

# Caustic crossings as a new probe of dark matter

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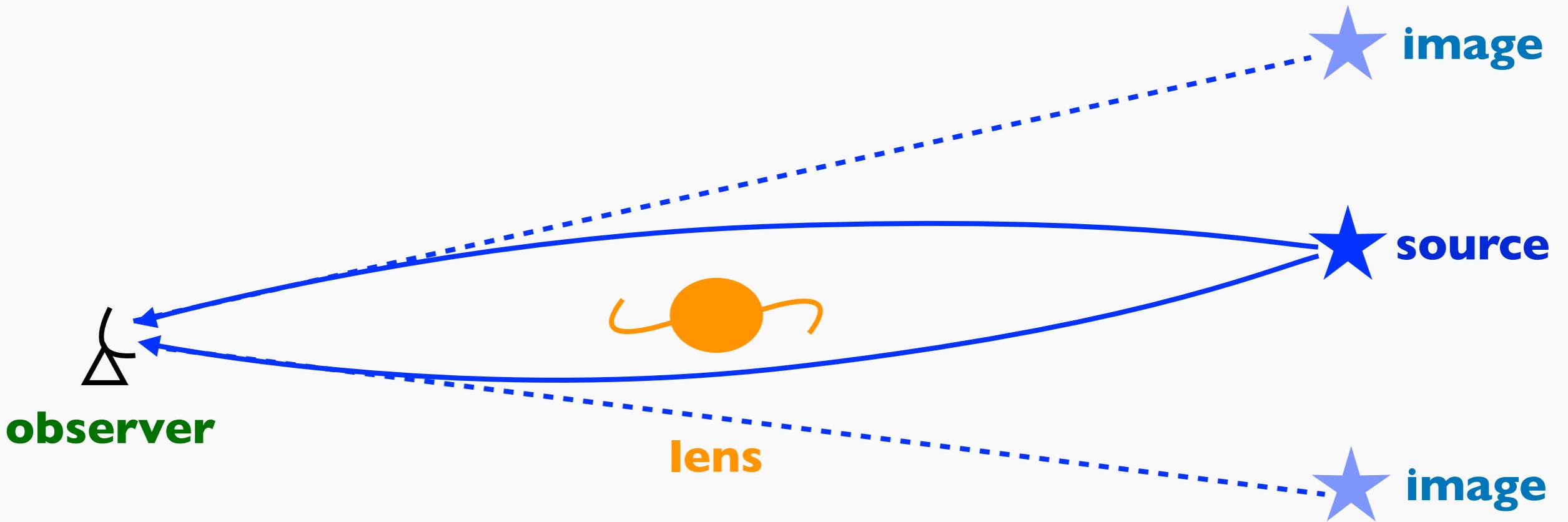
CHIBA  
UNIVERSITY



Center for  
Frontier  
Science

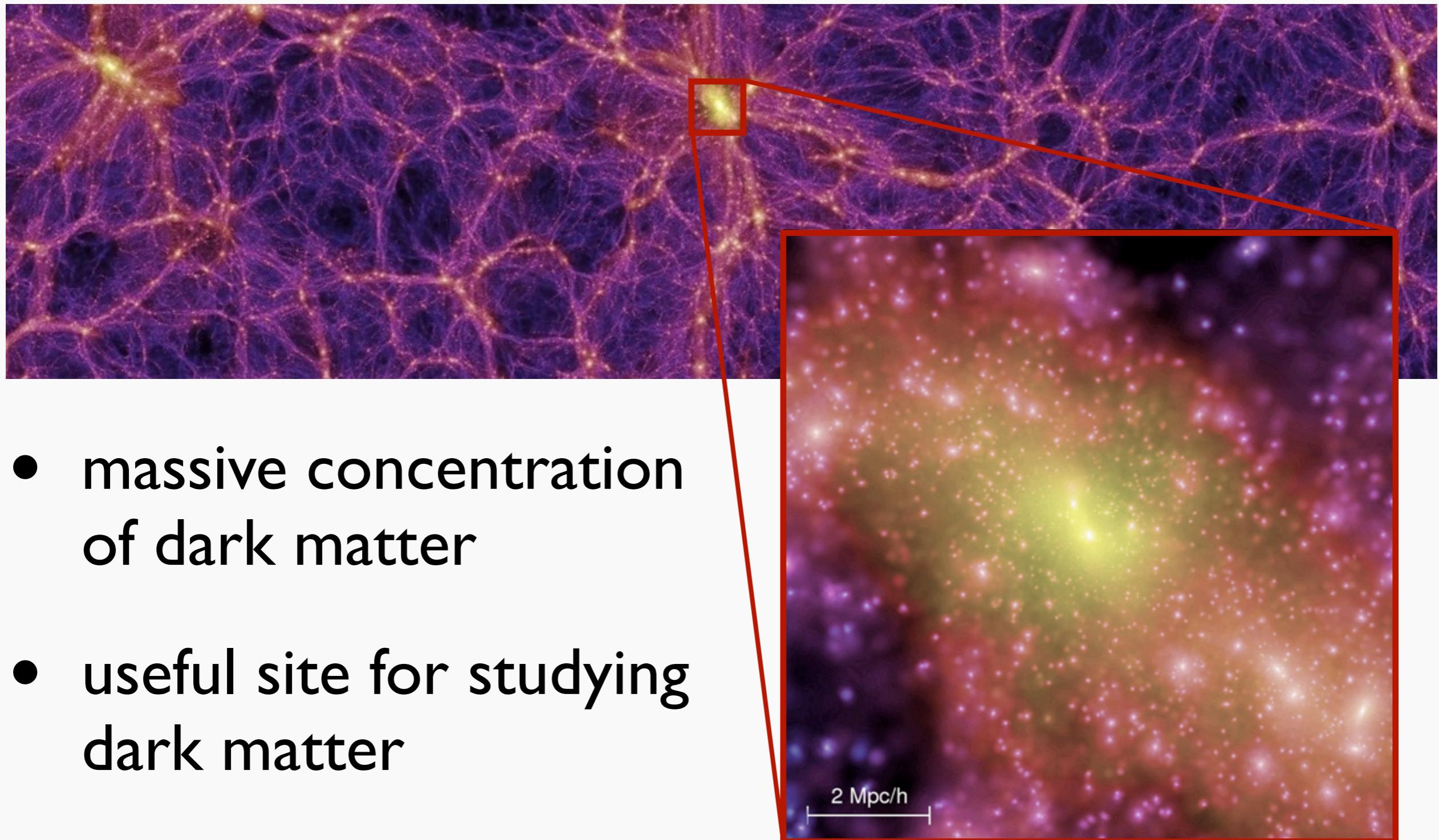
2024/10/15 Cosmic Indicators of Dark Matter@Tohoku

# Strong gravitational lensing

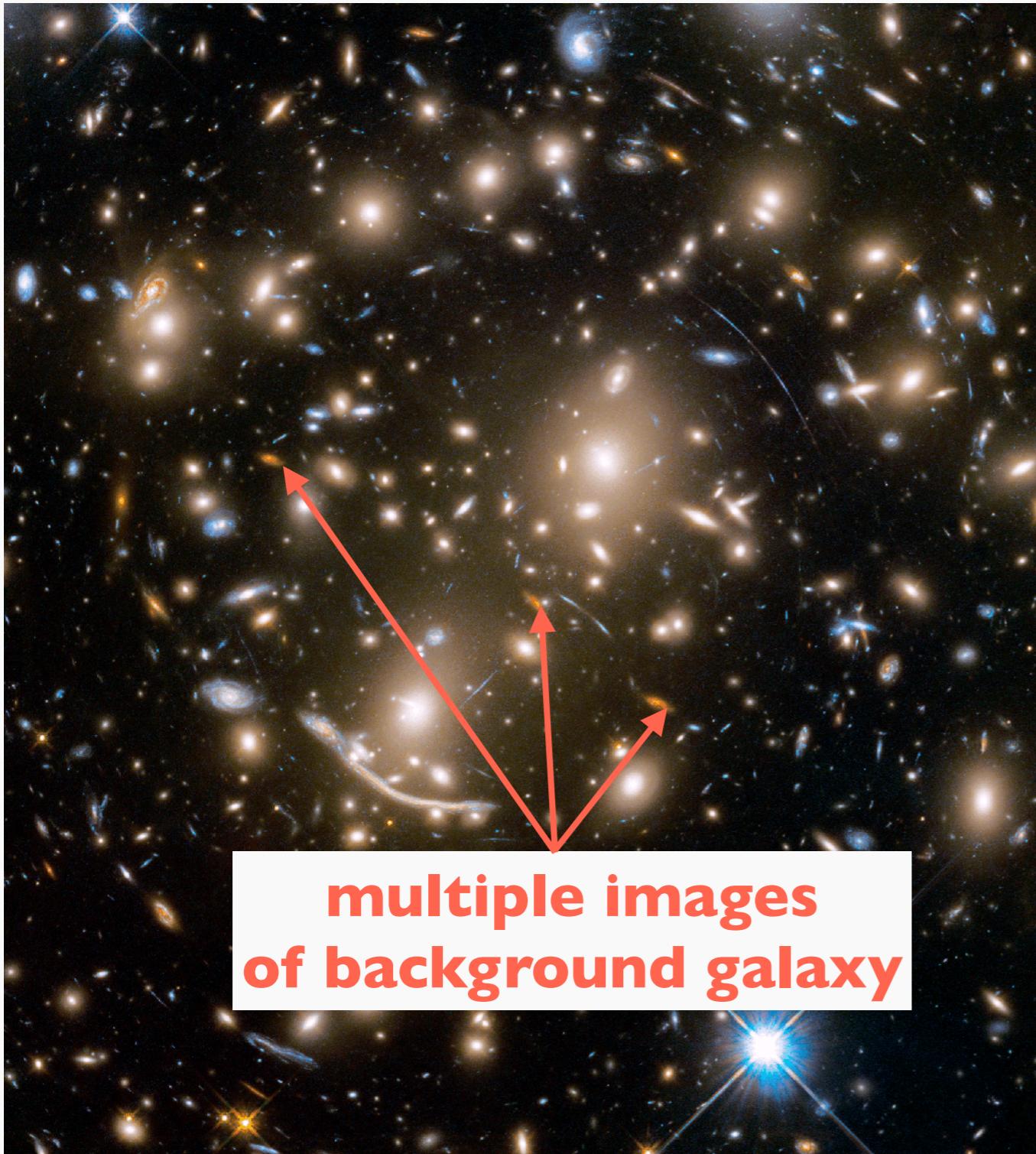


- multiple solution of **image position**  $\vec{\theta}$  for lens equation  $\vec{\beta} = \vec{\theta} - \vec{\alpha}(\vec{\theta})$   
→ **multiple images**

