A Rao-Blackwellized Particle Filter for Superelliptical Extended Target Tracking

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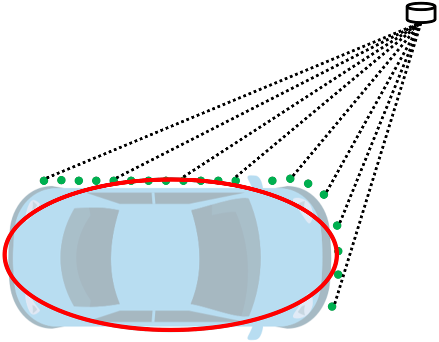
Outline

Introduction & Motivation



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Goals:

- Dynamic target with multiple **contour** measurements
- Measurements from at most two sides
- Extended target tracking with a parametric extent model

$\Rightarrow$ maybe a better shape than an ellipse?

Problem Formulation: Ellipses to Superellipses

Ellipse equation in $\mathbb{R}^2$:

$$(\mathbf{y}-\mathbf{c})^\intercal \mathbf{W}^{-1}(\mathbf{y}-\mathbf{c}) = 1$$

with centroid $\mathbf{c} \in \mathbb{R}^2$ & pos. def. $\mathbf{W}$.

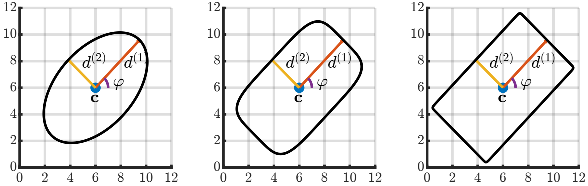
We can write the above equation as

$$\|\mathbf{D}^{-1}\mathbf{R}_\varphi^\intercal(\mathbf{y}-\mathbf{c})\|_2^2 = 1$$

where $\mathbf{W} = \mathbf{R}_\varphi \mathbf{D}^2 \mathbf{R}_\varphi^\intercal$ with $\mathbf{D} = \operatorname{diag}\left(d^{(1)}, d^{(2)}\right)$, $\mathbf{R}_\varphi$ is the rotation matrix by φ.

What if we used another norm ℓ_q?

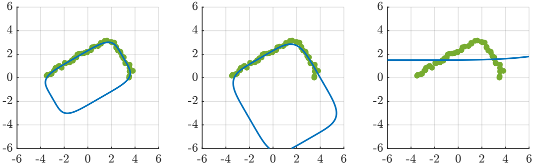
$$\|\mathbf{D}^{-1}\mathbf{R}_\varphi^\intercal(\mathbf{y}-\mathbf{c})\|_q^q = 1$$



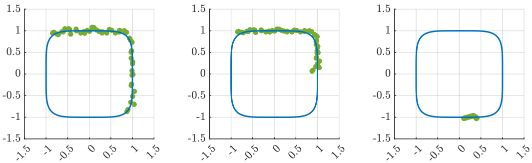
(a) $q = 2$ (b) $q = 5$ (c) $q = 50$

Problem Formulation: Scaling Constraints

Goal: For measurements $\{\mathbf{y}^m\}_{m=1}^{M}$, find $\underset{d^{(1)},d^{(2)},\varphi,\mathbf{c}}{\operatorname{argmin}} \sum_{m=1}^{M} \left(\|\mathbf{D}^{-1}\mathbf{R}_\varphi^\intercal(\mathbf{y}^m - \mathbf{c})\|_q^q - 1 \right)^2$



Cost $= 1.83$ | Cost $= 1.91$ | Cost $= 0.45$



All are viable according to the above cost function, and dangerously so!

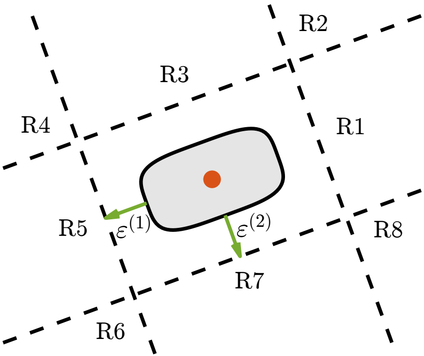
Scaling constaints to ensure the width & height of the measurements match the contour:

$$\min_m \left\{ \mathbf{u}_j^\intercal \mathbf{R}_\varphi^\intercal (\mathbf{y}^m - \mathbf{c}) \right\} = -d^{(j)}$$

$$\max_m \left\{ \mathbf{u}_j^\intercal \mathbf{R}_\varphi^\intercal (\mathbf{y}^m - \mathbf{c}) \right\} = d^{(j)}$$

for each dimension $j = 1, 2$.

