PHY 201 VAM Project

Jacob Harris, Alexis Lohman (ailohman@asu.edu) and Neil Karerakattil

April 24th, 2020

1 Problem 4

$$R_1(\varphi_1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos(\varphi_1) & -sin(\varphi_1) \\ 0 & sin(\varphi_1) & cos(\varphi_1) \end{bmatrix} \tag{1}$$

$$R_2(\varphi_2) = \begin{bmatrix} cos(\varphi_2) & 0 & sin(\varphi_2) \\ 0 & 1 & 0 \\ -sin(\varphi_2) & 0 & cos(\varphi_2) \end{bmatrix} \tag{2}$$

$$R_3(\varphi_3) = \begin{bmatrix} cos(\varphi_3) & -sin(\varphi_2) & 0 \\ sin(\varphi_2) & cos(\varphi_2) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3}$$

$$R_3(\varphi_3)R_2(\varphi_2)R_1(\varphi_1)$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos(\varphi_1) & -sin(\varphi_1) \\ 0 & sin(\varphi_1) & cos(\varphi_1) \end{bmatrix} \times \begin{bmatrix} cos(\varphi_2) & 0 & sin(\varphi_2) \\ 0 & 1 & 0 \\ -sin(\varphi_2) & 0 & cos(\varphi_2) \end{bmatrix} \times$$

$$\begin{bmatrix} cos(\varphi_3) & -sin(\varphi_2) & 0 \\ sin(\varphi_2) & cos(\varphi_2) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

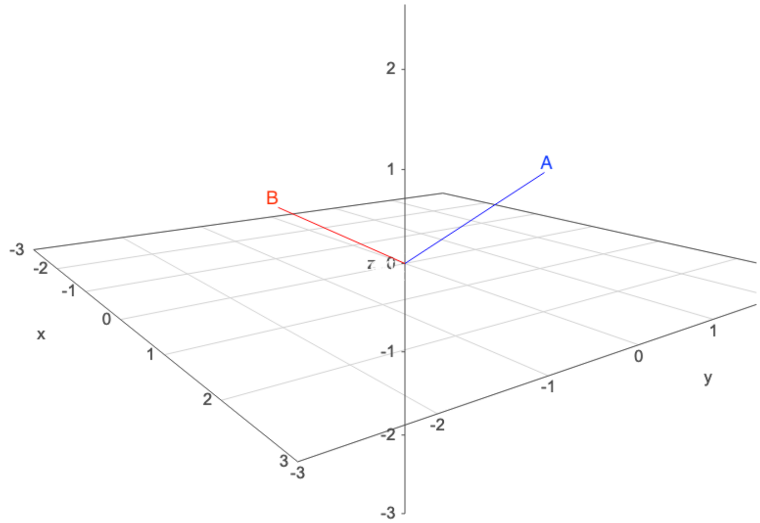
$$= \begin{bmatrix} cos(\varphi_2)cos(\varphi_3) & -sin(\varphi_3) & sin(\varphi_2)cos(\varphi_3) \\ cos(\varphi_2)sin(\varphi_3) & cos(\varphi_3) & sin(\varphi_2)sin(\varphi_3) \\ -sin(\varphi_2) & 0 & cos(\varphi_2) \end{bmatrix} \times \begin{bmatrix} cos(\varphi_3) & -sin(\varphi_2) & 0 \\ sin(\varphi_2) & cos(\varphi_2) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} cos(\varphi_2)cos(\varphi_3) & cos(\varphi_3)sin(\varphi_1)sin(\varphi_2) - cos(\varphi_1)sin(\varphi_3) & sin(\varphi_1)sin(\varphi_3) + cos(\varphi_1)cos(\varphi_3)sin(\varphi_2) \\ cos(\varphi_2)sin(\varphi_3) & cos(\varphi_1)cos(\varphi_3) + sin(\varphi_1)sin(\varphi_2)sin(\varphi_3) & cos(\varphi_1)sin(\varphi_2)sin(\varphi_3) - sin(\varphi_1)cos(\varphi_3) \\ -sin(\varphi_2) & sin(\varphi_1)cos(\varphi_2) & cos(\varphi_1)cos(\varphi_2) \end{bmatrix}$$

