Indirect measurements of nuclear cross sections: tempering experimental results with theory

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September 9, 2019

IM RELEASE: LLNL-PRES-789322

This work was performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under contract DE-AC52-07NA27344. Lawrence Livermore National Security, LLC

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I will give a summary of the major results of my summer projects

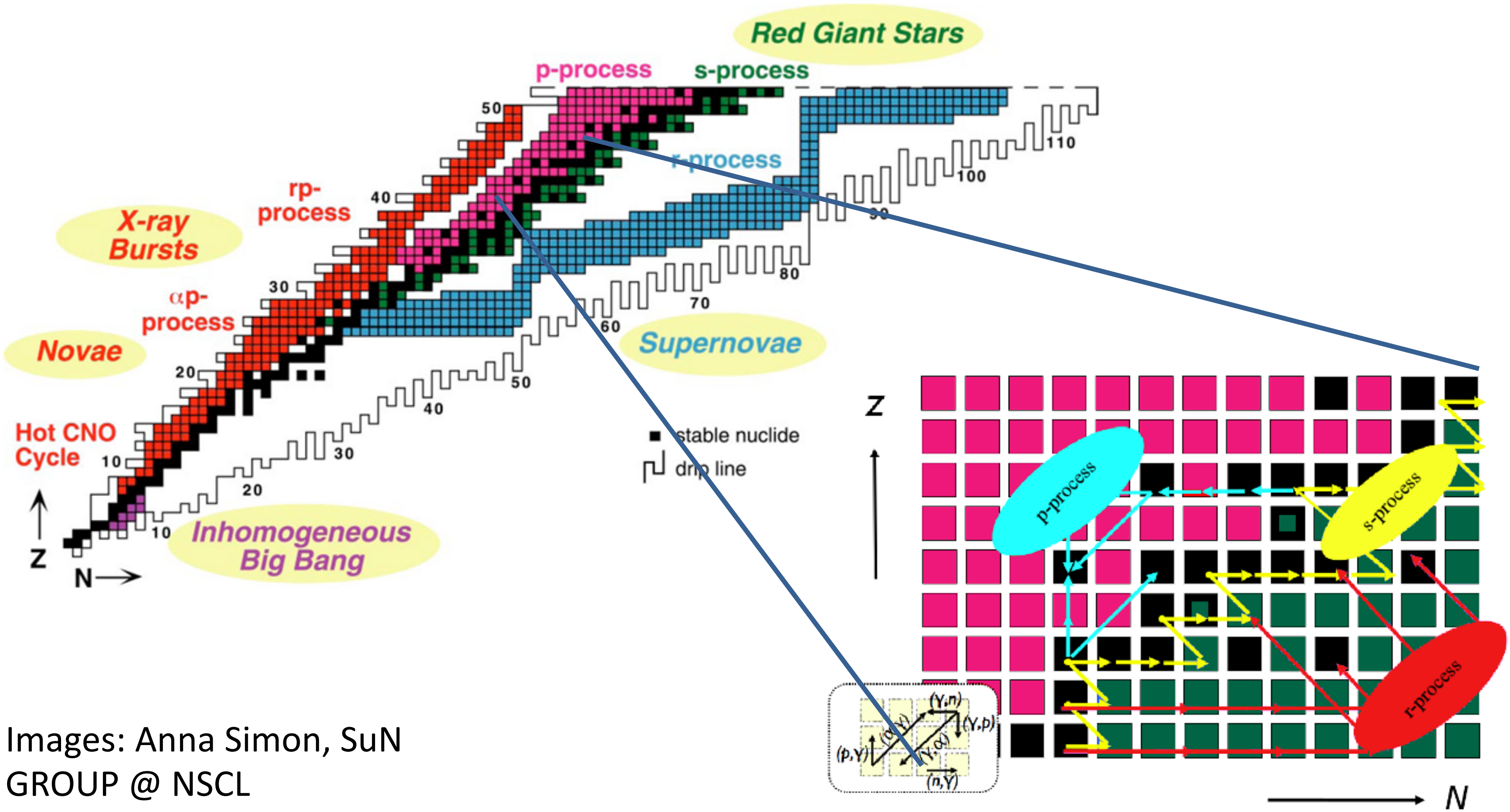
PART 1:

- Use of in inverse-reactions for indirect measurements at University of Notre Dame
- Applying our theory capabilities to improve predictive power of indirect measurements

PART 2:

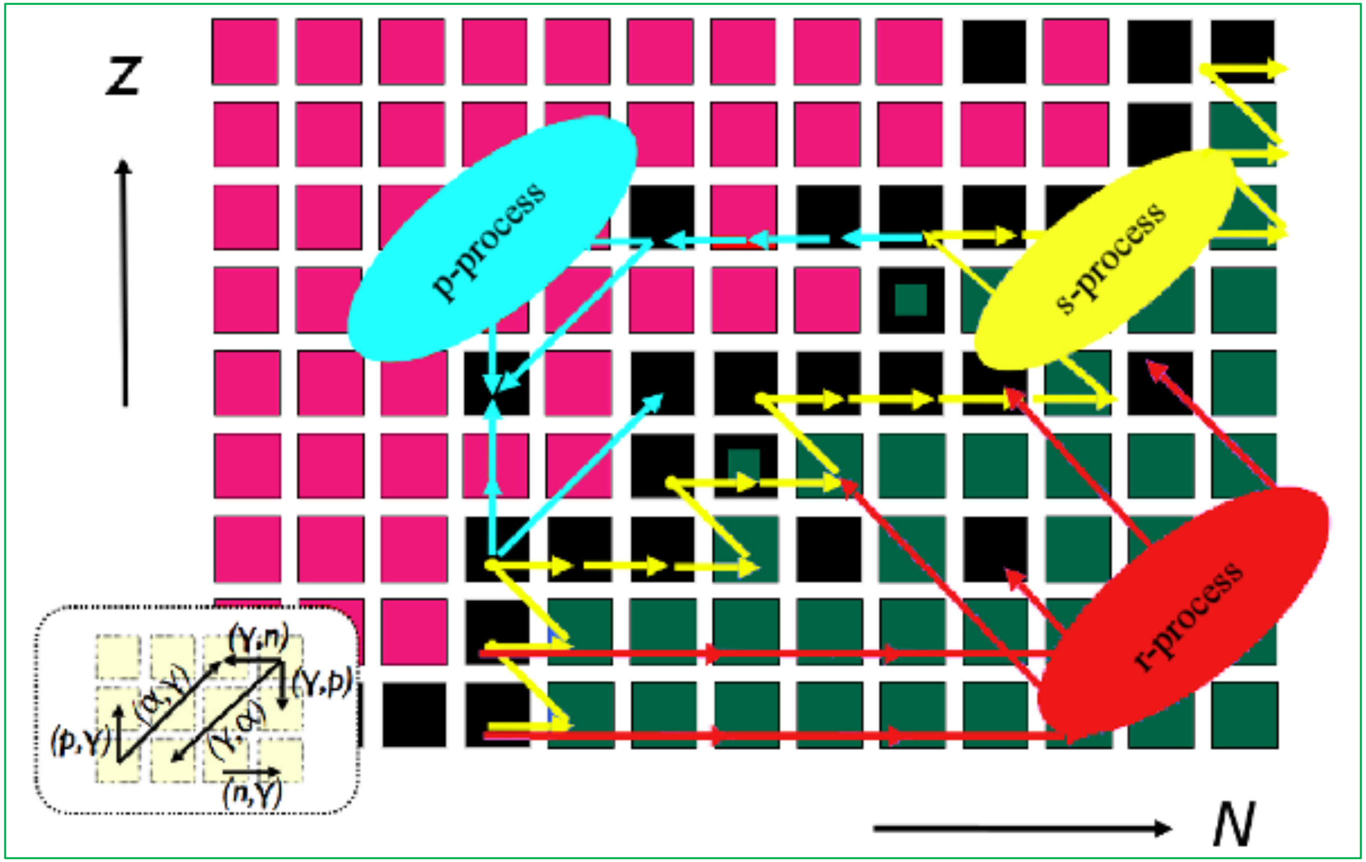
- Sensitivity of an approximation method used for indirect measurements
- Applying this surrogate method to recent data

All of the elements in the universe came from a handful of astrophysical processes



Images: Anna Simon, SuN GROUP @ NSCL
groups.nscl.msu.edu/SuN/

Proton-rich elements formed in p-process



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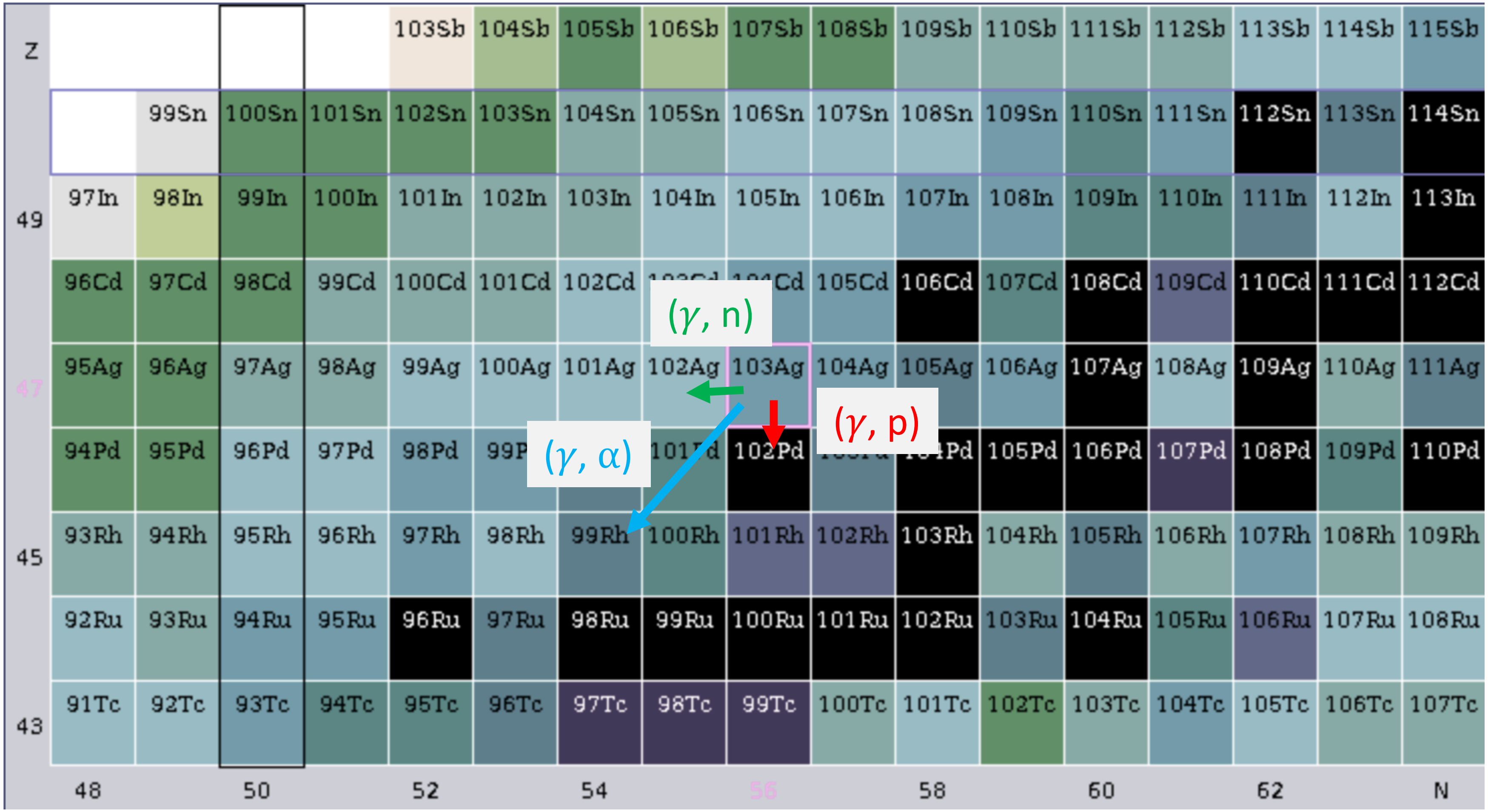
The details of the p-process(es) remain a mystery: unable to predict abundances



Images: NNDC chart of nuclides

Measuring (p,γ) can tell us about (γ,p)

- Astrophysics application need better cross sections
- Orlando Olivas-Gomez from University of Notre Dame
- Indirect measurement of (γ,p) cross sections from (p,γ) experiment

(p,γ) = Proton capture
(γ,p) = photo-disintegration of a photon

UND group measured proton capture cross sections

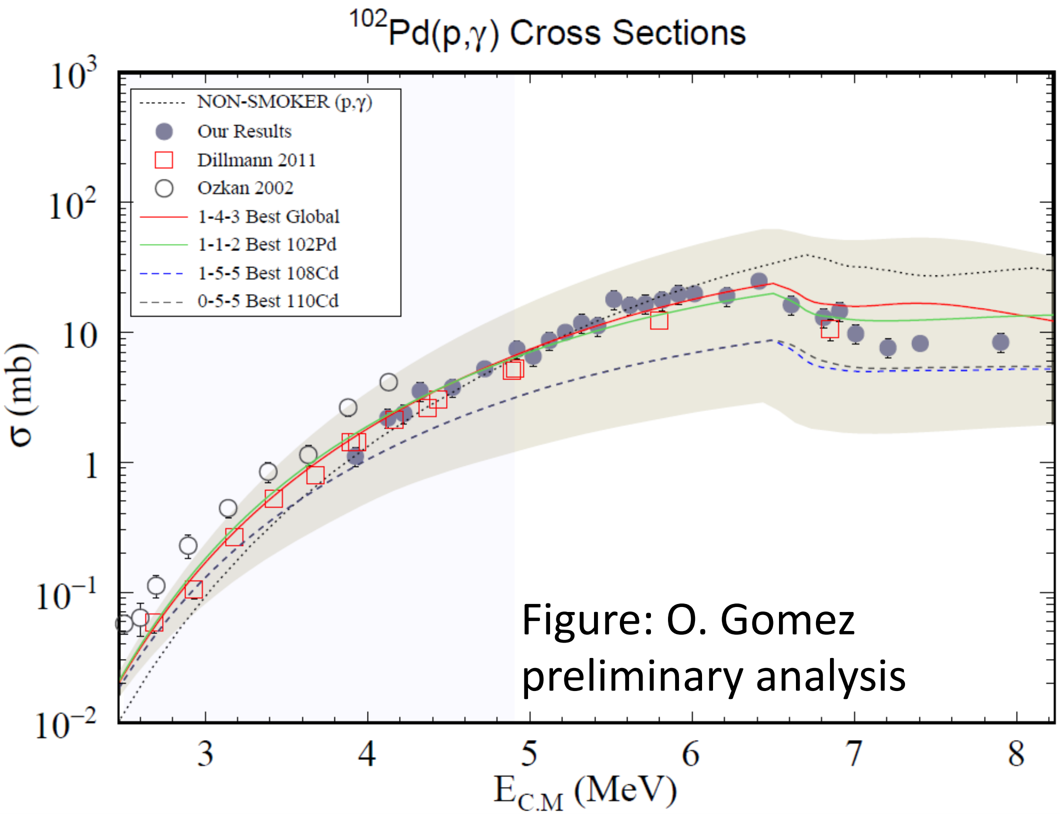
- Original idea:
  - Find the best theoretical model to describe (p, γ) data
  - Use that model to predict (γ,p) cross sections

Targets:

102Pd

108Cd

110Cd (Never done before!)



This is where I come in…

I wrote a code for this!

