A FORTRAN PROGRAM FOR EXPERIMENTAL WIMP ANALYSIS

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Model-Independent WIMP Scattering Responses and Event Rates

Contents

This is the manual for the FORTRAN version of the model-independent WIMP scattering response and event rate code, which was originally written in Mathematica and described in arXiv:1308.6288.

1. Primary functions

1.1. **Differential event rate.**

$$\frac{dR_D}{dE_R} = \frac{dR_D}{d\vec{q}^2}(q) = N_T n_\chi \int_{v_{min}}^{\infty} \frac{d\sigma(v,q)}{d\vec{q}^2} \tilde{f}(\vec{v}) v d^3v \tag{1}$$

where q is the WIMP-nucleon momentum transfer, N_T is the number of target nuclei, $n_\chi = \rho_\chi/m_\chi$ is the local dark matter density, σ is the WIMP-nucleon cross section, and $\tilde{f}$ is dark matter velocity distribution in the lab-frame. $\tilde{f}(\vec{v})$ is obtained by boosting the Galactic-frame distribution $f(\vec{v})$,

$$\tilde{f}(\vec{v}) = f(\vec{v} + \vec{v}_{earth}), \tag{2}$$

where $\vec{v}_{earth}$ is the velocity of the earth in the galactic rest frame. The simplest model is a three-dimensional Maxwell distribution:

$$f(\vec{v}) \propto e^{-\vec{v}^2/v_0^2}, \tag{3}$$

where v_0 is some scaling factor (typically taken to be around 220 km/s).

In order to evaluate the integral in (**??**), we make the conversion to spherical coordinates, and take special care to deal with the velocity boost in (**??**). Assuming a $1/v^2$ velocity dependence of the cross section term (see section **??**), we need to evaluate an integral of the form

$$I = \int_{v_{min}}^{v_{max}} d^3v \frac{f(\vec{v} + \vec{v}_{earth})}{v} = \int_{v_{min}}^{v_{max}} d^3v \frac{1}{v} e^{-(\vec{v}+\vec{v}_{earth})^2/v_0^2} \tag{4}$$

Noting that $(\vec{v} + \vec{v}_{earth})^2 = \vec{v}^2 + \vec{v}_{earth}^2 + 2vv_{earth}\cos(\theta)$, with $|\vec{v}| \equiv v$ and θ defining the angle between the two vectors, it's convenient to make the substitution $d^3v = d\phi d(\cos\theta)v^2 dv$:

$$\begin{aligned}
I &= \int_0^{2\pi} d\phi \int_{v_{min}}^{v_{max}} dv \int_{-1}^{1} d(\cos\theta) e^{-2vv_{earth}\cos\theta/v_0^2} v^2 \frac{1}{v} e^{-(\vec{v}^2+\vec{v}_{earth}^2)/v_0^2} \\
&= 2\pi \int_{v_{min}}^{v_{max}} dv v e^{-(\vec{v}^2+\vec{v}_{earth}^2)/v_0^2} \left(-\frac{v_0^2}{2vv_{earth}} e^{-2vv_{earth}\cos\theta/v_0^2} \right)_{-1}^{1} \\
&= \frac{\pi v_0^2}{v_{earth}} \int_{v_{min}}^{v_{max}} dv e^{-(\vec{v}^2+\vec{v}_{earth}^2)/v_0^2} \left(-e^{-2vv_{earth}/v_0^2} + e^{+2vv_{earth}/v_0^2} \right) \\
&= \frac{\pi v_0^2}{v_{earth}} \int_{v_{min}}^{v_{max}} dv \left(-e^{(v+v_{earth})^2/v_0^2} + e^{(v-v_{earth})^2/v_0^2} \right) \\
&= \frac{\pi v_0^2}{v_{earth}} \int_{v_{min}}^{v_{max}} dv \left(g(v - v_{earth}) - g(v + v_{earth}) \right)
\end{aligned} \tag{5}$$

where in the last equality, we have defined a one-dimensional Gaussian form

$$g(v) \propto e^{-v^2/v_0^2}. \tag{6}$$

The final expression for I can be trivially generalized to other spherically-symmetric velocity-dependent forms of the differential cross section. What's important is the reduction of the velocity-boosted d^3v integral to a radial integral which can be carried out with one-dimensional quadrature:

$$\int_{v_{min}}^{v_{max}} d^3v\sigma(v)e^{-(\vec{v}+\vec{v}_{earth})^2/v_0^2} = \frac{\pi v_0^2}{v_{earth}} \int_{v_{min}}^{v_{max}} dv\sigma(v)v^2 \left(g(v - v_{earth}) - g(v + v_{earth})\right). \tag{7}$$