Problem Formulation: Visibility Conditions with Tolerance



Only one side visible
"Barely visible second side"

How to know which side is visible? How to know which side is *sufficiently* visible?

To see $d^{(1)}$:

$$\left|\mathbf{u}_2^\intercal \mathbf{R}_\varphi^\intercal (\mathbf{s}-\mathbf{c})\right| > d^{(2)} + \varepsilon^{(2)}$$

To see $d^{(2)}$:

$$\left|\mathbf{u}_1^\intercal \mathbf{R}_\varphi^\intercal (\mathbf{s}-\mathbf{c})\right| > d^{(1)} + \varepsilon^{(1)}$$

Define binary variables for convenience:

$$b^{(j)} := \mathbb{1}\left(\left|\mathbf{u}_{j'}^\intercal \mathbf{R}_\varphi^\intercal (\mathbf{s}-\mathbf{c})\right| > d^{(j')} + \varepsilon^{(j')}\right)$$

Method: Conditional Linearity

Rewrite the superellipse equation: $\|\mathbf{D}^{-1}\mathbf{R}_{\varphi}^{\mathsf{T}}(\mathbf{y}-\mathbf{c})\|_q^q = 1$

This is equivalent to writing

$$\boldsymbol{\lambda}^{\mathsf{T}} \left|\mathbf{R}_{\varphi}^{\mathsf{T}}(\mathbf{y}-\mathbf{c})\right|^q = 1$$

where $\boldsymbol{\lambda} = [\lambda^{(1)} \ \lambda^{(2)}]^{\mathsf{T}} := \left[\left({}^{1}\!/_{d^{(1)}}\right)^q \ \left({}^{1}\!/_{d^{(2)}}\right)^q \right]^{\mathsf{T}}$

For a measurement set $\{\mathbf{y}^m\}_{m=1}^M$, define $\tilde{\mathbf{y}} := \mathbf{R}_{\varphi}^{\mathsf{T}}(\mathbf{y}-\mathbf{c})$, then we can also write

$$\mathbf{H}(\varphi, \mathbf{c})\boldsymbol{\lambda} = \mathbf{1}_{M\times 1}$$

where $\mathbf{H}(\varphi, \mathbf{c}) = [|\tilde{\mathbf{y}}^1|^q \ \cdots \ |\tilde{\mathbf{y}}^M|^q]^{\mathsf{T}}$

To use the conditional linearity, define the dynamic target's state vector at time k as

$$\mathbf{x}_k := \Big[\underbrace{\varphi_k \ c_k^{(1)} \ c_k^{(2)} \ \dot{c}_k^{(1)} \ \dot{c}_k^{(2)}}_{:=\mathbf{x}_k^n} \ \underbrace{\lambda_k^{(1)} \ \lambda_k^{(2)}}_{:=\mathbf{x}_k^l} \Big]^{\mathsf{T}}$$

Method: State Space Model

State dynamics: Constant velocity

$$
\begin{aligned}
\mathbf{x}^n_{k+1} &= \mathbf{F}^n\mathbf{x}^n_k + \mathbf{G}^n\mathbf{w}^n_k \\
\mathbf{x}^l_{k+1} &= \mathbf{x}^l_k \quad + \mathbf{w}^l_k
\end{aligned}
$$

with $\mathbf{w}^n_k \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}^n_k)$ and $\mathbf{w}^l_k \sim \mathcal{N}\left(\mathbf{0}, \mathbf{Q}^l_k\right)$.

Measurement model:

$$
\begin{aligned}
1 &= \boldsymbol{\lambda}^{\mathsf{T}}_k \left|\mathbf{R}^{\mathsf{T}}_{\varphi_k}(\mathbf{y}^m_k - \mathbf{c}_k)\right|^q + e^m_k \\
\mathbf{1}_{M\times 1} &= \mathbf{H}(\varphi, \mathbf{c})\boldsymbol{\lambda} \qquad\qquad + \mathbf{e}_k
\end{aligned}
$$

with $e^m_k \sim \mathcal{N}\left(0, r^h_k\right)$ and $\mathbf{e}_k \sim \mathcal{N}\left(\mathbf{0}_{M\times 1}, r^h_k \mathbf{I}_{M_k}\right)$ for $\mathcal{Y}_k = \{\mathbf{y}^m_k\}^{M_k}_{m=1}$.