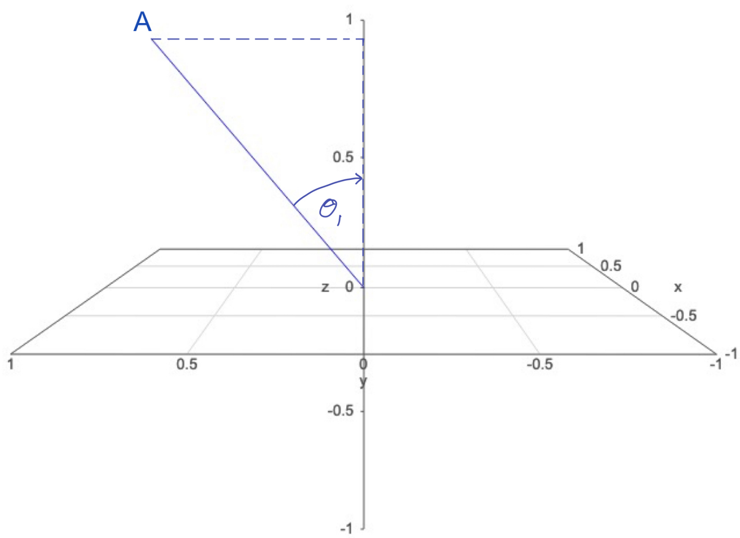
2 Problem 5

$$A = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} B = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \tag{4}$$

Looking to align $\vec{A}$ along the y-axis and $\vec{B}$ along the z-axis.



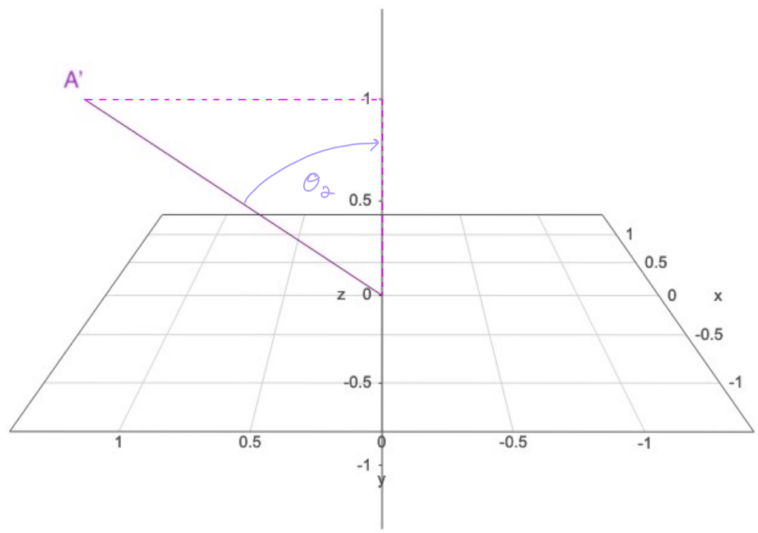
Where the magnitudes of $\vec{A} = \sqrt{3}$ and $\vec{B} = \sqrt{6}$. These values can be used to check the magnitudes throughout the rotations.



$$cos(\theta_1) = \{\frac{1}{\vec{A}}\} = \frac{1}{\sqrt{2}} = sin(\theta_1)$$

It is needed to rotate $\vec{A}$ about the z-axis CCW in order to eventually line up with the y-axis.

$$R_3(\theta_1) = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \vec{A'} {=} R_3(\theta_1) \times \vec{A} = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ \sqrt{2} \\ 1 \end{bmatrix}$$



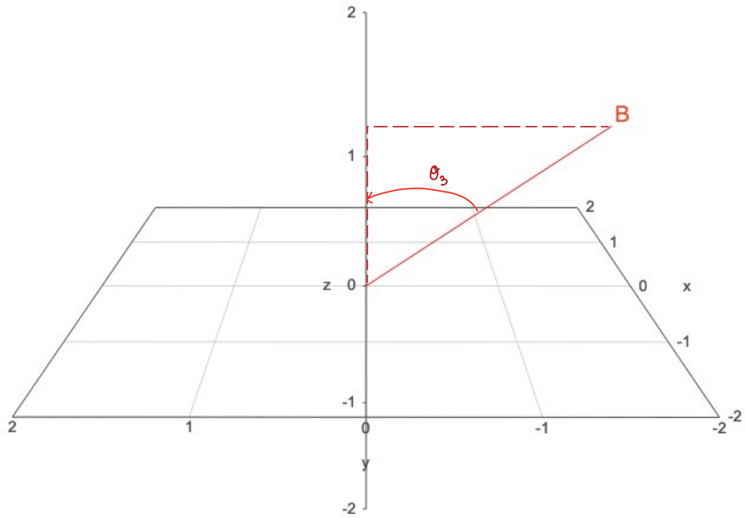
With $\vec{A'}$, $\cos(\theta_2) = \sqrt{\frac{2}{3}}$ and $\sin(\theta_2) = \frac{1}{\sqrt{3}}$. $\vec{A'}$ will now be rotated CW about the x-axis.

