problem set 6 part 1

October 6, 2020

[80]:
```python
import numpy as np
import matplotlib.pyplot as plt
import matplotlib.colors as mcolors

datafile = 'stellar.csv'
data = np.loadtxt(datafile, delimiter=',',  usecols=(1,2,3,4,5,6))
print(data.shape)

print('Problem Set 6')
print('Alexis Lohman')
print('Due 10/7 10:45AM')
print('PART 1')

L = data[:,4];
M = data[:,2];
R = data[:,1];
T = data[:,0];

L_log = np.log10(L)
M_log = np.log10(M)
R_log = np.log10(R)
T_log = np.log10(T)

print('#1')
plt.figure(1);
plt.plot(M_log, L_log, '.')
plt.xlabel('$Mass_{log}$ (Solar Masses)')
plt.ylabel('$Luminosity_{log}$ (Solar Luminosity)')
plt.title('Mass v. Luminosity')
plt.show()

print('#2')
plt.figure(2);
plt.plot(M_log, R_log, '.')
plt.xlabel('$Mass_{log}$ (Solar Masses)')
plt.ylabel('$Radius_{log}$ (Solar Radius)')
plt.title('Mass v. Radius')
```

```
plt.show()

plt.figure(3);
plt.plot(M_log, T_log, '.')
plt.xlabel('$Mass_{log}$ (Solar Masses)')
plt.ylabel('$Temperature_{log}$ (K)')
plt.title('Mass v. Temperature')
plt.show()

print('#3')
plt.figure(4);
plt.plot(M_log[3: -4], L_log[3: -4], '.')
plt.xlabel('$Mass_{log}$ (Solar Masses)')
plt.ylabel('$Luminosity_{log}$ (Solar Luminosity)')
fit = np.polyfit(M_log, L_log, 1)
M_log_sub = M_log[3: -4]
linefit = fit[0]*M_log_sub + fit[1]
plt.plot(M_log_sub, linefit)
plt.title('Best Fit: Mass v. Luminosity')
plt.show()
print('Polyfit Results for Mass v. Luminosity:', fit)

print('#4')
plt.figure(5);
plt.plot(M_log[3: -4], R_log[3: -4], '.')
plt.xlabel('$Mass_{log}$ (Solar Masses)')
plt.ylabel('$Radius_{log}$ (Solar Radius)')
fit = np.polyfit(M_log, R_log, 1)
M_log_sub = M_log[3: -4]
linefit = fit[0]*M_log_sub + fit[1]
plt.plot(M_log_sub, linefit)
plt.title('Best Fit: Mass v. Radius')
plt.show()
print('Polyfit Results for Mass v. Radius:', fit)

plt.figure(6);
plt.plot(M_log[3: -4], T_log[3: -4], '.')
plt.xlabel('$Mass_{log}$ (Solar Masses)')
plt.ylabel('$Temperature_{log}$ (K)')
fit = np.polyfit(M_log, T_log, 1)
M_log_sub = M_log[3: -4]
linefit = fit[0]*M_log_sub + fit[1]
plt.plot(M_log_sub, linefit)
plt.title('Best Fit: Mass v. Temperature')
plt.show()
print('Polyfit Results for Mass v. Temperature:', fit)
```

```python
print('#5')
#Using equation L = 4*pi*R**2*sigma*T**4, R = np.sqrt(L/(4*pi*sigma*t**4))

def Rad(Lum, Temp):
    Radius = (Lum/(4*np.pi*(5.67e-8)*(Temp*Temp*Temp*Temp)))**0.5
    return Radius

L_array = np.array(L)
T_array = np.array(T)
R_ft = Rad(L_array, T_array)

print('Calculated radius in solar R are:', R_ft,'.', 'Experimental radius␣
↪values are:', R, '.')
print('The calculated values are much smaller than the experimental raidus␣
↪values by a magnitude of 4.')