<u>My work with the theory side of indirect measurements</u>

- Fit nuclear reaction model parameters with MCMC
- Indirect measurement
  -> data
  -> parameter estimation
  -> predict desired reaction
- Surrogate method

<u>Tech specs</u>

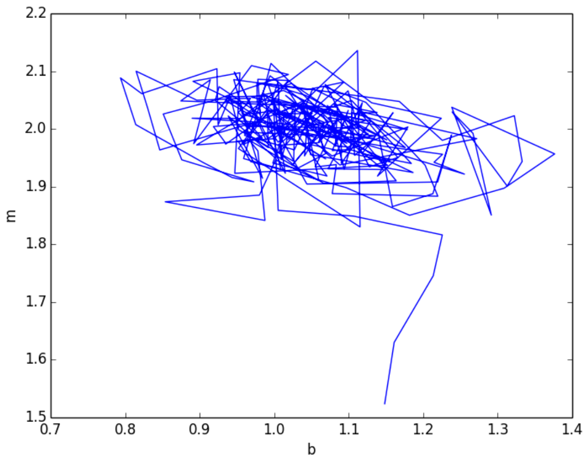
- Python, multiple Hauser-Feshbach reaction codes, multiple observables, simultaneous fitting, parallel sampling, 2000+ LOC

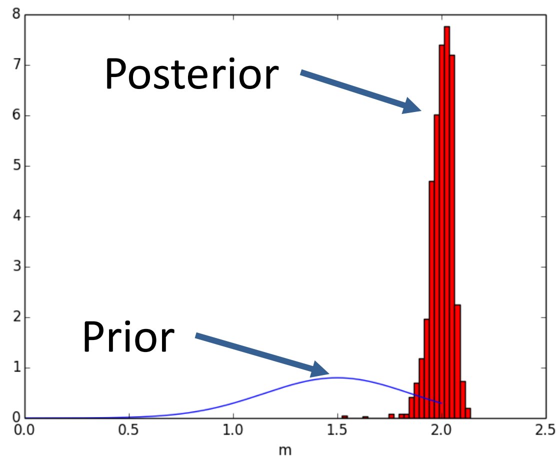
<u>Notable improvements this summer</u>

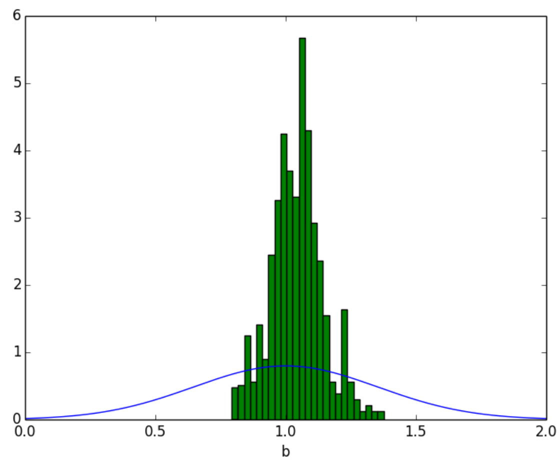
- Ability to include prior distributions for parameters
- Compatibility with open-source Hauser-Feshbach model code, EMPIRE

Applying MCMC to linear regression

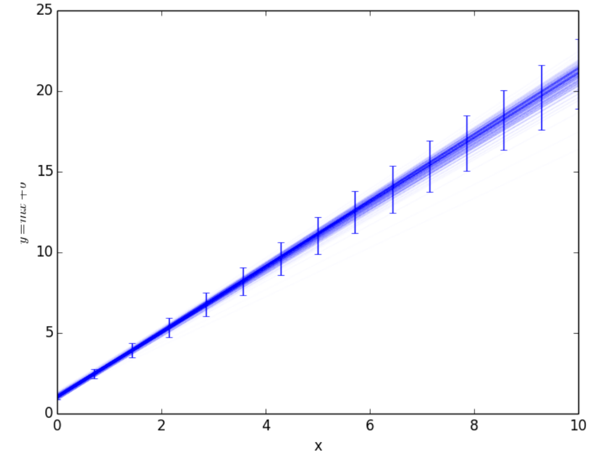
Works by censoring a random walk







y = m x + b



I can improve the resulting theory with MCMC

- Original idea:
  - Find the best theoretical model to describe (p, γ) data
  - Use that best model to predict (γ,p) cross sections
- What this will improve:
  - Extract more information from the data
  - Probability distributions for parameters
  - Uncertainty bars

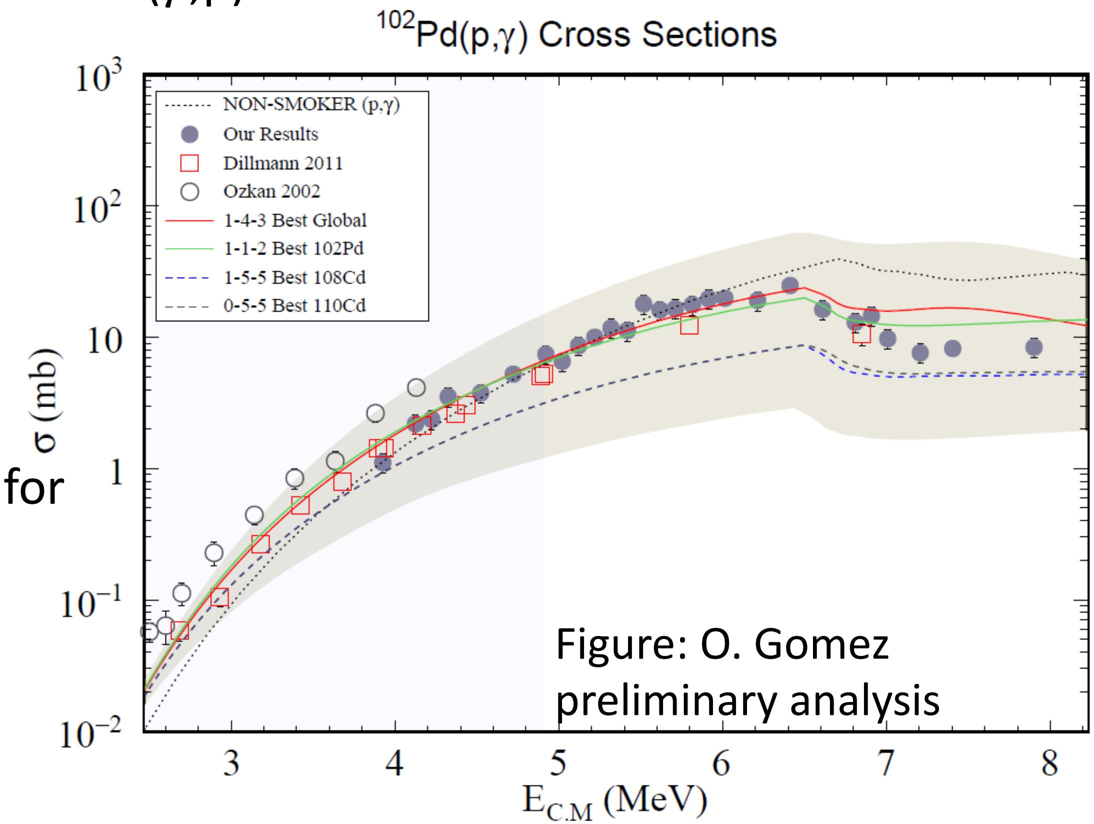
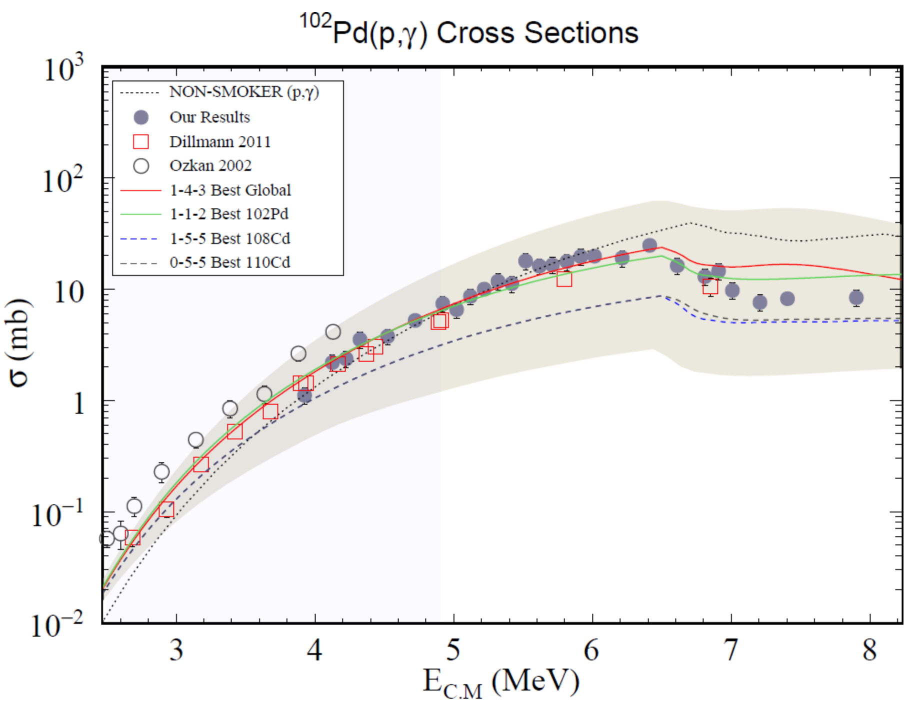
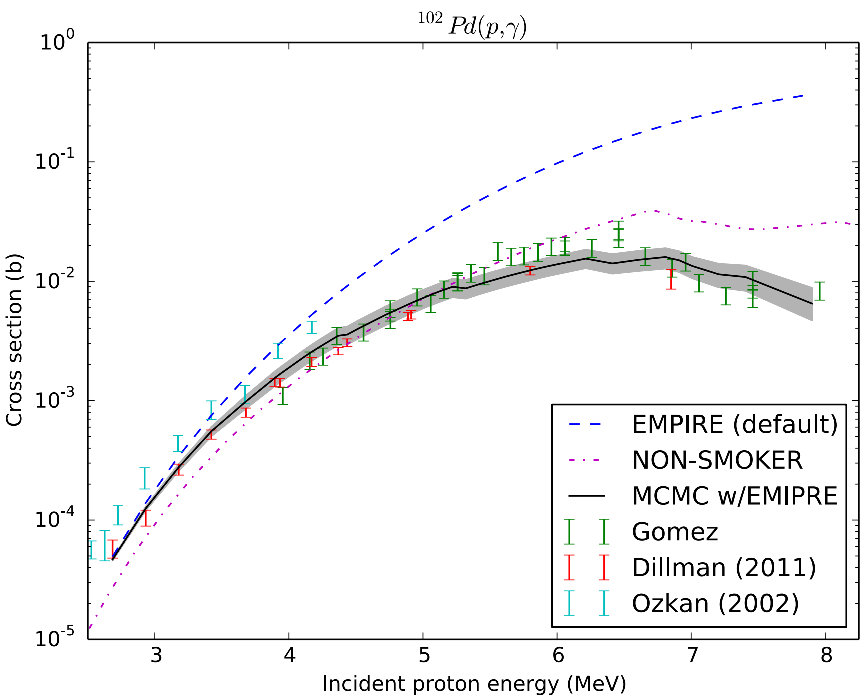


Figure: O. Gomez preliminary analysis

From nearest-facsimile to probability distribution



Selecting the best 'default' option



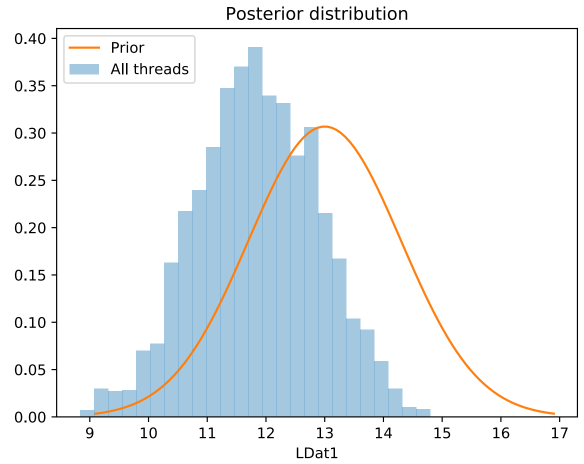
Tuning a model to constraints given by the data

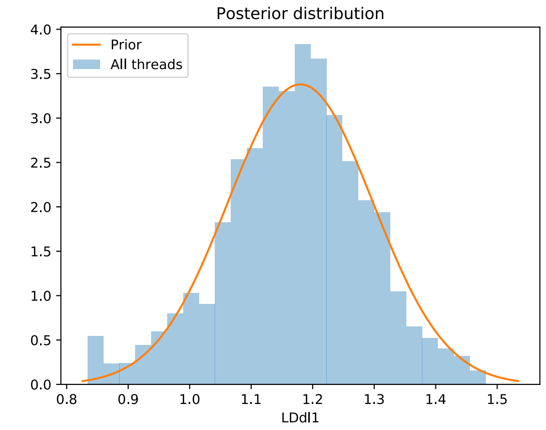
I examined nuclear structure model parameters

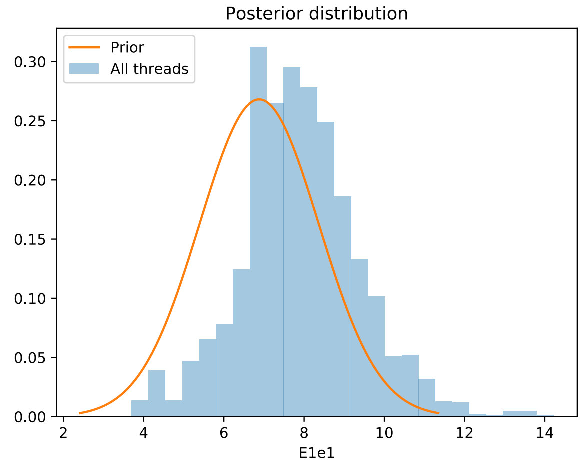
- Gamma-ray strength functions
- Nuclear level densities
- Priors mostly from Reference Input Parameter Library (RIPL-3)

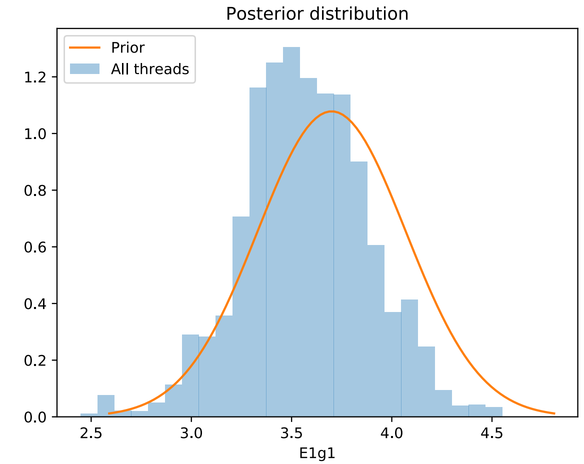
Nuclear reaction model parameter estimation

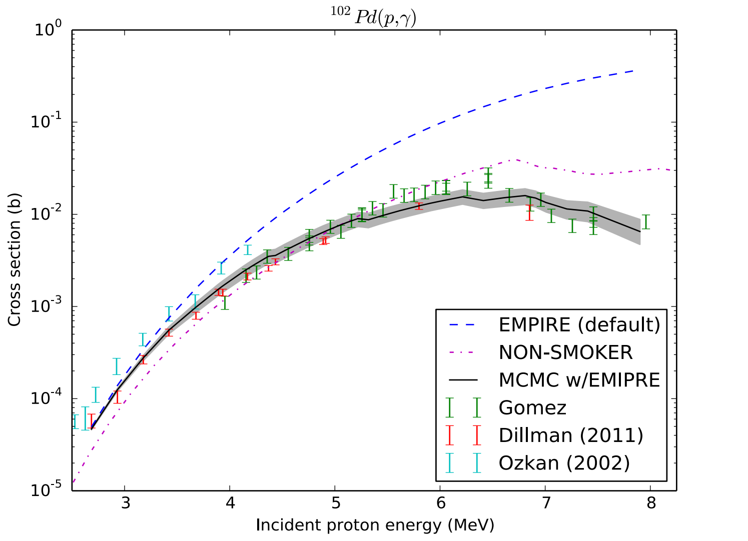
Parameter distributions from the 102Pd cross section fitting