$$\vec{A''} {=} R_3(\theta_2) \times \vec{A'} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} \\ 0 & -\frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}} \end{bmatrix} \times \begin{bmatrix} 0 \\ \sqrt{2} \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ \sqrt{(3)} \\ 0 \end{bmatrix}$$

Therefore, $\vec{A''} = R_3(\theta_1) \times R_3(\theta_2) \times \vec{A'}$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} \\ 0 & -\frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}} \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ \sqrt{(3)} \\ 0 \end{bmatrix}$$

Using MATLAB to check the findings, the results are the same.



Following the process used to align $\vec{A}$, $\vec{B}$ will need to be rotated CCW about the z-axis, CW about the x-axis and finally CW about the y-axis.

$\vec{B''} = R_3(\theta_3) \times R_3(\theta_3) \times \vec{B'}$ where, $\vec{B'} = R_3(\theta_3) \times R_3(\theta_3) \times \vec{B}$.

Solving for $\vec{B'}$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} \\ 0 & -\frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}} \end{bmatrix} \times \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{3}{\sqrt{2}} \\ 0 \\ \frac{3}{\sqrt{6}} \end{bmatrix}$$

Where $\cos(\theta_3) = \sqrt{1}2$ and $\sin(\theta_3) = \frac{\sqrt{3}}{2}$. $\vec{B'}$ will now be rotated CW about the y-axis.

$$R_3(\theta_3) = \begin{bmatrix} \frac{1}{2} & 0 & -\frac{\sqrt{3}}{2} \\ 0 & 1 & 0 \\ \frac{\sqrt{3}}{2} & 0 & \frac{1}{2} \end{bmatrix}$$

$$\vec{B''} = R_3(\theta_3) \times \vec{B'} = \begin{bmatrix} \frac{1}{2} & 0 & -\frac{\sqrt{3}}{2} \\ 0 & 1 & 0 \\ \frac{\sqrt{3}}{2} & 0 & \frac{1}{2} \end{bmatrix} \times \begin{bmatrix} \frac{3}{\sqrt{2}} \\ 0 \\ \frac{3}{\sqrt{6}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \sqrt{6} \end{bmatrix}$$

In order to find the rotation matrix, C, that will align $\vec{A}$ and $\vec{B}$ along with the y-axis and z-axis, respectively, it's necessary to multiply $R_3(\theta_1) \times R_3(\theta_2) \times R_3(\theta_3)$.

$$C = \begin{bmatrix} \frac{1}{2} & 0 & -\frac{\sqrt{3}}{2} \\ 0 & 1 & 0 \\ \frac{\sqrt{3}}{2} & 0 & \frac{1}{2} \end{bmatrix} \times \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} \\ 0 & -\frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}} \end{bmatrix}$$

$$C = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{6}} & -\sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} \end{bmatrix}$$

3 Problem 6

Find a new basis in which $\vec{A}$ and $\vec{B}$ from Problem 5 are aligned with $\hat{x}_2$ and $\hat{x}_3$.

The rotation matrix found in problem 5 is the same rotation matrix needed to transform the basis vectors onto the vectors A and B. If the matrix R rotates the vectors A and B onto the y and z axis respectively, then the same rotation will transform the y and z axis onto vectors A and B respectively. This is because a rotation of the vectors A and B is identical to doing a passive rotation of the basis vectors.

$$\begin{bmatrix} \hat{x}_1' \\ \hat{x}_2' \\ \hat{x}_1' \end{bmatrix} = R \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{x}_1 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{6}} & -\sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} \end{bmatrix} \times \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{x}_1 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}}\hat{x}_1 - \frac{1}{\sqrt{2}}\hat{x}_3 \\ \frac{1}{\sqrt{3}}\hat{x}_1 + \frac{1}{\sqrt{3}}\hat{x}_2 + \frac{1}{\sqrt{3}}\hat{x}_3 \\ \frac{1}{\sqrt{6}}\hat{x}_1 - \sqrt{\frac{2}{3}}\hat{x}_2 + \frac{1}{\sqrt{3}}\hat{x}_3 \end{bmatrix}$$

4 Problem 7