+ **Visibility Conditions**
+ **Scaling Constraints**
+ **Positivity Constraints on λ_k**

Method: Inference

Rao-Blackwellized Particle Filter: $p(\mathbf{x}_k^l, \mathbf{x}_{0:k}^n | \mathcal{Y}_{1:k}) = \underbrace{(\mathbf{x}_k^l | \mathbf{x}_{0:k}^n, \mathcal{Y}_{1:k})}_{\text{Kalman Filter}} \underbrace{p(\mathbf{x}_{0:k}^n | \mathcal{Y}_{1:k})}_{\text{Particle Filter}}$

Particle Filter:

$$p(\mathbf{x}_{0:k}^n | \mathcal{Y}_{1:k}) \approx \sum_{i=1}^{N} w_{k,(i)} \delta(\mathbf{x}_{0:k}^n - \mathbf{x}_{0:k,(i)}^n)$$

Kalman Filter:

$$p(\mathbf{x}_k^l | \mathbf{x}_{0:k,(i)}^n, \mathcal{Y}_{1:k}) = \mathcal{N}\left(\mathbf{x}_k^l; \boldsymbol{\mu}_{k|k,(i)}, \boldsymbol{\Sigma}_{k|k,(i)}\right)$$

Particle Sampling:

$$\mathbf{x}_{k+1,(i)}^n \sim \mathcal{N}\left(\mathbf{F}^n \mathbf{x}_{k,(i)}^n, \mathbf{G}^n \mathbf{Q}^n (\mathbf{G}^n)^\mathsf{T}\right)$$

Kalman Filter Time Update:

$$\boldsymbol{\mu}_{k+1|k,(i)} = \boldsymbol{\mu}_{k|k,(i)}$$
$$\boldsymbol{\Sigma}_{k+1|k,(i)} = \boldsymbol{\Sigma}_{k|k,(i)} + \mathbf{Q}^l$$

Method: Particle Filter Weight Update

$$
\begin{aligned}
\tilde{w}_{k,(i)} = & w_{k-1,(i)} \\
& \times \prod_{m=1}^{M_k} \mathcal{N}\left(\boldsymbol{\mu}_{k|k-1,(i)}^{\mathsf{T}} |\tilde{\mathbf{y}}_k^m|^q ; 1, r_k^h\right) && \alpha_k^h \\
& \times \prod_{j=1}^{2} \left\{ \begin{array}{l} \mathcal{N}\left(\min_m \left\{\mathbf{u}_j^{\mathsf{T}} \tilde{\mathbf{y}}_k^m\right\}; -\hat{d}_{k,(i)}^{(j)}, r_k^s\right) \times \\ \mathcal{N}\left(\max_m \left\{\mathbf{u}_j^{\mathsf{T}} \tilde{\mathbf{y}}_k^m\right\}; \hat{d}_{k,(i)}^{(j)}, r_k^s\right) \end{array} \right\}^{\hat{b}_{k,(i)}^{(j)}} && \alpha_k^s \\
& \times \mathbb{1}\left(\boldsymbol{\mu}_{k+1|k,(i)} > 0\right)
\end{aligned}
$$

Implicit measurement equation likelihood

Scaling constraints with visibility conditions

Positivity constraints

Method: Kalman Filter Measurement Update

$$\boldsymbol{\mu}_{k|k,(i)} = \boldsymbol{\mu}_{k|k-1,(i)} + \mathbf{K}_{k,(i)}\left(\mathbf{1}_{M_k \times 1} - \mathbf{H}_k \boldsymbol{\mu}_{k|k-1,(i)}\right)$$
$$\boldsymbol{\Sigma}_{k|k,(i)} = \boldsymbol{\Sigma}_{k|k-1,(i)} - \mathbf{K}_{k,(i)}\mathbf{S}_{k,(i)}\mathbf{K}_{k,(i)}^{\mathsf{T}}$$

where

$$\mathbf{K}_{k,(i)} = \mathbf{D}_{k,(i)}\boldsymbol{\Sigma}_{k|k-1,(i)}\mathbf{H}_k^{\mathsf{T}}\mathbf{S}_{k,(i)}^{-1}$$
$$\mathbf{S}_{k,(i)} = \mathbf{H}_k\boldsymbol{\Sigma}_{k|k-1,(i)}\mathbf{H}_k^{\mathsf{T}} + r_k^h \mathbf{I}_{M_k}$$
$$\mathbf{D}_{k,(i)} = \operatorname{diag}\left(\hat{b}_{k,(i)}^{(1)}, \hat{b}_{k,(i)}^{(2)}\right)$$

$\mathbf{D}_{k,(i)}$ prevents updates to the relevant $\lambda_k^{(j)}$ state if it is not visible.