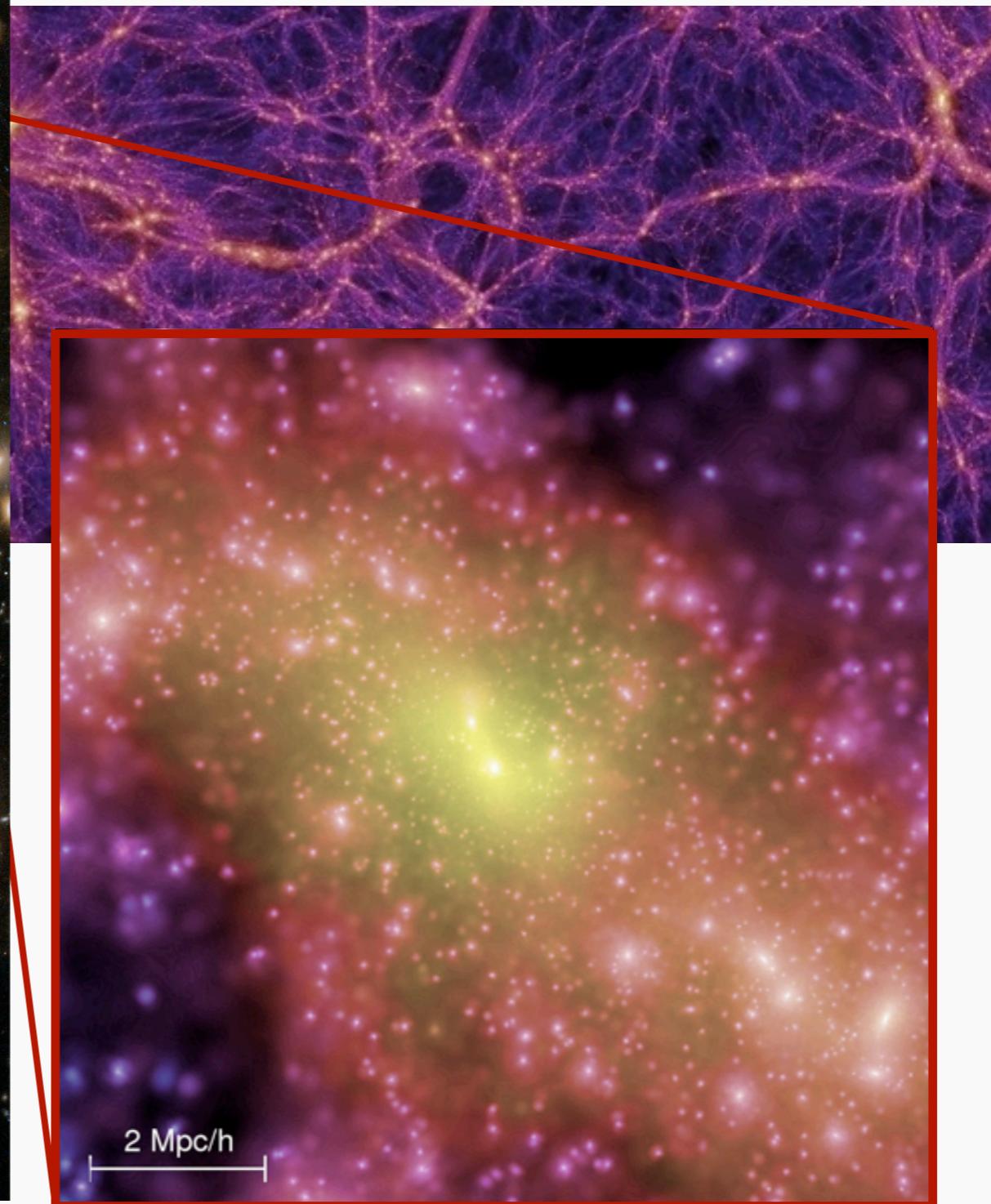
# Gravitational lensing by cluster



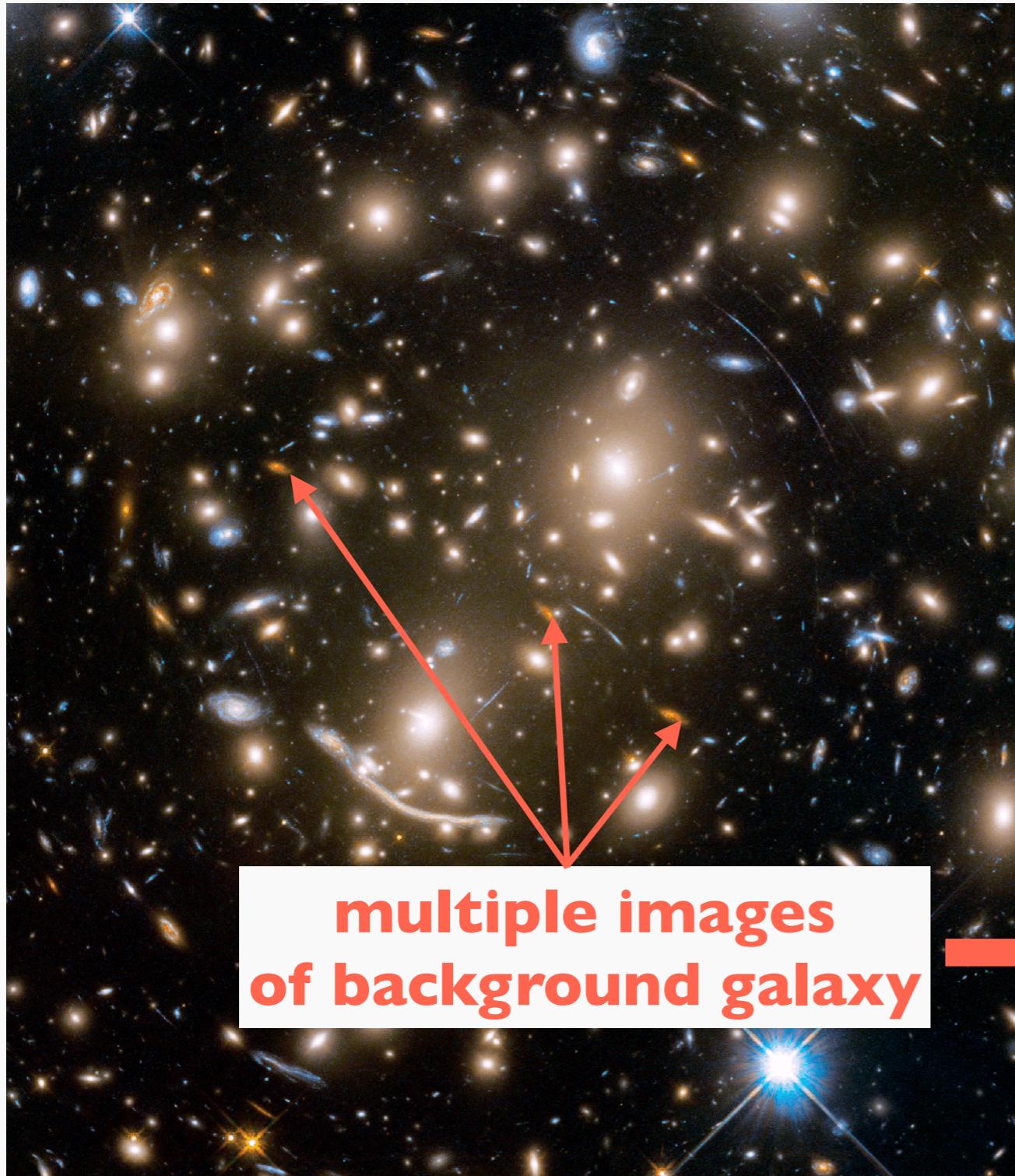
# Gravitational lensing by cluster



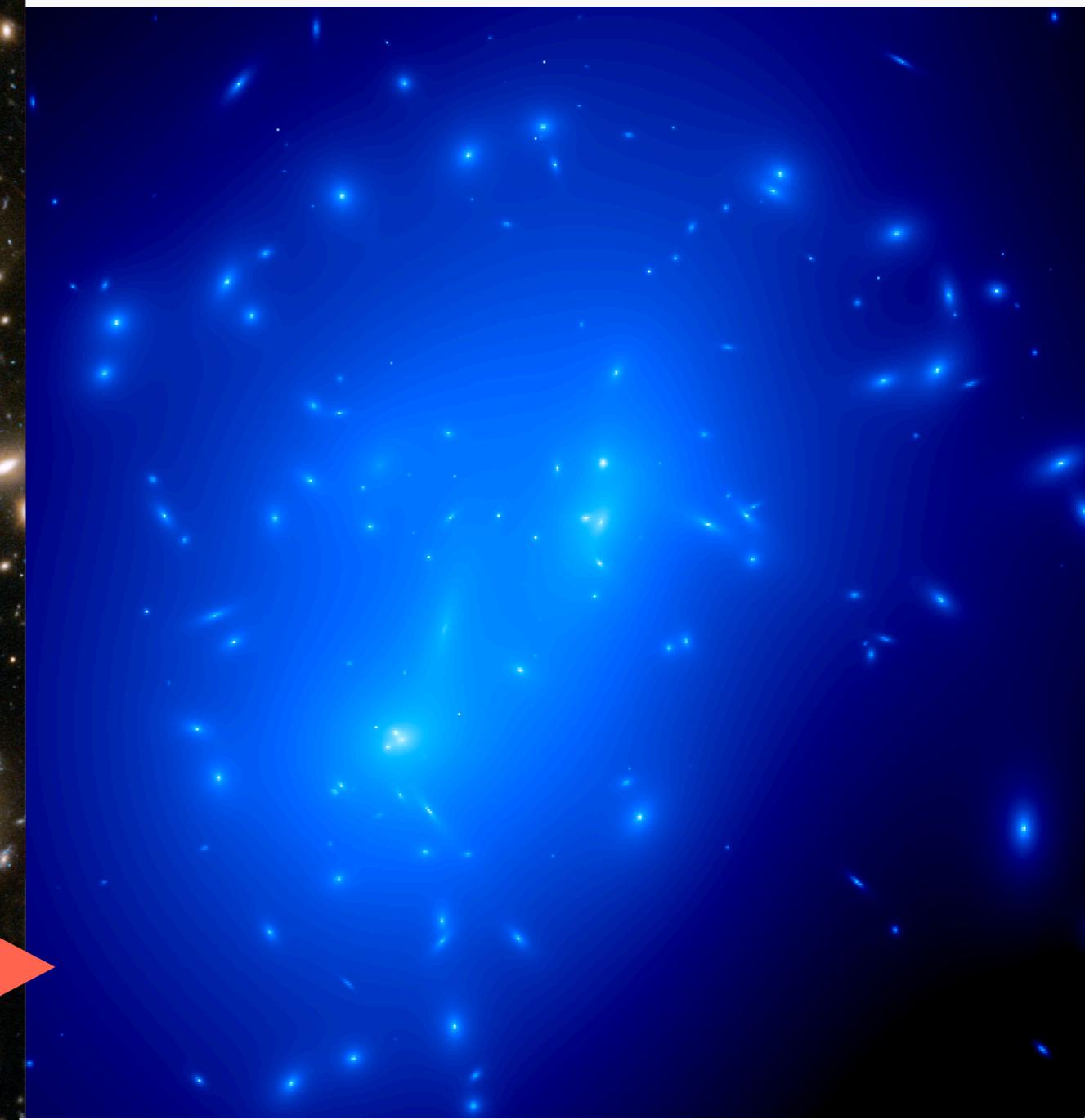
Abell 370, NASA/STScI



# Gravitational lensing by cluster



Abell 370, NASA/STScI



**reconstructed matter distribution**  
(Kawamata, MO+2016)

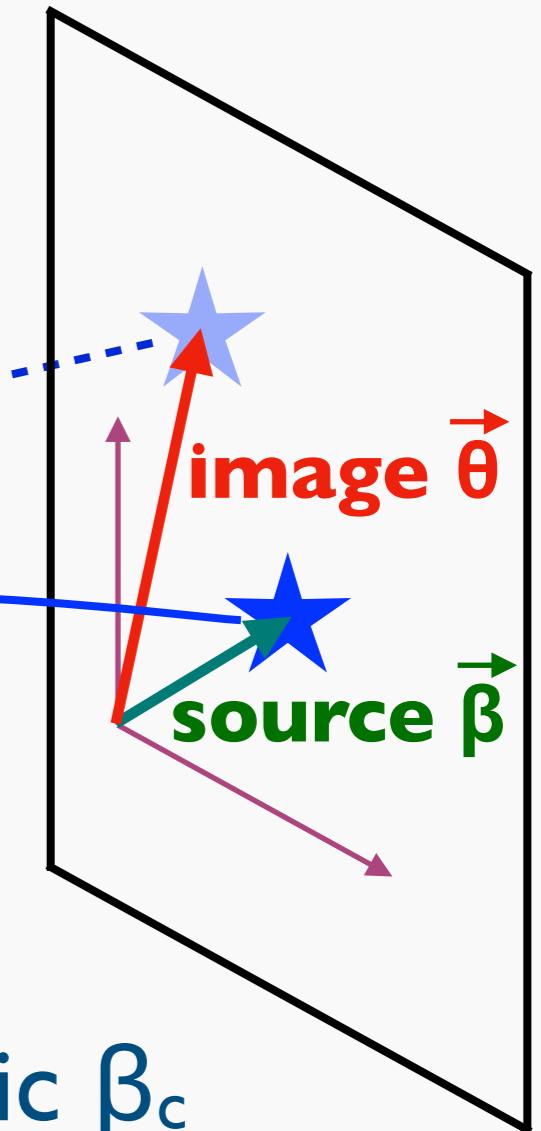
# Critical curve and caustic

lens equation: mapping btw source and image

$$\vec{\beta} = \vec{\theta} - \vec{\alpha}(\vec{\theta})$$



**lens (dark matter)**



magnification  $\mu$

$$\mu = \left[ \det \left( \frac{\partial \vec{\beta}}{\partial \vec{\theta}} \right) \right]^{-1}$$

critical curve  $\theta_c$

$$\det \left( \frac{\partial \vec{\beta}}{\partial \vec{\theta}} \right) \Big|_{\vec{\theta}=\vec{\theta}_c} = 0$$

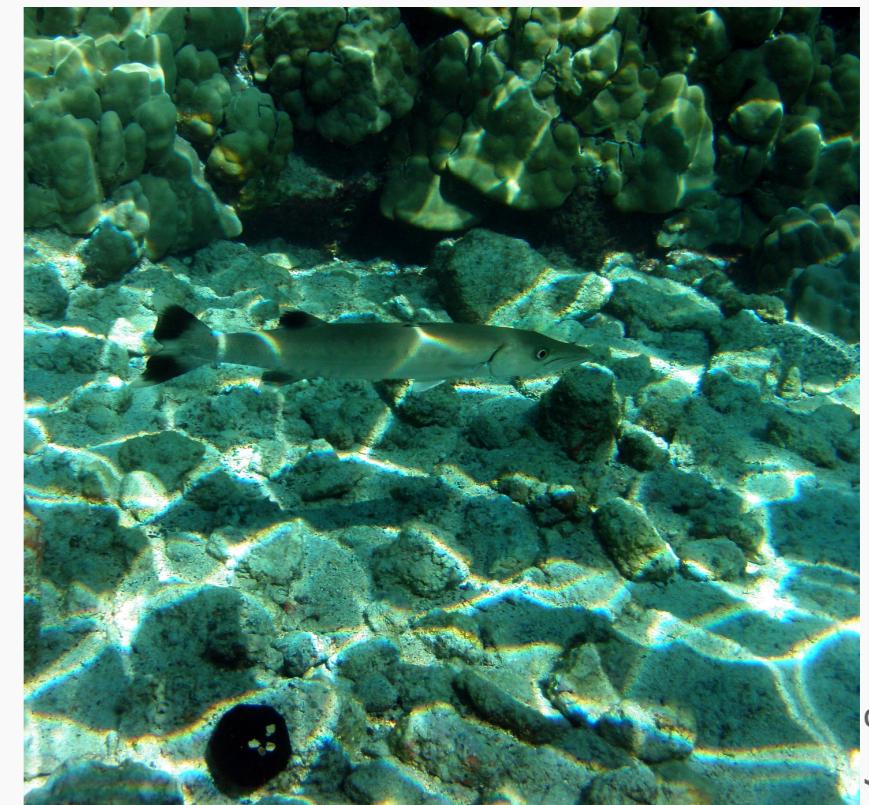
caustic  $\beta_c$

$$\vec{\beta}_c = \vec{\beta}(\vec{\theta}_c)$$

**near critical curve/caustic  $\rightarrow$  high magnification** 6

# Caustic

- concentration of reflected or refracted light
- in gravitational lensing, it is where
  - magnification of a point source formally diverges
  - a pair of multiple images appear/disappear



Brocken Inaglory

# Caustic crossing

critical curve  
(image plane)

caustic  
(source plane)

