```
#POLYTROPIC PROPERTIES: CENTRAL DENSITY, PRESSURE AND TEMPERATURE
#CODE AUTHORED BY ALEXIS LOHMAN
#AST 421 SPRING 2021
#APRIL 20TH, 2021

import math
pi=math.pi
##Finding the Anaylytical Solution using A, m0, and Z0 from Mathematica
n=1; M=2*10**(33); R=7*10**(10); A=4.48799*10**(-11); m0=3.14159; Z0=3.14159;
G=6.6743*10**(-11); mu=1.2*1.66*10**(-24); kb=1.38*10**(-16);

#Solving for Central Density (rho)
den=(M*A**3)/(4*pi*m0);
Density="{:e}".format(den)
print("The polytropic star has a mass of 2*10^(33) kg, a radius of 7*10^(10) cm, an
print("The central density is " + Density +" g/cm^3.")

#Solving for free Constant c
c=((4*pi*G)/((n+1)*A**2))*(den**((n-1)/n));

#Solving for Central Pressure
press=c*den**((1+n)/n);
Pressure="{:e}".format(press)
print("The central pressure is " + Pressure +" dyne/cm^2.")

#Solving for Central Temperature
T=(mu*press)/(kb*den);
Temperature="{:e}".format(T)
print("The central temperature is " + Temperature +" K.")
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The polytropic star has a mass of 2*10^(33) kg, a radius of 7*10^(10) cm, and
The central density is 4.579587e+00 g/cm^3.
The central pressure is 4.366509e+12 dyne/cm^2.
The central temperature is 1.376317e+04 K.
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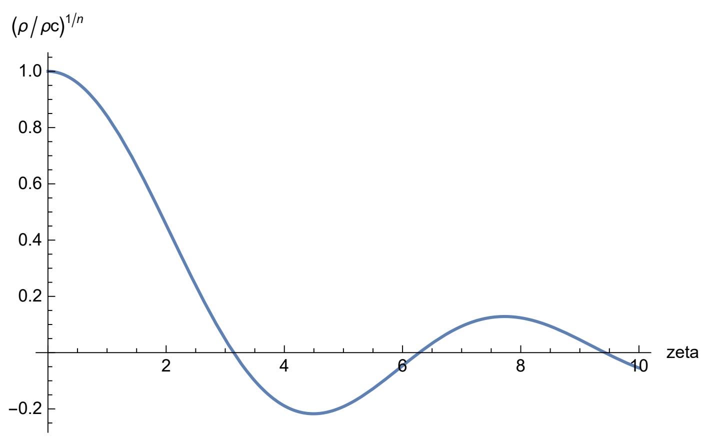
The code solves for the charateristics of a polytropic star with n=1, mass = 2x10^(33)kg and radius = 7x10^(10)cm. In order to solve for the properties of the polytropic star, it is neccessary to find the paramters A, m0 and Z0. These values were acheived by using Mathematica and NDSolve to acheive results for the Lane-Emden equation via the Interplotting function. From here, there is sufficent information to solve for the central pressure, density and temperature both with Mathematica and Python to get a concise anaylitical result.

Mathematica Produced Code

```
In[218]:= nn = 1; eps = 10^(-44); γ = 1/nn + 1;
diff = NDSolve[{ωω''[zz] + (2 / zz) ωω'[zz] + ωω[zz]^(nn) == 0,
    ωω[eps] == 1. - eps^2 (1 / 6), ωω'[eps] == -eps / 3}, ωω, {zz, eps, 20}];
Plot[ωω[t] /. diff // Evaluate, {t, eps, 10}, AxesLabel → {"zeta", "(ρ / ρc)^(1/n)"}]
trial = 2;
z0 = FindRoot[ωω[z1] /. diff[[1]], {z1, trial}]
z3 = z1 /. z0[[1]];
m0 = Integrate[(ωω[z2] /. diff[[1]]) (z2^2), {z2, 0, z3}];
N[m0]
mst = 1.00452034; rst = 1.00574713; teff = 5700; msu = 1.991 * 10^(33);
rsu = 6.96 * 10^(10); G = 6.674 * 10^(-8); μ = 1.2 (1.66 * 10^(-24)); kb = 1.38 * 10^(-16);
a = z3 / (rst (rsu))
density = (mst msu a^3) / (4 * pi * m0)
constant = (4 * pi * G * density^((nn - 1) / nn)) / (a^2 (nn + 1));
press = constant density^ γ
temp = (μ press) / (kb density)
Plot[density × ωω[t] /. diff // Evaluate, {t, eps, 10}, AxesLabel → {"zeta", "cgs ρ"}]
Plot[Evaluate[press ωω[t]^((nn+1)/nn) /.diff], {t, eps, 10},
 BaseStyle → {10, FontFamily → "Helvetica"}, AxesLabel → {"zeta", "cgs pressure"}]
Plot[Evaluate[temp ωω[t]^(1/nn) /.diff], {t, eps, 10},
 BaseStyle → {10, FontFamily → "Helvetica"}, AxesLabel → {"zeta", "temperature K"}]
```

Out[220]=

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Out[222]= {z1 → 3.14159}

Out[225]= 3.14159

Out[227]= 4.48799 × 10^-11
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Central Density 45.1987

Out[228]= $\dfrac{\text{pi} \int_0^{3.14159} z2^2 \,\text{InterpolatingFunction}[\text{Domain: } \{\{1.\times 10^{-44}, 20.\}\} \text{ Output: scalar}][z2]\, dz2}{}$

Central Pressure

1.35383×10^{17}

Out[230]=

$$\text{pi}\left(\int_0^{3.14159} z2^2\, \text{InterpolatingFunction}\left[\boxed{\text{Domain: } \{\{1.\times 10^{-44}, 20.\}\} \quad \text{Output: scalar}}\right][z2]\, dz2\right)^2$$

Central Temperature

4.32363×10^{7}

Out[231]=

$$\int_0^{3.14159} z2^2\, \text{InterpolatingFunction}\left[\boxed{\text{Domain: } \{\{1.\times 10^{-44}, 20.\}\} \quad \text{Output: scalar}}\right][z2]\, dz2$$