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AST 422

HW #4

9/23/20

① The Photoelectric effect can be an important heating mechanism for the grains of dust found in interstellar clouds (Section 12.1). The ejection of an electron leaves the grain with a (+) charge, which affects the rates at which other electrons and ions collide with and stick to the grain to produce the heating. This process is particularly effective for UV photons ($\lambda \approx 100$nm) striking the smaller dust grains. If the average E of the ejected electron is about 5 eV, estimate the work function of a typical dust grain.

$E_{avg} = 5\,eV \qquad \lambda \approx 100\,nm$

$K_{max} = \frac{hc}{\lambda} - \phi$

$\phi = \frac{hc}{\lambda} - K_{max}$

$hc = (3\times10^{8}\,m/s)(6.626\times10^{-34}\,Js)$

$= (1.9878\times10^{-25}\,\cancel{J}m)\left(\frac{6.242\times10^{18}\,eV}{\cancel{J}}\right)$

$= 1.241\times10^{-6}\,eVm$

$\phi \approx \frac{hc}{100nm} - 5eV$

$\phi \approx \frac{1.241\times10^{-6}\,eV\cancel{m}}{1\times10^{-7}\,\cancel{m}} - 5eV$

$\phi \approx 12.4eV - 5eV$

$\boxed{\phi \approx 7.41eV}$

② A one-electron atom is an atom with Z protons in the nucleus and with all but one of its electrons lost to ionization.

$\boxed{a}$ Starting with Coulomb's law, determine expressions for the orbital radii and energies for a Bohr model of the one-electron atom with Z protons.

$F = k\frac{q_1 q_2}{r^2} \qquad q_1 = -e \quad q_2 = Ze$

reduced mass of e^-

$F = K\frac{Ze(-e)}{r^2} = -\mu\frac{v^2}{r}$

KE of system $= \frac{1}{2}\mu v^2 = \frac{1}{2}\cancel{r}K\frac{Ze^2}{r^{\cancel{2}}} = \frac{KZe^2}{2r}$

$E_{TOT} = KE + PE$, so solve for Potential Energy

$$PE = \frac{kq_1q_2}{r} = \frac{kze(-e)}{r} = \frac{-kze^2}{r}$$

$$E_{TOT} = KE + PE = \frac{kze^2}{2r} - \frac{kze^2}{r}\cdot\frac{2}{2} = \frac{kze^2}{2r} - \frac{2kze^2}{2r} = -\frac{kze^2}{2r} = E_{TOT} = -KE$$

substitute using $L = n\hbar = mvr$

$$E = -KE = -\frac{1}{2}mv^2 = -\frac{1}{2}\frac{(mvr)^2}{mr^2} = -\frac{1}{2}\frac{(n\hbar)^2}{mr^2}$$

Solving for radii, r

$$\left(-\frac{1}{2}\frac{(n\hbar)^2}{mr^2} = \frac{kze^2}{2r}\right)\frac{2r^2}{kze^2}$$

$$r = \frac{-(n\hbar)^2}{mkze^2}$$

1b Find the radius of the ground-state orbit, the ground state energy, and the ionization energy of a single ionized helium (He II). $e = 2.71828$ $\hbar = 1.055\times10^{-34}\ J\cdot s$

$k = 1.381\times10^{-23}\ J K^{-1}$

$$r = -\frac{\hbar^2}{m\ ke^2}\frac{n^2}{z} = -\frac{(1.055\times10^{-34}\ J\cdot s)^2}{(4\times10^{8})(1.381\times10^{-23}\ JK^{-1})(1.602\times10^{-19})^2}\left(\frac{1}{2}\right) = 1.92\times10^{-35}\ m$$

c Repeat part (b) for doubly ionized helium.

$$r = -\frac{\hbar^2}{m\ ke^2}\frac{n^2}{z} = -\frac{(1.055\times10^{-34}\ J\cdot s)^2}{(9.11\times10^{-31} kg)(1.381\times10^{-23}\ JK^{-1})(1.602\times10^{-19})^2}\left(\frac{1}{3}\right) = 1.28\times10^{-35}\ m$$

③ A white dwarf is a very dense, with its ions and electrons packed extremely close together. Each electron may be considered to be located within a region of size $\Delta x = 1.5\times10^{-12}$ m. Use Heseinberg's U.P. to est. S_{min} of the electron. Do you think that the effects of relativity will be important for these stars?