```

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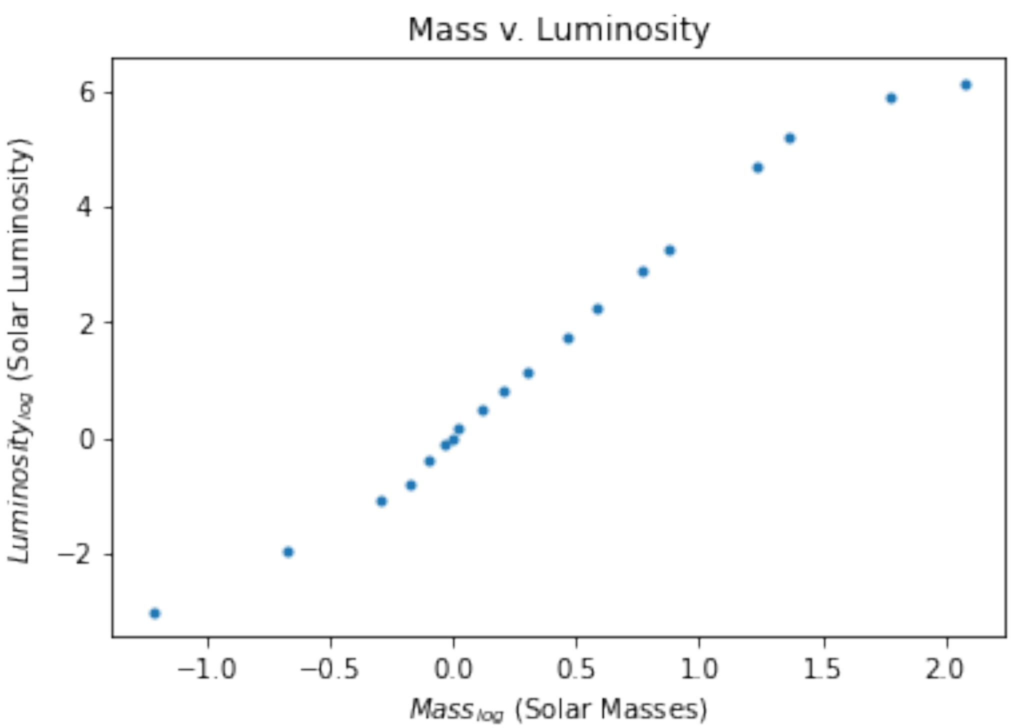
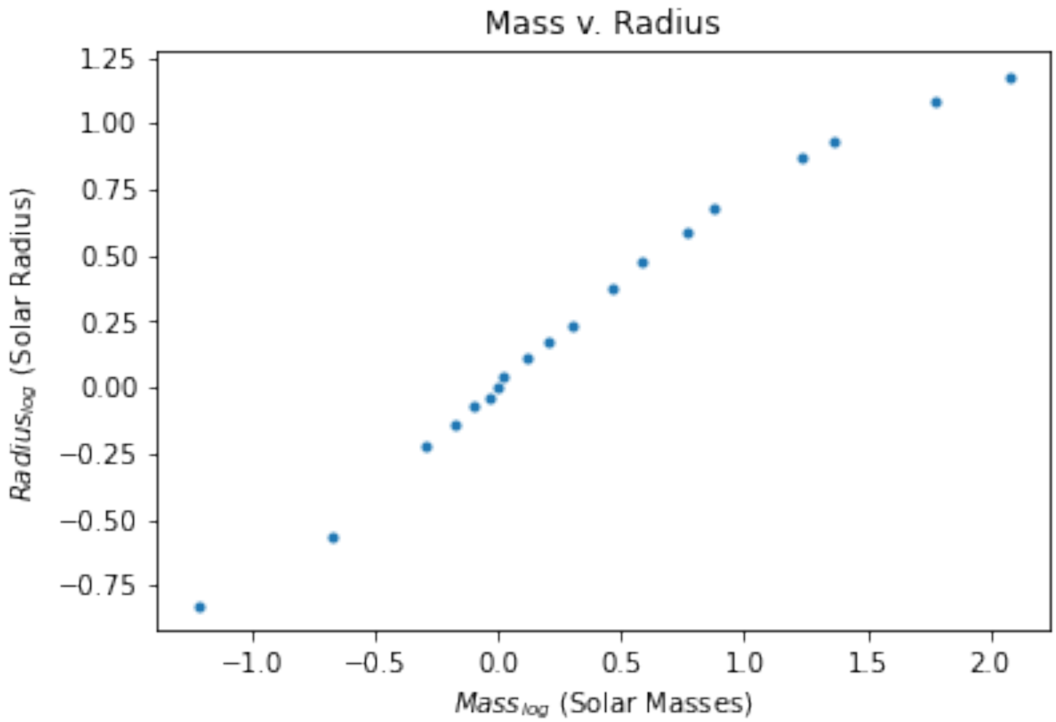
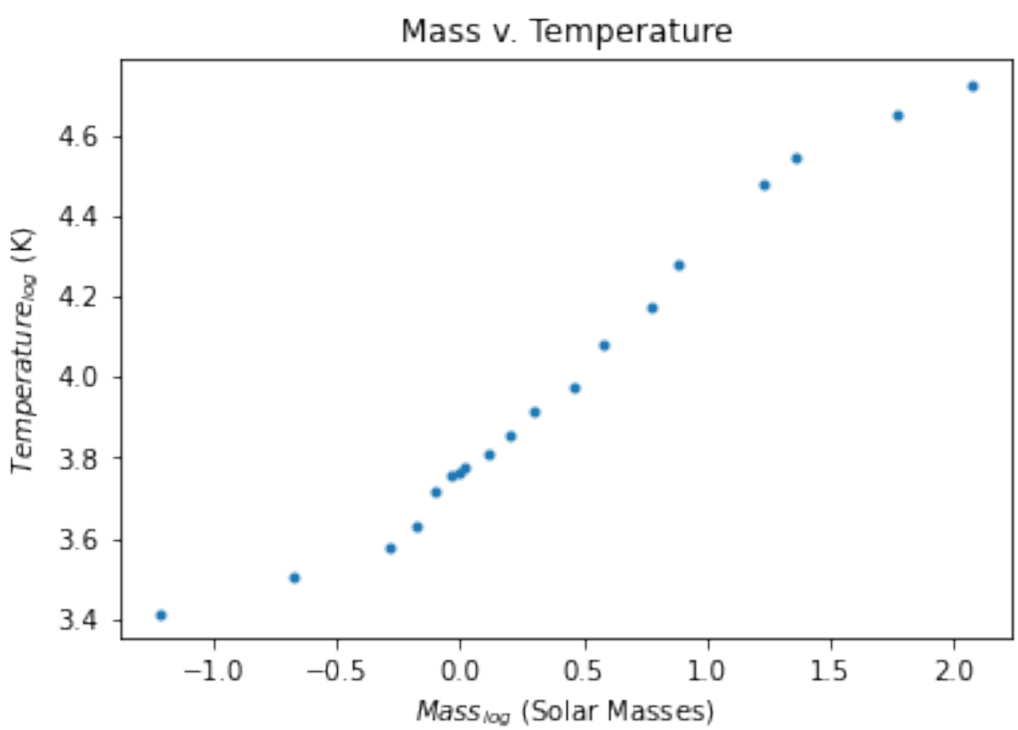
(19, 6)
Problem Set 6
Alexis Lohman
Due 10/7 10:45AM
PART 1
#1
```

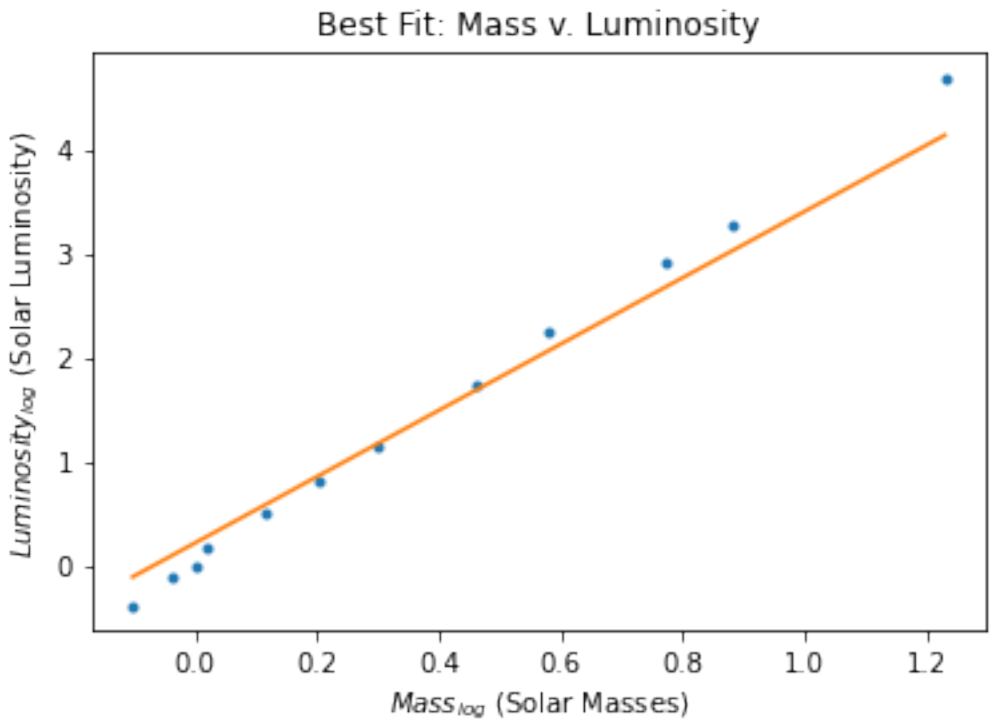


#2


Mass v. Radius
1.25
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
$Radius_{log}$ (Solar Radius)
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
$Mass_{log}$ (Solar Masses)


Mass v. Temperature
4.6
4.4
4.2
4.0
3.8
3.6
3.4
$Temperature_{log}$ (K)
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