The FORTRAN code uses equation (**??**) to evaluate the event rate integral in equation (**??**). Of course, analytic solutions of (**??**) exist in the form of error functions, but the above form allows for evaluation with quadrature, making it easy to later modify the velocity distribution (as long as it remains spherically symmetric). For example, adding a velocity cut-off is as easy as changing the limit on the quadrature, with no need to write a whole new subroutine for the analytic forms found in the Mathematica script.

1.2. **Differential cross section.**

$$\frac{d\sigma(v, E_R)}{dE_R} = 2m_T \frac{d\sigma(v)(v, \vec{q}^2)}{d\vec{q}^2} = 2m_T \frac{1}{4\pi v^2} T(v, q), \tag{8}$$

Where v is the velocity of the dark matter particles in the lab-frame, q is the momentum transfer of the scattering event, m_T is the mass of the target nucleus, and $T(v, q)$ is the transition or scattering probability. We can see here that the differential cross section has an explicit $1/v^2$ dependence, independent of any velocity dependence of $T(v, q)$.

1.3. **Transition probability / Scattering probability.**

The scattering probability is

$$T(v, q) = \frac{1}{2j_\chi + 1} \frac{1}{2j_T + 1} \sum_{spins} |\mathcal{M}(v, q)|^2 \tag{9}$$

where j_χ is the spin of the WIMP, j_T is the spin angular momentum of the target nucleus, and $\mathcal{M}$ Galilean invariant amplitude:

$$T(v, q) = \frac{4\pi}{2j_T + 1} \frac{1}{(4m_\chi)^2} \sum_{\tau=0}^{1} \sum_{\tau'=0}^{1} \sum_{i=1}^{8} R_i^{\tau\tau'}(v^2, q^2) W_i^{\tau\tau'}(q) \tag{10}$$

where m_χ is the mass of the dark matter particle and τ is an index used to sum over isospin couplings. The coefficients $R_i^{\tau,\tau'}$ are dark matter particle response functions, to be define in another section. The operators $W_i^{\tau\tau'}(q)$ are nuclear response functions, which are sums over matrix elements of nuclear operators constructed from Bessel spherical harmonics and vector spherical harmonics.

1.4. **Dark matter response functions.**

1.5. **Nuclear response functions.**

There are eight nuclear response functions $W_i^{\tau\tau'}(y)$ considered here. The unit-less variable y is defined

$$y = \left(\frac{qb}{2}\right)^2, \tag{11}$$

in terms of the harmonic oscillator size parameter b, which has a default value of

$$b^2 = 41.467/(45A^{-1./3} - 25A^{-2/3})\ fm^2. \tag{12}$$

$$W_M^{\tau\tau'}(y) = \sum_{even\ J} \langle j_T | M_{J\tau}(y) | j_T \rangle^2 \tag{13}$$

$$W_{\Sigma''}^{\tau\tau'}(y) = \sum_{even\ J} \langle j_T | \Sigma''_{J\tau}(y) | j_T \rangle^2 \tag{14}$$

1.6. **Nuclear operators.**

of which there are six, are nuclear operators constructed from Bessel spherical harmonics and vector spherical harmonics, and are evaluated here on the ground state of the target nucleus.

2. Inputs and control file usage

This code has a twofold user-interface scheme. This first is through the interactive command-line interface. The program will prompt the user for the minimum necessary inputs to run a calculation with default parameter values. In the case of a differential event rate calculation, these would be the momentum transfer q, information about the target nucleus including the density matrices, and finally the coefficient of the WIMP-nucleus interaction. The coefficients are read in from the second user-interface scheme: the control file.