$m_e = 9.109\times10^{-31}$ kg

$\Delta x = 1.5\times10^{-12}$ m $\quad V_{min} = ?$ $\quad \hbar = 1.055\times10^{-34}\ J\cdot s$

$$\Delta x \approx \frac{\hbar}{\Delta p} \Rightarrow \Delta p \approx \frac{\hbar}{\Delta x}$$

$$\Delta p \approx \frac{1.055\times10^{-34}\,J.s}{1.5\times10^{-12}m} \qquad \frac{kg\,m^{\cancel{2}}s^{-1}}{\cancel{m}} = kgms^{-1}$$

$$\Delta p \approx 7.03\times10^{-23}\,kgms^{-1}$$

where $P_{min} \approx \Delta p$ so

$$V_{min} = \frac{P_{min}}{m_e}$$

$$\frac{7.72\times10^{7}}{3\times10^{8}} \approx 26\%$$

$$V_{min} = \frac{7.03\times10^{-23}\,kgms^{-1}}{9.11\times10^{-31}\,kg}$$

$$V_{min} = 7.72\times10^{7}\,m/s$$

The effects of relativity are important as this speed is ~26% as fast as c.

④ An electron spends roughly 10^{-8}s in the first excited state of the H atom before making a spontaneous downward transition to the ground state.

[a] Use Heseinberg's U.P. (Eq. 5.20) to determine the uncertainty ΔE in the energy of the first excited state.

$$\Delta E \Delta t \approx \hbar \qquad \Delta t = 10^{-8}s \qquad \hbar = 1.055\times10^{-34}\,J.s$$

$$\Delta E \approx \frac{\hbar}{\Delta t}$$

$$\Delta E \approx \frac{1.055\times10^{-34}\,J.\cancel{s}}{10^{-8}\,\cancel{s}}$$

$$\Delta E \approx 1.055\times10^{-26}\,J$$

[b] Calculate uncertainty $\Delta\lambda$ in the wavelength of the photon involved in a transition between the ground & first excited of the hydrogen atom. Why can you assume that $\Delta E = 0$ for the ground state?

$$\Delta E = \frac{hc}{\lambda} \longrightarrow \frac{dE}{d\lambda} = \frac{-hc}{\lambda^2} \xrightarrow[\text{to solve for } d\lambda]{\text{Rearrange}} \frac{dE\,\lambda^2}{-hc} = d\lambda$$

where $\Delta E = dE$ & $\Delta\lambda = d\lambda$ $\longrightarrow$ $\Delta E \dfrac{-\lambda^2}{hc} = \Delta\lambda$

when at the ground state, $\Delta E = 0$

$\Delta E = 6.58 \times 10^{-8}$ J
$\lambda = 1.216 \times 10^{-7}$

$$\Delta \lambda_1 = \frac{\lambda^2}{hc}(0) = \boxed{0}$$

$$\Delta \lambda_2 = \frac{\lambda^2}{hc}\Delta E = \frac{(1.216 \times 10^{-7}\,m)^2}{(6.626 \times 10^{-34}\,J \cdot s)(3 \times 10^8\,m/s)}(6.58 \times 10^{-8}\,J)$$

$$\Delta \lambda_2 = \boxed{4.894 \times 10^{3}\,m}$$

⑤ The members of a class of stars known as Ap stars are distinguished by their strong global magnetic fields (usually a few tenths of one tesla). The star HD215441 has an unusually strong magnetic field of 3.4 T. Find the frequencies and wavelengths of the three components of the H α spectral line produced by the normal Zeeman effects for this magnetic field.

$$V = V_0 = V_0 \pm \frac{eB}{4\pi M} \rightarrow \Delta V = \frac{eB}{4\pi m_e}$$

$$V_0 = \frac{c}{\lambda} = \frac{3 \times 10^8\,m/s}{6.5628 \times 10^{-7}\,m} = 4.571 \times 10^{14}\,Hz$$

$$\Delta V = \frac{(1.602 \times 10^{-19}\,C)(3.4\,T)}{4\pi(9.11 \times 10^{-31}\,kg)} = 4.759 \times 10^{10}\,Hz$$

$B = 3.4T$ $\quad f = ?$ $\quad \lambda = ?$ $\quad m_e = 9.11 \times 10^{-31}$ kg $\quad f = \frac{c}{\lambda}$