Method: Shape Adaptation

Can we also learn the superellipse exponent q?

State dynamics: Almost stationary dynamics for q_k, identical for the rest.

Augment the state vector to include q_k: $\mathbf{x}_k^* = [q_k \ \mathbf{x}_k^\mathsf{T}]^\mathsf{T}$

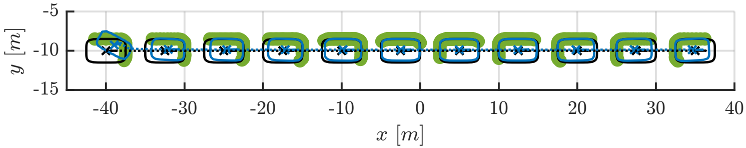
Measurement model:

$$1 = \boldsymbol{\lambda}_k^\mathsf{T} \left| \mathbf{R}_{\varphi_k}^\mathsf{T} (\mathbf{y}_k^m - \mathbf{c}_k) \right|^{q_k} + e_k^m$$

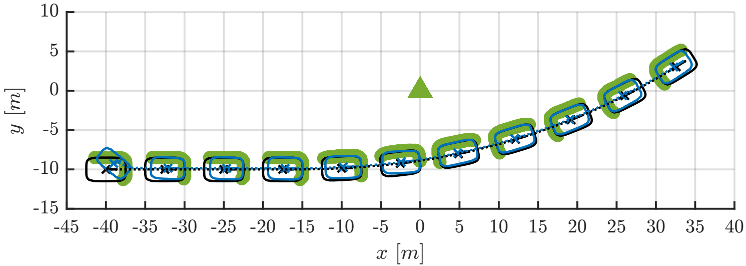
for $\mathcal{Y}_k = \{\mathbf{y}_k^m\}_{m=1}^{M_k}$.

+ **Visibility Conditions**
+ **Scaling Constraints**
+ **Positivity Constraints on $\boldsymbol{\lambda}_k$**
+ **Positivity Constraint on q_k**

Results: Simulation Results

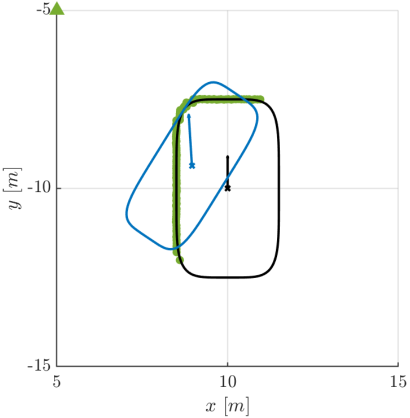


(a) Linear trajectory.

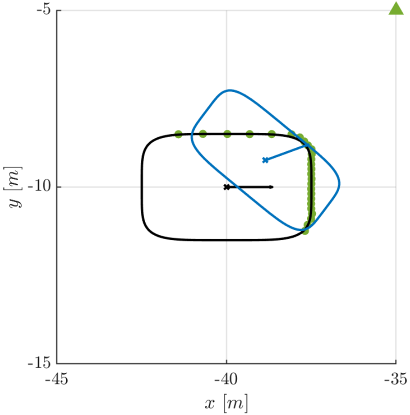


(b) Curved trajectory.

Results: Simulation Results



(a) U-Turn trajectory.



(b) Curved trajectory with drift.

Results: Shape Adaptation

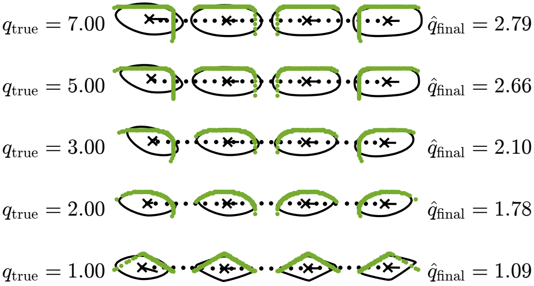


Figure: Learned q values with augmented state.

