CHAPTER 2: PROBABILITY THEORY AND DECISION MAKING

(Week 3-4: Lecture Notes)

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1. INTRODUCTION: UNCERTAINTY AND RISK

There is never 100% certainty in engineering projects.

- Will the bridge withstand the wind load?
- When will the circuit fail?
- Will the software crash under heavy traffic?

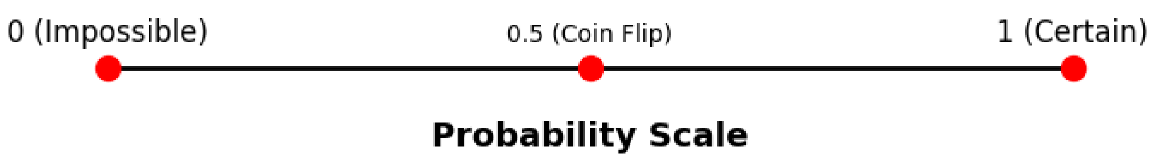
These are all **"Random (Stochastic) Events"**.

Probability Theory helps us manage this uncertainty by assigning a value between 0 and 1.

- **0:** Impossible.
- **1:** Certain.

In [1]:

```
import matplotlib.pyplot as plt

# Probability Scale Visualization
plt.figure(figsize=(8, 2))
plt.plot([0, 1], [0, 0], color='black', linewidth=2)
plt.scatter([0, 0.5, 1], [0, 0, 0], color='red', s=100, zorder=5)

plt.text(0, 0.1, "0 (Impossible)", ha='center', fontsize=12)
plt.text(1, 0.1, "1 (Certain)", ha='center', fontsize=12)
plt.text(0.5, 0.1, "0.5 (Coin Flip)", ha='center', fontsize=10)
plt.text(0.5, -0.3, "Probability Scale", ha='center', fontsize=14, fontweight='b

plt.ylim(-0.5, 0.5)
plt.axis('off')
plt.show()
```



2. BASIC CONCEPTS AND SET THEORY

To understand probability, we first define the "Universe".

2.1. Sample Space (S)

The set of all possible outcomes.

- *Example (Coin):* $S = \{Heads, Tails\}$
- *Example (Dice):* $S = \{1, 2, 3, 4, 5, 6\}$
- *Example (Production):* $S = \{Good, Defective\}$

2.2. Event (A, B, C...)

A subset of the sample space we are interested in.

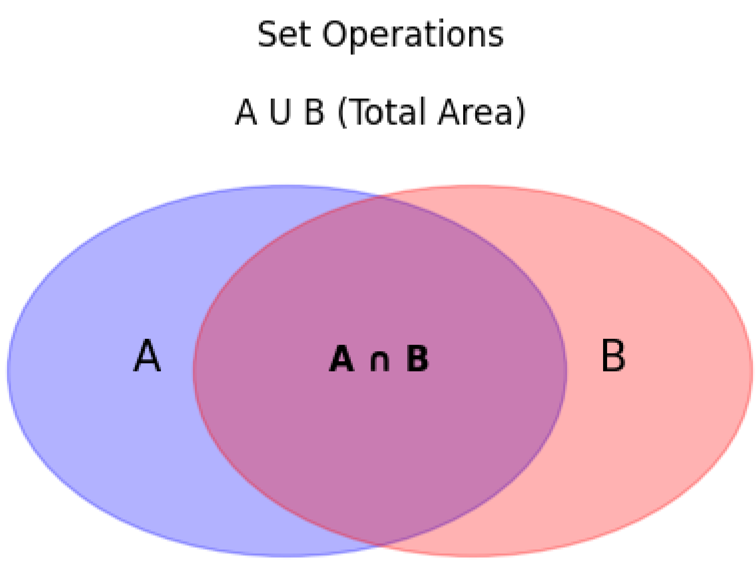
- *Example:* Rolling an even number. $A = \{2, 4, 6\}$

2.3. Set Operations

1. **Union (U):** "A OR B". (At least one happens).
    - $A \cup B$
2. **Intersection (∩):** "A AND B". (Both happen).
    - $A \cap B$
3. **Complement (A'):** "NOT A".
    - A' or A^c

