A FORTRAN PROGRAM FOR EXPERIMENTAL WIMP ANALYSIS

Based on the Mathematica package by Haxton-Berkeley group

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In search of WIMPs (Weakly Interacting Massive Particles)

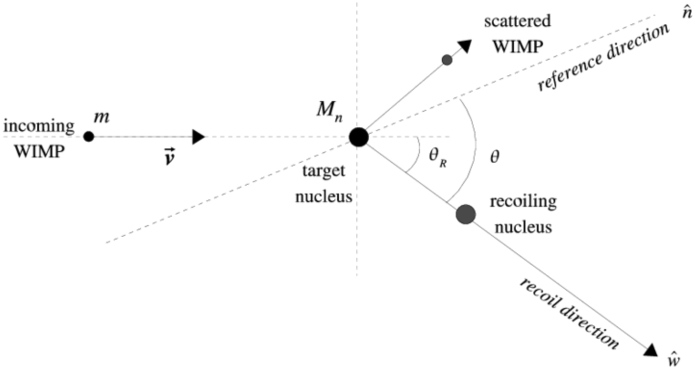

Fig. from [Alenazi and Gondolo, 2008]

Predict event rate

$$\frac{dR_D}{dE_R} = \frac{dR_D}{d\vec{q}^2}(q) = N_T n_\chi \int_{v_{min}}^{\infty} \frac{d\sigma(v,q)}{d\vec{q}^2} v \; \tilde{f}(\vec{v}) \; d^3 v \qquad (1)$$

q is the WIMP-nucleon momentum transfer
N_T is the number of target nuclei,
$n_\chi = \rho_\chi / m_\chi$ is the local dark matter density,
σ is the WIMP-nucleon cross section
$\tilde{f}(\vec{v})$ is dark matter velocity distribution in the lab-frame

Mathematica Package for (WIMP) Experimental Analysis

Model-independent WIMP Scattering Responses and Event Rates:
A Mathematica Package for Experimental Analysis

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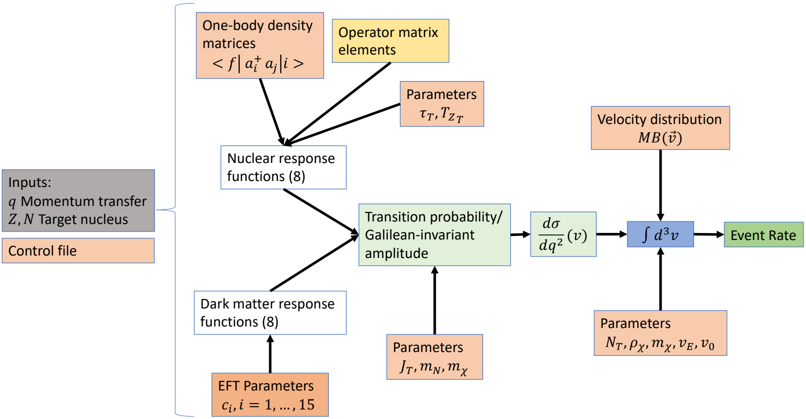
References [Anand et al., 2014] [Fitzpatrick et al., 2013].

Reasons to write a FORTRAN version:

1. Speed
2. Interfacing with a parameter estimate workflow (e.g. MCMC)
3. Portable, *readable*, expandable, free

Code flow structure



Assume normal WIMP wind... boosted of course

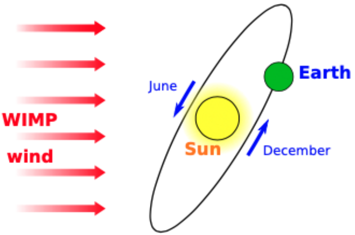
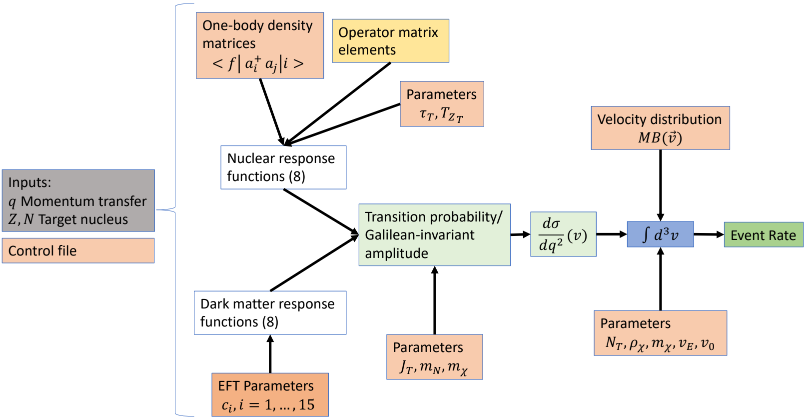