$Mass_{log}$ (Solar Masses)

#3



Polyfit Results for Mass v. Luminosity: [3.19211598 0.22130372]
#4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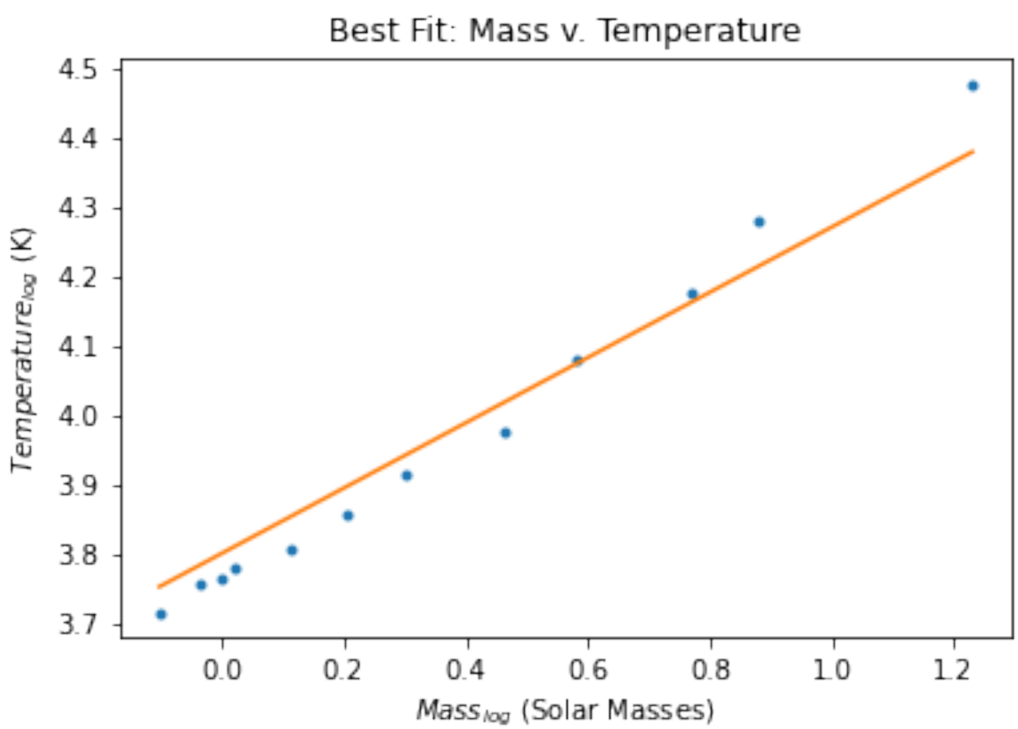
Polyfit Results for Mass v. Radius: [0.65084596 0.00758054]
```



```
Polyfit Results for Mass v. Temperature: [0.47052618 3.80050493]
#5
Calculated radius in solar R are: [4.99017547e-04 5.19986486e-04 3.98741934e-04
2.94337794e-04
 1.43045150e-04 1.51691088e-04 1.10376673e-04 9.64613216e-05
 6.59234385e-05 5.82633194e-05 5.17390654e-05 4.03038624e-05
 3.52166103e-05 3.24091300e-05 2.83936678e-05 2.48148844e-05