$e = 2.71828$

$\lambda = 656.281$ nm $\quad V_0 =$

$$f_1 = \frac{c}{\lambda} = \frac{(3 \times 10^8\,m/s)}{(656.281 \times 10^{-9}\,m)} = \boxed{4.571 \times 10^{14}\,Hz}$$

$$f_2 = V_0 = \boxed{4.571 \times 10^{14}\,Hz}$$

$$f_3 = V_0 - \Delta V = \boxed{4.570 \times 10^{14}\,Hz}$$

$$\lambda_1 = \frac{c}{f_1} = \boxed{656.281\,nm}$$

$$\lambda_2 = \frac{c}{f_2} = \frac{(3\times10^8)}{(4.571\times10^{14})} = \boxed{656.311\,nm}$$

$$\lambda_3 = \frac{c}{f_3} = \frac{(3\times10^8)}{(4.570\times10^{14})} = \boxed{656.445\,nm}$$

$$R = \frac{\lambda}{\Delta\lambda} = \frac{\lambda_1}{\lambda_3 - \lambda_1} = \frac{656.281\,nm}{656.445\,nm - 656.281\,nm} = \boxed{3.768\times10^3}$$

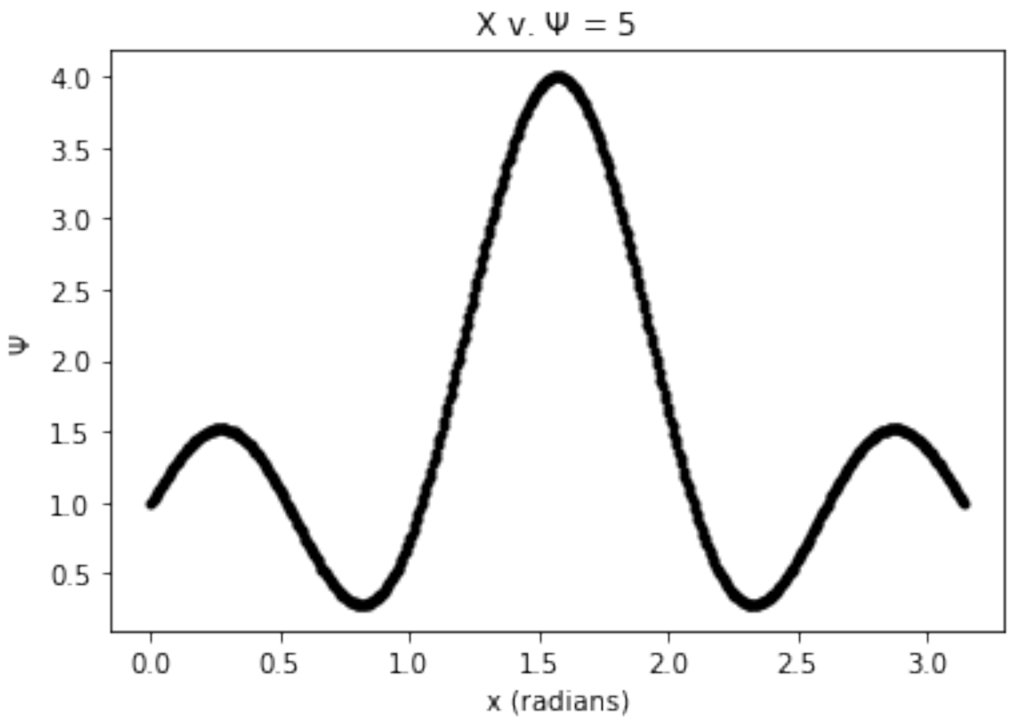
The R-value needed to view all three of the lines is 3.768×10^3.

