A FORTRAN PROGRAM FOR EXPERIMENTAL WIMP ANALYSIS

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Model-Independent WIMP Scattering Responses and Event Rates

Contents

This is the manual for the FORTRAN version of the model-independent WIMP scattering response and event rate code, which was originally written in Mathematica and described in arXiv:1308.6288.

1. Quickstart guide

This program primarily computes two quantities. The first is the WIMP-nucleus differential event rate spectra:

$$\frac{dR_D}{dE_R} = \frac{dR_D}{d\vec{q}^2}(q) = N_T \frac{\rho_\chi}{m_\chi} \int_{v_{min}}^{v_{escape}} \frac{2m_T}{4\pi v^2} \frac{1}{2j_\chi+1} \frac{1}{2j_T+1} \sum_{spins} |\mathcal{M}(v,q)|^2 \tilde{f}(\vec{v}) v d^3 v \tag{1}$$

This quantity has units of events/GeV and is implicitly multiplied by an effective exposure of 1 Kilogram-Day of target nuclei. This is done by taking $N_t = 1\ kilogram \cdot day/m_T$, where m_T is the mass of the target nucleus in GeV. Recoil energies E_R are given in keV.

The cross section is determined using a user-defined WIMP-nucleus interaction within a non-relativistic effective field theory (EFT) framework. The interaction is specified by 16 coupling coefficients defining an interaction:

$$\sum_{x=p,n} \sum_{i=1}^{16} c_i^x \mathcal{O}_i \tag{2}$$

The second is the integral of the first over either energy or momentum-transfer, for a range specified by the user.

1.1. Event rate spectra (events per GeV).

The program will prompt the user for the minimum necessary inputs to run a calculation with default parameter values, including the name of a control file which contains the EFT coefficients, and optionally, additional customizations to the calculation parameters.

After selecting the option to compute an event rate spectra, there are six further lines of input. These will be explained by an example:

```
 Enter the target proton number
54
 Enter the target neutron number
77
 Enter name of control file (.control):
xe131
...
  Enter shell-model space file name (.sps)
GCN5082
...
  Enter name of one-body density file (.dres)
xe131gcn
...
 What is the range of recoil energies in kev?
 Enter starting energy, stoping energy, step size:
0.0001 250. 1.0
```

The first two entries are self-evident: we specifiy the number of protons and neutrons in the target nucleus. In this case, 54 and 74, respectively, for ^{131}Xe.

Third is the name of the control file containing the EFT coefficients and other, optional, settings. The '.control' file extension is omitted. This contents of this file will be explained in more detail later.

Fourth is a file which gives context for the Shell Model used to describe the nuclear wave functions. Future releases of this code will infer this information, but for now the user must provide the filename containing the single-particle space quantum numbers. The '.sps' file extension is omitted.

Fifth is the file containing the nuclear wave functions in the form of one-body density matrices. Only the ground-state need be included. The '.dres' file extension is omitted.

Sixth and finally are three numbers specifying the range of recoil energies E_R that the differential scattering rate should be computed for.

The event rate spectra will be written to a file, and as a side effect of the calculation, the total event rate for the energy range requested will be estimatted by numerical integration. Note that the accuracy of this result will depend on the choice of the step size.

1.2. **Control file.** Each EFT parameter is written on its own line in [mycontrolfile].control, with four values: the keyword "coefnonrel", the operator number (integer 1..16), the coupling type ("p"=proton, "n"=neutron, "s"=scalar, "v"=vecctor), and the coefficient value. For example,

```
coefnonrel    1    s     3.1
```

would set $c_1^0 = 3.1$. We take the isospin convention:

$$\begin{aligned} c^0 &= c^p + c^n \\ c^1 &= c^p - c^n \end{aligned} \tag{3}$$

Thus, the previous example is equivalent to:

```
coefnonrel    1    p     1.55
coefnonrel    1    n     1.55
```

The control file also serves a more general but optional function: to set any parameter in the program to a custom value. Simply add an entry to the control file with two values: the first should be the keyword for the parameter and the second should be the value to set that parameter to. For example, to set the velocity of the earth in the galactic frame to 240 km/s, you should add the line:

```
vearth  240.0
```

1.3. **Total integrated events.** Inputs 2 - 6 for this calculation will be almost identical to those required for the event rate spectra calculation. The main difference is in the sixth and final input:

```
 Using adaptive numerical integration to determine  total integrated event rate.
 What are the limits of integration for recoil energies in kev?
 Enter starting energy, stoping energy, relative error:
0.0 250.0 0.001
```

Here, we have again been asked for a starting and stopping recoil energy, but this time the third value is the desired relative error of the integrated spectra. In this example, 0.001 corresponds to a desired uncertainty of 0.1%.