 2.32049875e-05 1.21338864e-05 5.54187649e-06] . Experimental radius values are:
[15.   12.    8.5   7.4   4.8   3.9   3.    2.4   1.7   1.5   1.3   1.1
  1.    0.92  0.85  0.72  0.6   0.27  0.15] .
The calculated values are much smaller than the experimental raidus values by a
magnitude of 4.
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[ ]:
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Part II

(6) $L \propto M^a$

Eq. 10.6 $\rightarrow \frac{dP}{dr} = -\frac{GM_r\rho}{r^2} = -\rho g \Rightarrow \frac{dP}{dr} \approx -\frac{P}{R}$

Eq. $\rightarrow \frac{dT}{dV} = \frac{\ell(r)}{4\pi r^2}\frac{3}{16}\frac{K\rho}{\sigma T^3} \Rightarrow \frac{dT}{dV} \approx -\frac{T}{R}$ $\qquad \rho \propto \frac{M}{R^3}$

$-\frac{P}{R} = \frac{GM\rho}{R^3} = -\frac{GM}{R^2}\frac{M}{R^3} = -\frac{GM^2}{R^5}$ $\qquad \frac{P}{R} \propto \frac{M^2}{R^5}$

$P = \frac{\rho K T}{M} \rightarrow T \propto \frac{P}{\rho} \propto \frac{M^2}{R^4}\frac{R^3}{M} \propto \frac{M}{R}$

$L \propto -R^2 T^3 \frac{dT}{dV}\frac{1}{K\rho} \propto R^2 \frac{M^3}{R^3}\frac{M}{R^2}\frac{R^3}{M} \propto M^3 \Rightarrow \boxed{L \propto M^3}$

(7) This compares very nicely. In question 3 I got 3.192 and in question 6 I got 3. these values are very close.