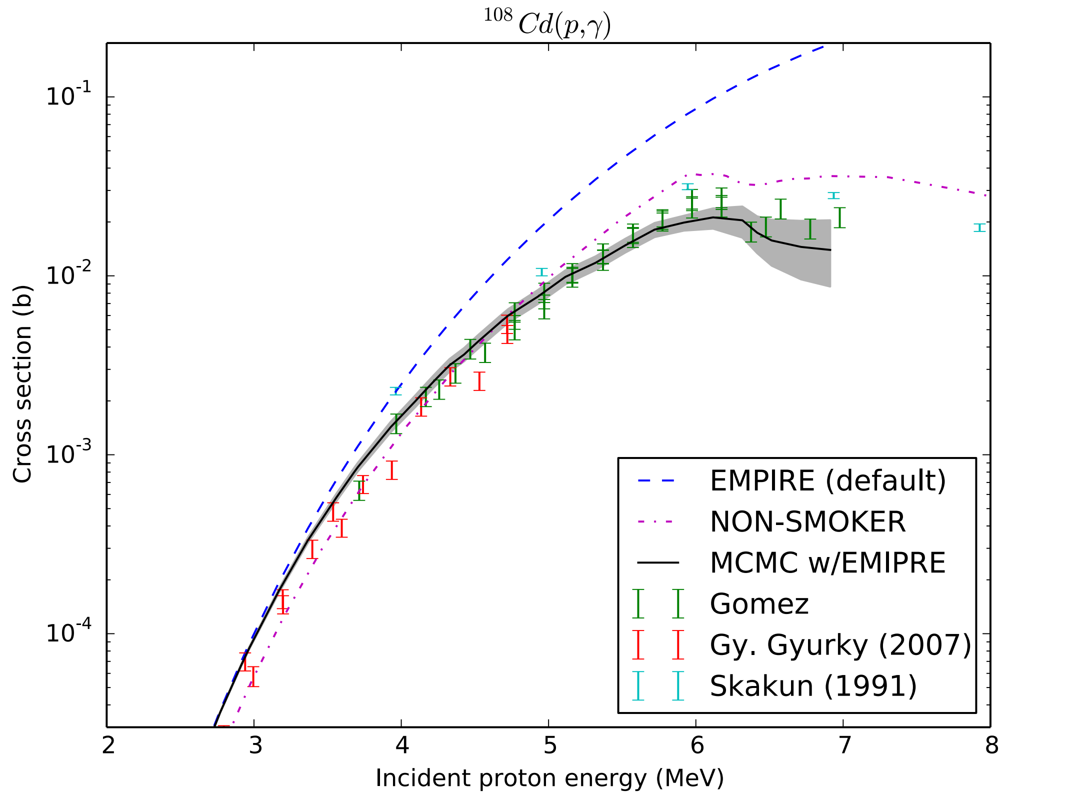




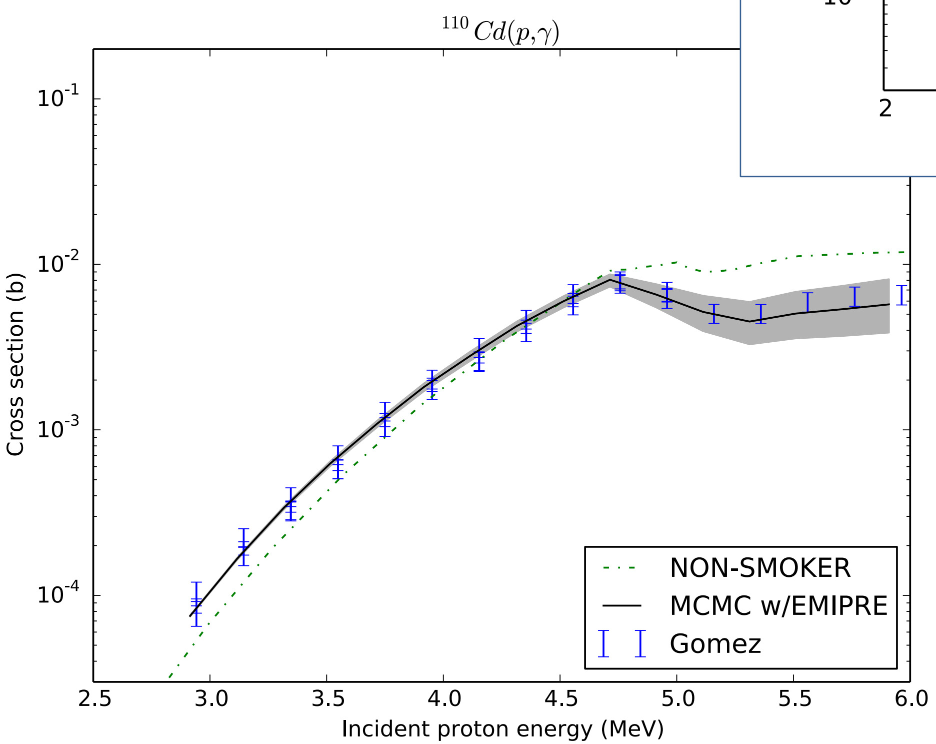




$^{102}Pd(p,\gamma)$
Cross section (b)
Incident proton energy (MeV)
EMPIRE (default)
NON-SMOKER
MCMC w/EMIPRE
Gomez
Dillman (2011)
Ozkan (2002)



$^{108}Cd(p,\gamma)$
Cross section (b)
Incident proton energy (MeV)
EMPIRE (default)
NON-SMOKER
MCMC w/EMIPRE
Gomez
Gy. Gyurky (2007)
Skakun (1991)



$^{110}Cd(p,\gamma)$
Cross section (b)
Incident proton energy (MeV)
NON-SMOKER
MCMC w/EMIPRE
Gomez

With parameters that reflect the data, now we can find the desired cross sections

Steps

1. Find the best theoretical model to describe (p, γ) data
2. Use that model to predict (γ,p) cross sections

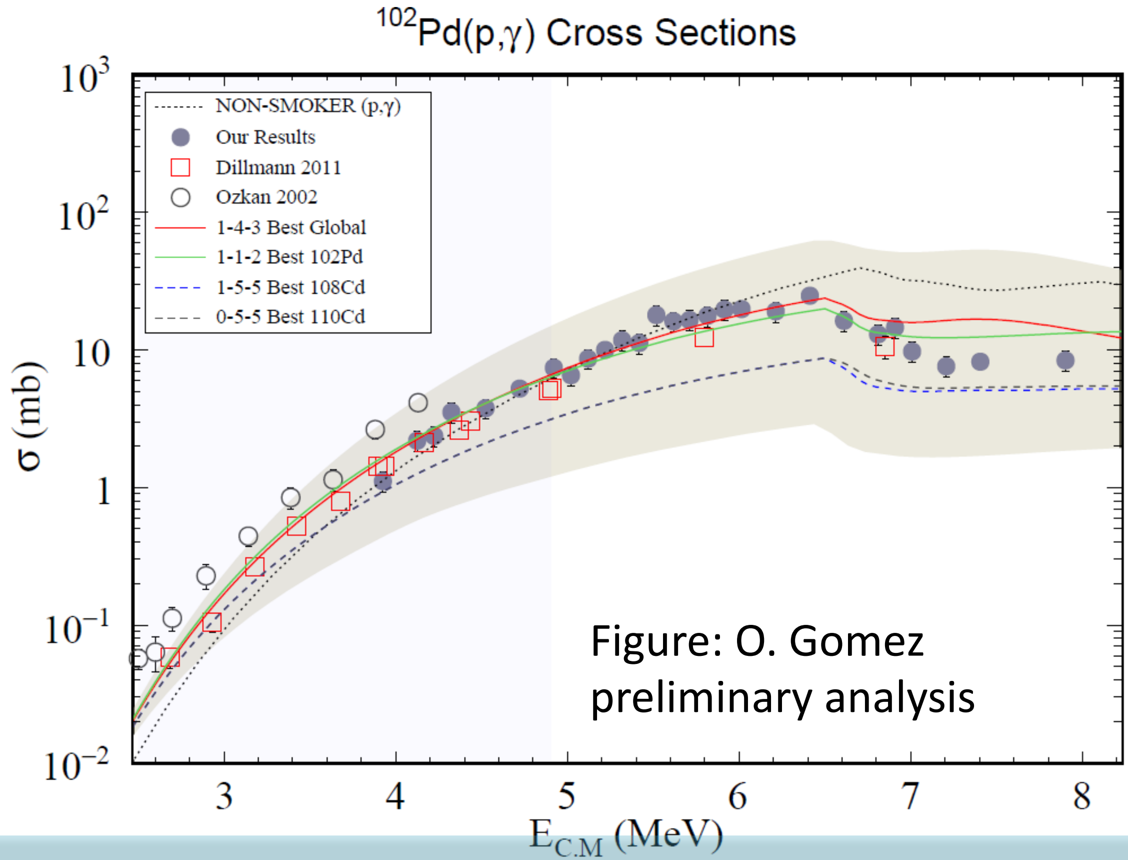
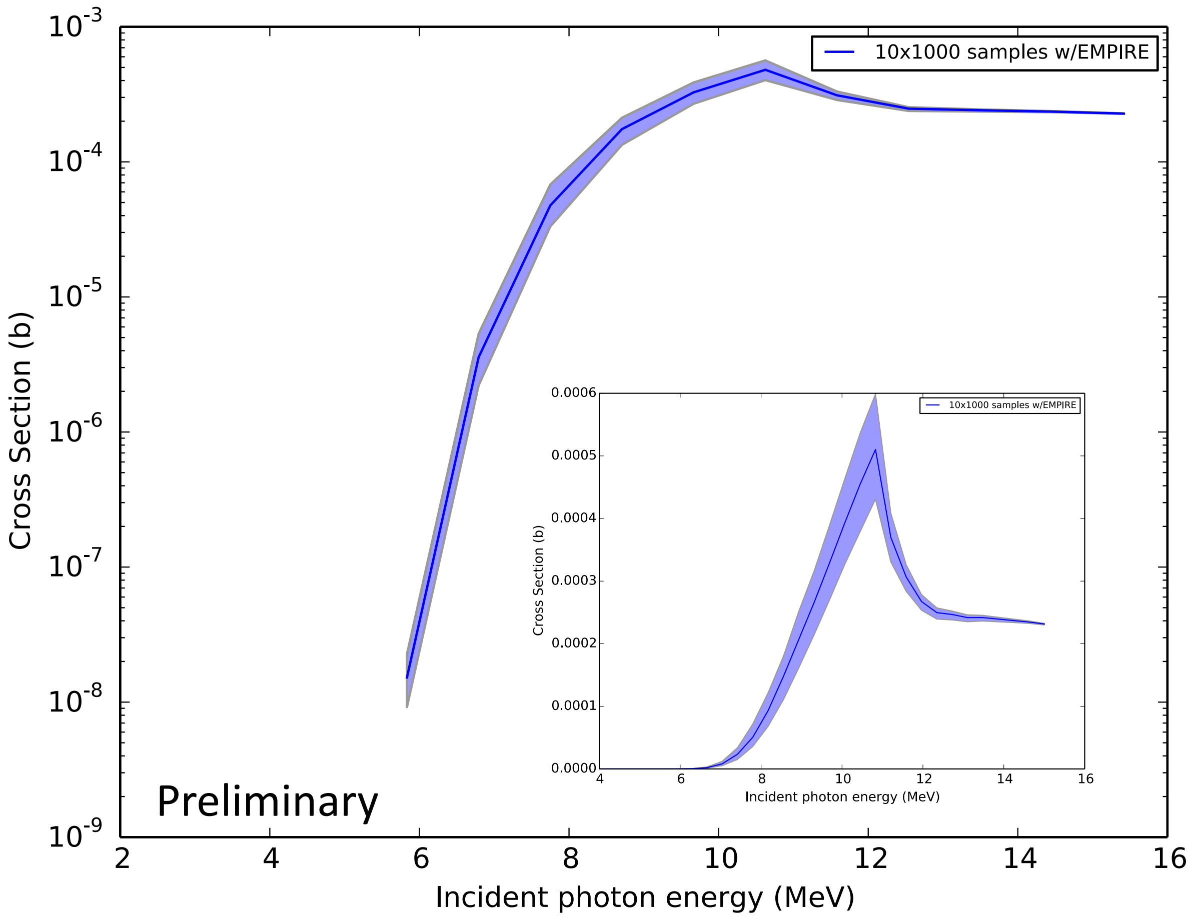


Figure: O. Gomez preliminary analysis

Ag103 (γ,p) cross section from (p, γ)-constrained parameters

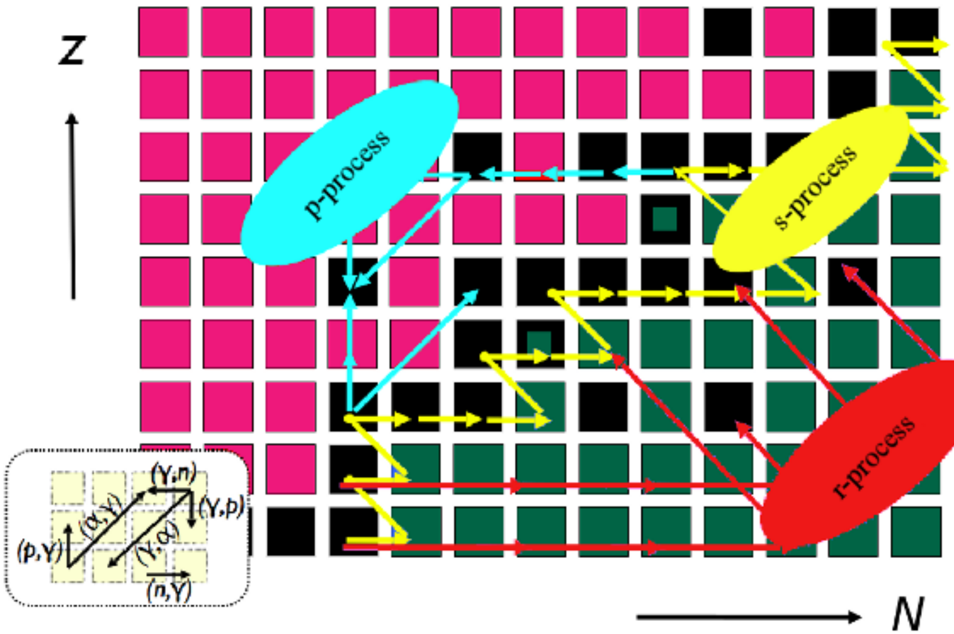
Distribution (blue band) reflects information encoded in experimental error bars



Improved cross sections means improved astrophysics models

<u>Take aways</u>

- Re-evaluated cross sections may change understanding of the p-process
- Error bars tell us how much we can infer from the new measurements