An important difference between this calculation and the event rate spectra calculation is that in this evaluation, the spectra is not written to file. This is because an adaptive integration routine is used to keep the number of function evaluations to a minimum. As a result, this calculation will be much faster than computing the entire spectra.

2. Primary functions

2.1. Differential event rate.

$$\frac{dR_D}{dE_R} = \frac{dR_D}{d\vec{q}^2}(q) = N_T n_\chi \int_{v_{min}}^{\infty} \frac{d\sigma(v,q)}{d\vec{q}^2} \tilde{f}(\vec{v}) v d^3v \tag{4}$$

where q is the WIMP-nucleon momentum transfer, N_T is the number of target nuclei, $n_\chi = \rho_\chi/m_\chi$ is the local dark matter density, σ is the WIMP-nucleon cross section, and $\tilde{f}$ is dark matter velocity distribution in the lab-frame. $\tilde{f}(\vec{v})$ is obtained by boosting the Galactic-frame distribution $f(\vec{v})$,

$$\tilde{f}(\vec{v}) = f(\vec{v} + \vec{v}_{earth}), \tag{5}$$

where $\vec{v}_{earth}$ is the velocity of the earth in the galactic rest frame. The simplest model is a three-dimensional Maxwell distribution:

$$f(\vec{v}) \propto e^{-\vec{v}^2/v_0^2}, \tag{6}$$

where v_0 is some scaling factor (typically taken to be around 220 km/s).

In order to evaluate the integral in (4), we make the conversion to spherical coordinates, and take special care to deal with the velocity boost in (5). Assuming a $1/v^2$ velocity dependence of the cross section term (see section 2.2), we need to evaluate an integral of the form

$$I = \int_{v_{min}}^{v_{max}} d^3v \frac{f(\vec{v} + \vec{v}_{earth})}{v} = \int_{v_{min}}^{v_{max}} d^3v \frac{1}{v} e^{-(\vec{v}+\vec{v}_{earth})^2/v_0^2} \tag{7}$$

Noting that $(\vec{v} + \vec{v}_{earth})^2 = \vec{v}^2 + \vec{v}_{earth}^2 + 2vv_{earth}\cos(\theta)$, with $|\vec{v}| \equiv v$ and θ defining the angle between the two vectors, it's convenient to make the substitution $d^3v = d\phi d(\cos\theta) v^2 dv$:

$$\begin{aligned}
I &= \int_0^{2\pi} d\phi \int_{v_{min}}^{v_{max}} dv \int_{-1}^{1} d(\cos\theta) e^{-2vv_{earth}\cos\theta/v_0^2} v^2 \frac{1}{v} e^{-(\vec{v}^2+\vec{v}_{earth}^2)/v_0^2} \\
&= 2\pi \int_{v_{min}}^{v_{max}} dv v e^{-(\vec{v}^2+\vec{v}_{earth}^2)/v_0^2} \left(-\frac{v_0^2}{2vv_{earth}} e^{-2vv_{earth}\cos\theta/v_0^2} \right)_{-1}^{1} \\
&= \frac{\pi v_0^2}{v_{earth}} \int_{v_{min}}^{v_{max}} dv e^{-(\vec{v}^2+\vec{v}_{earth}^2)/v_0^2} \left(-e^{-2vv_{earth}/v_0^2} + e^{+2vv_{earth}/v_0^2} \right) \\
&= \frac{\pi v_0^2}{v_{earth}} \int_{v_{min}}^{v_{max}} dv \left(-e^{(v+v_{earth})^2/v_0^2} + e^{(v-v_{earth})^2/v_0^2} \right) \\
&= \frac{\pi v_0^2}{v_{earth}} \int_{v_{min}}^{v_{max}} dv \left(g(v - v_{earth}) - g(v + v_{earth}) \right)
\end{aligned} \tag{8}$$

where in the last equality, we have defined a one-dimensional Gaussian form

$$g(v) \propto e^{-v^2/v_0^2}. \tag{9}$$

The final expression for I can be trivially generalized to other spherically-symmetric velocity-dependent forms of the differential cross section. What's important is the reduction of the velocity-boosted d^3v integral to a radial integral which can be carried out with one-dimensional quadrature:

$$\int_{v_{min}}^{v_{max}} d^3v \sigma(v) e^{-(\vec{v}+\vec{v}_{earth})^2/v_0^2} = \frac{\pi v_0^2}{v_{earth}} \int_{v_{min}}^{v_{max}} dv \sigma(v) v^2 \left(g(v - v_{earth}) - g(v + v_{earth}) \right). \tag{10}$$