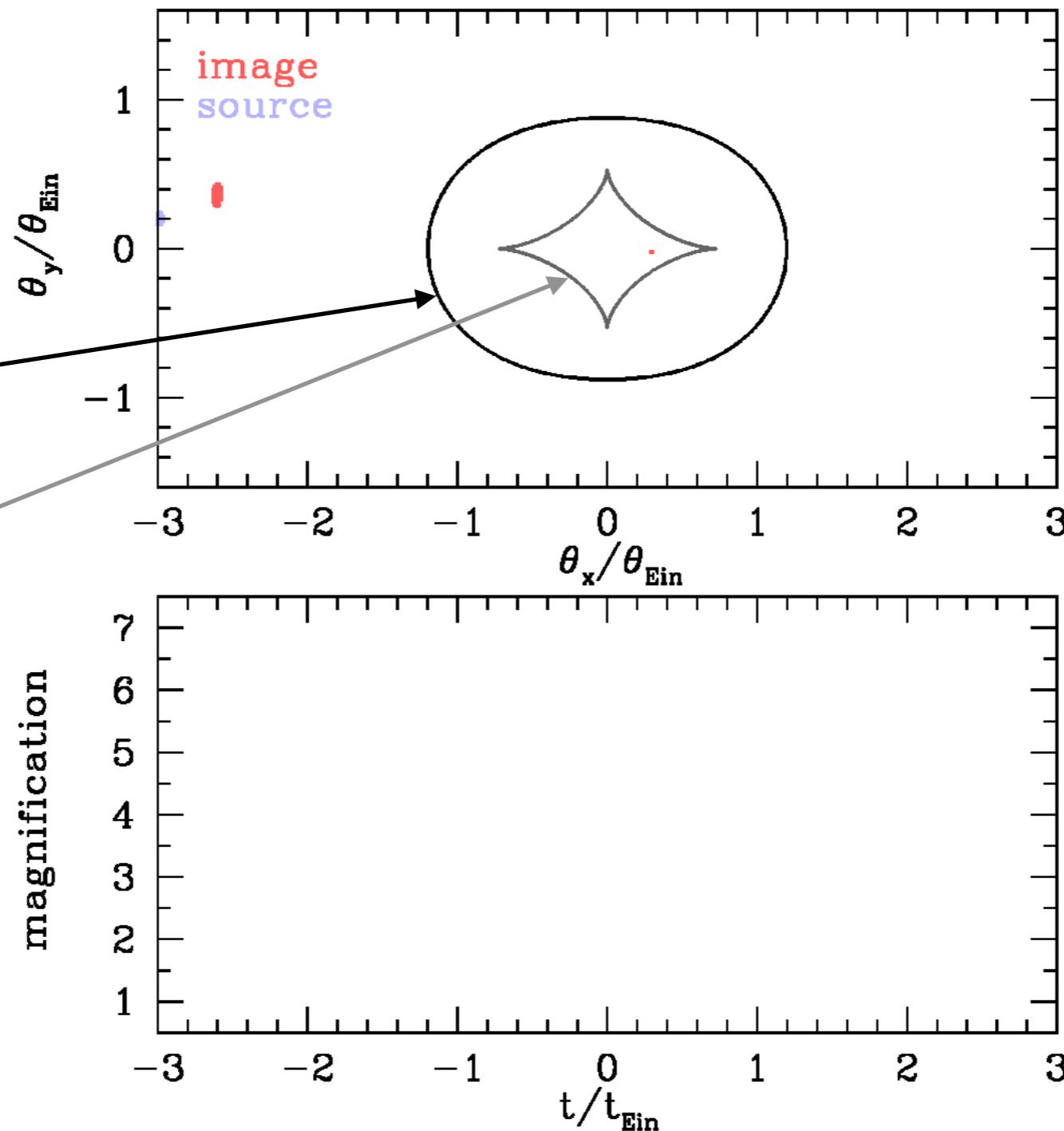
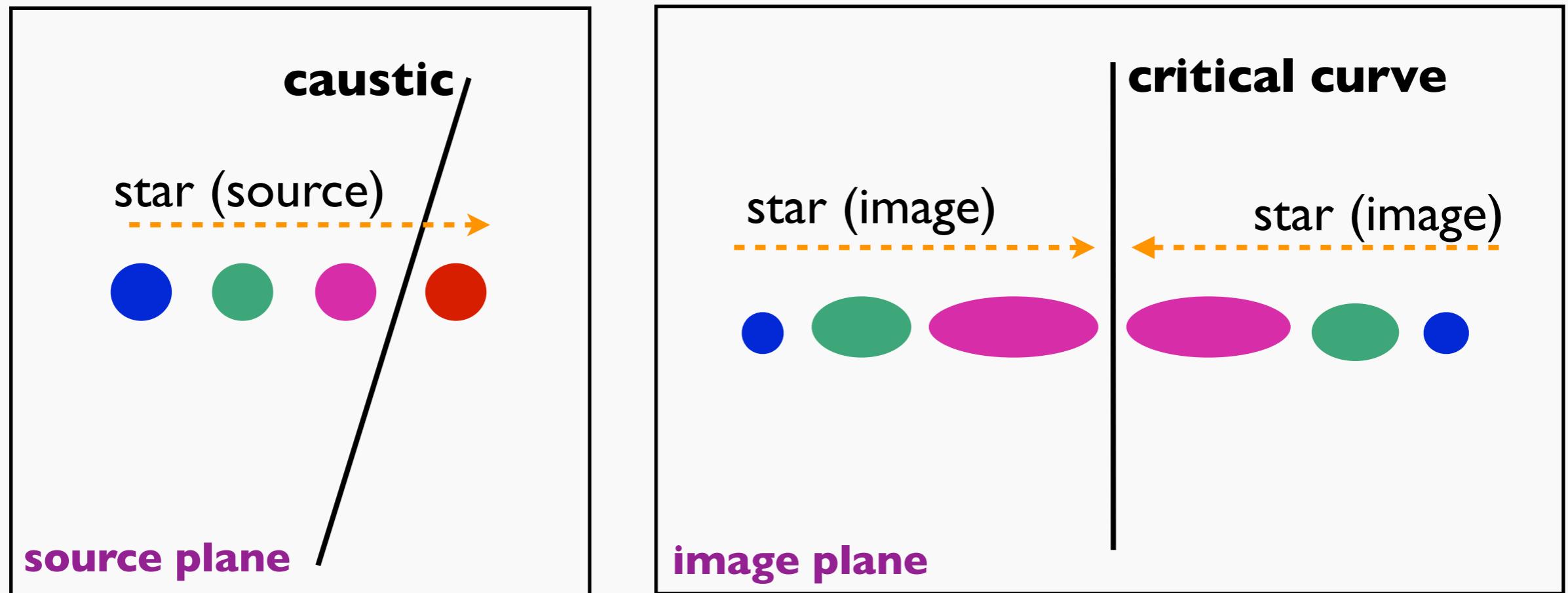
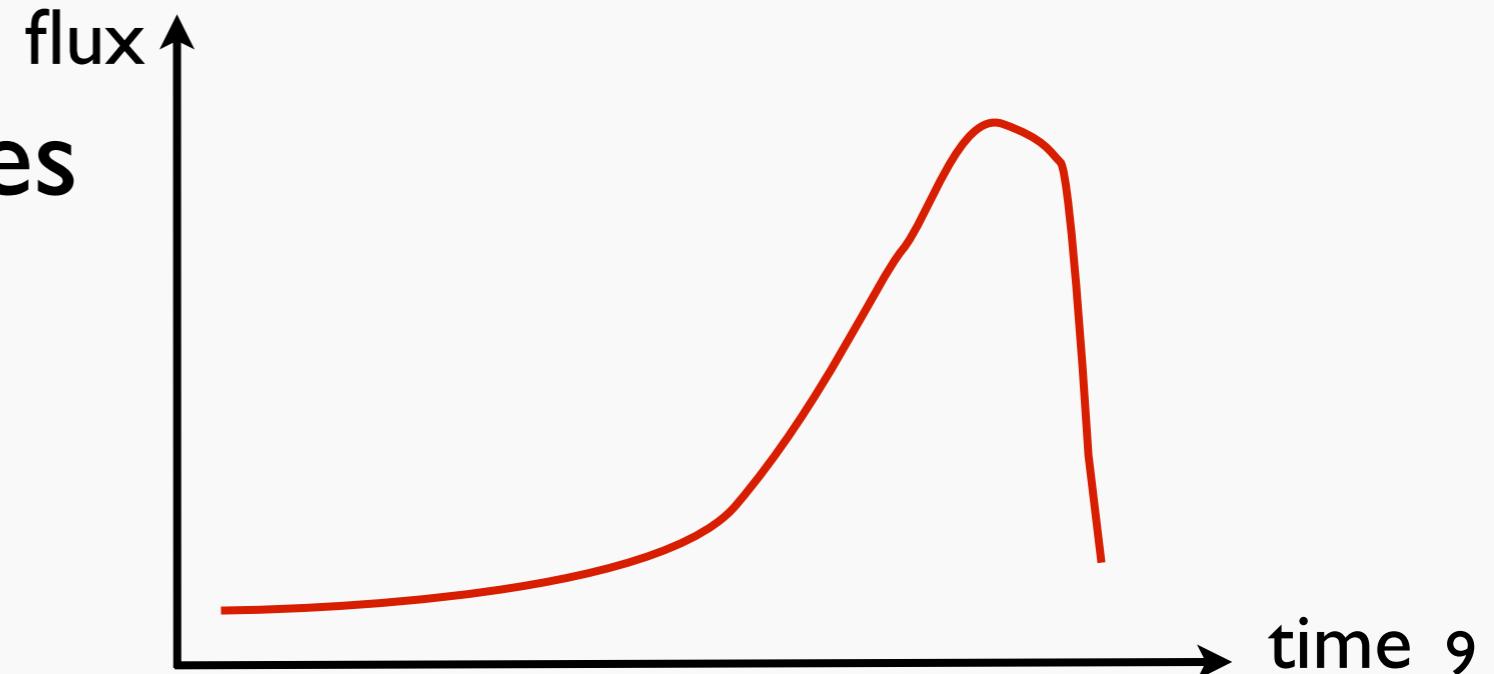


image  
(observed)  
source  
(not observed)

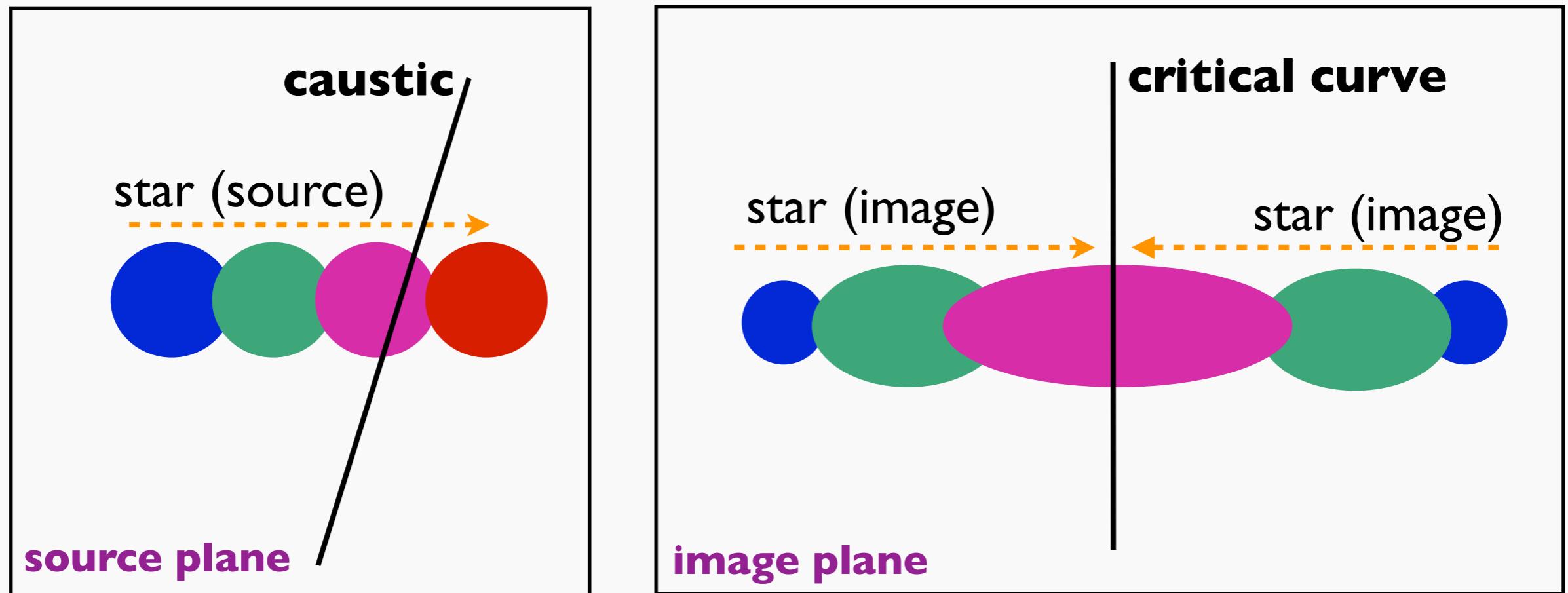
# Caustic crossing



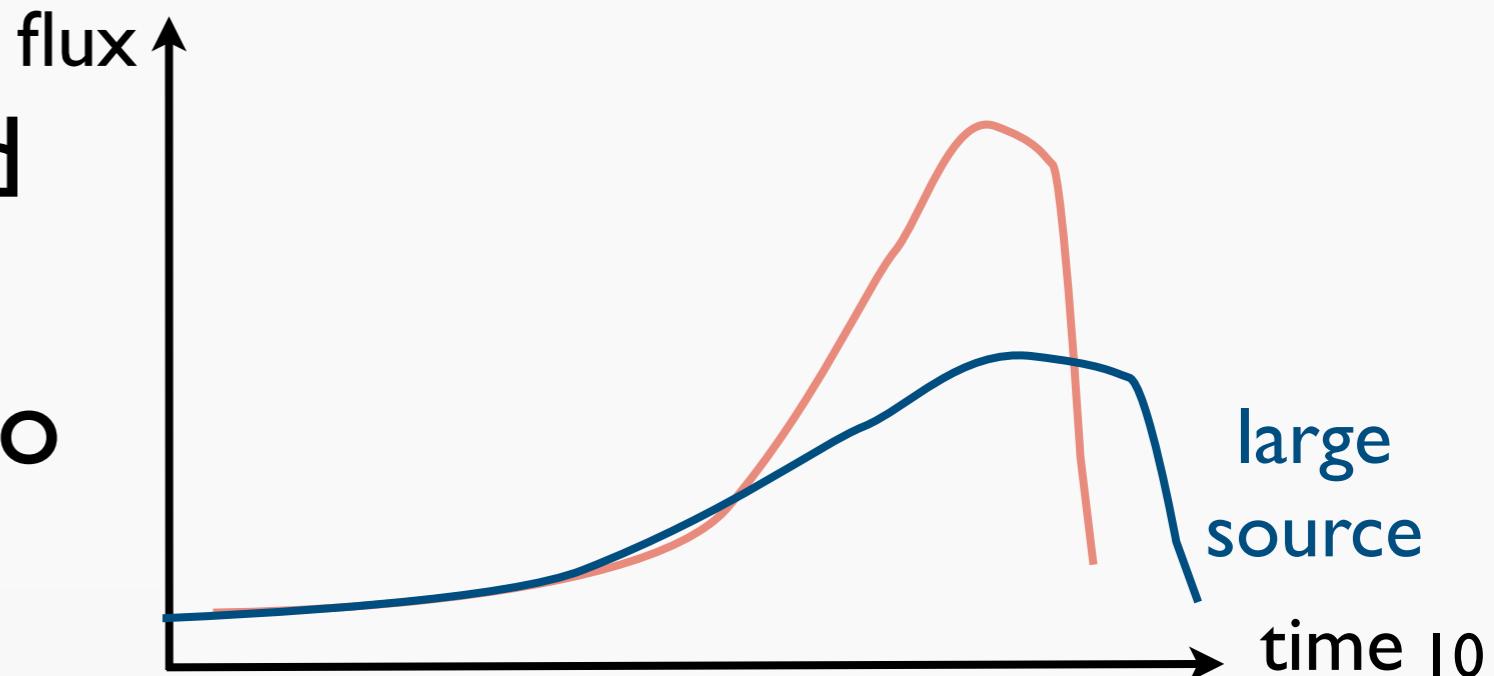
- two multiple images disappear  
→ **asymmetric light curve**