Fig. from [Freese et al., 2013]

$$\tilde{f}(\vec{v})d^3v = f(\vec{v} + \vec{v}_{earth})d^3v, \tag{2}$$

$$f(\vec{v})d^3v \propto e^{-\vec{v}^2/v_0^2}d^3v \tag{3}$$

Code flow structure



The cross section is determined from the Galilean invariant amplitude from EFT

$$\frac{d\sigma(v,q)}{dq^2} = 2m_T \frac{1}{4\pi v^2} T(v,q), \tag{4}$$

$$T(v,q) = \frac{4\pi}{2j_T+1} \frac{1}{(4m_\chi)^2} \sum_{\tau=0}^{1} \sum_{\tau'=0}^{1} \sum_{i=1}^{8} R_i^{\tau\tau'}(v^2,q^2) W_i^{\tau\tau'}(q) \tag{5}$$

- τ is isospin
- j_T target nucleus spin
- 8 terms: products of WIMP response functions R and nuclear response functions W
- Why 8? Starting from 15 general operators in LO, NLO, N^2LO in momenta and velocities, GIA expanded in spherical and vector spherical harmonics and averaged over spins, left with 8 combinations of 6 nuclear operators.

Code comparison

	Mathematica	FORTRAN
Files	1	25
Lines of code	1600*	4000
Subroutines	?	40 +
Si28 Eventrate runtime	1.8 s	0.12 s
Interface	Mathematica notebook	Interactive + control file