Images: Anna Simon, SuN GROUP @ NSCL
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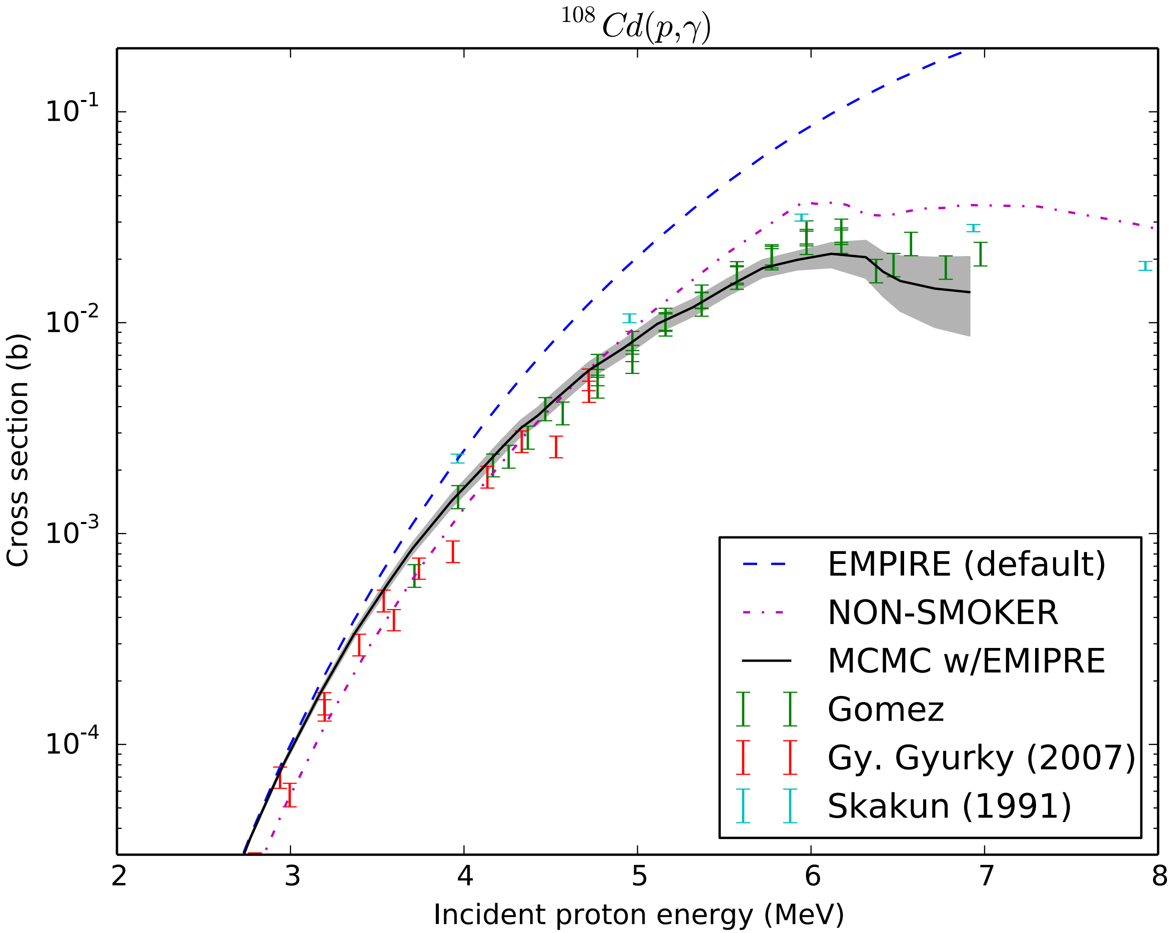
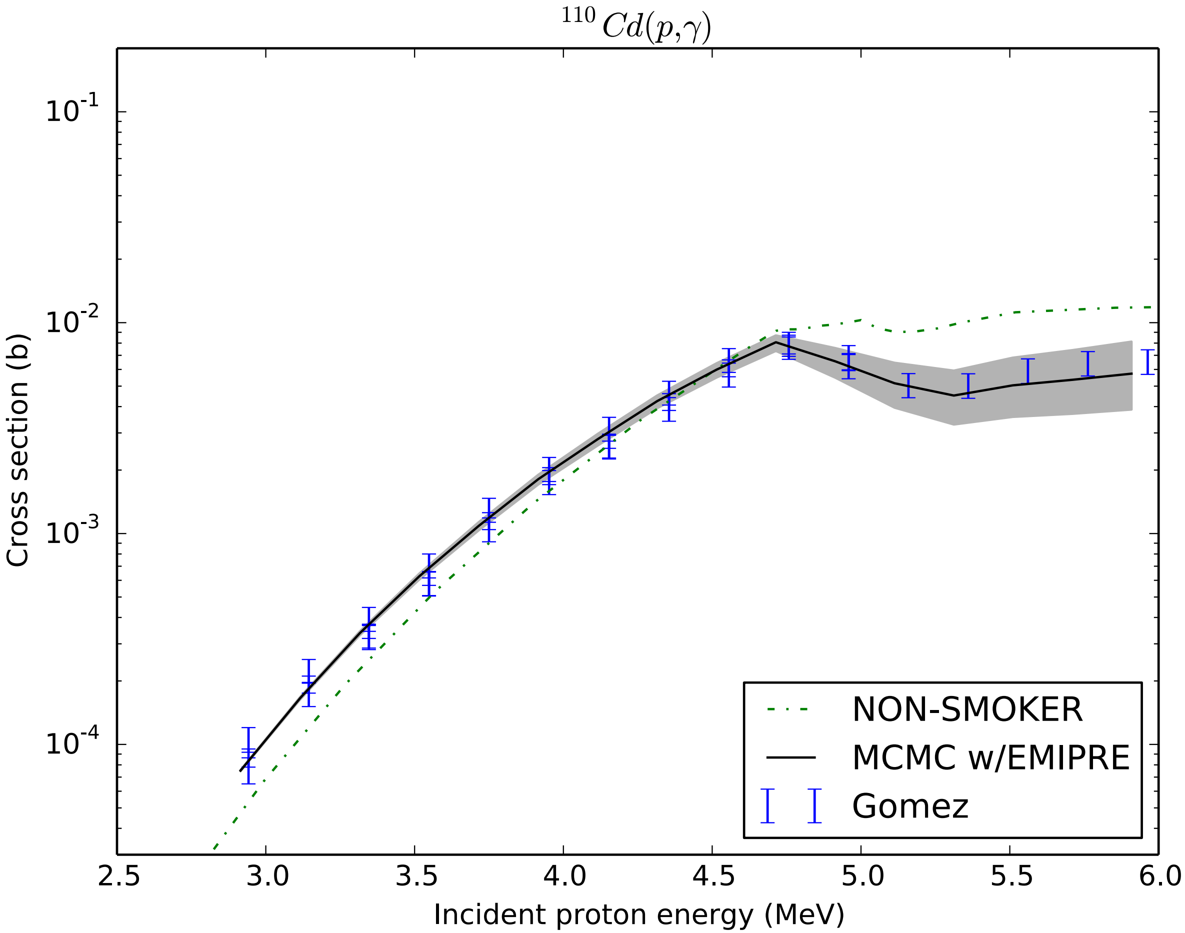
p + 108Cd → 109In + γ



p + 110Cd → 111In + γ

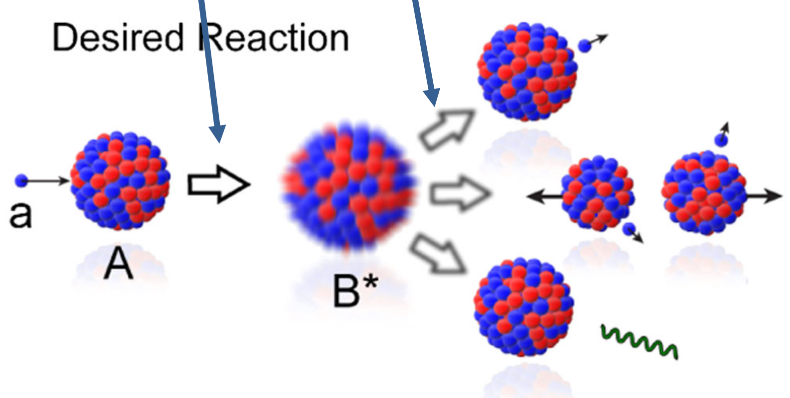


Hauser-Feshbach Model

Full Hauser-Feshbach formulation:

$$\frac{d\sigma^{HF}_{\alpha\chi}(E_\alpha)}{dE_{\chi'}} = \pi\frac{\lambda_\alpha^2}{2\pi}\sum_{J\pi}\omega_\alpha^J\sum_{lsl's'I'}\frac{T^J_{\alpha ls}(E_\alpha)T^J_{\chi'l's'}(E_{\chi'})\rho_{I'}(U')}{\sum'_{\chi''l''s''}T^J_{\chi''l''s''}(E_{\chi''})+\sum_{\chi''l''s''I''}\int T^J_{\chi''l''s''}(E_{\chi''})\rho_{I''}(U'')dE_{\chi''}} \tag{8}$$

$$\sigma_{a,b}(E) = \sum_{J\pi}\sigma_a^{CN}(E,J\pi)P_b(E,J\pi)$$

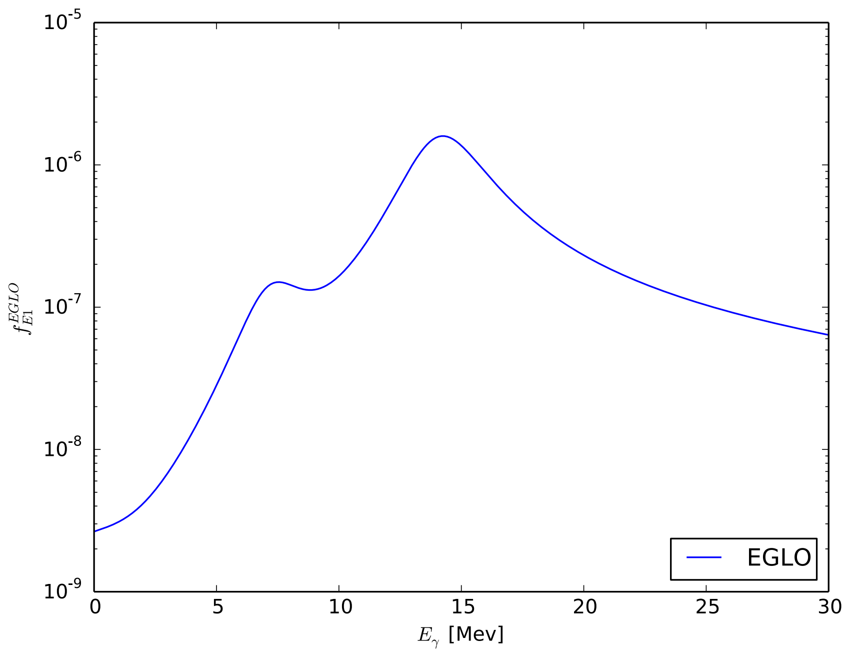


Gamma-ray strength functions

$$f_{E1}^{EGLO}(E_\gamma) = 8.674 \times 10^{-8} \left\{ \sum_{i=1}^{2} \Gamma_{E1i}\sigma_{E1i} \frac{E_\gamma \bar{\Gamma}(E_\gamma, T)}{(E_{E1i}^2 - E_\gamma^2)^2 + (E_\gamma \bar{\Gamma}(E_\gamma, T))^2} + \frac{0.7}{E_{E1i}^3}\bar{\Gamma} \right\}$$

The energy dependent width is

$$\bar{\Gamma}(E_\gamma, T) = \left[k_0 + (1 - k_0) \frac{(E_\gamma - \epsilon)}{(E_{E1i} - \epsilon)} \right] \frac{\Gamma_{E1i}(E_\gamma^2 + 4\pi T^2)}{E_{E1i}^2}.$$



Nuclear level density

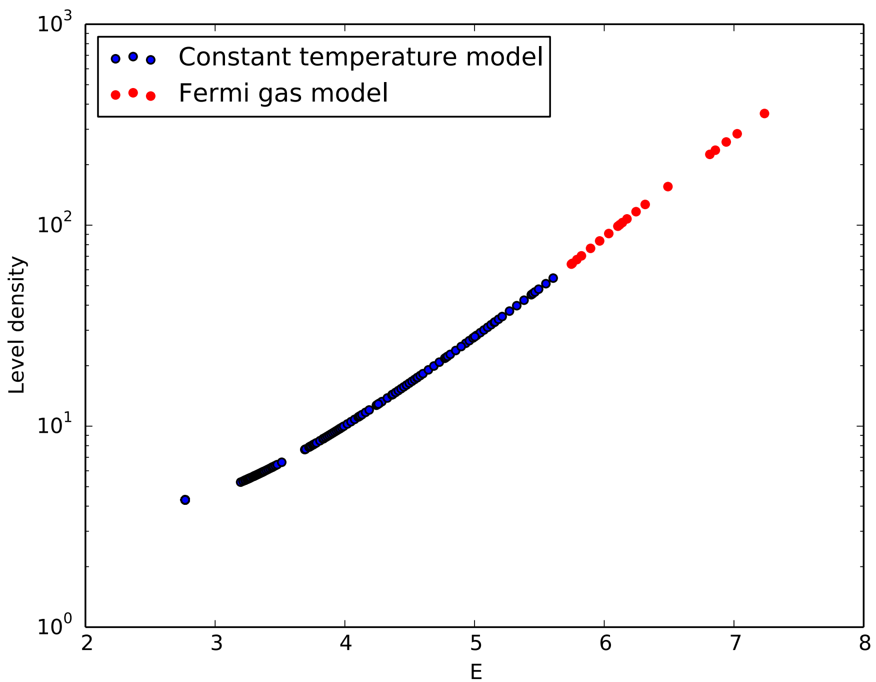
$$\rho(E_x) = \begin{cases} \rho^{CT}(E_x) = \frac{1}{T}\exp\left(\frac{E_x - E_0}{T}\right) & E_x \geq U_x \\ \rho^{FG}(E_x) \propto \frac{1}{\sigma^{3/2} a^{1/4} U^{5/4}} \exp\left(2\sqrt{aU} - \frac{(J+1/2)^2}{2\sigma^2}\right) & E_x > U_x \end{cases}$$

where

$$\sigma^2 = \lambda\sqrt{aU}A^{2/3} = 0.146\sqrt{aU}A^{2/3}$$

$$a = \tilde{a}[1 + (1 - \exp(-\gamma U))\delta W/U] = E_x/T^2$$

$$U = E_x - \Delta$$



PART 1:

- Collaboration with University of Notre Dame experimentalists using in inverse-reactions for <u>indirect measurements</u>
- Applying our theory capabilities to improve predictive power of their measurements

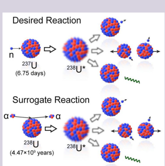
PART 2:

- Sensitivity of an approximation method used for <u>indirect measurements</u>
- Applying this surrogate method to recently new data

Indirect Measurements are Necessary for Radioactive Target Measurements

Reactions on *radioactive targets* are difficult or impossible to measure.

Theory of compound nuclear reactions limited by available nuclear structure.



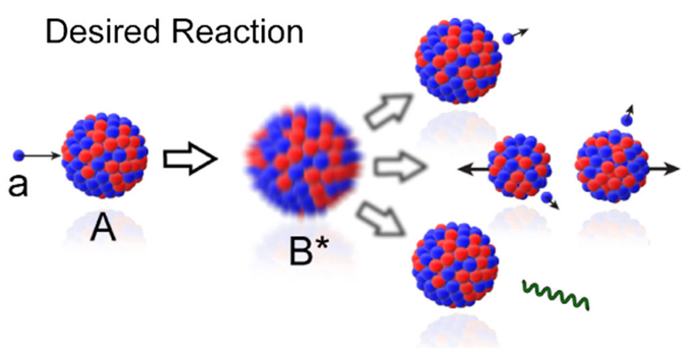
The Surrogate Method allows indirect measurements of cross sections by combining surrogate data and theory.