Problem Set 4 - Problem 5.18 C&O

September 23, 2020

[6]:
```python
import numpy as np
import matplotlib.pyplot as plt

# ----------------------------------------------------------------------------
# Write a function to generate wave function (Psi) given N terms
# ----------------------------------------------------------------------------

def wavefunction(x, N):
  psi = np.zeros(x.shape);
  for n in range(1, N+1, 2):
    D = (-1)**((n-1)/2)*np.sin(n*x);
    psi += 2/(N+1) + D
  return psi;

# ----------------------------------------------------------------------------
# Part a
# ----------------------------------------------------------------------------

x_rad = np.linspace(0, np.pi, num=500);


psi_5 = wavefunction(np.array(x_rad), 5)
plt.figure(1);
plt.clf();
plt.plot(x_rad, psi_5,'k.');
plt.xlabel('x (radians)');
plt.ylabel('$\Psi$');
plt.title('X v. $\Psi$ = 5');
plt.show()
plt.savefig('figure_1.png');


# ----------------------------------------------------------------------------
# Part b
# ----------------------------------------------------------------------------

psi_11 = wavefunction(np.array(x_rad), 11)
```

```
plt.figure(2);
plt.clf();
plt.plot(x_rad, psi_11,'k.');
plt.ylabel('$\Psi$');
plt.xlabel('x (radians)');
plt.title('X v. $\Psi$ = 11');
plt.show()
plt.savefig('figure_2.png');

# -----------------------------------------------------------------------------
# Part c
# -----------------------------------------------------------------------------

psi_21 = wavefunction(np.array(x_rad), 21)
plt.figure(3);
plt.clf();
plt.plot(x_rad, psi_21,'k.');
plt.ylabel('$\Psi$');
plt.xlabel('x (radians)');
plt.title('X v. $\Psi$ = 21');
plt.show()
plt.savefig('figure_3.png');


# -----------------------------------------------------------------------------
# Part d
# -----------------------------------------------------------------------------

psi_41 = wavefunction(np.array(x_rad), 41)
plt.figure(2);
plt.clf();
plt.plot(x_rad, psi_41,'k.');
plt.ylabel('$\Psi$');
plt.xlabel('x (radians)');
plt.title('X v. $\Psi$ = 41');
plt.show()
plt.savefig('figure_4.png');

# -----------------------------------------------------------------------------
# Part e
# -----------------------------------------------------------------------------
# The particular will be known with least uncertainty when N is greatest. As
# more points are added to the data set, the more obvious the position is,␣
↪giving
# less uncertainty. Based upon this reaosning, N=41 would give the positioning
# with the least uncertainty. Conversely, N=41 would also give you momentum with
# the least uncertainty, as well.
```



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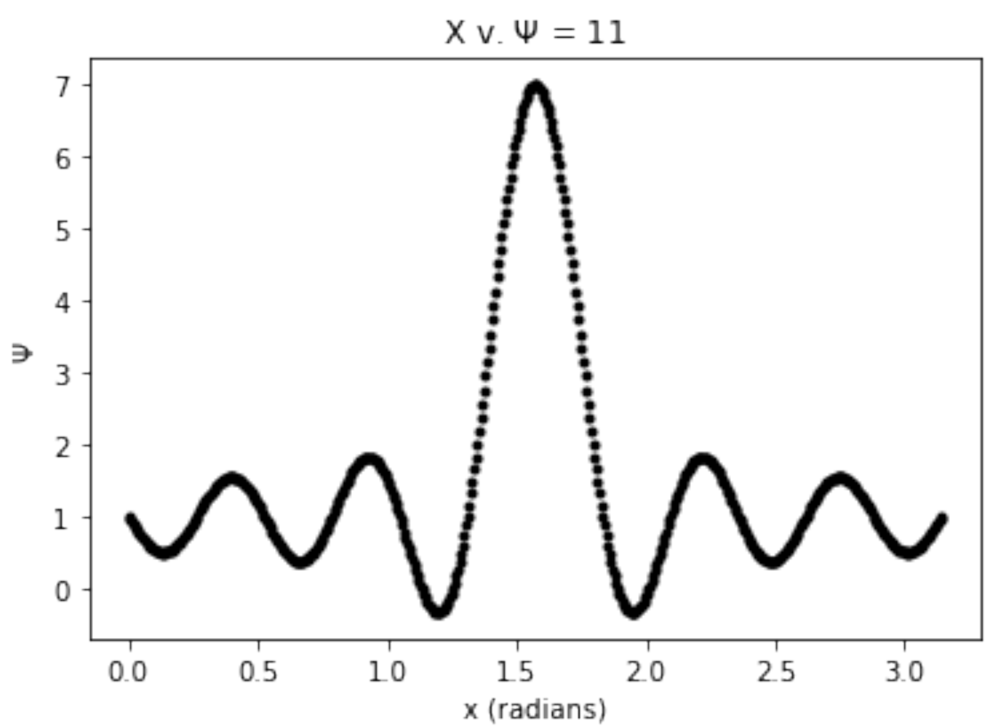