In [2]:

```
import matplotlib.pyplot as plt

# Venn Diagram
fig, ax = plt.subplots(figsize=(6, 4))

circle_a = plt.Circle((0.4, 0.5), 0.3, color='blue', alpha=0.3, label='A')
circle_b = plt.Circle((0.6, 0.5), 0.3, color='red', alpha=0.3, label='B')

ax.add_patch(circle_a)
ax.add_patch(circle_b)

ax.text(0.25, 0.5, "A", fontsize=15, ha='center')
ax.text(0.75, 0.5, "B", fontsize=15, ha='center')
ax.text(0.5, 0.5, "A ∩ B", fontsize=12, ha='center', fontweight='bold')
ax.text(0.5, 0.9, "A U B (Total Area)", fontsize=12, ha='center')

ax.set_xlim(0, 1)
ax.set_ylim(0, 1)
ax.axis('off')
plt.title("Set Operations")
plt.show()
```



3. AXIOMS OF PROBABILITY

Three fundamental rules:

- **Rule 1:** $0 \leq P(A) \leq 1$
- **Rule 2:** $P(S) = 1$ (Total probability is 100%)
- **Rule 3 (Addition Rule):** If events are **Disjoint** (Mutually Exclusive), just sum them up.
  - $P(A \cup B) = P(A) + P(B)$

3.1. General Addition Rule

If events intersect, we must subtract the double-counted part.

Formula: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Example: Class stats:

- 50% speak English. $P(E) = 0.50$
- 40% speak German. $P(G) = 0.40$
- 20% speak both. $P(E \cap G) = 0.20$

Q: Probability of speaking English OR German?

Sol: $0.50 + 0.40 - 0.20 = 0.70$ (70%)

In [3]:
```
# Data
p_eng = 0.50
p_ger = 0.40
p_both = 0.20

# Calculation
```

```python
p_or = p_eng + p_ger - p_both

print(f"English OR German: {p_or*100}%")
```

```
English OR German: 70.0%
```

4. CONDITIONAL PROBABILITY

This is crucial for engineering. It means "Updating probability based on new info".

Definition: Probability of A given B has occurred. ($P(A|B)$)

Formula:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Example (Quality Control):

- 5% Paint Defect (B).
- 10% Size Defect (E).
- 2% BOTH defects ($B \cap E$).

Q: We know a part has a **Paint Defect**. What is the chance it also has a Size Defect?

Sol: $P(E|B) = 0.02/0.05 = 0.40$ (40%)

Note: The probability jumped from 10% to 40% because we have extra info.

In [4]:
```python
# Given
p_paint_error = 0.05
p_intersection = 0.02

# Conditional: P(Size | Paint)
p_cond = p_intersection / p_paint_error

print(f"Probability of Size Defect given Paint Defect: {p_cond*100}%")
```

```
Probability of Size Defect given Paint Defect: 40.0%
```

5. INDEPENDENCE

Two events are **Independent** if knowing one doesn't change the probability of the other.

Test: If $P(A|B) = P(A)$, they are independent.

Multiplication Rule (Only for Independent Events):

$$P(A \cap B) = P(A) \times P(B)$$

Example (Redundancy): Plane has 2 engines. Failure rate for each is 0.001 (Independent).

- Chance of BOTH failing:
- $0.001 \times 0.001 = 0.000001$ (1 in a million).

In [5]:
```
p_fail = 0.001

# Independent events -> Multiply
p_total_fail = p_fail * p_fail

print(f"Single Engine Failure: {p_fail}")
print(f"Total System Failure:  {p_total_fail:.6f}")
```

```
Single Engine Failure: 0.001
Total System Failure:  0.000001
```

6. BAYES' THEOREM

Going from "Result" back to "Cause".

Formula:

$$P(B_k|A) = \frac{P(A|B_k) \cdot P(B_k)}{P(A)}$$

Example: Which Supplier?

- **Supplier 1 (T1):** 60% volume, 1% defect rate.
- **Supplier 2 (T2):** 40% volume, 2% defect rate.

Event: We picked a part and it is **DEFECTIVE**. **Q:** Probability it came from T2?