# Caustic crossing

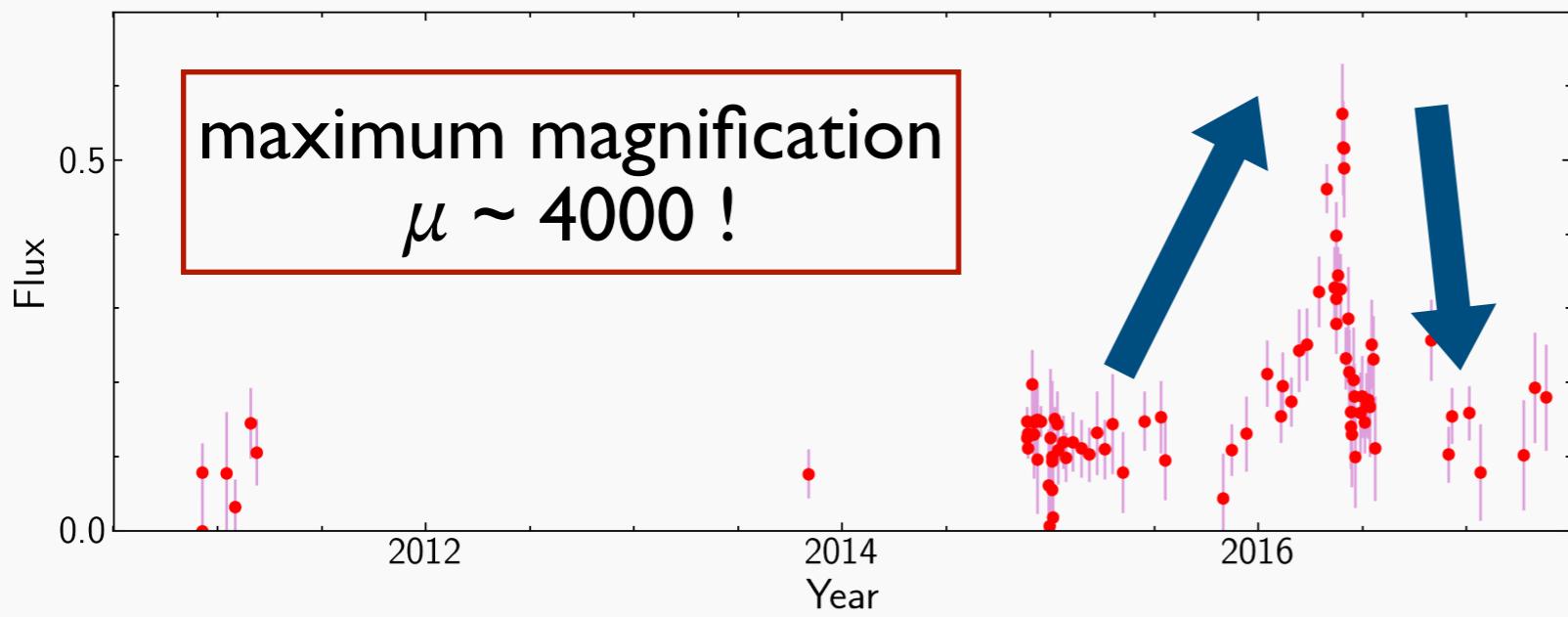
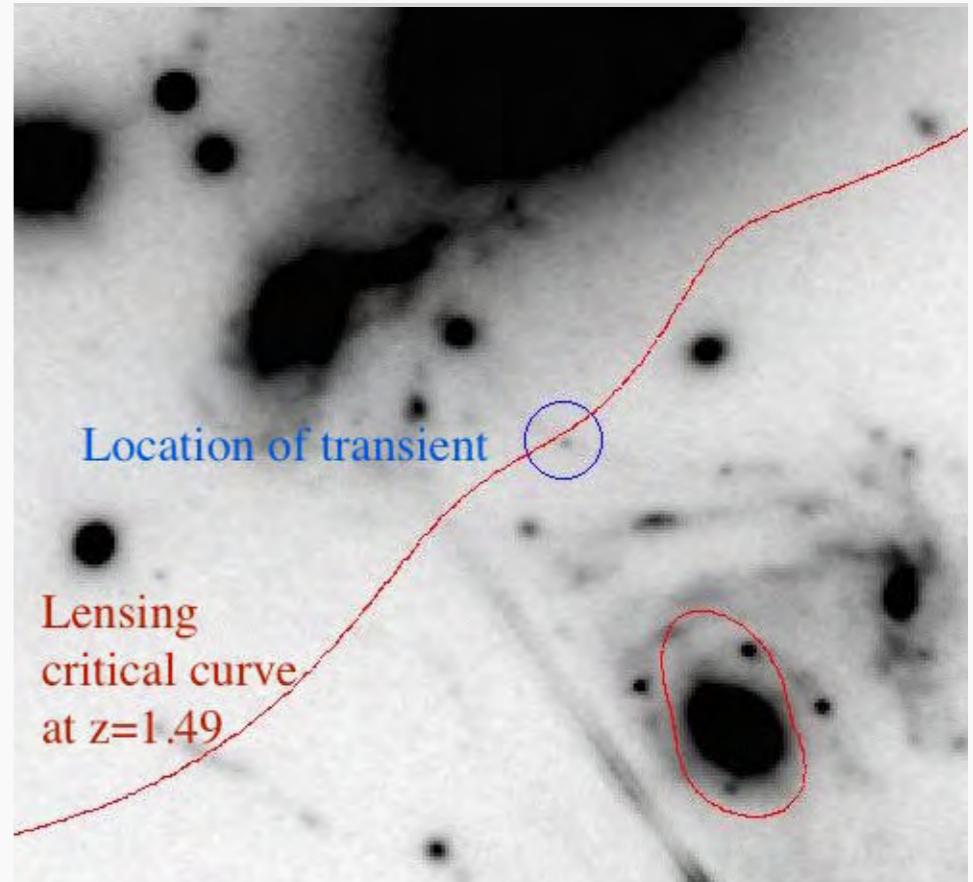
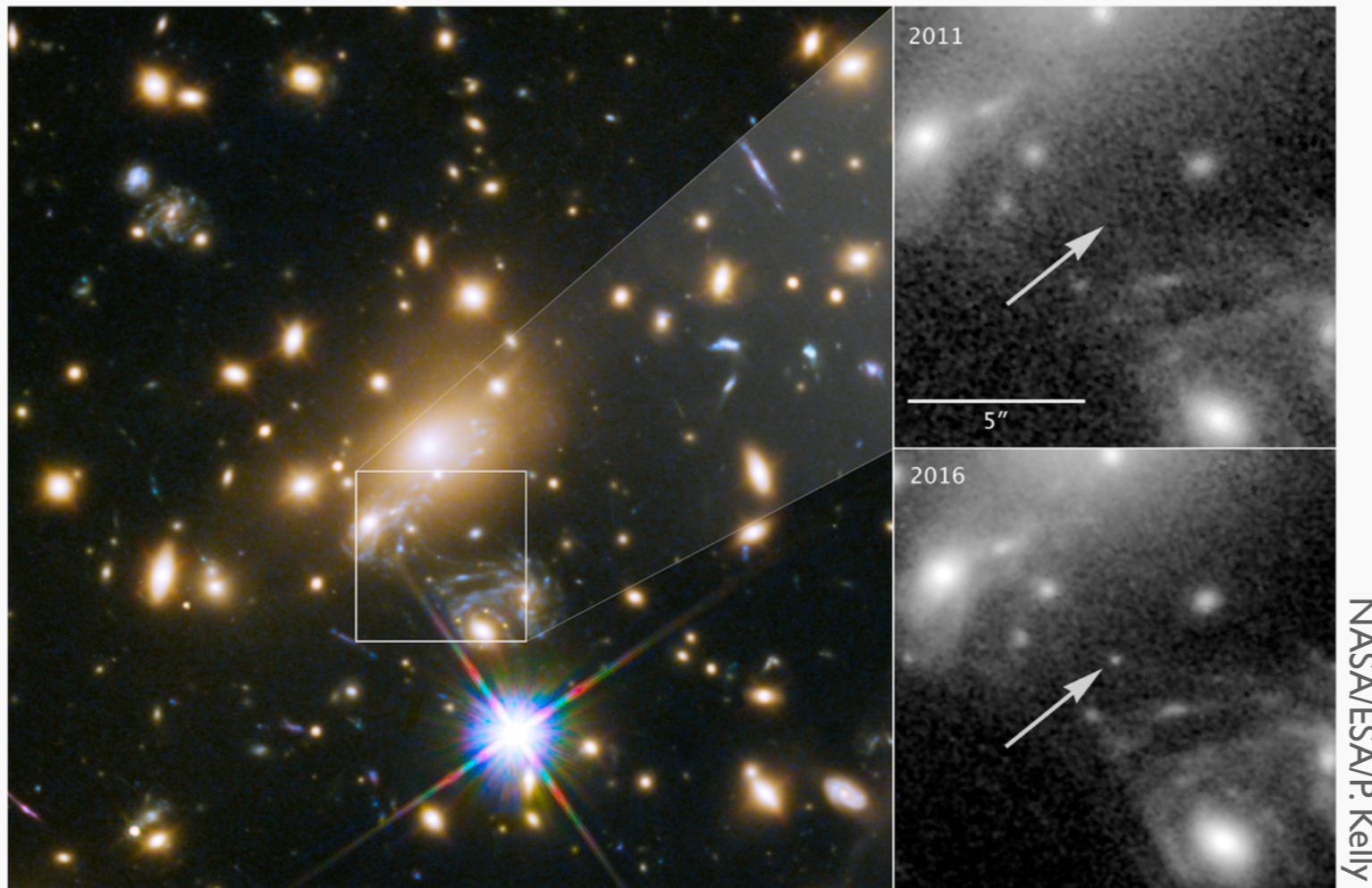