The FORTRAN code uses equation (10) to evaluate the event rate integral in equation (4) with quadrature. Analytic solutions of (10) exist in the form of error functions; we use the above form since it makes easy to later modify the velocity distribution (as long as it remains spherically symmetric). For example, adding a velocity cut-off is as easy as changing the limit on the quadrature, with no need to write a whole new subroutine for the analytic forms found in the Mathematica script.

2.2. **Differential cross section.**

$$\frac{d\sigma(v,E_R)}{dE_R} = 2m_T \frac{d\sigma(v)(v,\vec{q}^2)}{d\vec{q}^2} = 2m_T \frac{1}{4\pi v^2} T(v,q), \tag{11}$$

Where v is the velocity of the dark matter particles in the lab-frame, q is the momentum transfer of the scattering event, m_T is the mass of the target nucleus, and $T(v,q)$ is the transition or scattering probability. We can see here that the differential cross section has an explicit $1/v^2$ dependence, independent of any velocity dependence of $T(v,q)$.

2.3. **Transition probability / Scattering probability.** The scattering probability is

$$T(v,q) = \frac{1}{2j_\chi+1}\frac{1}{2j_T+1}\sum_{spins}|\mathcal{M}(v,q)|^2 \tag{12}$$

where j_χ is the spin of the WIMP, j_T is the spin angular momentum of the target nucleus, and $\mathcal{M}$ Galilean invariant amplitude:

$$T(v,q) = \frac{4\pi}{2j_T+1}\frac{1}{(4m_\chi)^2}\sum_{\tau=0}^{1}\sum_{\tau'=0}^{1}\sum_{i=1}^{8} R_i^{\tau\tau'}(v^2,q^2)W_i^{\tau\tau'}(q) \tag{13}$$

where m_χ is the mass of the dark matter particle and τ is an index used to sum over isospin couplings. The coefficients $R_i^{\tau,\tau'}$ are dark matter particle response functions, to be define in another section. The operators $W_i^{\tau\tau'}(q)$ are nuclear response functions, which are sums over matrix elements of nuclear operators constructed from Bessel spherical harmonics and vector spherical harmonics.

2.4. **Dark matter response functions.** There are 8 dark matter response functions which group 15 operator coefficients c_i^τ according the pair of nuclear response functions which they multiply.

$$R_M^{\tau\tau'}(v,q) = \frac{1}{4}cl(j_\chi)\left((v^2-(\frac{q}{2\mu_t})^2)(c_5^\tau c_5^{\tau'}q^2 + c_8^\tau c_8^{\tau'}) + c_{11}^\tau c_{11}^{\tau'}q^2\right) \tag{14}$$

$$+ (c_1^\tau + c_2^\tau(v^2-(\frac{q}{2\mu_t})^2))(c_1^{\tau'} + c_2^{\tau'}(v^2-(\frac{q}{2\mu_t})^2)) \tag{15}$$

$$R_{\Sigma''}^{\tau\tau'}(v,q) = \frac{1}{16}cl(j_\chi)\left(c_6^\tau c_6^{\tau'}q^4 + (c_{13}^\tau c_{13}^{\tau'}q^2 + c_{12}^\tau c_{12}^{\tau'})(v^2 - q^2/(2\mu_T)^2) + 2c_4^\tau c_6^{\tau'}q^2 + c_4^\tau c_4^{\tau'}\right) + \frac{1}{4}c_{10}^\tau c_{10}^{\tau'}q^2 \tag{16}$$

$$R_{\Sigma'}^{\tau\tau'}(v,q) = \frac{1}{32}cl(j_\chi)\left(2c_9^\tau c_9^{\tau'}q^2 + (c_{15}^\tau c_{15}^{\tau'}q^4 + c_{14}^\tau c_{14}^{\tau'}q^2 - 2c_{12}^\tau c_{15}^{\tau'}q^2 + c_{12}^\tau c_{12}^{\tau'})(v^2 - q^2/(2\mu_T)^2) + 2c_4^\tau c_4^{\tau'}\right) \tag{17}$$