The simplest and necessary use case of the control file is to read in the coefficients of the WIMP-nucleon interaction. Each coefficient is stored on its one line in a file which ends in .control, with 4 entries each: the keyword "coefnonrel", a 0 or 1 indicating proton or neutron coupling, the index of the coefficient to set, and finally the value of the coefficient. These entries can be separated by tabs or spaces. For example, to set $c_5^{(n)} = 3.1$, the corresponding line in the control file would be:

```
coefnonrel    1    5     3.1
```

The control file also serves a more general but optional function: to set any parameter in the program to a custom value. To use this functionality is simple: simply add an entry to the control file with two values: the first should be the keyword for the parameter (see section **??**) and the second should be the value to set that parameter to. For example, to set the velocity of the earth in the galactic frame to 240 km/s, you should add the line:

```
vearth  240.0
```

3. Keywords

4. Example: si28 differential scattering rate

```
 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
 Welcome to our FORTAN 90 implementation of the WIMP-nucleon scattering code.
 Based on the Mathematica script described in arXiv:1308.6288 (2003).
   VERSION 1.1 UPDATED: 2020.05.23 @ SDSU
   Dev. contact: cjohnson@sdsu.edu
                 ogorton@sdsu.edu
 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

 Select an option:
 [1] Differential scattering rate per detector per unit recoil energy.
 [2] Scattering probability.
 [3] Differential cross section per recoil energy.
 [4] (Future feature) Total cross section.
 [5] (Future feature) Total scattering rate per detector.
 [6] (Future featuer)
1
 Enter q, the three-momentum transfer of the scattering reaction:
1

 Enter the neutron number
14

 Enter the proton number
14
 Setting default parameter values.

 Enter name of control file (.control):
../si28
 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
 Reading control file.
 Possible keywords:
 coefnonrel
 vearth
```

```
 dmdens
 quadtype
 intpoints
 gev
 femtometer
 dmmass
 vescape
 ntarget
 weakmscale
 vscale
 mnucleon
 dmspin

 # Coefficient matrix
 # Ommitted values ar
 # c_i^t
 # i = 1,...,16
 # t = 0 protons, 1 n
 #control name  t i c
 Set non-relativistic coefficient: op           1 p/n           1 c   3.1000000000000001
 Set non-relativistic coefficient: op           3 p/n           1 c   3.1000000000000001
 Set non-relativistic coefficient: op           4 p/n           1 c   3.1000000000000001
 Set non-relativistic coefficient: op           5 p/n           1 c   3.1000000000000001
 Set non-relativistic coefficient: op           6 p/n           1 c   3.1000000000000001
 Set non-relativistic coefficient: op           7 p/n           1 c   3.1000000000000001
 Set non-relativistic coefficient: op           8 p/n           1 c   3.1000000000000001
 Invalid keyword "fakekeyword". Ignoring.
 vearth: Set velocity of earth in galactic frame set to   232.00000000000000
 dmdens: Set local dark matter density to   1.0000000000000000
 intpoints: Set number of integral lattice points to        1000
 End of control file.

  Enter shell-model space file name (.sps)
../sd
 Shell-Model space file name ../sd
           1           0           3           2
           2           0           5           2
           3           1           1           0
           4           0           1           1
           5           0           3           1
           6           0           1           0

  Enter name of one-body density file (.res)
../si28w
           6
 Fill core? [y/n]
y
 Filling core orbitals.
 Printing density matrix.
 # spo =           6
 J=           0
           1           1  0.475710005       0.00000000
           2           2   2.66933990       0.00000000
           3           3  0.703809977       0.00000000
           4           4   2.00000000       2.00000000
```

```
          5           5   2.82842708       2.82842708
          6           6   2.00000000       2.00000000
J=           1
J=           2
J=           3
J=           4
J=           5
J=           6
J=           7
J=           8
J=           9
J=          10
b[dimless]=   1.8506094217300415
b[fm]=   9.3783331984848814
y=   21.988283395450917
mN  0.93827199935913086
jchi  0.50000000000000000
mchi   50.000000000000000
Jiso=           0
Tiso=           0
Mtiso=           0
ap,an          14          14
Miso=   28.000000000000000
muT=   17.222406817915434
q=   1.0000000000000000
vdist_min =    2.9031946886766143E-002
vdist_max =    2640.0000000000000
nt   1.0000000000000000
rhochi   1.0000000000000000
mchi   50.000000000000000
v0=   220.00000000000000
ve=   232.00000000000000
Integral lattice size =         1000
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Event rate =    1.3425254633130306E-021
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```