- maximum mag. and width of the light curve is sensitive to source size





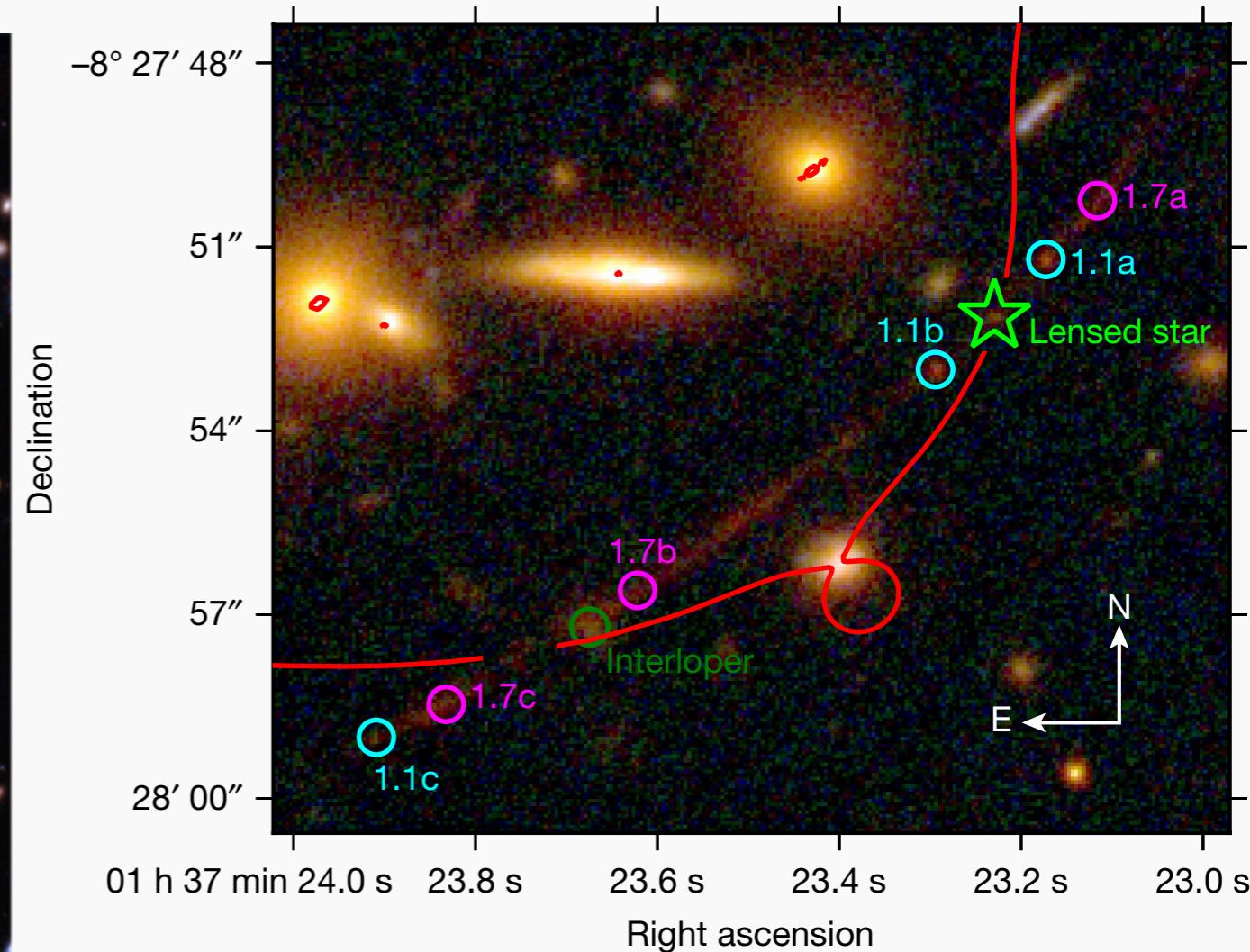
# Discovery of Icarus



- single star  
(blue supergiant)  
at  $z=1.5$



# Discovery of Earendel



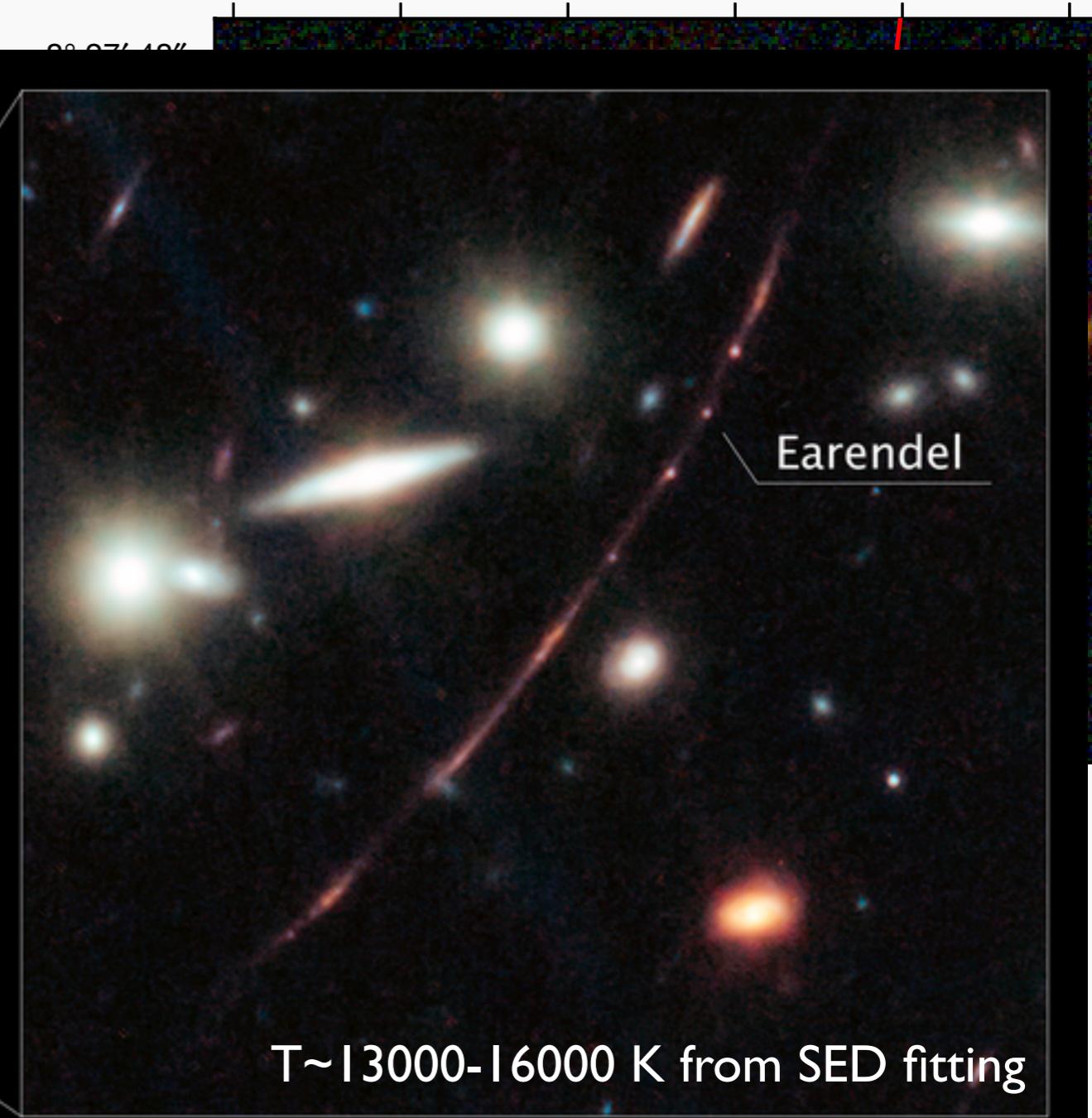
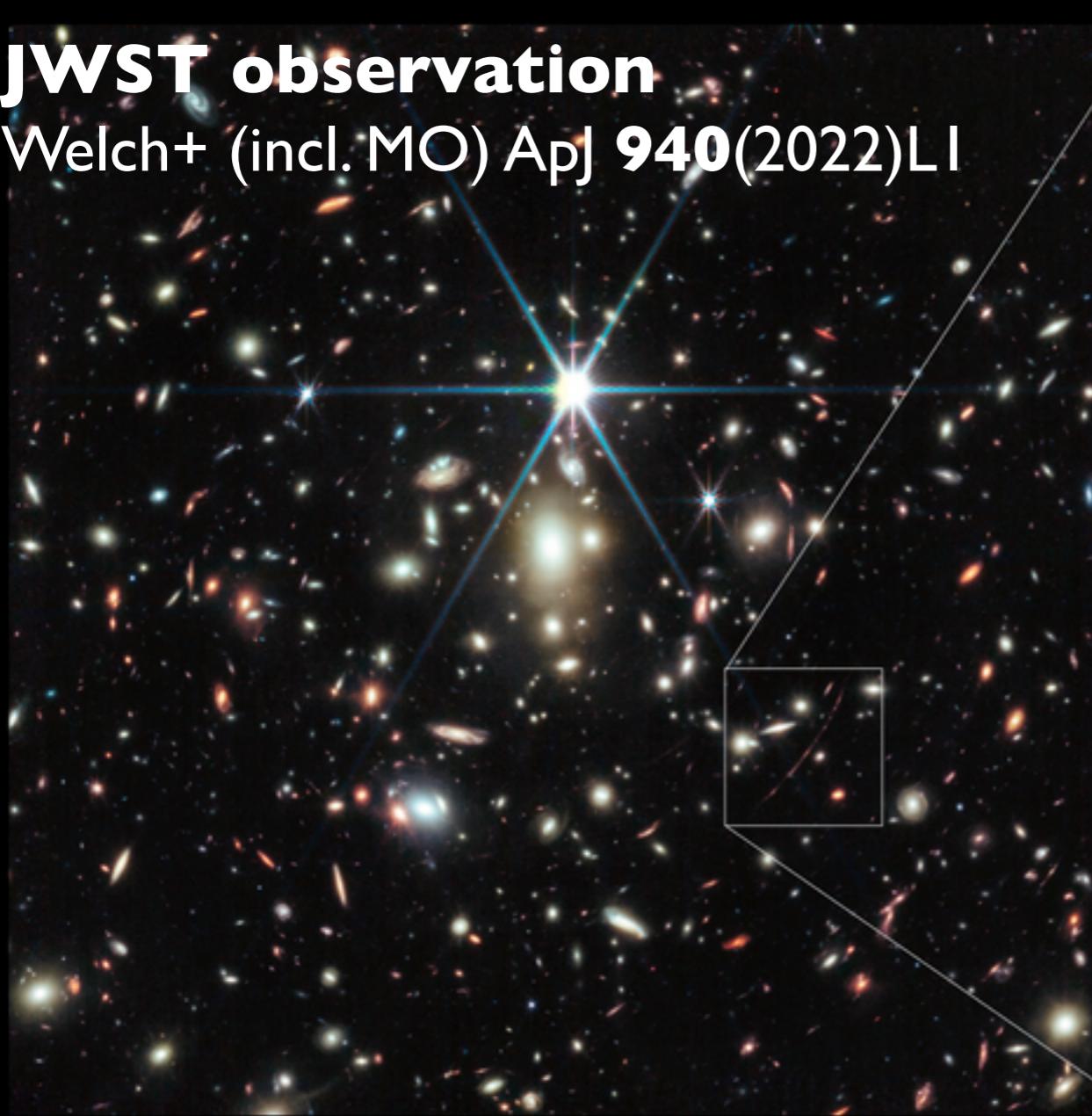
- single star at  $z=6.2$
- magnification  
 $\mu \sim 10000$  (?)



# Discovery of Earendel

**JWST observation**

Welch+ (incl. MO) ApJ 940(2022)L1



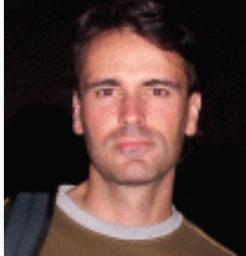
$\mu \sim 10000$  (?)

# Interpretation of caustic crossings

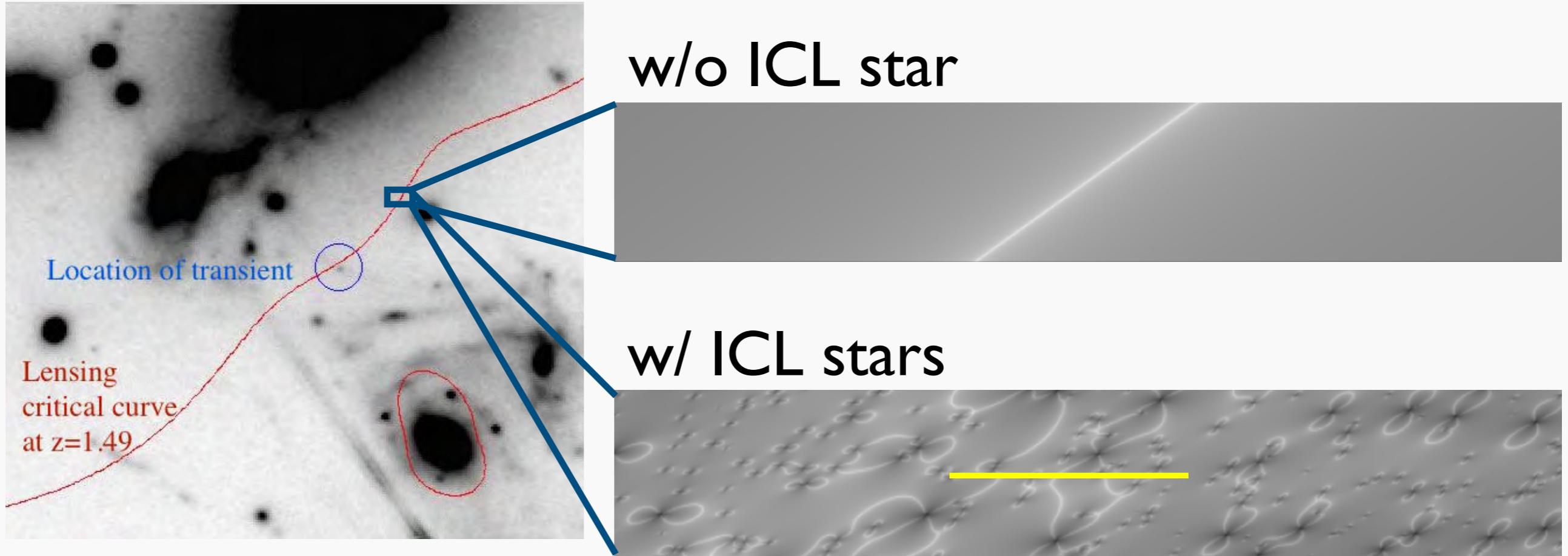
- caustic crossings look very simple, yet in fact they are not that simple because the mass distribution is not completely smooth
- non-smoothness due to stars responsible for **intra-cluster star (ICL)**
- tidal stripping of cluster member galaxies explains ICL



NASA/ESA/IAC/HFF team, STScI



# ‘Destruction’ of critical curve

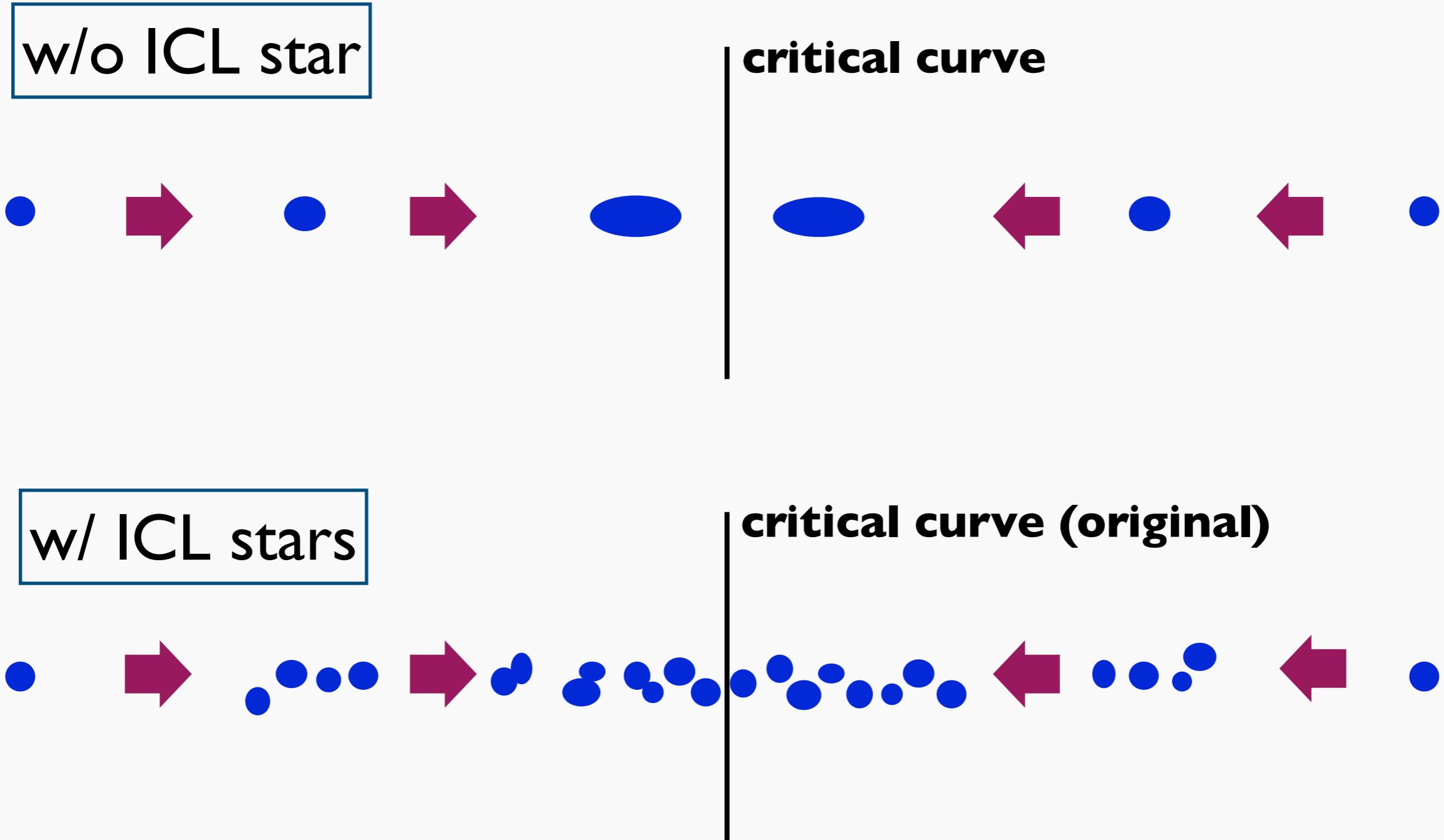


- destruction of critical curve due to overlapping Einstein radii of ICL stars

$$\tau = \frac{\Sigma}{M} \pi (\sqrt{\mu_t} \theta_E D_{ol})^2$$

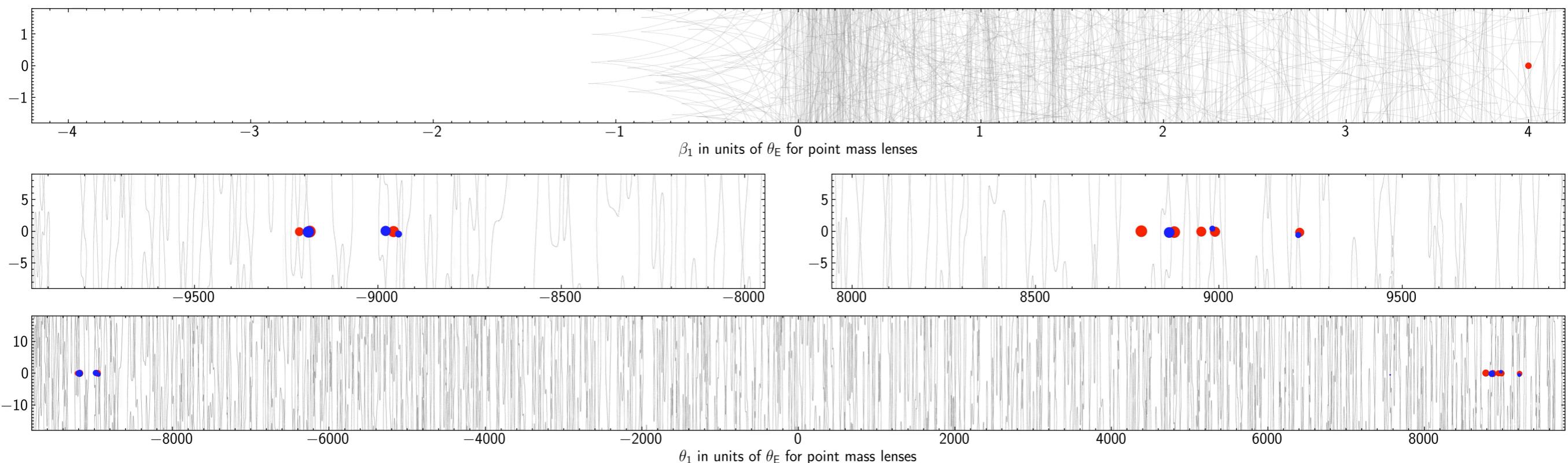
$\tau \gtrsim 1 \rightarrow \text{saturation}$