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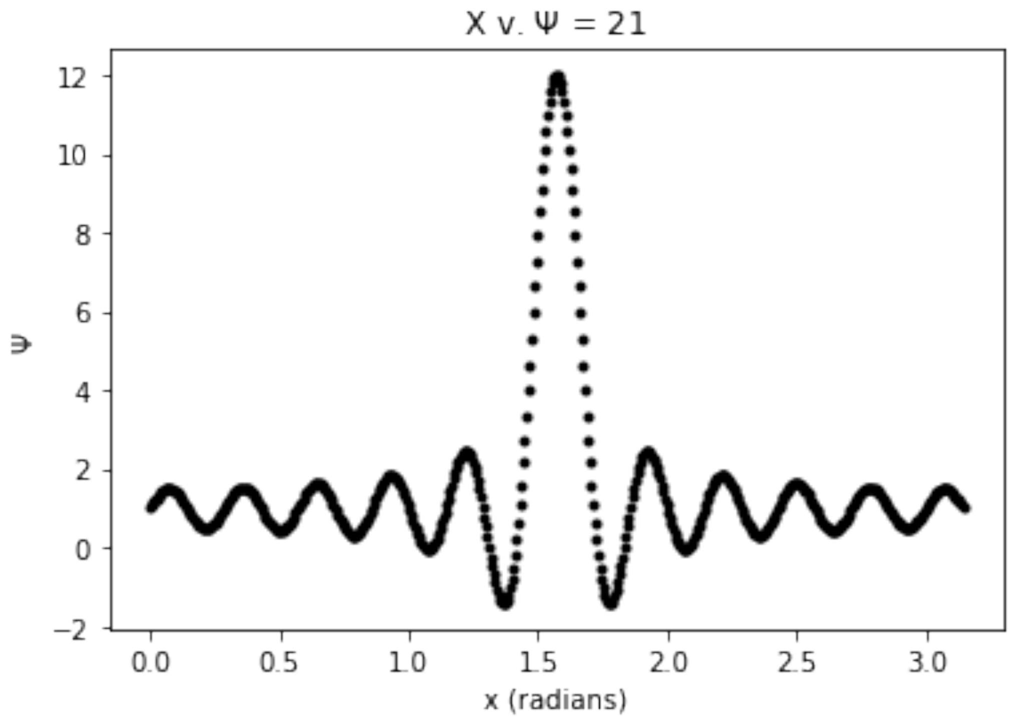

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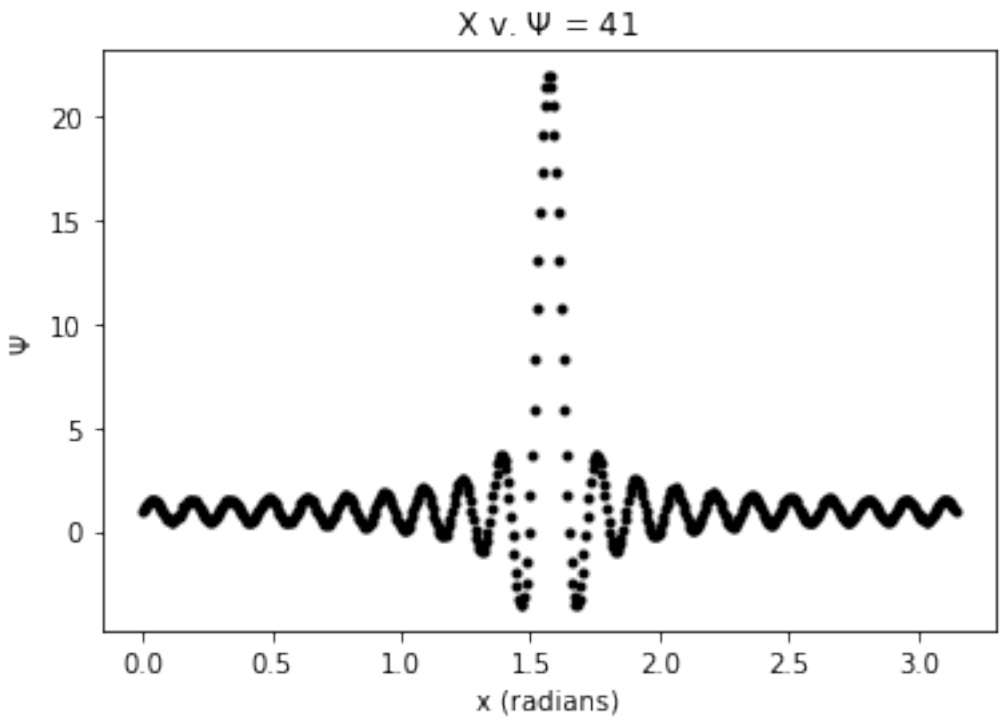


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