Hauser-Feshbach Theory models compound nuclear reactions in two stages

1. Formation of the compound nucleus (CN)

2. Decay of the CN

$$\sigma_{\alpha\beta} = \sigma_{\alpha}^{CN} G_{\beta}^{CN}$$



* Conservation of energy, spin and parity

$$\sigma_{\alpha\beta}(E) = \sum_{J\pi} \sigma_{\alpha}^{CN}(E, J\pi) G_{\beta}^{CN}(E, J\pi)$$

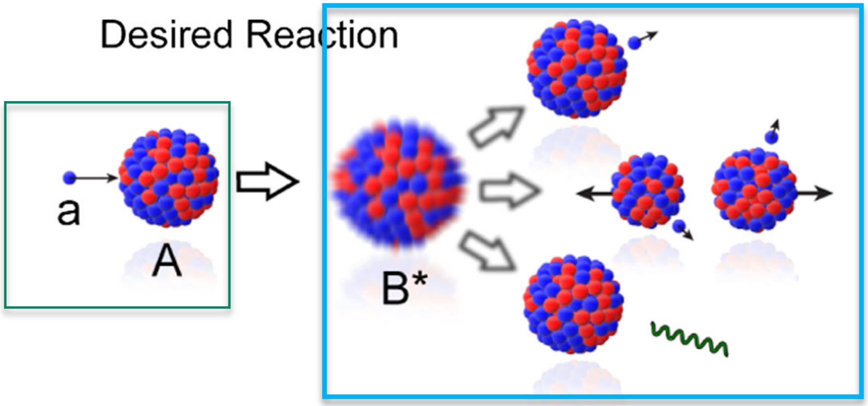
The Surrogate Idea: requires a lot of theory

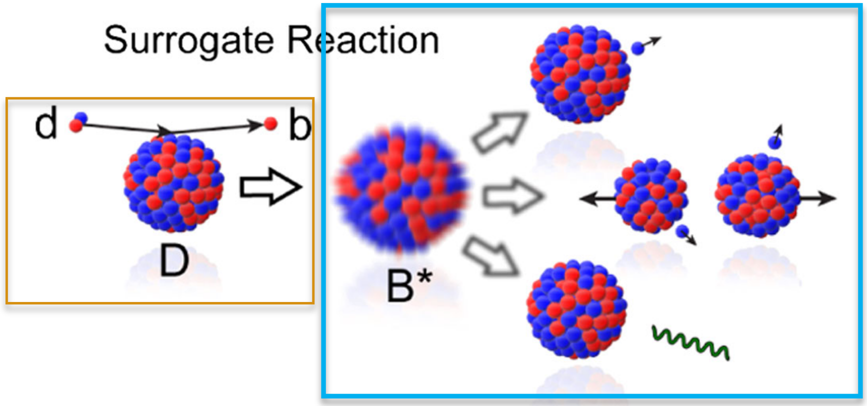
Desired Reaction

$$\sigma_{\alpha\beta}(E) = \sum_{J\pi} \sigma_{\alpha}^{CN}(E,J\pi) G_{\beta}^{CN}(E,J\pi)$$

Surrogate Reaction

$$P_{\delta\beta}(E) = \sum_{J\pi} F_{\delta}^{CN}(E,J\pi) G_{\beta}^{CN}(E,J\pi)$$





The Weiskopf-Ewing Limit applies under special conditions

$$\sigma_{\alpha\beta}(E) = \sum_{J\pi} \sigma_{\alpha}^{CN}(E, J\pi) G_{\beta}^{CN}(E, J\pi)$$

E (CN) must be high ->
decay into continuum of states

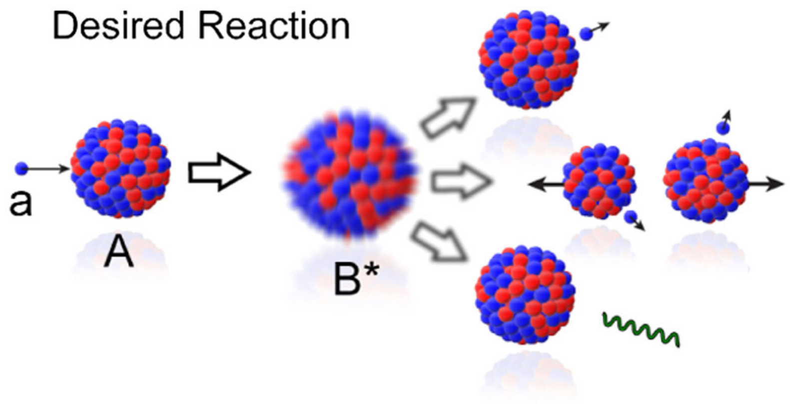
- Works for neutron induced fission (n,f)
- Fails for neutron capture

$$\sigma_{\alpha\beta}^{WE}(E) = \sigma_{\alpha}'^{CN}(E) G_{\beta}^{CN}(E)$$

The Weiskopf-Ewing Limit Greatly Simplifies the Surrogate Method

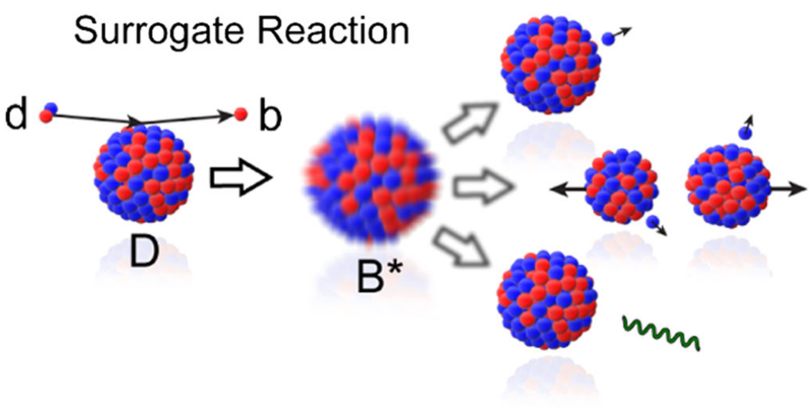
Desired Reaction

$$\sigma_{\alpha\beta}^{WE}(E) = \sigma_{\alpha}'^{CN}(E) G_{\beta}^{CN}(E)$$



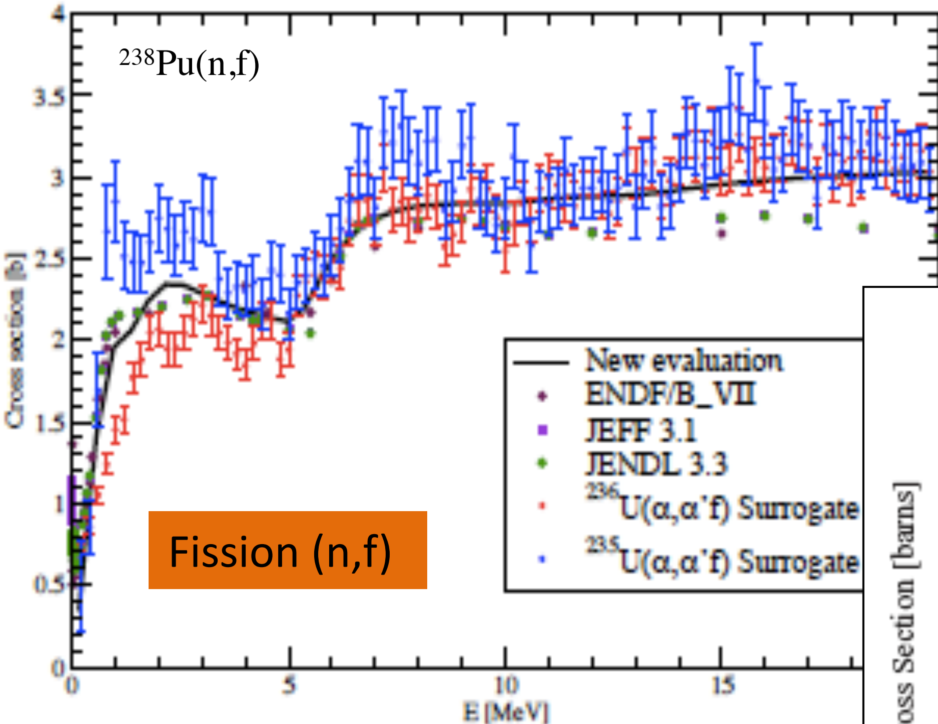
Surrogate Reaction

$$P_{d\beta}^{WE}(E) = G_{\beta}^{CN}(E)$$

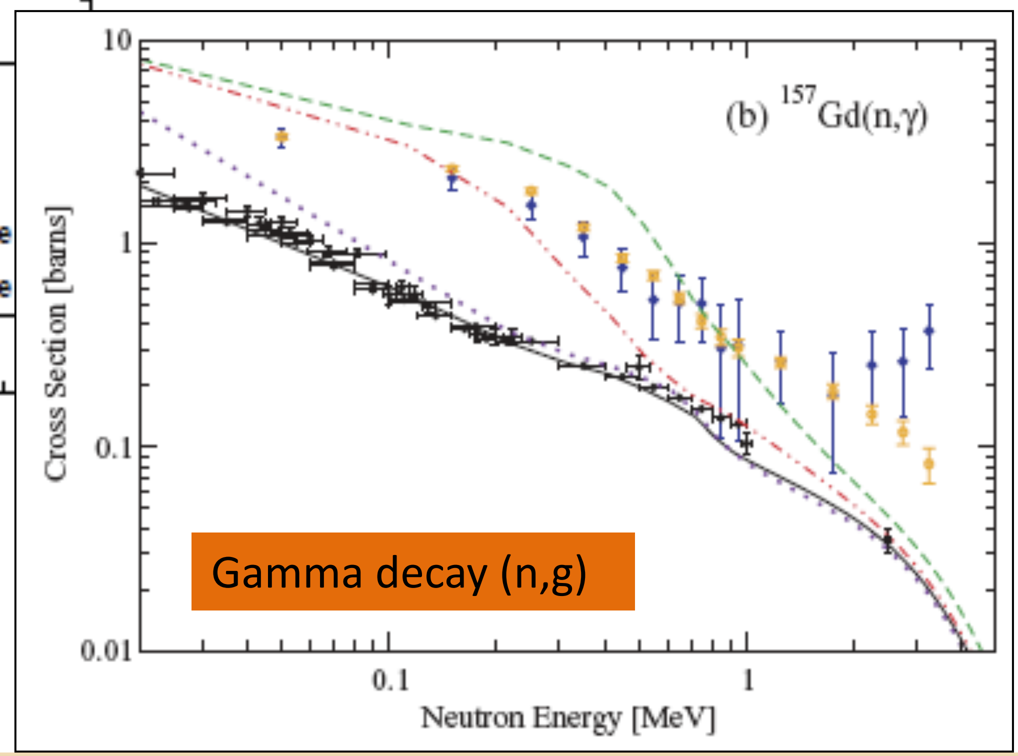


Does the Weisskopf-Ewing approximation work? Yes, for fission (n,f), not for gamma decay (n,g).

J. Escher and F.S. Dietrich, PRC 81 (2010) 024612
N. Scielzo, J. Escher, et al., PRC 81 (2010) 034608

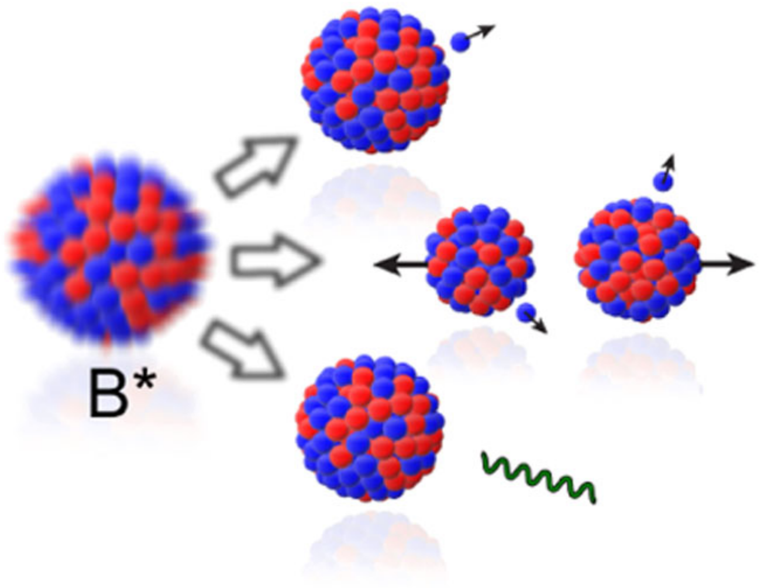


Ressler, Burke, Escher,..
PRC 83 (2011) 054610

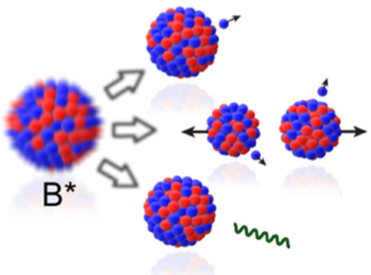


What about other neutron induced reactions?

- Remains untested for (n,n’) and (n,2n) reactions
- We can test the model sensitivity to Spin and Parity

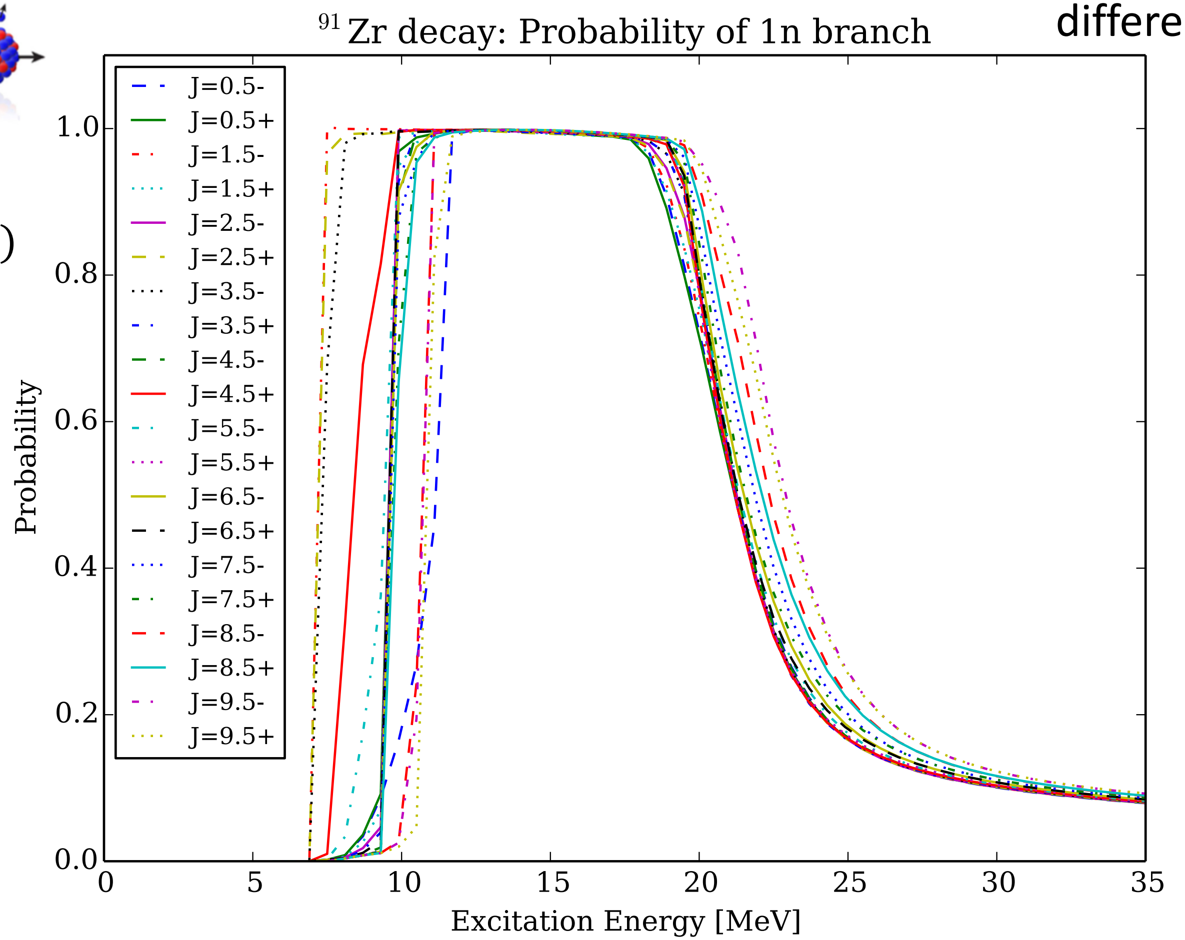


I tested the Weisskopf-Ewing approximation for the (n,n’) reaction on 90Zr targets



$$P_{d\beta}^{WE}(E) = G_{\beta}^{CN}(E)$$

How does a nucleus with different spins decay?