$$+\frac{1}{8}\left(c_3^\tau c_3^{\tau'}q^2 + c_7^\tau c_7^{\tau'}\right)(v^2 - q^2/(2\mu_T)^2) \tag{18}$$

$$R_{\Phi''}^{\tau\tau'}(v,q) = \frac{q^2}{(4m_N)^2}cl(j_\chi)\left(c_{12}^\tau - c_{15}^\tau q^2\right)\left(c_{12}^{\tau'} - c_{15}^{\tau'}q^2\right) + \frac{q^2}{(4m_N)^2}q^2 c_3^\tau c_3^{\tau'} \tag{19}$$

$$R_{\tilde{\Phi}'}^{\tau\tau'}(v,q) = \frac{q^2}{(4m_N)^2}cl(j_\chi)\left(c_{12}^\tau c_{12}^{\tau'}q^2 + c_{12}^\tau c_{12}^{\tau'}\right) \tag{20}$$

$$R_{\Delta}^{\tau\tau'}(v,q) = \frac{q^2}{(2m_N)^2}cl(j_\chi)\left(c_5^\tau c_5^{\tau'}q^2 + c_8^\tau c_8^{\tau'}\right) + 2\frac{q^2}{m_N^2}c_2^\tau c_2^{\tau'}(v^2 - q^2/(2\mu_T)^2) \tag{21}$$

$$R_{\Delta\Sigma'}^{\tau\tau'}(v,q) = \frac{q^2}{(2m_N)^2}cl(j_\chi)\left(c_4^\tau c_5^{\tau'} - c_8^\tau c_9^{\tau'}\right) - \frac{q^2}{m_N}c_2^\tau c_3^{\tau'}(v^2 - q^2/(2\mu_T)^2) \tag{22}$$

$$R_{\Phi''M}^{\tau\tau'}(v,q) = \frac{q^2}{4m_N}cl(j_\chi)c_{11}^\tau\left(c_{12}^{\tau'} - c_{12}^{\tau'}q^2\right) + \frac{q^2}{m_N}c_3^{\tau'}\left(c_1^\tau + c_2^\tau(v^2 - q^2/(2\mu_T)^2)\right) \tag{23}$$

As a shorthand we have introduced the notation

$$cl(j) = 4j(j+1)/3. \tag{24}$$

2.5. **Nuclear response functions.** There are eight nuclear response functions $W_i^{\tau\tau'}(y)$ considered here. The unit-less variable y is defined

$$y = \left(\frac{qb}{2}\right)^2, \tag{25}$$

in terms of the harmonic oscillator size parameter b, which has a default value of

$$b^2 = 41.467/(45A^{-1./3} - 25A^{-2/3})\ fm^2. \tag{26}$$

$$W_M^{\tau\tau'}(y) = \sum_{even\ J} \langle j_T|\, M_{J\tau}(y)\, |j_T\rangle \, \langle j_T|\, M_{J\tau'}(y)\, |j_T\rangle \tag{27}$$

$$W_{\Sigma''}^{\tau\tau'}(y) = \sum_{odd\ J} \langle j_T|\, \Sigma''_{J\tau}(y)\, |j_T\rangle \, \langle j_T|\, \Sigma''_{J\tau'}(y)\, |j_T\rangle \tag{28}$$

$$W_{\Sigma'}^{\tau\tau'}(y) = \sum_{odd\ J} \langle j_T|\, \Sigma'_{J\tau}(y)\, |j_T\rangle \, \langle j_T|\, \Sigma'_{J\tau'}(y)\, |j_T\rangle \tag{29}$$

$$W_{\Phi''}^{\tau\tau'}(y) = \sum_{even\ J} \langle j_T|\, \Phi''_{J\tau}(y)\, |j_T\rangle \, \langle j_T|\, \Phi''_{J\tau'}(y)\, |j_T\rangle \tag{30}$$

$$W_{\tilde{\Phi}'}^{\tau\tau'}(y) = \sum_{even\ J} \langle j_T|\, \tilde{\Phi}'_{J\tau}(y)\, |j_T\rangle \, \langle j_T|\, \tilde{\Phi}'_{J\tau'}(y)\, |j_T\rangle \tag{31}$$

$$W_{\Delta}^{\tau\tau'}(y) = \sum_{odd\ J} \langle j_T|\, \Delta_{J\tau}(y)\, |j_T\rangle \, \langle j_T|\, \Delta_{J\tau'}(y)\, |j_T\rangle \tag{32}$$

$$W_{\Delta\Sigma'}^{\tau\tau'}(y) = \sum_{odd\ J} \langle j_T|\, \Delta_{J\tau}(y)\, |j_T\rangle \, \langle j_T|\, \Sigma'_{J\tau'}(y)\, |j_T\rangle \tag{33}$$

$$W_{\Phi''M}^{\tau\tau'}(y) = \sum_{even\ J} \langle j_T|\, \Phi''_{J\tau}(y)\, |j_T\rangle \, \langle j_T|\, M_{J\tau'}(y)\, |j_T\rangle \tag{34}$$