Mathematica: Sample code

dmformfactor-temp.m — v1.1 (git: master)

```
PhiPPsummand2[y_,{np_,lp_,jp_},{n_,l_,j_},jcap_]:=If[l==0,0,QNorm[l-1]*
Sqrt[l]*ThreeJ[{lp,0},{jcap+1,0},{l-1,0}]*BesselElementPlus[y,{np,lp},{n,l},jcap+1]*Sum[minus^(jcap-lcap)*JNorm[lcap]*SixJ[{jcap+1,1,lcap},{1,jcap,1}]*SixJ[{jcap+1,1,lcap},{l,lp,l-1}]*NineJSymbol[{lp,l,lcap},{1/2,1/2,1},{jp,j,jcap}],{lcap,jcap,jcap+1}]]
PhiPPsummand3[y_,{np_,lp_,jp_},{n_,l_,j_},jcap_]:=If[jcap==0,0,QNorm[l+1]*Sqrt[l+1]*ThreeJ[{lp,0},{jcap-1,0},{l+1,0}]*BesselElementMinus[y,{np,lp},{n,l},jcap-1]*Sum[minus^(jcap-lcap+1)*JNorm[lcap]*SixJ[{jcap-1,1,lcap},{1,jcap,1}]*SixJ[{jcap-1,1,lcap},{l,lp,l+1}]*NineJSymbol[{lp,l,lcap},{1/2,1/2,1},{jp,j,jcap}],{lcap,jcap-1,jcap}]]
PhiPPsummand4[y_,{np_,lp_,jp_},{n_,l_,j_},jcap_]:=If[jcap==0,0,If[l==0,0,QNorm[l-1]*
Sqrt[l]*ThreeJ[{lp,0},{jcap-1,0},{l-1,0}]*BesselElementPlus[y,{np,lp},{n,l},jcap-1]*Sum[minus^(jcap-lcap)*JNorm[lcap]*SixJ[{jcap-1,1,lcap},{1,jcap,1}]*SixJ[{jcap-1,1,lcap},{l,lp,l-1}]*NineJSymbol[{lp,l,lcap},{1/2,1/2,1},{jp,j,jcap}],{lcap,jcap-1,jcap}]]]
PhiPPJX[y_,{np_,lp_,jp_},{n_,l_,j_},jcap_]:=PhiPPoverall[y,{np,lp,jp},{n,l,j},jcap]*(QNorm[jcap+1]*Sqrt[jcap+1]*(PhiPPsummand1[y,{np,lp,jp},{n,l,j},jcap]+PhiPPsummand2[y,{np,lp,jp},{n,l,j},jcap])+If[jcap==0,0,QNorm[jcap-1]*Sqrt[jcap]*(PhiPPsummand3[y,{np,lp,jp},{n,l,j},jcap]+PhiPPsummand4[y,{np,lp,jp},{n,l,j},jcap])])
PhiPPJE[y_,{ncapp_,jp_},{ncap_,j_},jcap_]:=PhiPPJX[y,{Nodal[ncapp,jp],Lnumber[ncapp,jp],jp},{Nodal[ncap,j],Lnumber[ncap,j],j},jcap]
PhiPPJ[y_,{ncapp_,jp_},{ncap_,j_},jcap_]:=If[PhysicalConditionsNormal[ncapp,jp,ncap,j,jcap],PhiPPJE[y,{ncapp,jp},{ncap,j},jcap],0]

(**PhiTPJ**)
PhiPJX[y_,{np_,lp_,jp_},{n_,l_,j_},jcap_]:=PhiPPoverall[y,{np,lp,jp},{n,l,j},jcap]*(-QNorm[jcap+1]*Sqrt[jcap]*(PhiPPsummand1[y,{np,lp,jp},{n,l,j},jcap]+PhiPPsummand2[y,{np,lp,jp},{n,l,j},jcap])+If[jcap==0,0,QNorm[jcap-1]*Sqrt[jcap+1]*(PhiPPsummand3[y,{np,lp,jp},{n,l,j},jcap]+PhiPPsummand4[y,{np,lp,jp},{n,l,j},jcap])])
PhiPJE[y_,{ncapp_,jp_},{ncap_,j_},jcap_]:=PhiPJX[y,{Nodal[ncapp,jp],Lnumber[ncapp,jp],jp},{Nodal[ncap,j],Lnumber[ncap,j],j},jcap]
PhiPJ[y_,{ncapp_,jp_},{ncap_,j_},jcap_]:=If[PhysicalConditionsNormal[ncapp,jp,ncap,j,jcap],PhiPJE[y,{ncapp,jp},{ncap,j},jcap],0]
PhiTPJ[y_,{ncapp_,jp_},{ncap_,j_},jcap_]:=PhiPJ[y,{ncapp,jp},{ncap,j},jcap]+SigmaJ[y,{ncapp,jp},{ncap,j},jcap]/2


(***Operators of Abnormal Parity***)

(**DeltaJ**)
MJLDivQoverall[y_,{np_,lp_,jp_},{n_,l_,j_},jcap_,lcap_]:=minus^(lcap+j+1/2)*QNorm[lp]*QNorm[jp]*QNorm[j]*QNorm[jcap]*QNorm[lcap]*SixJ[{lp,jp,1/2},{j,l,jcap}]/Sqrt[4Pi]
MJLDivQsummand1[y_,{np_,lp_,jp_},{n_,l_,j_},jcap_,lcap_]:=-Sqrt[l+1] QNorm[l+1] SixJ[{lcap,1,jcap},{l,lp,l+1}] ThreeJ[{lp,0},{lcap,0},{l+1,0}] BesselElementMinus[y,{np,lp},{n,l},lcap]
MJLDivQsummand2[y_,{np_,lp_,jp_},{n_,l_,j_},jcap_,lcap_]:=Sqrt[l] QNorm[l-1] SixJ[{lcap,1,jcap},{l,lp,l-1}] ThreeJ[{lp,0},{lcap,0},{l-1,0}] BesselElementPlus[y,{np,lp},{n,l},lcap]
MJLDivQ[y_,{np_,lp_,jp_},{n_,l_,j_},jcap_,lcap_]:=MJLDivQoverall[y,{np,lp,jp},{n,l,j},jcap,lcap]*(MJLDivQsummand1[y,{np,lp,jp},{n,l,j},jcap,lcap]+MJLDivQsummand2[y,{np,lp,jp},{n,l,j},jcap,lcap])

DeltaPrime[y_,{np_,lp_,jp_},{n_,l_,j_},jcap_]:=-Sqrt[jcap]/QNorm[jcap]*MJLDivQ[y,{np,lp,jp},{n,l,j},jcap,jcap+1]+Sqrt[jcap+1]/QNorm[jcap]*MJLDivQ[y,{np,lp,jp},{n,l,j},jcap,jcap-1]
Deltaop[y_,{np_,lp_,jp_},{n_,l_,j_},jcap_]:=MJLDivQ[y,{np,lp,jp},{n,l,j},jcap,jcap]
DeltaJE[y_,{ncapp_,jp_},{ncap_,j_},jcap_]:=Deltaop[y,{Nodal[ncapp,jp],Lnumber[ncapp,jp],jp},{Nodal[ncap,j],Lnumber[ncap,j],j},jcap]
DeltaJ[y_,{ncapp_,jp_},{ncap_,j_},jcap_]:=If[PhysicalConditionsAbnormal[ncapp,jp,ncap,j,jcap],DeltaJE[y,{ncapp,jp},{ncap,j},jcap],0]

(**SigmaPJ**)
SigmaPrime[y_,{np_,lp_,jp_},{n_,l_,j_},jcap_]:=-Sqrt[jcap]/QNorm[jcap]*MJLSigma[y,{np,lp,jp},{n,l,j},jcap,jcap+1]+Sqrt[jcap+1]/QNorm[jcap]*MJLSigma[y,{np,lp,jp},{n,l,j},jcap,jcap-1]
SigmaPrimeJE[y_,{ncapp_,jp_},{ncap_,j_},jcap_]:=SigmaPrime[y,{Nodal[ncapp,jp],Lnumber[ncapp,jp],jp},{Nodal[ncap,j],Lnumber[ncap,j],j},jcap]
SigmaPJ[y_,{ncapp_,jp_},{ncap_,j_},jcap_]:=If[PhysicalConditionsAbnormal[ncapp,jp,ncap,j,jcap],SigmaPrimeJE[y,{ncapp,jp},{ncap,j},jcap],0]

(**SigmaPPJ**)
SigmaSecond[y_,{np_,lp_,jp_},{n_,l_,j_},jcap_]:=(Sqrt[jcap+1] MJLSigma[y,{np,lp,jp},{n,l,j},jcap,jcap+1]+Sqrt[jcap]
```