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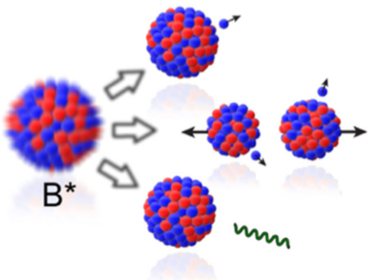
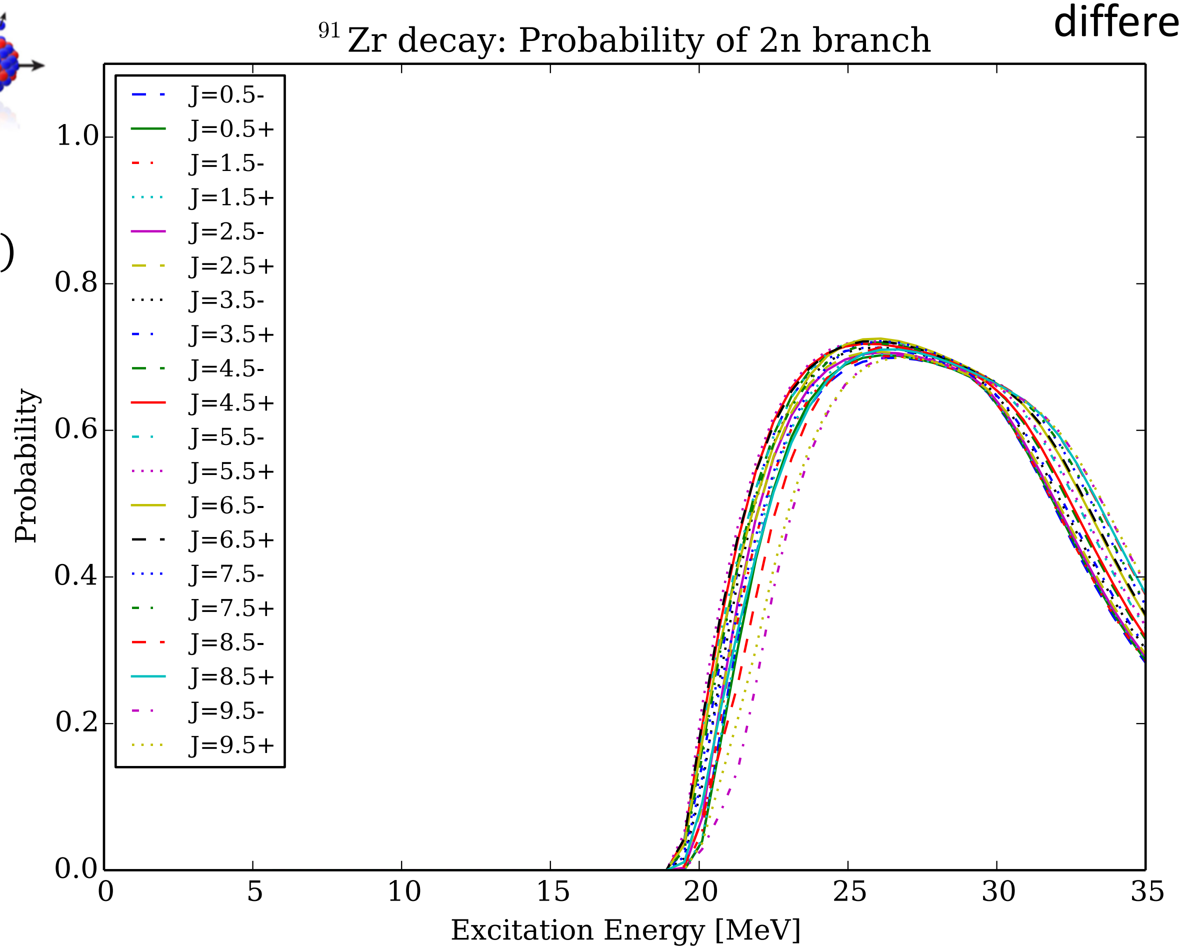
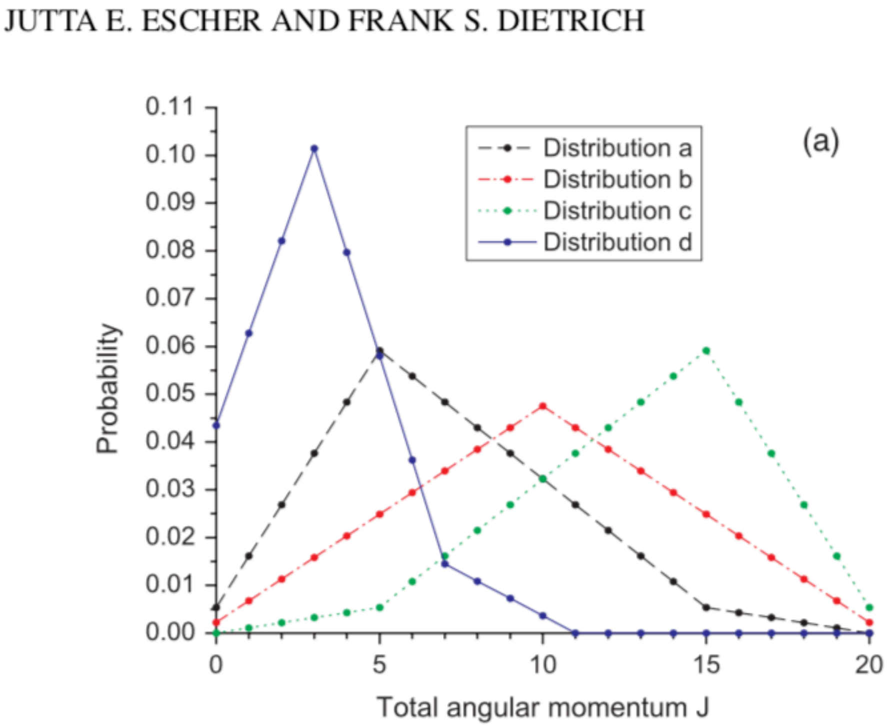


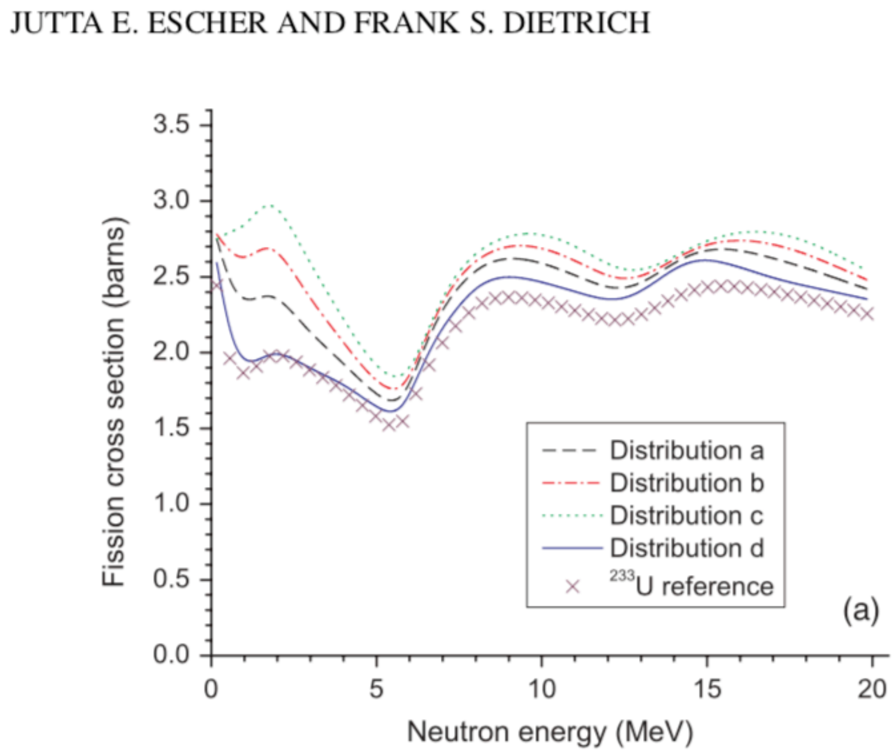
$$P_{d\beta}^{WE}(E) = G_{\beta}^{CN}(E)$$

How does a nucleus with different spins decay?



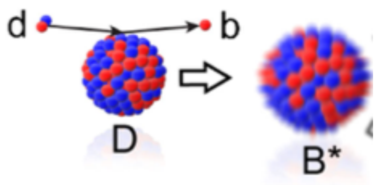
A reaction produces nuclei with a range of spins and parities!



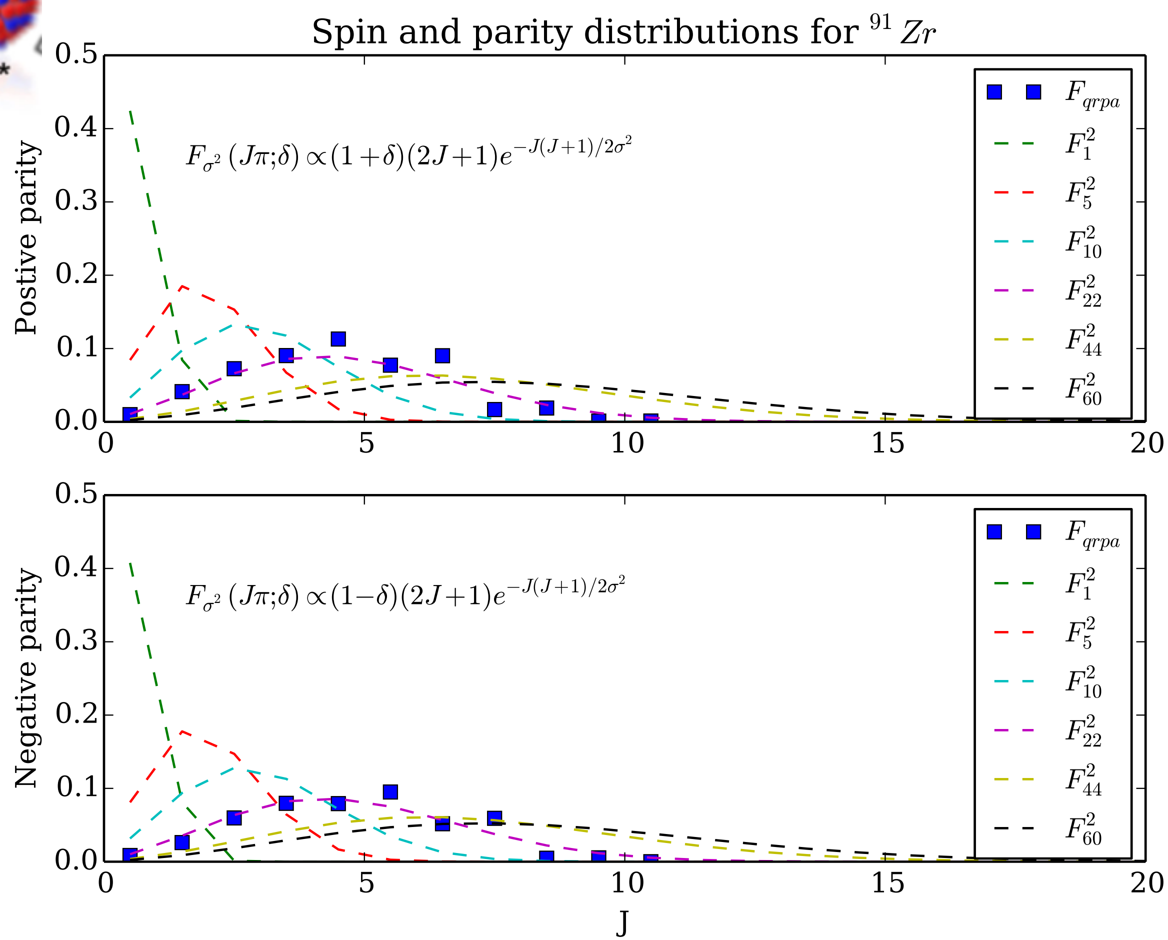


Escher, Dietrich, Phys. Rev. C 74, 054601 (2006)

I chose an analytic distribution with two parameters to occupy the compound nucleus



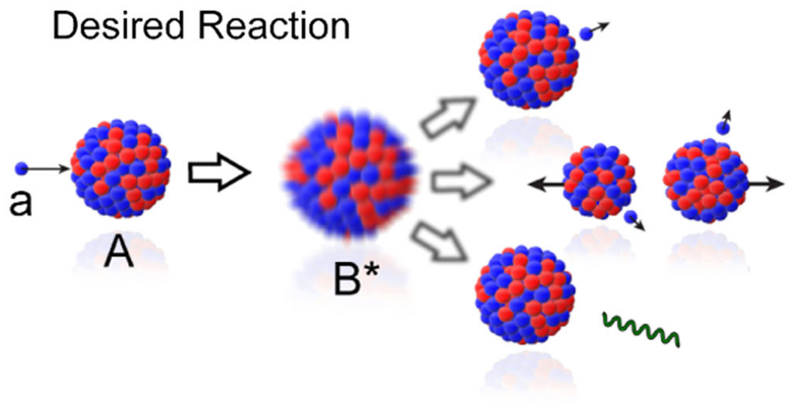
$n + {}^{90}Zr$



Full picture: Weisskopf-Ewing approximation

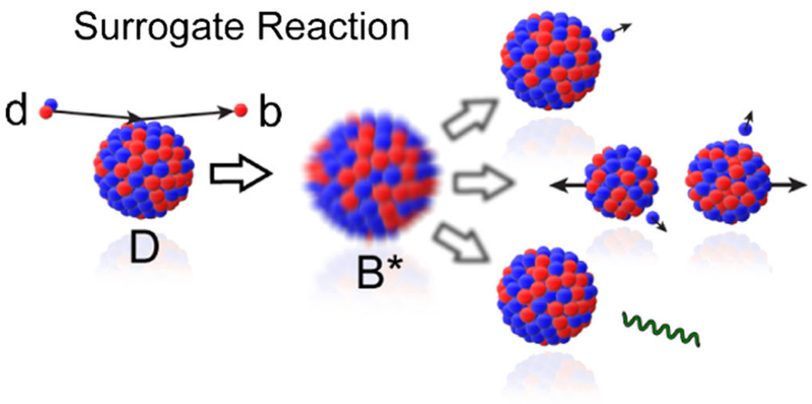
Desired Reaction

$$\sigma_{\alpha\beta}^{WE}(E) = \sigma_{\alpha}'^{CN}(E)G_{\beta}^{CN}(E)$$

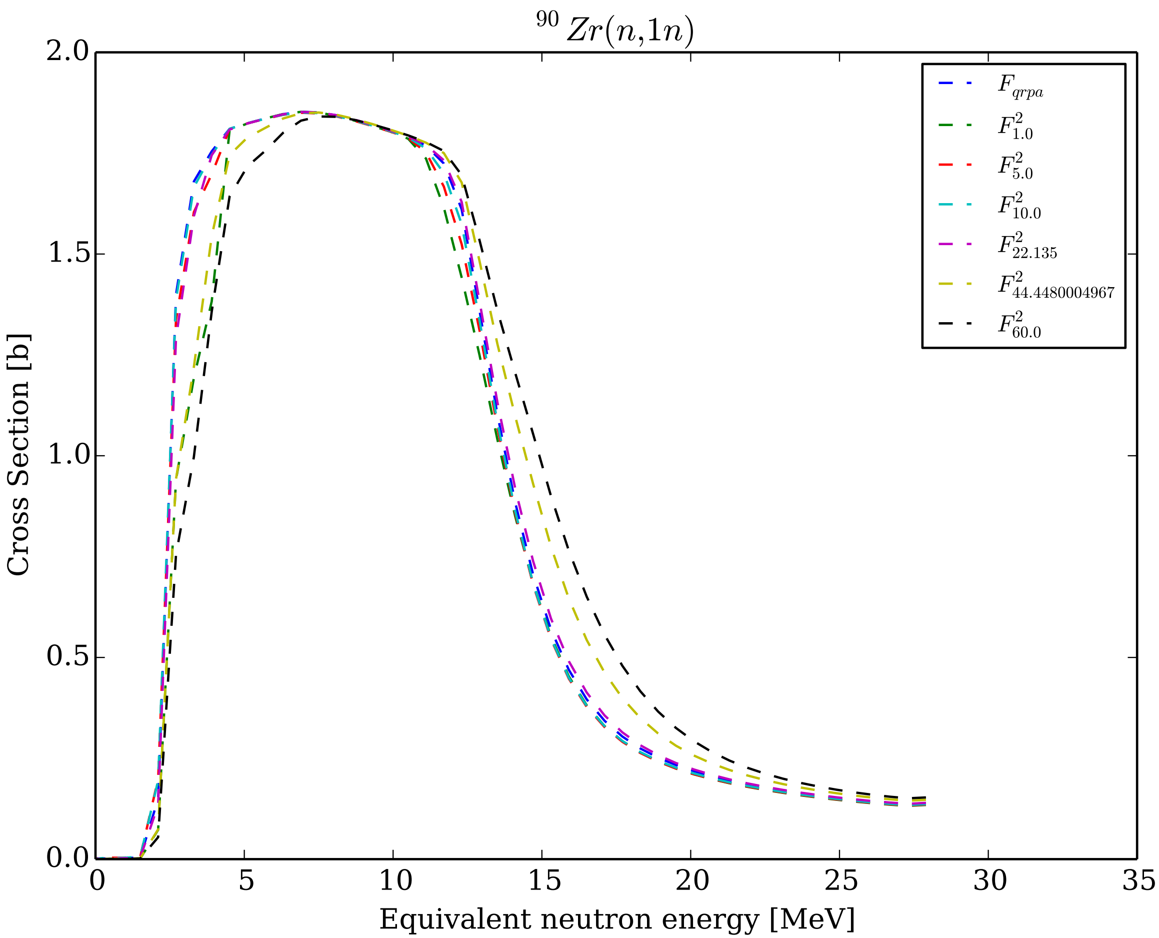


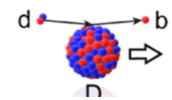
Surrogate Reaction

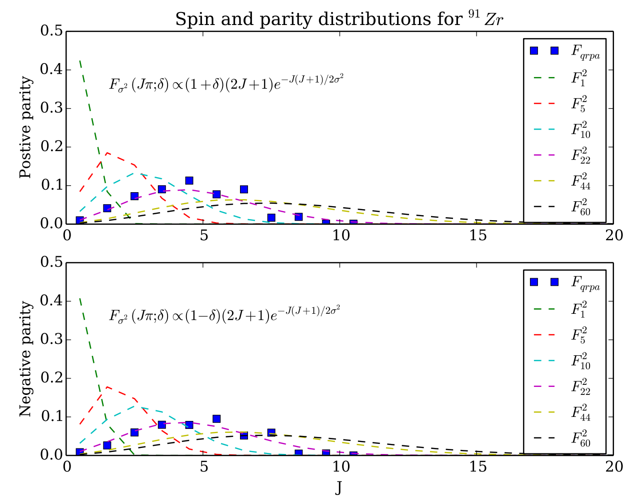
$$P_{d\beta}^{WE}(E) = G_{\beta}^{CN}(E)$$