2.6. **Nuclear operators and their matrix elements.** of which there are six, are nuclear operators constructed from Bessel spherical harmonics and vector spherical harmonics, and are evaluated here on the ground state of the target nucleus.

3. Example: si28 differential scattering rate

```
 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
 Welcome to our FORTAN 90 implementation of the WIMP-nucleon scattering code.
 Based on the Mathematica script described in arXiv:1308.6288 (2003).
   VERSION 1.1 UPDATED: 2020.05.23 @ SDSU
   Dev. contact: cjohnson@sdsu.edu
                 ogorton@sdsu.edu
 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

 Select an option:
 [1] Differential scattering rate per detector per unit recoil energy.
 [2] Scattering probability.
 [3] Differential cross section per recoil energy.
 [4] (Future feature) Total cross section.
 [5] (Future feature) Total scattering rate per detector.
 [6] (Future featuer)
1
 Enter q, the three-momentum transfer of the scattering reaction:
1

 Enter the neutron number
14

 Enter the proton number
14
 Setting default parameter values.
```

```
 Enter name of control file (.control):
../si28
 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
 Reading control file.
 Possible keywords:
 coefnonrel
 vearth
 dmdens
 quadtype
 intpoints
 gev
 femtometer
 dmmass
 vescape
 ntarget
 weakmscale
 vscale
 mnucleon
 dmspin

 # Coefficient matrix
 # Ommitted values ar
 # c_i^t
 # i = 1,...,16
 # t = 0 protons, 1 n
 #control name  t i c
 Set non-relativistic coefficient: op           1 p/n           1 c   3.1000000000000001
 Set non-relativistic coefficient: op           3 p/n           1 c   3.1000000000000001
 Set non-relativistic coefficient: op           4 p/n           1 c   3.1000000000000001
 Set non-relativistic coefficient: op           5 p/n           1 c   3.1000000000000001
 Set non-relativistic coefficient: op           6 p/n           1 c   3.1000000000000001
 Set non-relativistic coefficient: op           7 p/n           1 c   3.1000000000000001
 Set non-relativistic coefficient: op           8 p/n           1 c   3.1000000000000001
 Invalid keyword "fakekeyword". Ignoring.
 vearth: Set velocity of earth in galactic frame set to   232.00000000000000
 dmdens: Set local dark matter density to   1.0000000000000000
 intpoints: Set number of integral lattice points to        1000
 End of control file.

  Enter shell-model space file name (.sps)
../sd
 Shell-Model space file name ../sd
           1           0           3           2
           2           0           5           2
           3           1           1           0
           4           0           1           1
           5           0           3           1
           6           0           1           0

  Enter name of one-body density file (.res)
../si28w
           6
 Fill core? [y/n]
y
 Filling core orbitals.
 Printing density matrix.
 # spo =           6
 J=           0
           1           1  0.475710005       0.00000000
```

```
          2           2   2.66933990       0.00000000
          3           3  0.703809977       0.00000000
          4           4   2.00000000       2.00000000
          5           5   2.82842708       2.82842708
          6           6   2.00000000       2.00000000
 J=           1
 J=           2
 J=           3
 J=           4
 J=           5
 J=           6
 J=           7
 J=           8
 J=           9
 J=          10
 b[dimless]=   1.8506094217300415
 b[fm]=   9.3783331984848814
 y=   21.988283395450917
 mN  0.93827199935913086
 jchi  0.50000000000000000
 mchi   50.000000000000000
 Jiso=           0
 Tiso=           0
 Mtiso=           0
 ap,an          14          14
 Miso=   28.000000000000000
 muT=   17.222406817915434
 q=   1.0000000000000000
 vdist_min =    2.9031946886766143E-002
 vdist_max =    2640.0000000000000
 nt   1.0000000000000000
 rhochi   1.0000000000000000
 mchi   50.000000000000000
 v0=   220.00000000000000
 ve=   232.00000000000000
 Integral lattice size =         1000
 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
 Event rate =    1.3425254633130306E-021
 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```