# Caustic crossing w/ ICL

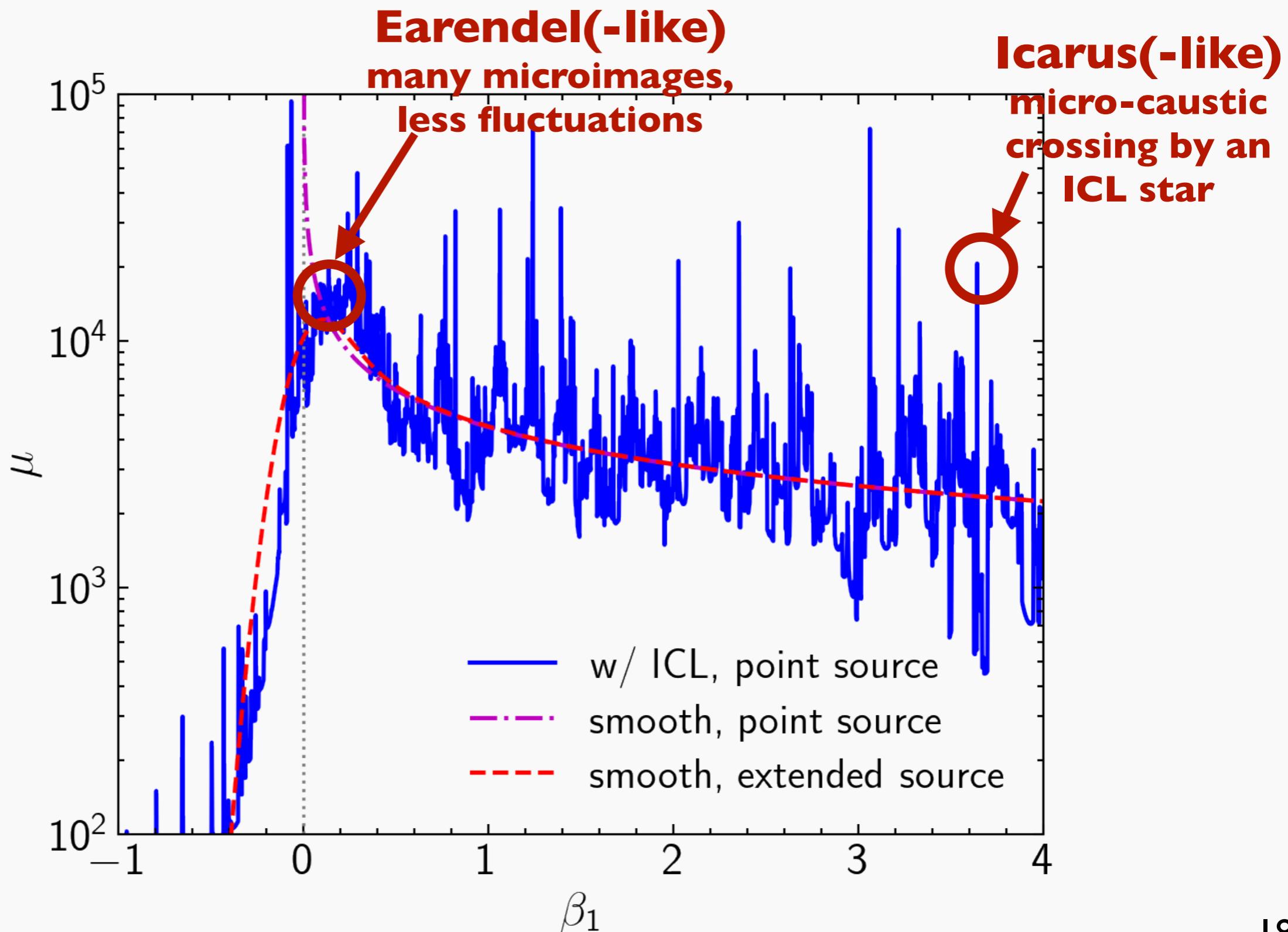


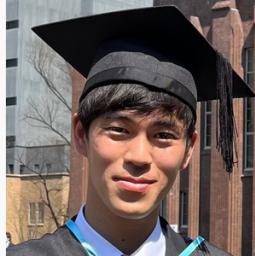


# Simulation



# Caustic crossing lightcurves

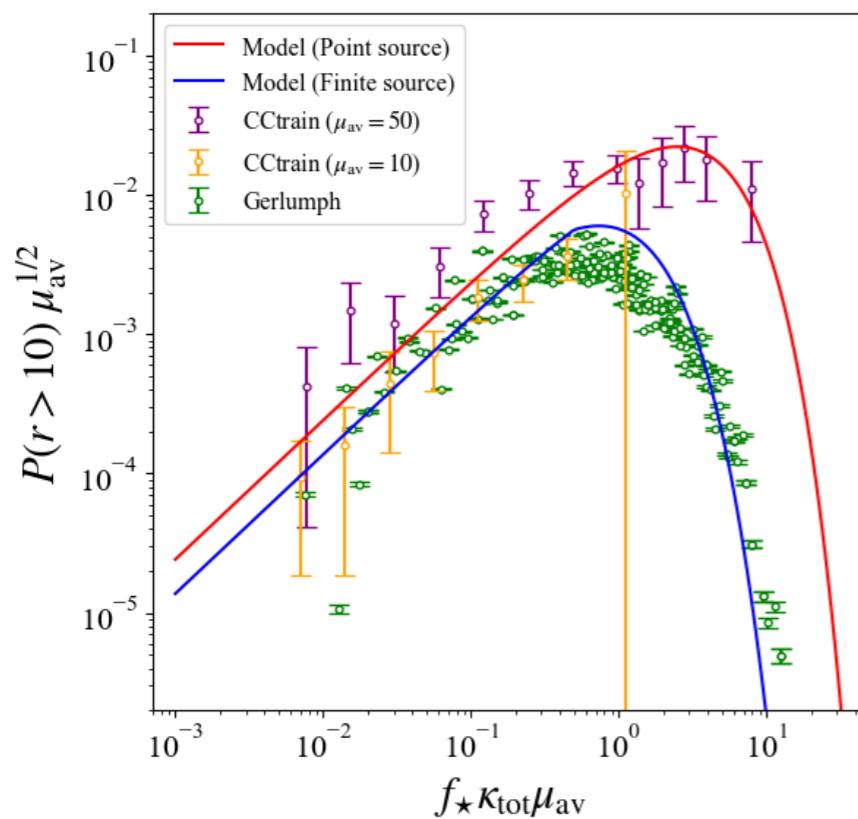




# Analytic model

- Assumption: caustic crossing probability is proportional to number of independent micro-critical curves  $N_\star^{\text{indep}} \leftarrow \text{Rayleigh dist.}$

$$\begin{aligned} \frac{dP}{d \log_{10} r} &\propto N_\star^{\text{indep}} \sqrt{\mu_{\text{av}}} r^{-2} S(r; r_{\max}) \\ &\propto f_\star \kappa_{\text{tot}} \exp(-f_\star \kappa_{\text{tot}} \mu_{\text{av}}) \sqrt{\mu_{\text{av}}} r^{-2} S(r; r_{\max}) \end{aligned}$$



parameter dependence  
in ray-tracing sim is  
well reproduced!

$\mu_{\text{av}}$  : mean magnif.

$S$  : finite source  
size effect

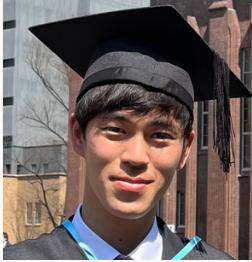
$f_\star$  : ICL fraction

$\kappa_{\text{tot}}$  : convergence

$r = \mu / \mu_{\text{av}}$

# Probing DM with caustic crossings

- caustic crossing probability is sensitive to mass fraction  $f_\star$  of compact objects
  - **primordial black holes** (PBH)
- caustic crossings appear near critical curves of clusters, which are sensitive to small-scale dark matter distribution
  - **warm dark matter** (WDM)  
**fuzzy dark matter** (FDM)

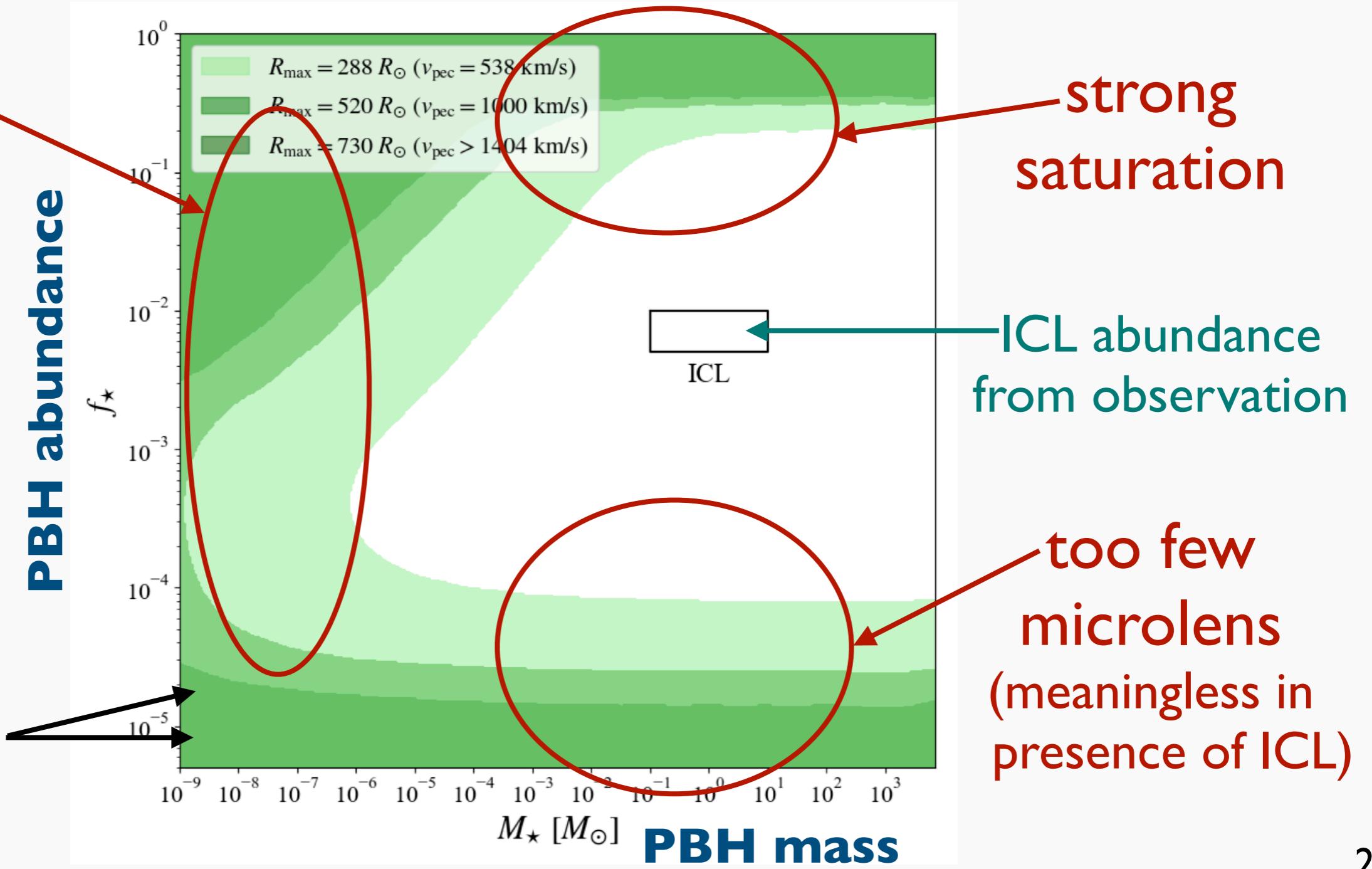


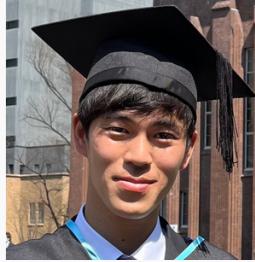
# Constraint on PBH

- constraints from event rate (w/o ICL)