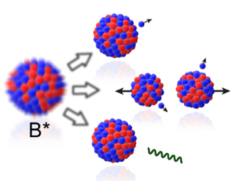


The tests I preformed suggest that the WE approximation is valid for these reactions

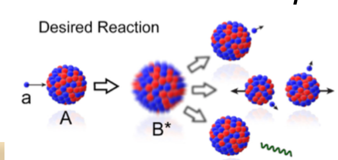




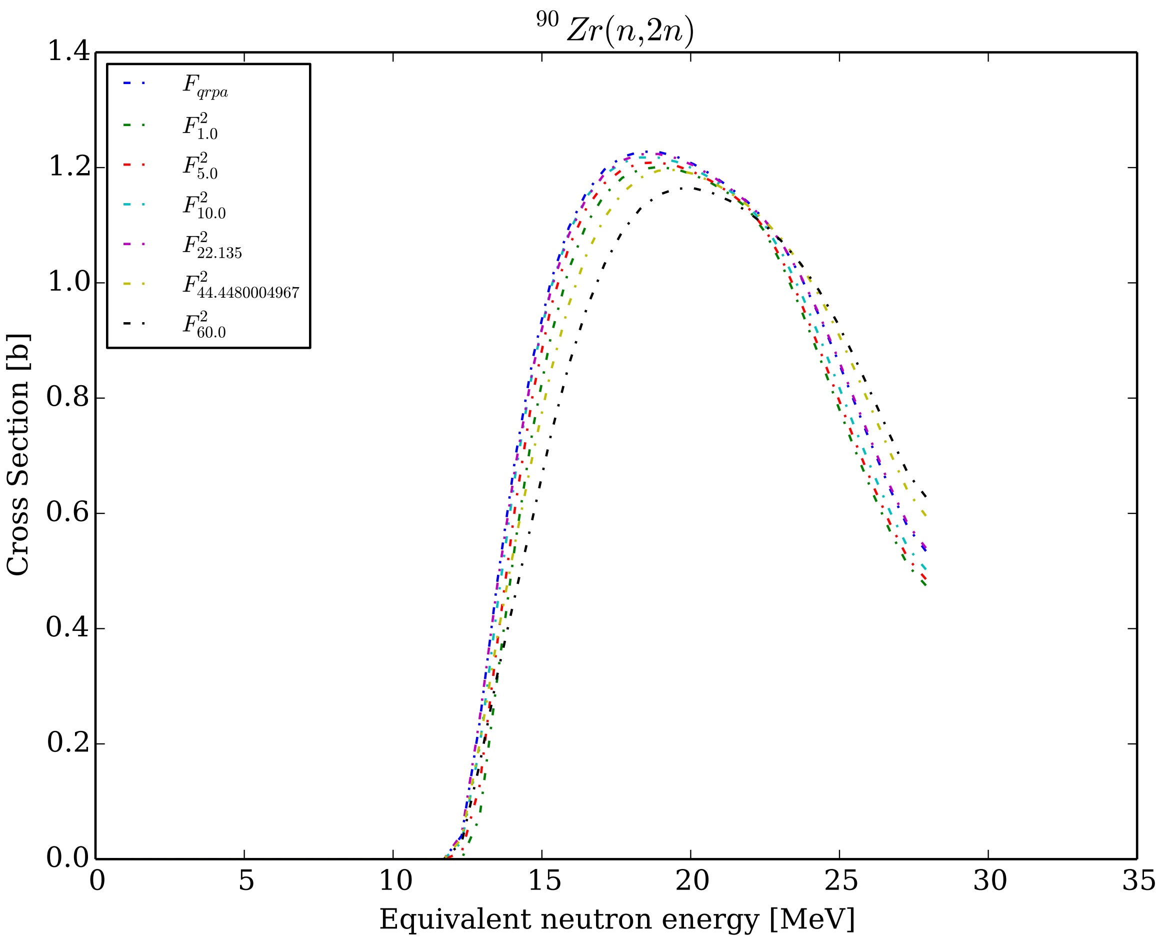


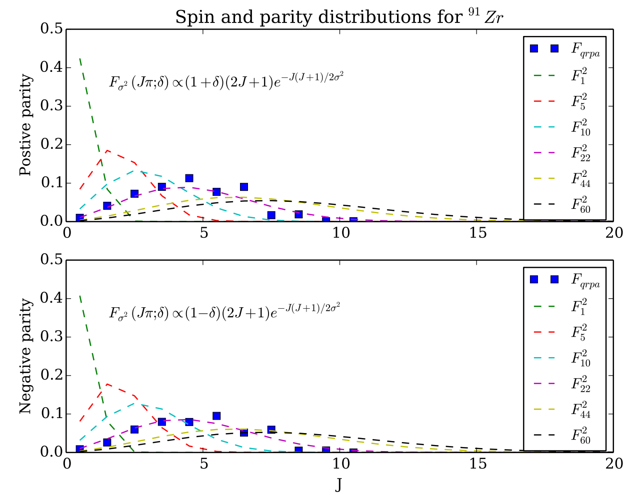


$$\sigma^{WE}_{\alpha\beta}(E) = \sigma'^{CN}_{\alpha}(E) G^{CN}_{\beta}(E)$$



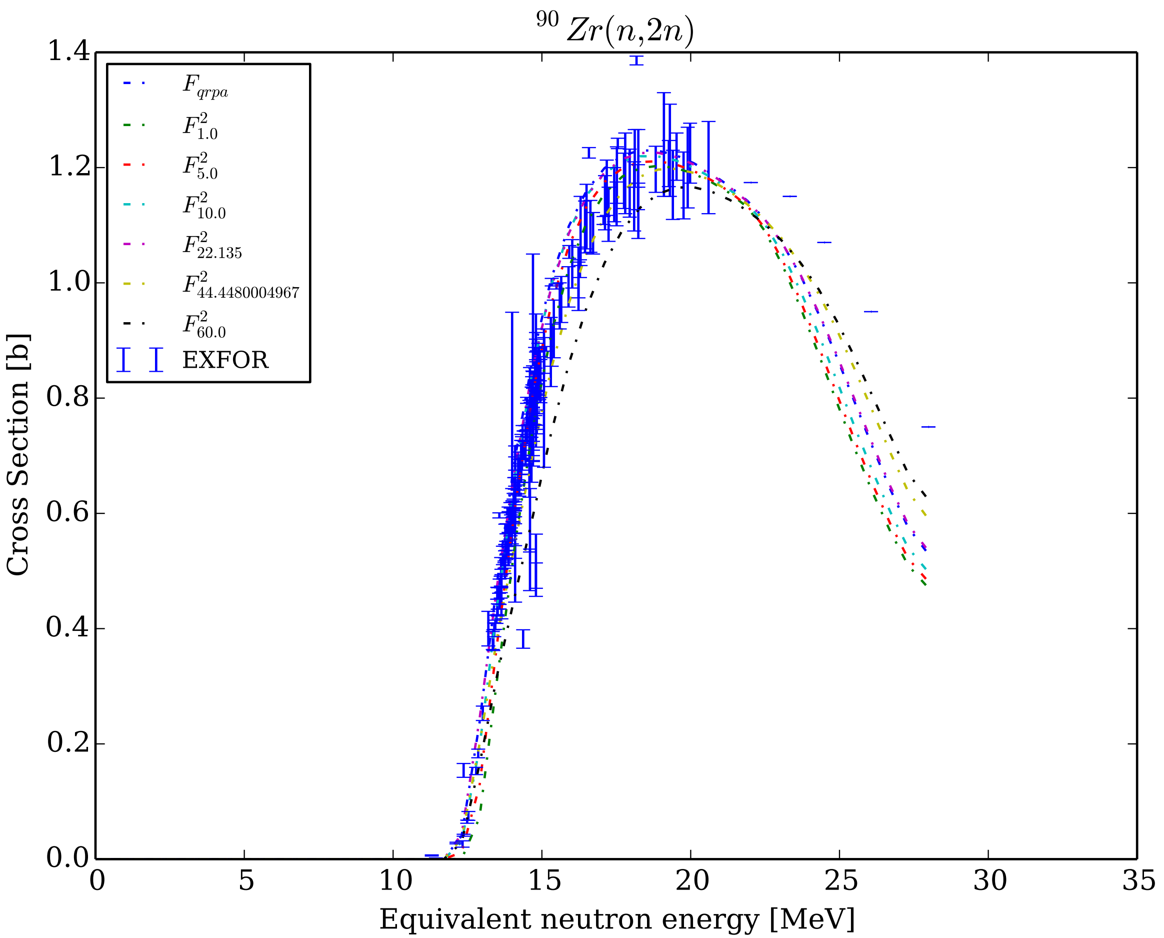
The tests I preformed suggest that the WE approximation is valid for these reactions

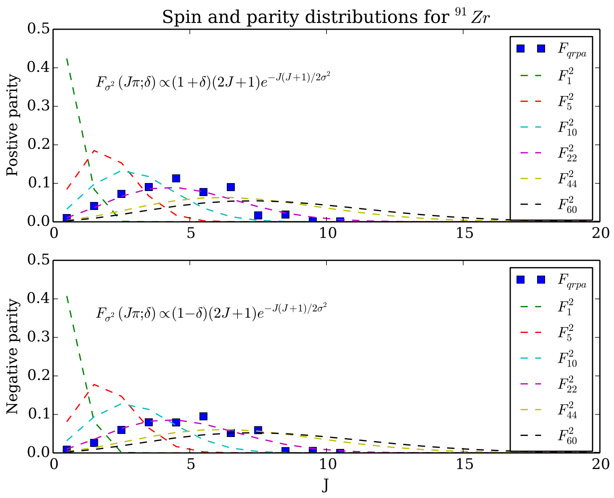




$$\sigma^{WE}_{\alpha\beta}(E) = \sigma'^{CN}_{\alpha}(E) G^{CN}_{\beta}(E)$$

Variance of the predictions fall within current experimental error





$$\sigma_{\alpha\beta}^{WE}(E) = \sigma_{\alpha}^{\prime CN}(E) G_{\beta}^{CN}(E)$$

I tested the WE approximation for (n,n) and (n,2n) on 90Zr

1. Generate artificial spin and parity distributions

   $$F(J,\pi)$$

2. Create simulated surrogate data

   $$P_{d\beta}^{WE}(E) = G_{\beta}^{CN}(E)$$

3. Compute the WE limit using the simulated surrogate data

   $$\sigma_{\alpha\beta}^{WE}(E) = \sigma_{\alpha}'^{CN}(E) P_{d\beta}^{CN}(E)$$

The simulated predictions $\sigma_{\alpha\beta}^{WE}$ are sufficiently similar for reasonable $F(J,\pi)$ → The WE limit is valid

Results are promising

Conclusions:

- (n,n') and (n,2n) reactions on Zr are much less sensitive to spin and parity than capture reactions
- Whether this is sufficient depends on the application
- Further investigations are required, but results are promising

Next steps:

- Investigate effects of pre-equilibrium
- Apply this theory to new Gadolinium data

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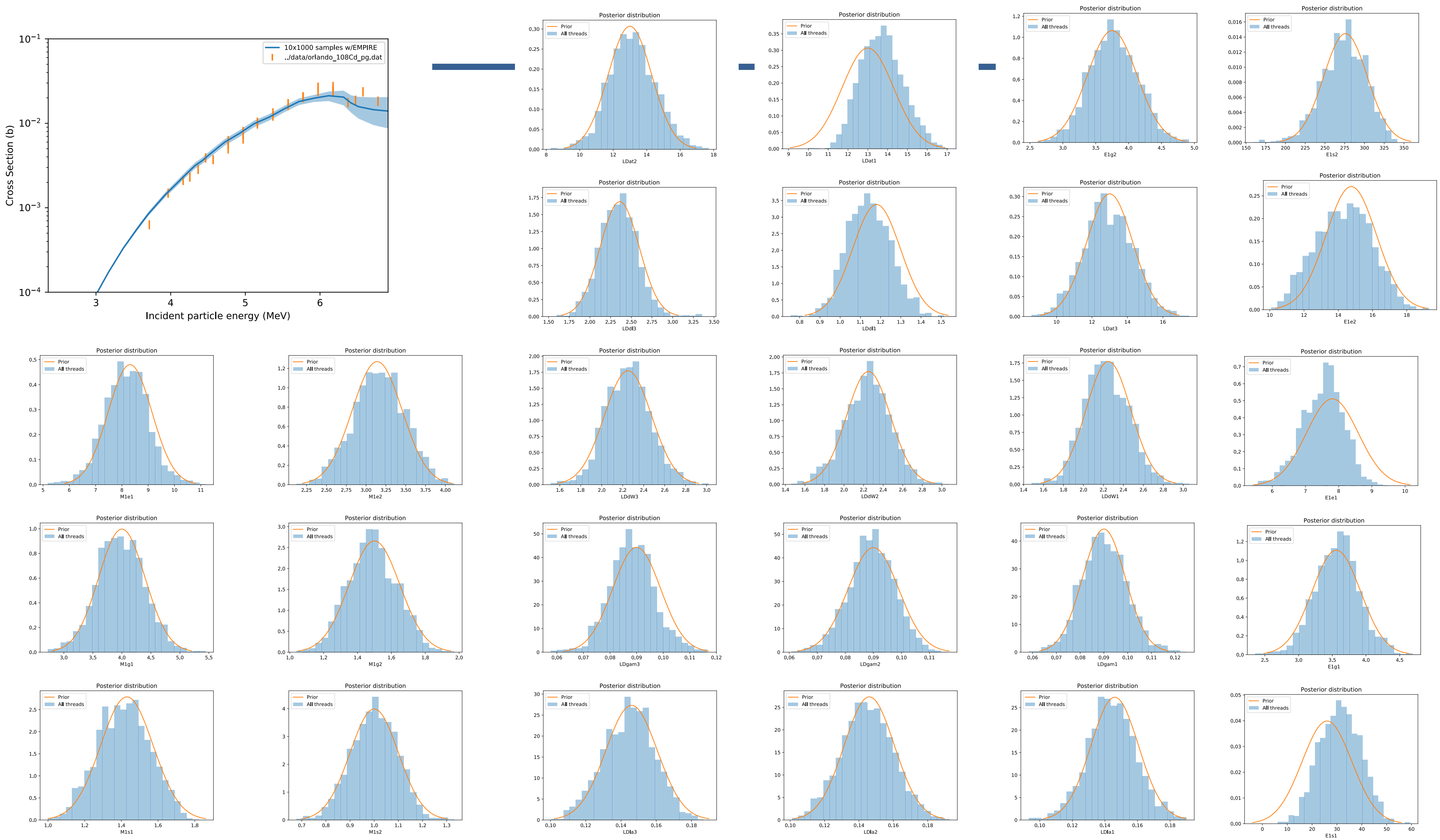

