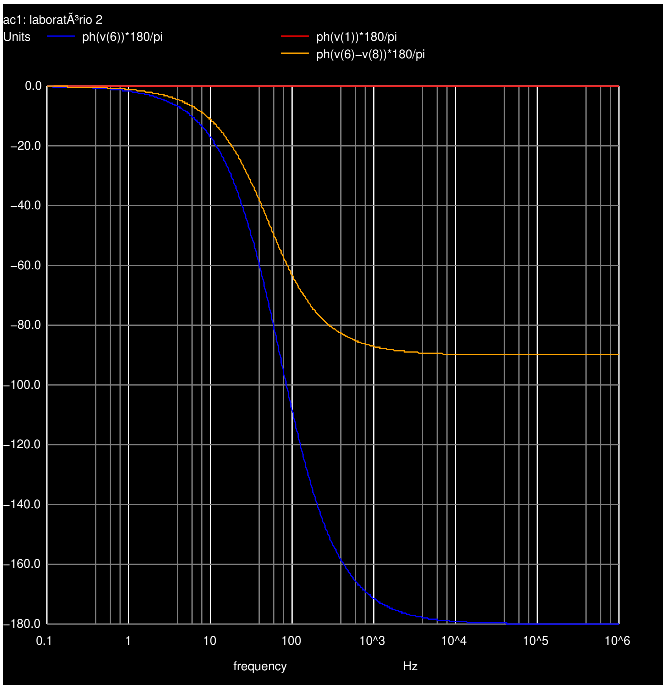

Figure 9: Phase in response to frequency changes.

4 Octave and Ngspice Comparison

4.1 Analysis for t<0

Name	Value [V]
V_0	0.000000
V_1	5.103557
V_2	4.832610
V_3	4.274744
V_5	4.870590
V_6	5.723435
V_7	-1.944932
V_8	-2.897433

Name	Value [A]
$@I_1$	-0.000261
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$@I_5$	0.000273
$@I_6$	-0.000942
$@I_7$	-0.000942

Name	Value [A or V]
@cd[i]	0.000000e+00
@gib[i]	-2.73490e-04
@rr1[i]	2.609385e-04
@rr2[i]	-2.73490e-04
@rr3[i]	-1.25516e-05
@rr4[i]	1.202716e-03
@rr5[i]	-2.73490e-04
@rr6[i]	9.417776e-04
@rr7[i]	9.417776e-04
v(1)	5.103557e+00
v(2)	4.832610e+00
v(3)	4.274744e+00
v(5)	4.870590e+00
v(6)	5.723435e+00
v(7)	-1.94493e+00
v(8)	-2.89743e+00
v(9)	0.000000e+00

Table 7: Voltages and currents obtained by node analysis in Octave on the left and by operating point in Ngspice on the right.

4.2 Natural solution for t>0

Name	Value [A], [V], [Ohm]
$@I_x$	0.002765
V_x	8.620868
R_{eq}	3118.377902

Name	Value [A or V]
@rr5[i]	-2.76454e-03
v(6)	8.620868e+00

Table 8: On the left, results from Octave, and on the right Ngspice. These tables give us the current and voltage between nodes 6 and 8. All the currents and voltages in other nodes/branches are null.

5 Conclusion

The objective of this laboratory assignment as been accomplished as can be seen by the results obtained. The Ngspice simulator is a very powerful tool, as we can see by aproximating very acurately the theorical model.

As can be seen by comparing the tables and figures, both Octave maths tool and circuit simulator Ngspice data match, almost perfectly, the differences are explained by the rounding of the last decimal case. Once again, the model used can be seen to accurately describe the results we obtain, as it was to be expected.