finite source size effect

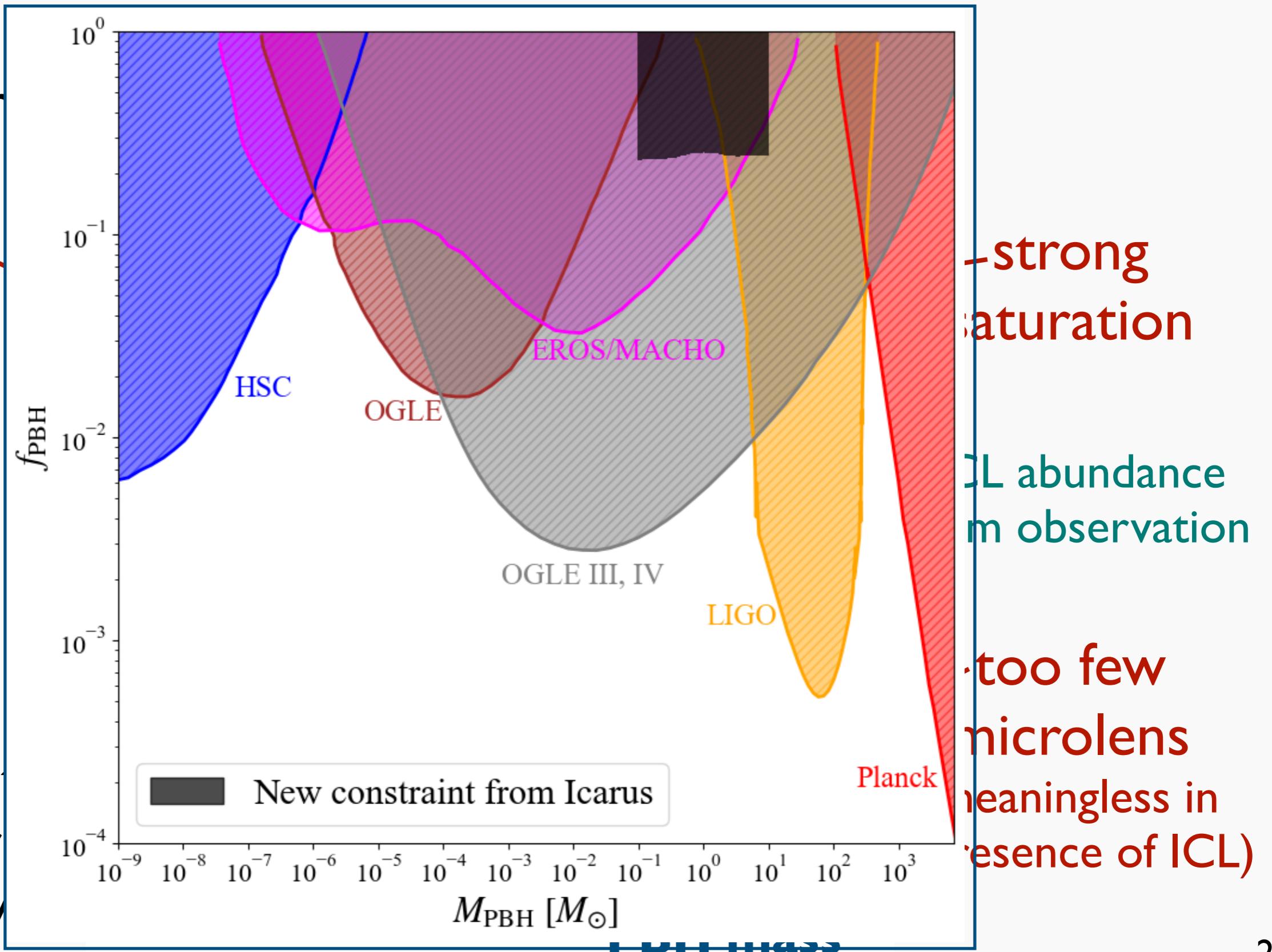
different cluster velocity





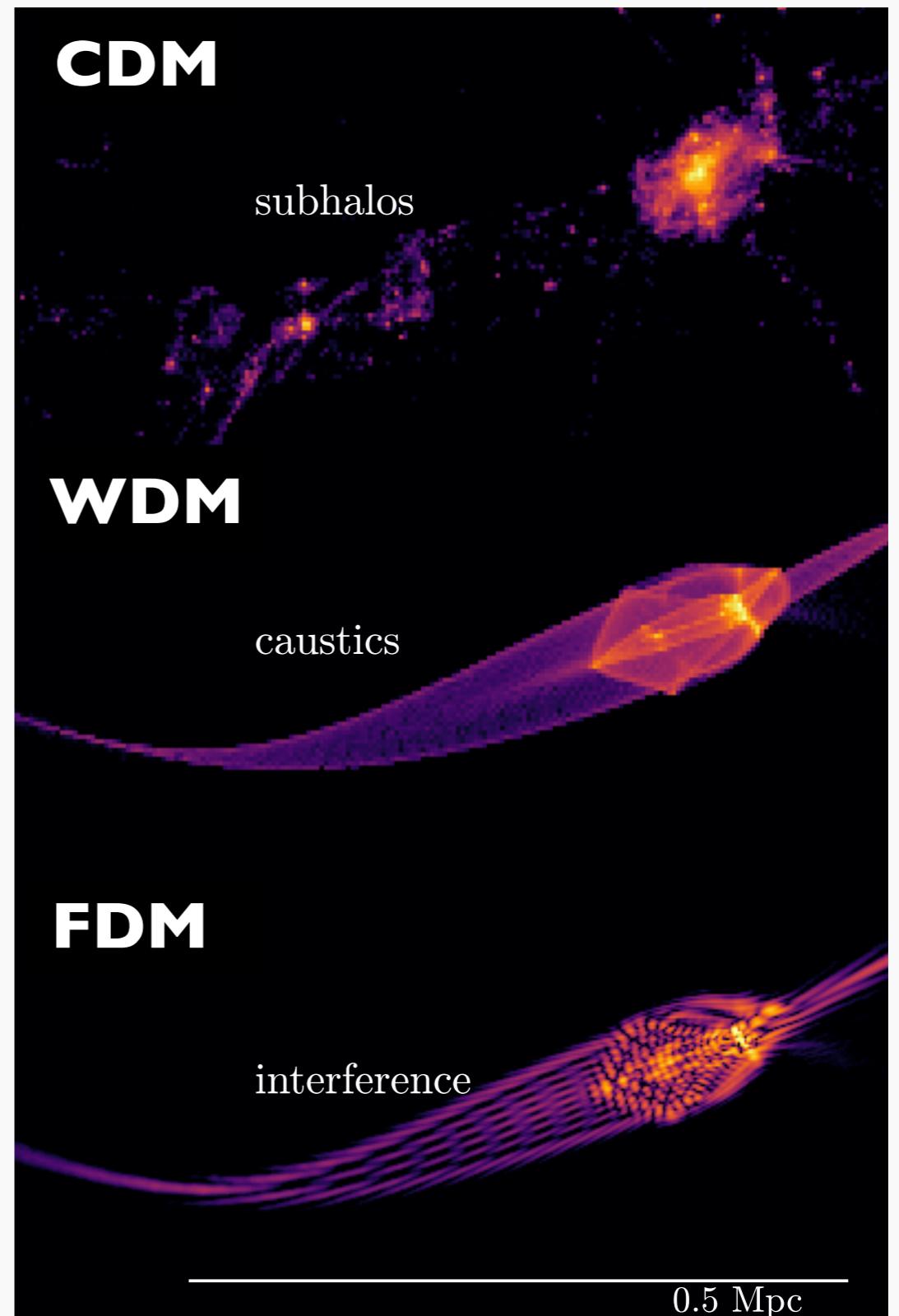
# Constraint on PBH

- core collapse supernova constraint
- finite source size effect
- different cluster velocity



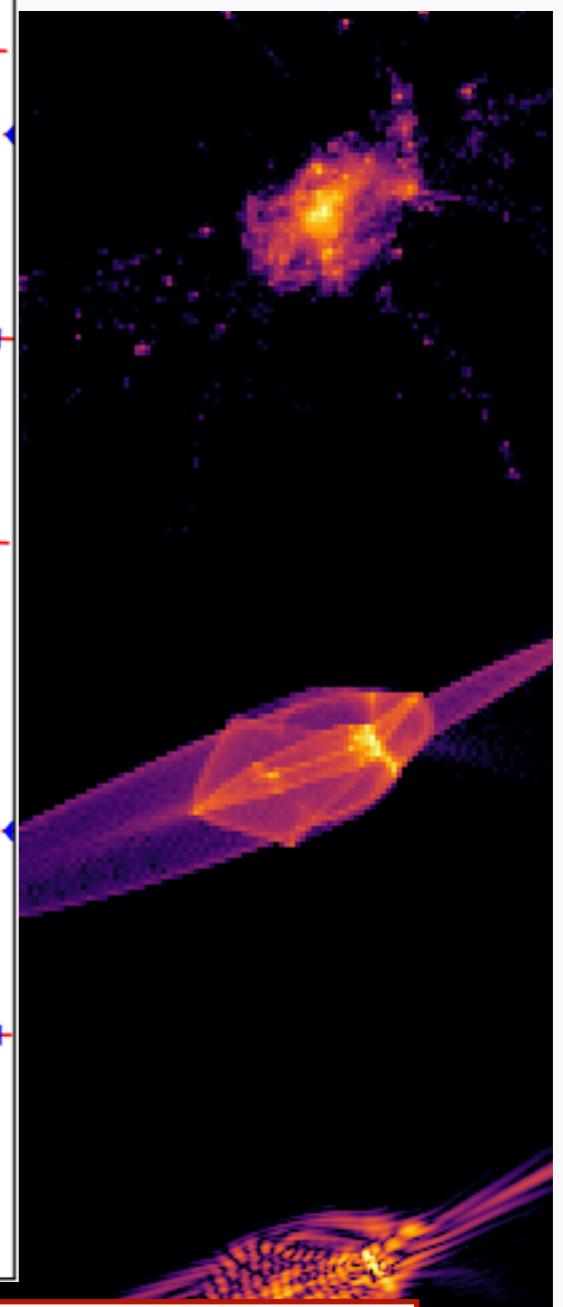
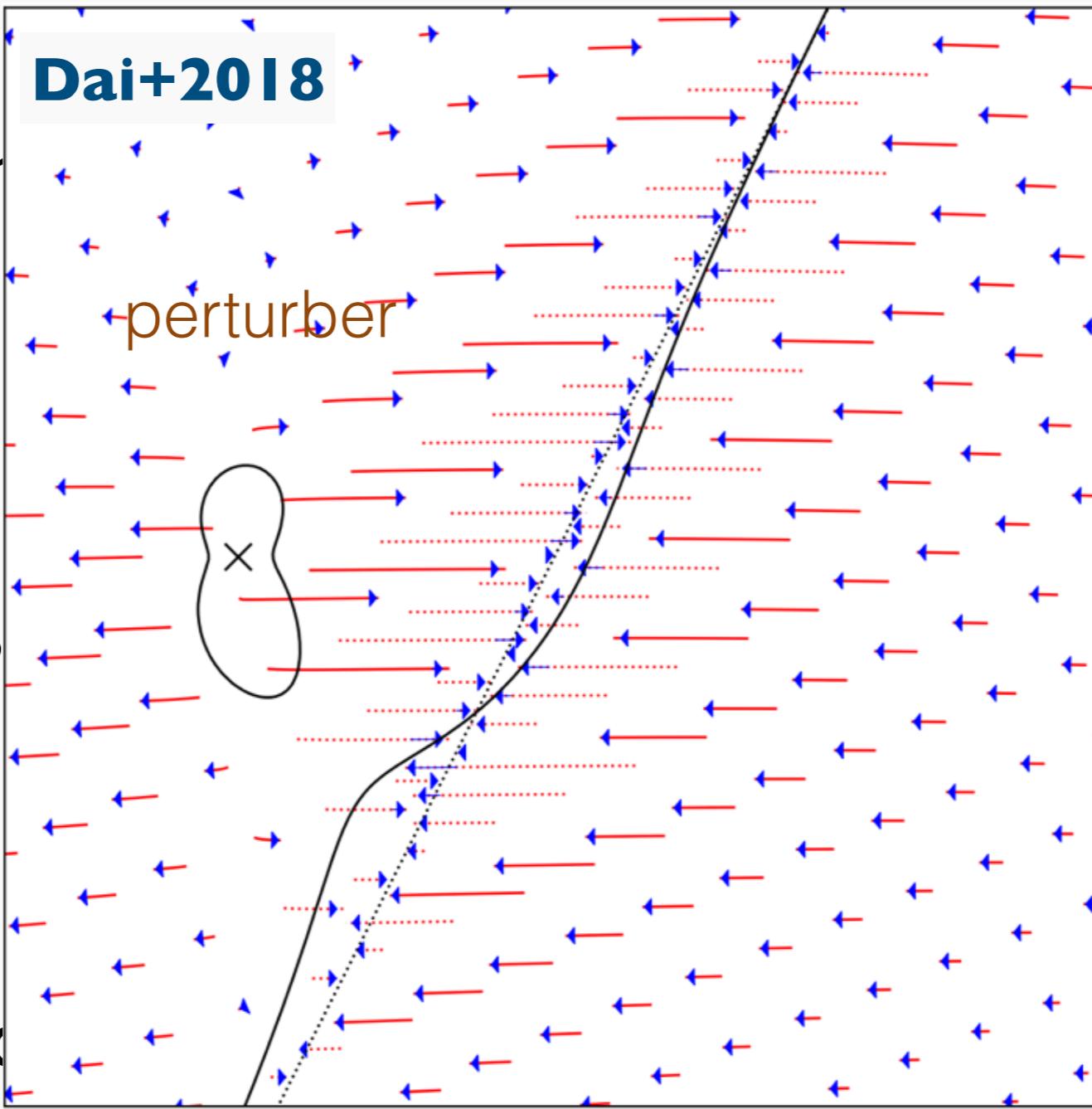
# Critical curve and dark matter