10x1000 samples w/EMPIRE
../data/orlando_108Cd_pg.dat
Cross Section (b)
Incident particle energy (MeV)
Posterior distribution
Prior
All threads
LDat2
LDat1
E1g2
E1s2
LDdl3
LDdl1
LDat3
E1e2
M1e1
M1e2
LDdW3
LDdW2
LDdW1
E1e1
M1g1
M1g2
LDgam3
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LDgam1
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M1s1
M1s2
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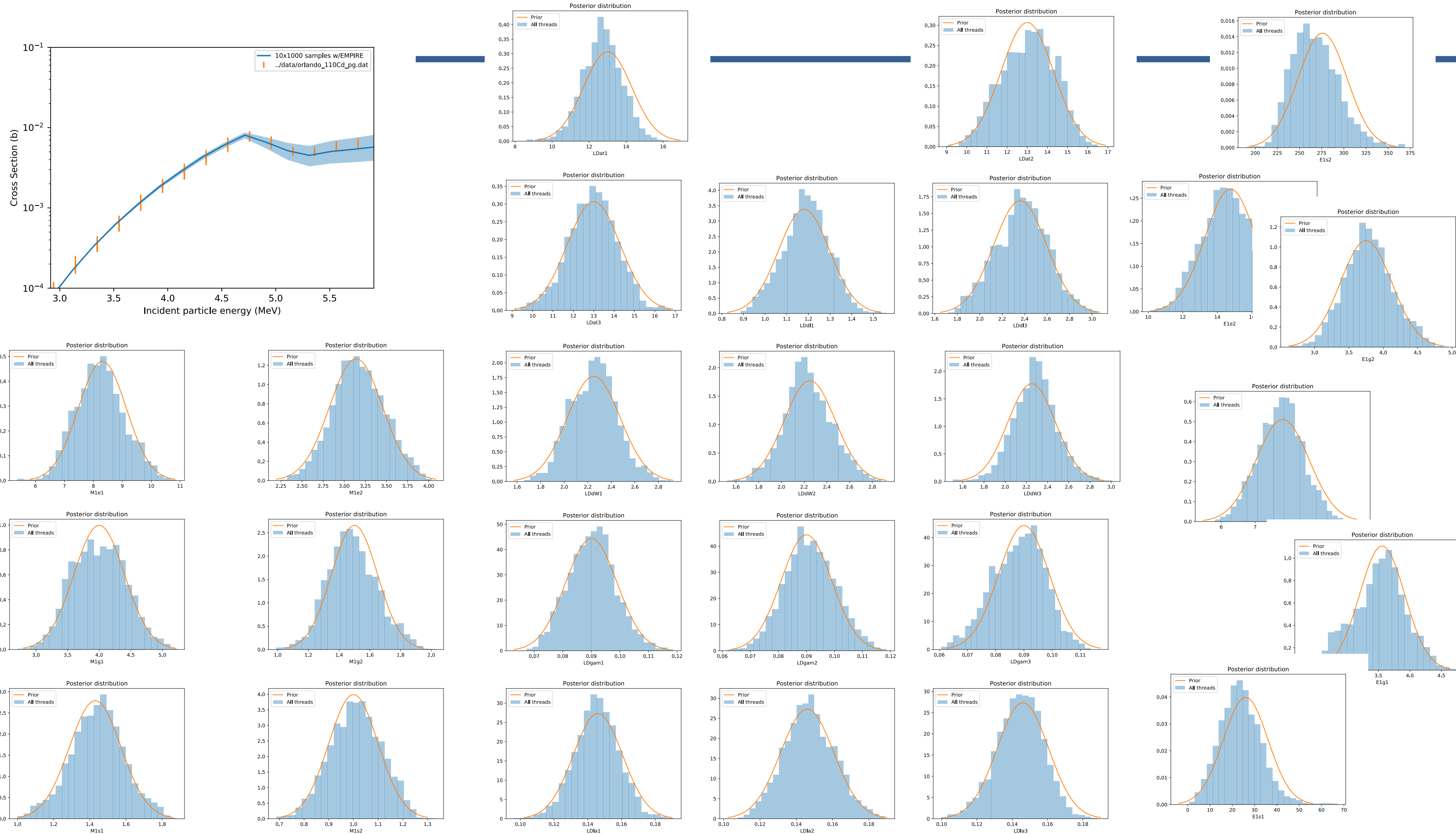
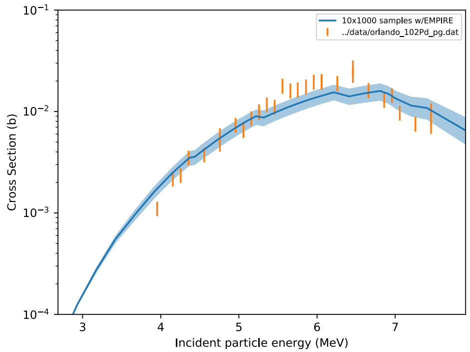
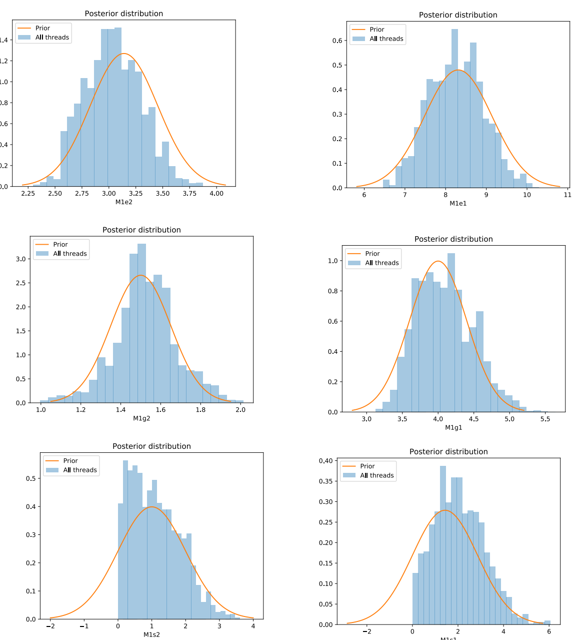
LDla2
LDla1
E1s1



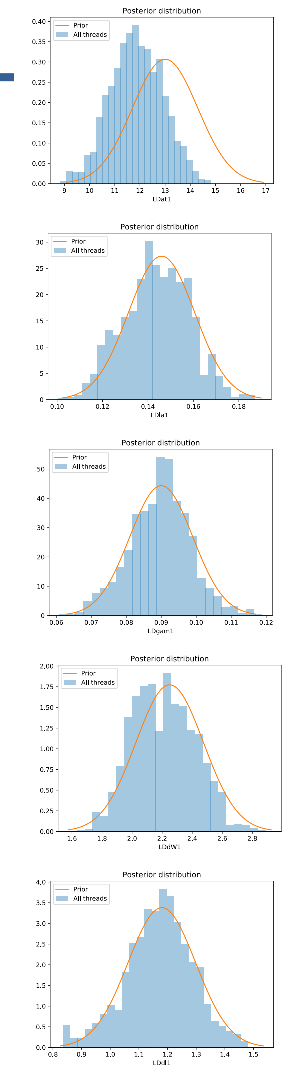
10x1000 samples w/EMPIRE
../data/orlando_110Cd_pg.dat
Cross Section (b)
Incident particle energy (MeV)
Posterior distribution
Prior
All threads
LDat1
LDat2
E1s2
LDat3
LDdl1
LDdl3
E1e2
E1g2
M1e1
M1e2
LDdW1
LDdW2
LDdW3
M1g1
M1g2
LDgam1
LDgam2
LDgam3
E1g1
M1s1
M1s2
LDla1
LDla2
LDla3
E1s1



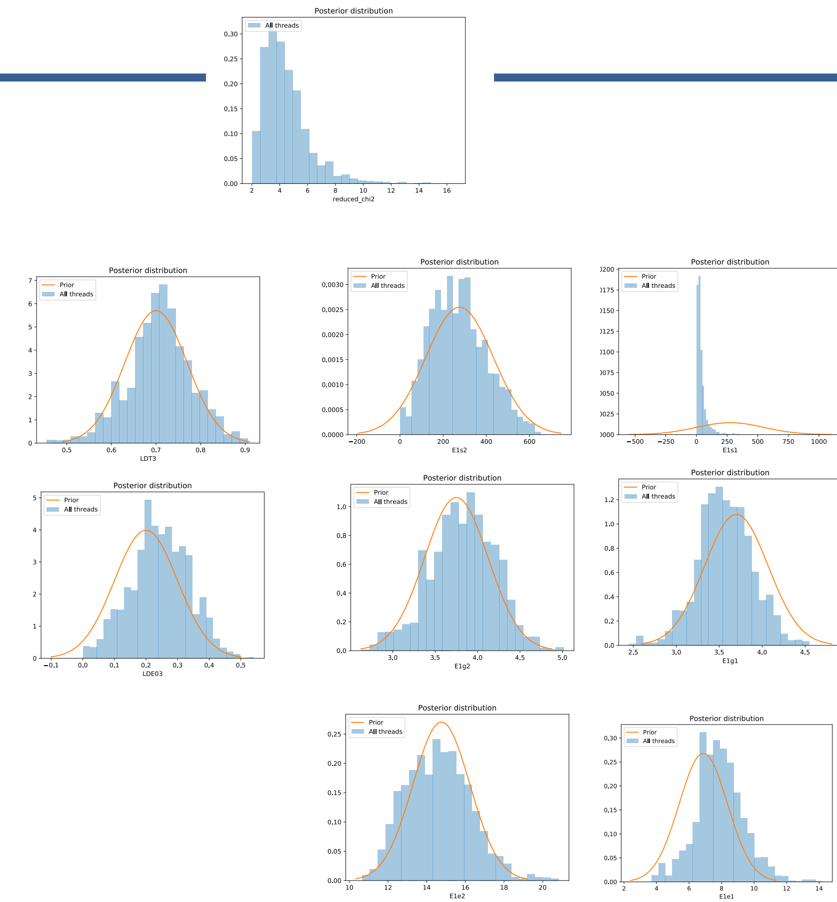
10x1000 samples w/EMPIRE
../data/orlando_102Pd_pg.dat
Cross Section (b)
Incident particle energy (MeV)



Posterior distribution
Prior
All threads
M1e2
M1e1
M1g2
M1g1
M1s2
M1s1



Posterior distribution
Prior
All threads
LDat1
LDla1
LDgam1
LDdW1
LDdl1



Posterior distribution
Prior
All threads
reduced_chi2
LDT3
LDE03
E1s2
E1s1
E1g2
E1g1
E1e2
E1e1

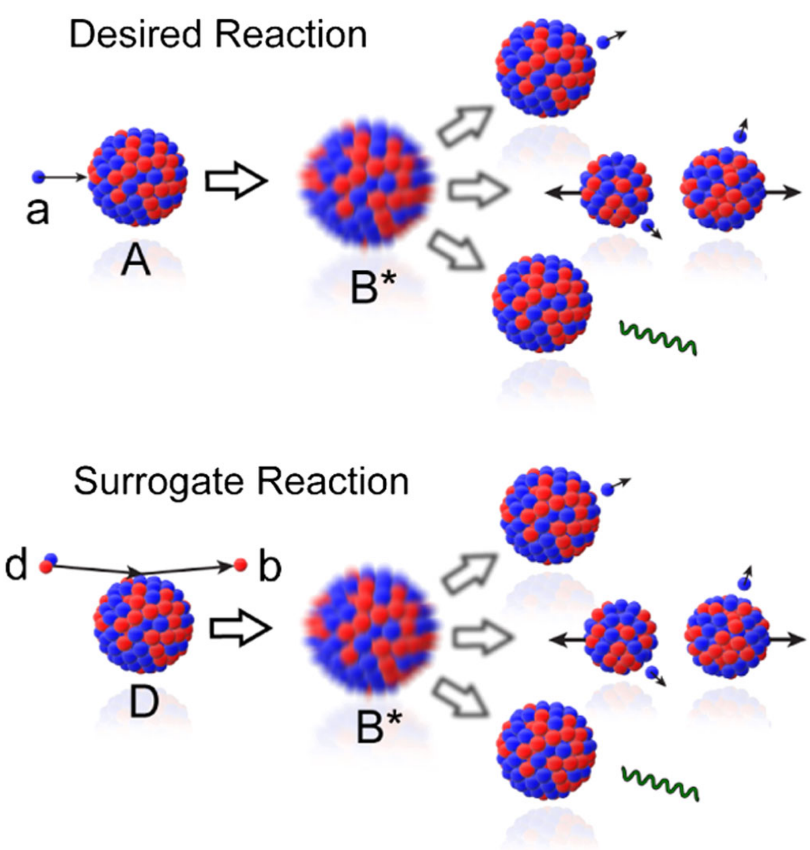
The Surrogate Idea

Desired Reaction

$$\sigma_{\alpha\beta}(E) = \sum_{J\pi} \sigma_{\alpha}^{CN}(E, J\pi) G_{\beta}^{CN}(E, J\pi)$$

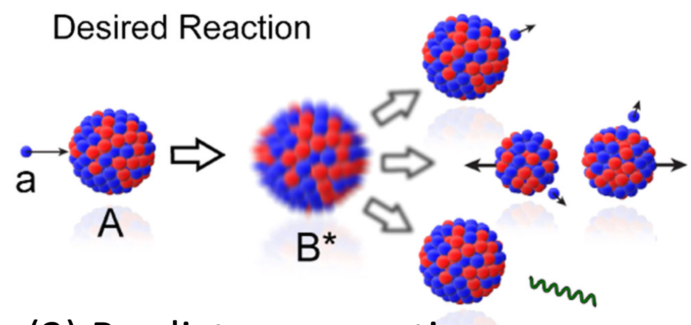
Hard part

Surrogate Reaction



The (General) Surrogate Method Combines Surrogate Data and Theory

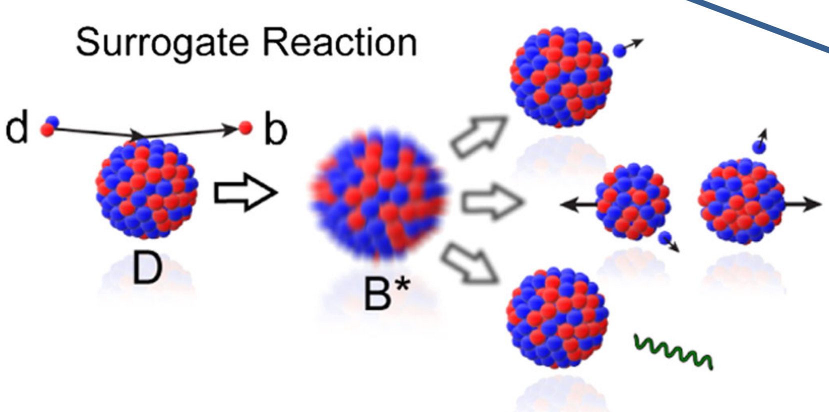
- $\sigma_{\alpha\beta}(E) = \sum_{J\pi} \sigma_{\alpha}^{CN}(E, J\pi) G_{\beta}^{CN}(E, J\pi)$
  - $\sigma_{\alpha}^{CN}(E, J\pi)$: well constrained
  - $G_{\beta}^{CN}(E, J\pi)$: lack of nuclear structure info



(3) Predict cross section

- $P_{\delta\beta}(E) = \sum_{J\pi} F_{\delta}^{CN}(E, J\pi) G_{\beta}^{CN}(E, J\pi)$

(2) Constrain parameters



(1) Measure surrogate