Line: 1 Mathematica Tab Size: 4

Fortran: Sample code

lib — vim dmresponse.f90 — 155×53

```fortran
!----------------------------------------------------------------------------
function dmrMJ(tau1, tau2, q, v, jchi)
    ! O. Gorton 2020.01.
    ! import modules from modules.f90
    use masses

    implicit none

    ! outputs
    REAL(kind=8) :: dmrMJ

    ! inputs
    integer, INTENT(IN) :: tau1, tau2
    REAL(kind=8), INTENT(IN) :: q
    REAL(kind=8), INTENT(IN) :: v
    REAL(kind=8), INTENT(IN) :: jchi

    ! functions called
    !REAL(kind=8) :: Cl

    dmrMJ = 0.25*Cl(jchi) * ( &
            (cvec(tau1)%c(5)*cvec(tau2)%c(5)*q*q + cvec(tau1)%c(8)*cvec(tau2)%c(8)) &
            * (v*v - q*q/(4*muT*muT)) &
            + cvec(tau1)%c(11)*cvec(tau2)%c(11)*q*q &
        ) + (cvec(tau1)%c(1) + cvec(tau1)%c(2) * (v*v - q*q/(4*muT*muT))) * ( &
            cvec(tau2)%c(1) + cvec(tau2)%c(2) * (v*v - q*q/(4*muT*muT)) &
        )

end function dmrMJ

!----------------------------------------------------------------------------
function dmrPhiPPJ(tau1, tau2, q, v, jchi)
    ! O. Gorton 2020.01.

    ! import modules from modules.f90
    use masses

    implicit none

    ! outputs
    REAL(kind=8) :: dmrPhiPPJ

    ! inputs
    REAL(kind=8), INTENT(IN) :: q
    REAL(kind=8), INTENT(IN) :: v
    REAL(kind=8), INTENT(IN) :: jchi
    integer, INTENT(IN) :: tau1, tau2

    ! functions called
    !REAL(kind=8) :: Cl

    dmrPhiPPJ = q*q/(16*mN*mN) * Cl(jchi) * (cvec(tau1)%c(12) &
```

87,1 14%

Fortran example input Si28

Annotated input file

```
1       ! Compute differential scattering rate
3.0     ! q : 3-momentum of scattering event
14      ! N neutrons
14      ! Z protons
si28    ! Control filename (Req. EFT coefficients)
sd      ! Single particle space file (BIGSTICK .sps)
si28w   ! One-body density file (BIGSTICK .res)
y       ! Yes, fill core nucleons
```

Fortran example control file Si28

Control file "si28.control"

```
# Coefficient matrix (non-relativistic)
# Ommitted values are assumed to be 0.0.
# c_i^t
# i = 1,...,16
# t = 0 protons, 1 neutrons
#control name    t      i       c_i^t
coefnonrel      1      5       3.1
fakekeyword   3.141
vearth 232.0
dmdens 1.0
intpoints 500
```

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Directional recoil rates for wimp direct detection.
Physical Review D, 77(4).

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