Part III

central density

$\rho(r) = \rho_c\left(1-\frac{r}{R}\right) = \frac{3M}{\pi R^3}\left(1-\frac{r}{R}\right)$

(8) ρ_c in terms of M & R

$dM = 4\pi\rho(r)r^2 dr$

$dM = 4\pi\rho_c\left(1-\frac{r}{R}\right)r^2 dr$

$\int dM = \int 4\pi\left(\rho_c - \frac{\rho_c r}{R}\right)r^2 dr$

$\int_{r=0}^{r=R} dM = 4\pi\int_{r=0}^{r=R}\left(\rho_c r^2 - \frac{\rho_c r^3}{R}\right)dr$

$M = 4\pi\left(\frac{\rho_c r^3}{3} - \frac{\rho_c r^4}{4R}\right)\Big|_0^R$

$M = 4\pi\left(\frac{\rho_c R^3}{3} - \frac{\rho_c R^3}{4}\right)$

$M = \frac{4\pi\rho_c R^3}{3} - \pi\rho_c R^3$

$M = \pi\rho_c R^3\left(\frac{4}{3}-1\right)$

$\left(M = \frac{\pi\rho_c R^3}{3}\right)\frac{3}{\pi R^3}$

$\boxed{\rho_c = \frac{3M}{\pi R^3}}$

(9) $dm(r) = 4\pi\rho(r) r^2 dr$

$dm(r) = 4\pi \rho_c (1 - \frac{r}{R}) r^2 dr$

$dm(r) = 4\cancel{\pi} \frac{3M}{\cancel{\pi} R^3} (1 - \frac{r}{R}) r^2 dr$

$\int dm(r) = \int \frac{12M}{R^3} (1 - \frac{r}{R}) r^2 dr$ $\quad r^2 - \frac{r^3}{R}$

$$\boxed{m(r) = \frac{12M}{R^3}\left(\frac{r^3}{3} - \frac{r^4}{4R}\right)}$$

(10) $dU_g = -G \frac{M_r m}{r}$

$U_g = -G\left(\frac{12M}{R^3}\right)\int_0^R \left(\frac{r^3}{3} - \frac{r^4}{4R}\right) 4\pi r \rho_c dr$

$U_g = -4\pi G\left(\frac{12M}{R^3}\right)\int_0^R \left(\frac{r^3}{3} - \frac{r^4}{4R}\right) r \rho_c dr$

$U_g = -4\cancel{\pi} G\left(\frac{12M}{R^3}\right)\left(\frac{3M}{\cancel{\pi} R^3}\right)\int_0^R \left(\frac{r^4}{3} - \frac{r^5}{4R}\right) dr$

$U_g = \frac{-144 M^2 G}{R^6}\left(\frac{r^5}{15} - \frac{r^6}{24R}\right)\Bigg|_0^R$

$U_g = \frac{-144 M^2 G}{R^6}\left(\frac{R^5}{15} - \frac{R^5}{24}\right)$

$U_g = \frac{-144 M^2 G}{R^6}\left(\frac{R^5}{40}\right)$

$$U_g = \frac{-144 M^2 G}{40 R} = \boxed{-\frac{18 M^2 G}{5R}}$$

(11) $\int dP = \int -\frac{G M_r \rho_r}{r^2} dr$

substitute for M_r $m(r)$ & ρ_r $\rho(r)$

$P = \frac{-G}{r_2}\int_0^R \left(\frac{12M}{R^3}\left(\frac{r^3}{3} - \frac{r^4}{4R}\right)\right)\left(\frac{3M}{\pi R^3}\left(1 - \frac{r}{R}\right)\right) dr$

$P = \frac{-36 G M^2}{R^3}\int_0^R \left(\frac{1}{r^2}\right)\left(\frac{r^3}{3} - \frac{r^4}{4R}\right)\left(1 - \frac{r}{R}\right) dr$

$P = \frac{-36 G M^2}{R^3}\int_0^R \left(\frac{r}{3} - \frac{r^2}{4R}\right)\left(\frac{1}{r^2} - \frac{1}{rR}\right) dr$

$P = \frac{-36 G M^2}{R^3}\int_0^R \left(\frac{1}{3r} - \frac{1}{3R} - \frac{1}{4R} + \frac{r}{4R^2}\right) dr$

$P = \frac{-36 G M^2}{R^3}\left(3\ln(r) - \frac{1}{3R} - \frac{1}{4R} + \frac{r^2}{8R^2}\right)\Bigg|_0^R$

$$\boxed{P = \frac{-36 G M^2}{R^3}\left(3\ln(R) - \frac{1}{3R} - \frac{1}{4R} + \frac{1}{8}\right)}$$

$m(r) = \frac{12M}{R^3}\left(\frac{r^3}{3} - \frac{r^4}{4R}\right)$

$\rho(r) = \frac{3M}{\pi R^3}\left(1 - \frac{r}{R}\right)$