Step 1: Total Defect Probability (Denominator)
$P(D) = (0.60 \times 0.01) + (0.40 \times 0.02) = 0.014$

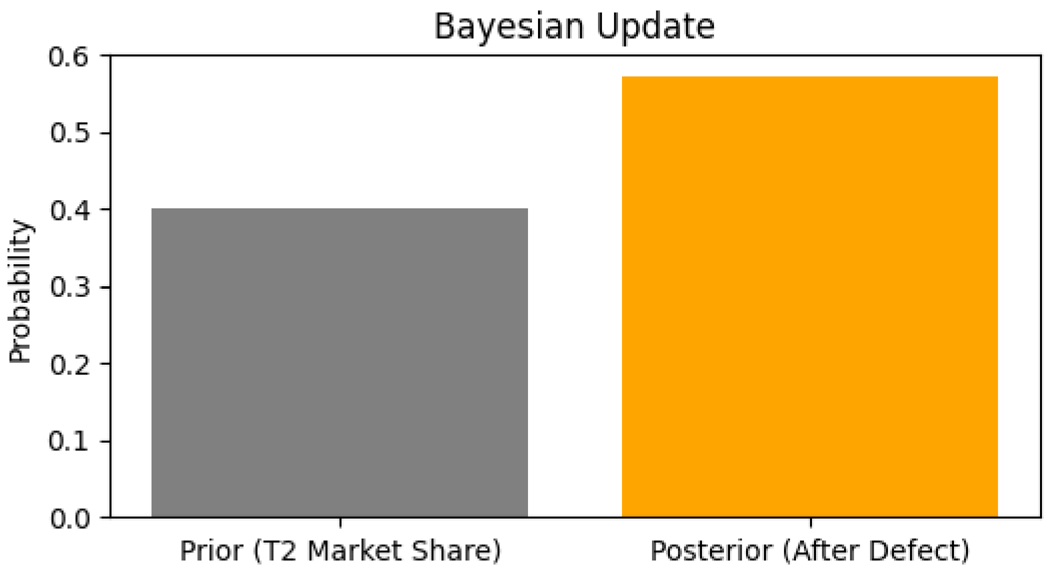
Step 2: Bayes $P(T2|D) = \frac{0.40 \times 0.02}{0.014} \approx 0.57$ (57%)

Interpretation: T2 is more likely to be the culprit.

In [6]:
```
import matplotlib.pyplot as plt

# Data
p_t1 = 0.60
p_t2 = 0.40
p_def_t1 = 0.01
p_def_t2 = 0.02

# 1. Total Defect
p_total_def = (p_t1 * p_def_t1) + (p_t2 * p_def_t2)

# 2. Bayes (Posterior)
p_t2_given_def = (p_t2 * p_def_t2) / p_total_def

print(f"Total Defect Rate: {p_total_def:.4f}")
print(f"Probability (T2 | Defective): {p_t2_given_def*100:.2f}%")

# Plot
```

```
plt.figure(figsize=(6, 3))
plt.bar(['Prior (T2 Market Share)', 'Posterior (After Defect)'], [p_t2, p_t2_giv
plt.title("Bayesian Update")
plt.ylabel("Probability")
plt.show()
```

```
Total Defect Rate: 0.0140
Probability (T2 | Defective): 57.14%
```



7. EXPECTED VALUE (E[X])

The average outcome in the long run.

Formula: $E[X] = \sum[x \cdot P(x)]$

Decision Example:

1. **Risky (A):** 20% chance to win 1M TL. ($E[A] = 200k$)
2. **Safe (B):** 80% chance to win 200k TL. ($E[B] = 160k$)

Mathematically, **A** is better. But in real life, variance (risk) matters too.

In [7]:
```
# Strategy A
ev_a = 1000000 * 0.20

# Strategy B
ev_b = 200000 * 0.80

print(f"Expected Value A: {ev_a}")
print(f"Expected Value B: {ev_b}")
```

```
Expected Value A: 200000.0
Expected Value B: 160000.0
```

8. Lecture Summary

1. **OR vs AND:** "OR" $\rightarrow$ Sum (+). "AND" $\rightarrow$ Multiply ($\times$).
2. **Given That:** Use $P(A|B)$. Known event goes to denominator.

3. **Bayes:** Draw a tree diagram first.