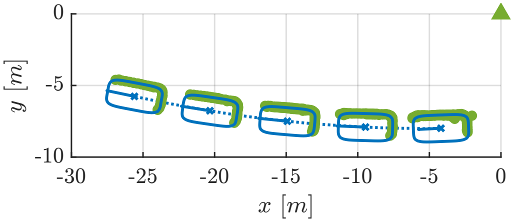
Model finds a suitable superellipse coefficient q to represent the data

but favors lower q values due to the implicit measurement equation:

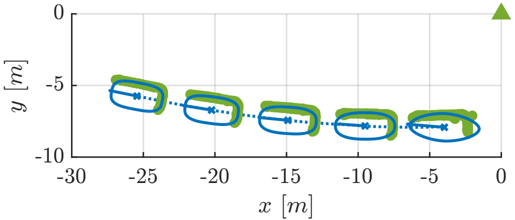
$$\|\mathbf{D}_k^{-1}\mathbf{R}_{\varphi_k}^{\mathsf{T}}(\mathbf{y} - \mathbf{c}_k)\|_{q_k}^{q_k} = 1$$

$\Rightarrow$ Higher q_k means eaiser deviation from 1 due to exponentiation of the norm!

Results: Real Data Application



(a) Real data with the original state vector, $q = 5$.



(b) Real data with the augmented state vector.

Faster convergence in shape due too higher number of measurements!

Final Words

Conclusions:

The proposed model can successfully track a dynamic target with contour measurements.

The implicit measurement model works, at the cost of additional constraints.

Even though the visibility conditions are derived for sufficiently rectangular targets, they seem to be working fine for non-rectangular targets.

Future Work:

Explicit measurement equation instead of an implicit one: Predicted measurements!

Better visibility conditions for insufficiently rectangular targets (especially with $0 < q < 2$).

Other parametric extent representations?

Thank you!

Motivation

ETT with a parametric extent model that better fits vehicles

Contour measurements (e.g. Lidar sensor)

Sensor-object geometry-aware inference

Superellipses

A generalization of ellipses

$$\|\mathbf{D}^{-1}\mathbf{R}_{\varphi}^{\mathsf{T}}(\mathbf{y}-\mathbf{c})\|_q^q = 1$$

$\mathbf{D}$ diagonal matrix of half-lengths, $\mathbf{R}_{\varphi}$ rotation matrix by φ radians, $\mathbf{c}$ centroid, $q > 0$ exponent.

Scale Constraints

To prevent ill-fitting contour estimates

$$\min_m \left\{\mathbf{u}_j^{\mathsf{T}}\mathbf{R}_{\varphi}^{\mathsf{T}}(\mathbf{y}^m - \mathbf{c})\right\} = -d^{(j)}$$

$$\max_m \left\{\mathbf{u}_j^{\mathsf{T}}\mathbf{R}_{\varphi}^{\mathsf{T}}(\mathbf{y}^m - \mathbf{c})\right\} = d^{(j)}$$

Visibility Conditions

Determine half-length variable is visible by the sensor

$$b^{(j)} := \mathbb{1}\left(\left|\mathbf{u}_{j'}^{\mathsf{T}}\mathbf{R}_{\varphi}^{\mathsf{T}}(\mathbf{s}-\mathbf{c})\right| > d^{(j')} + \varepsilon^{(j')}\right)$$

State Space & Dynamics

$$\mathbf{x}_k = \begin{bmatrix}\varphi_k & c_k^{(1)} & c_k^{(2)} & \dot{c}_k^{(1)} & \dot{c}_k^{(2)} & \lambda_k^{(1)} & \lambda_k^{(2)}\end{bmatrix}^{\mathsf{T}}$$

with constant velocity dynamics on the centroid.
For an adaptible shape, augment the state with q_k and almost stationary dynamics.

Measurement Model

$$1 = \boldsymbol{\lambda}_k^{\mathsf{T}}\left|\mathbf{R}_{\varphi_k}^{\mathsf{T}}(\mathbf{y}_k^m - \mathbf{c}_k)\right|^q + e_k^m$$

$$\mathbf{1}_{M\times 1} = \mathbf{H}(\varphi, \mathbf{c})\boldsymbol{\lambda} + \mathbf{e}_k$$

to exploit conditional linearity + visibility conditions + scaling & positivity constraints.

State Space & Inference

RBPF with constraints

$$\tilde{w}_{k,(i)} = w_{k-1,(i)} \times \alpha_{k,(i)}^h \times \alpha_{k,(i)}^s \times \mathbb{1}(\cdot)$$

Visibility conditions modify the Kalman gain matrix

$$\mathbf{K}_{k,(i)} = \mathbf{D}_{k,(i)}\boldsymbol{\Sigma}_{k|k-1,(i)}\mathbf{H}_k^{\mathsf{T}}\mathbf{S}_{k,(i)}^{-1}$$

Results

Works, with simulated & real data!