- many small clumps
- smooth, no clumps
- clumps due to quantum interference



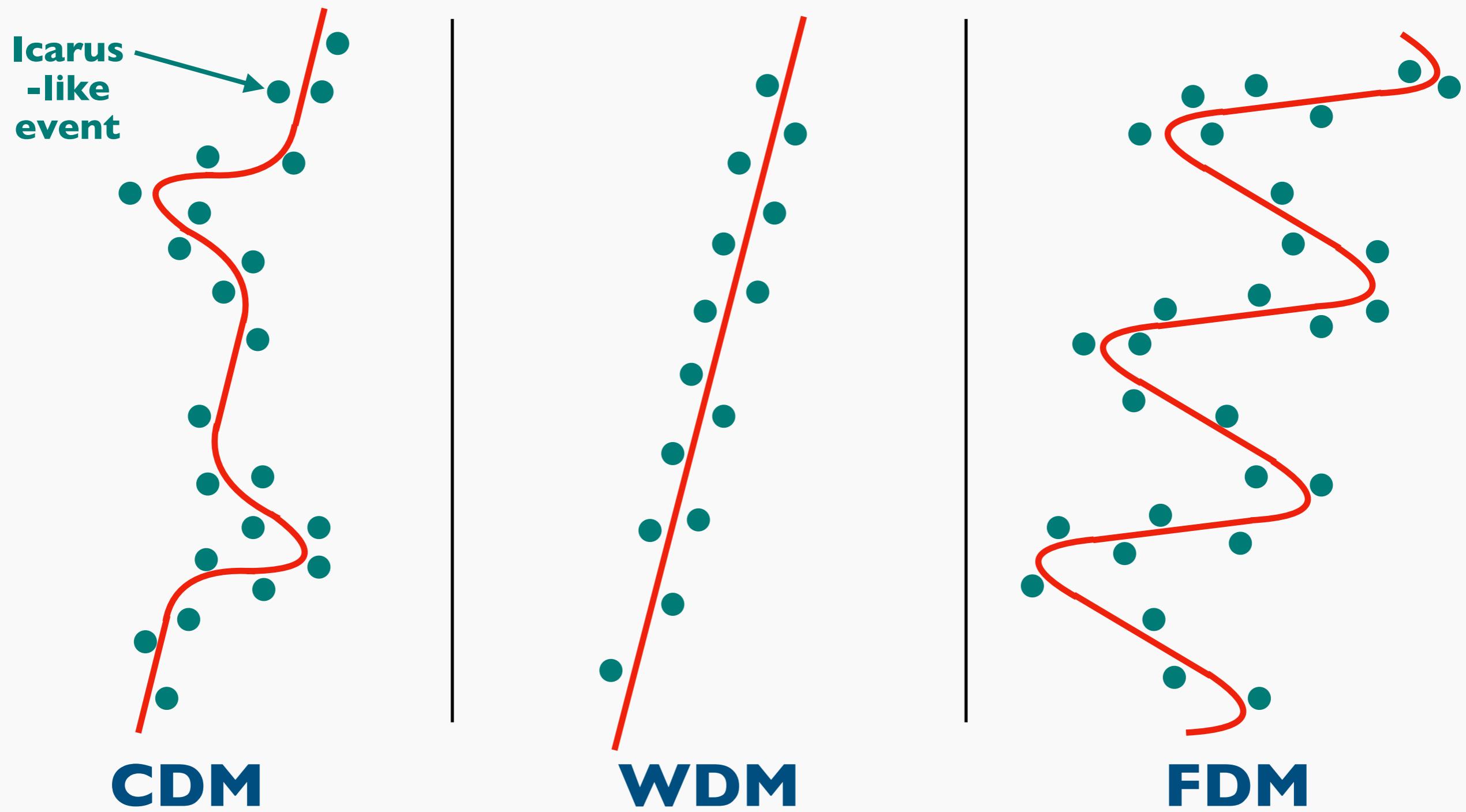
# Critical curve and dark matter

- many small clumps
- smooth, continuous
- clumps distort



**distortion of critical curve by small clumps**

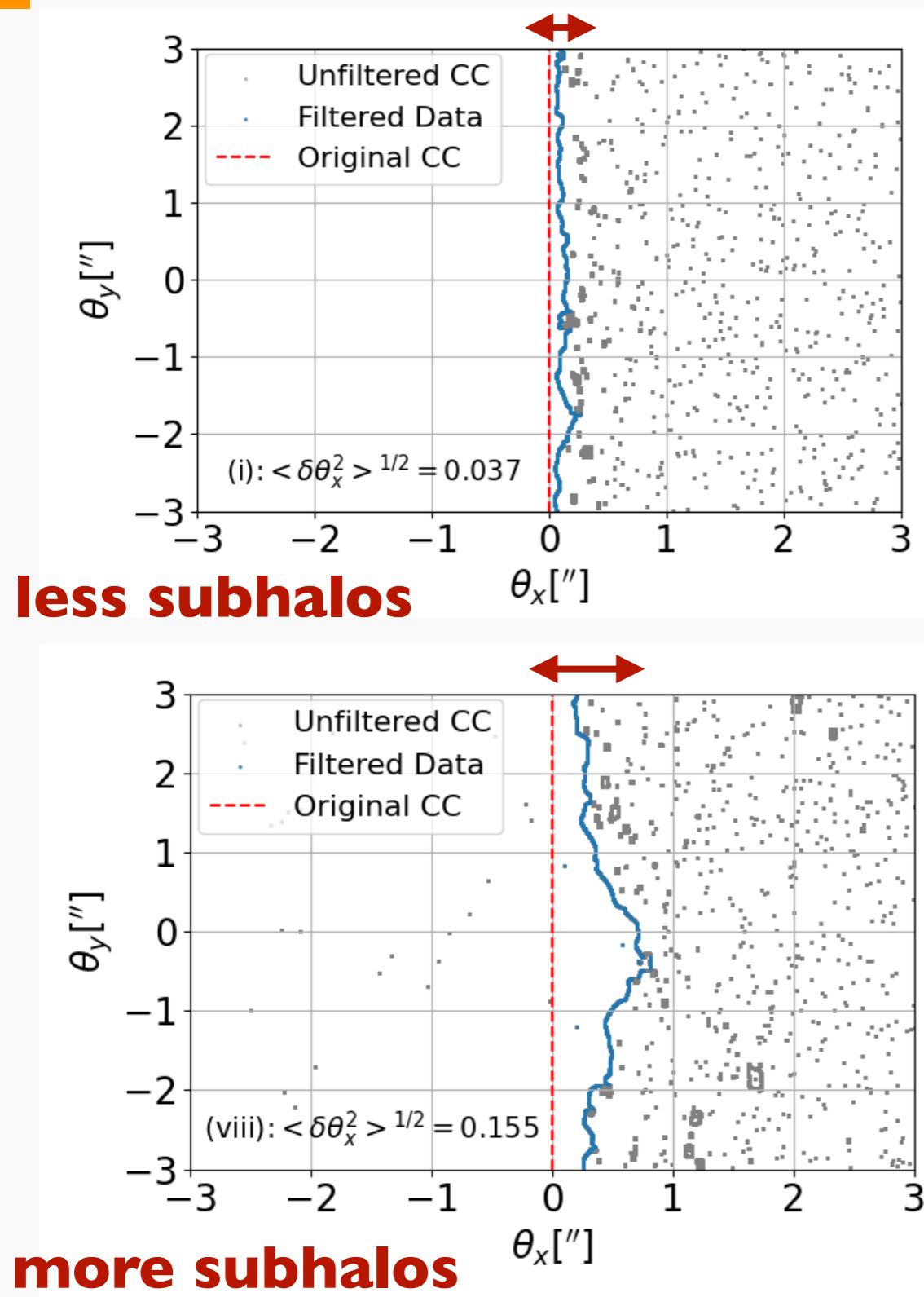
# Critical curve and caustic crossings



- can measure critical curve shape with many caustic crossings



# Critical curve fluctuations



- derive **an analytic formula** that connects  $P(k)$  of critical curve fluctuations with  $P(k)$  of DM small-scale density fluctuations!

**critical curve fluctuations**

$$P_{\delta\theta_x} = \frac{3}{2\epsilon^2} P_{\delta\kappa}$$

$\epsilon \sim 1/\theta_{\text{Ein}}$

**DM small-scale density fluctuations**

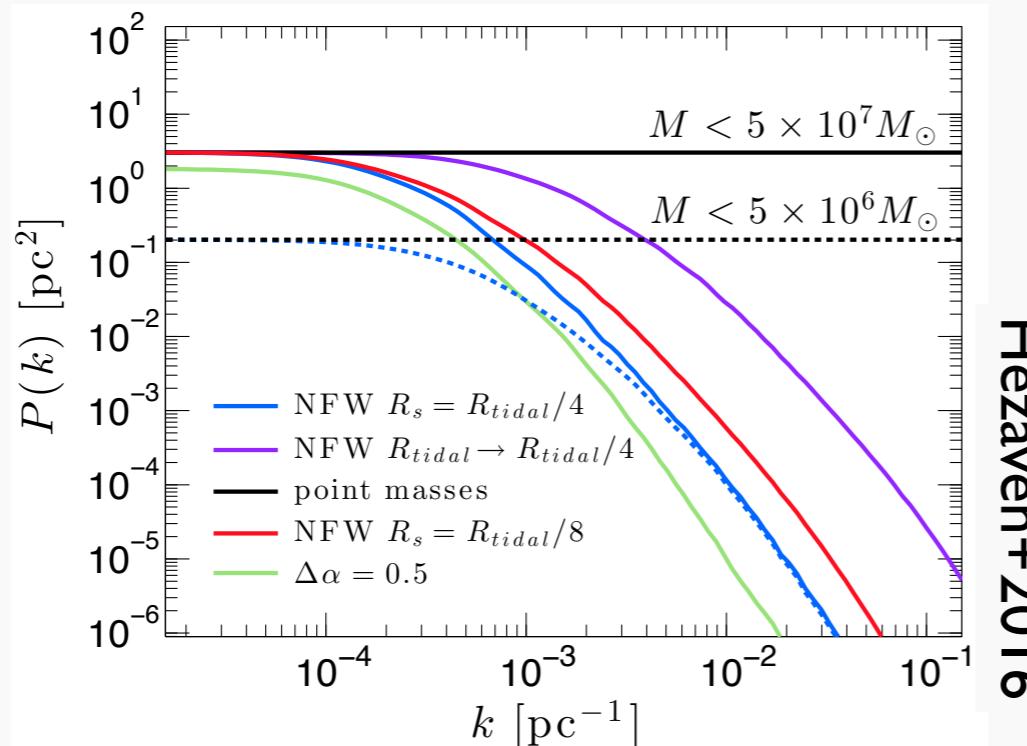
- formula validated with simple simulations

# $P(k)$ of CDM and WDM

- can be calculated with halo-model approach  
(e.g, Hezaveh+2016)

$$P(k) = \int dM \frac{dn}{dM} |\tilde{u}(k)|^2$$

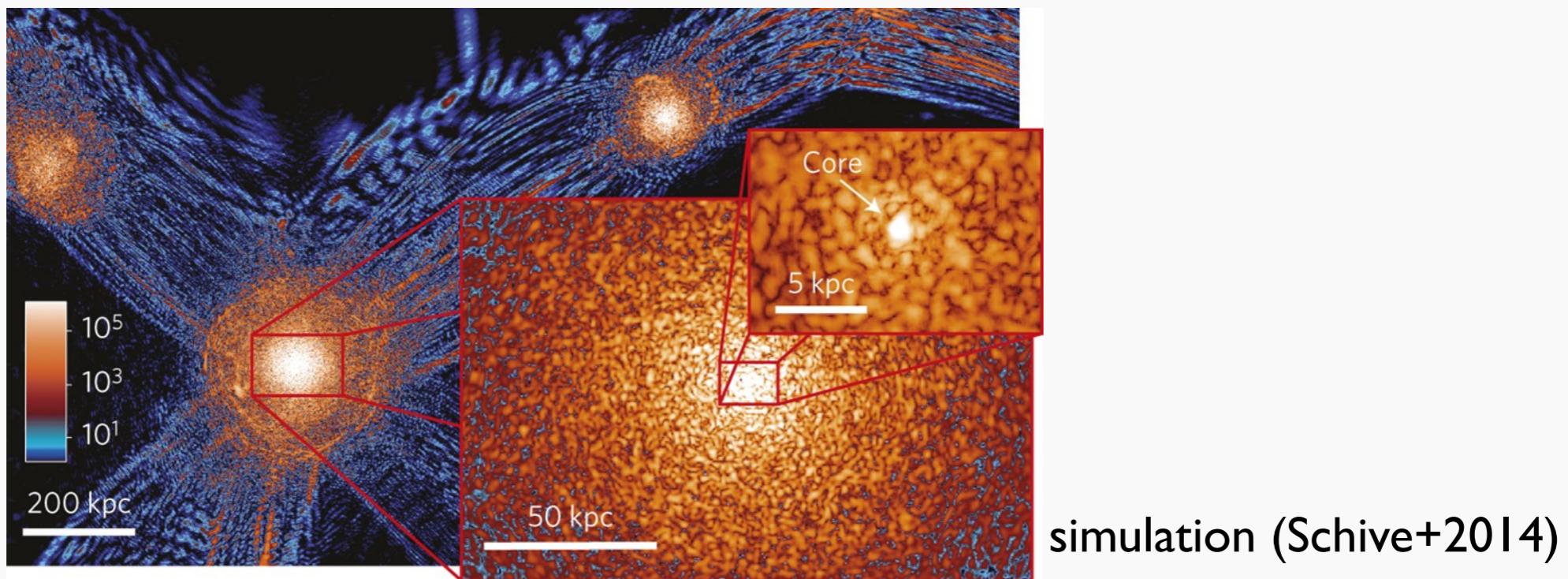
**subhalo mass function**   **Fourier transform of NFW profile**

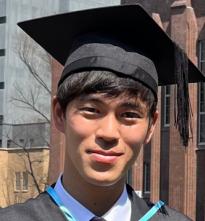


Hezaveh+2016

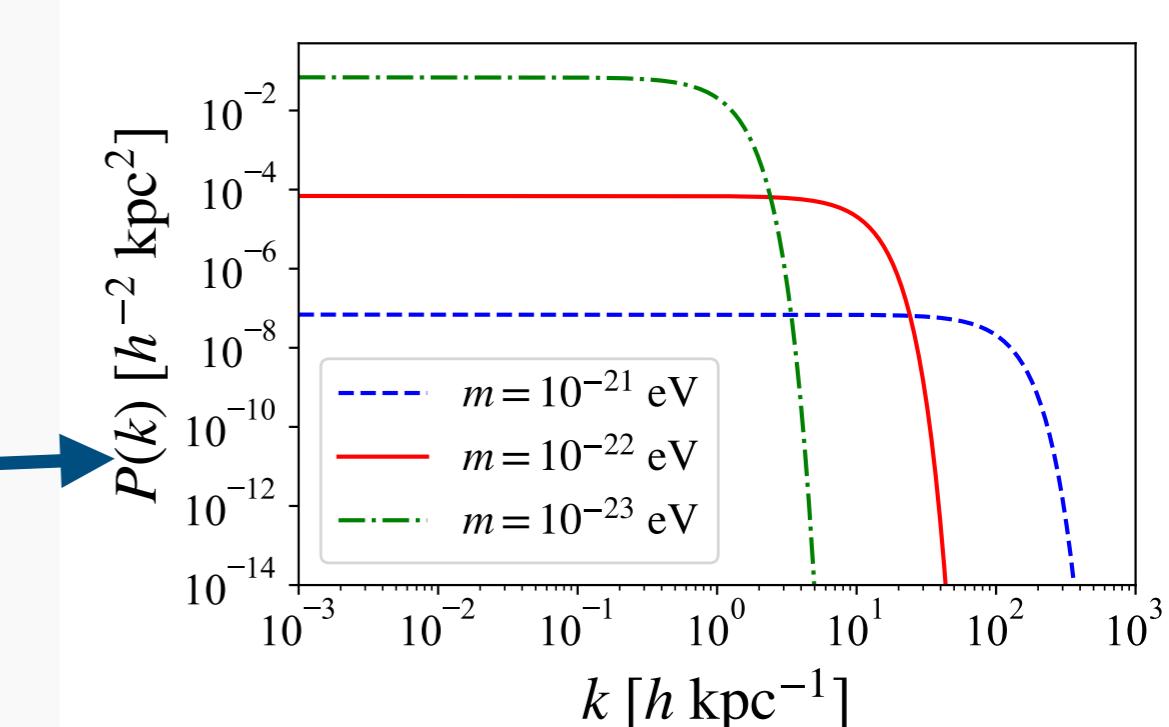
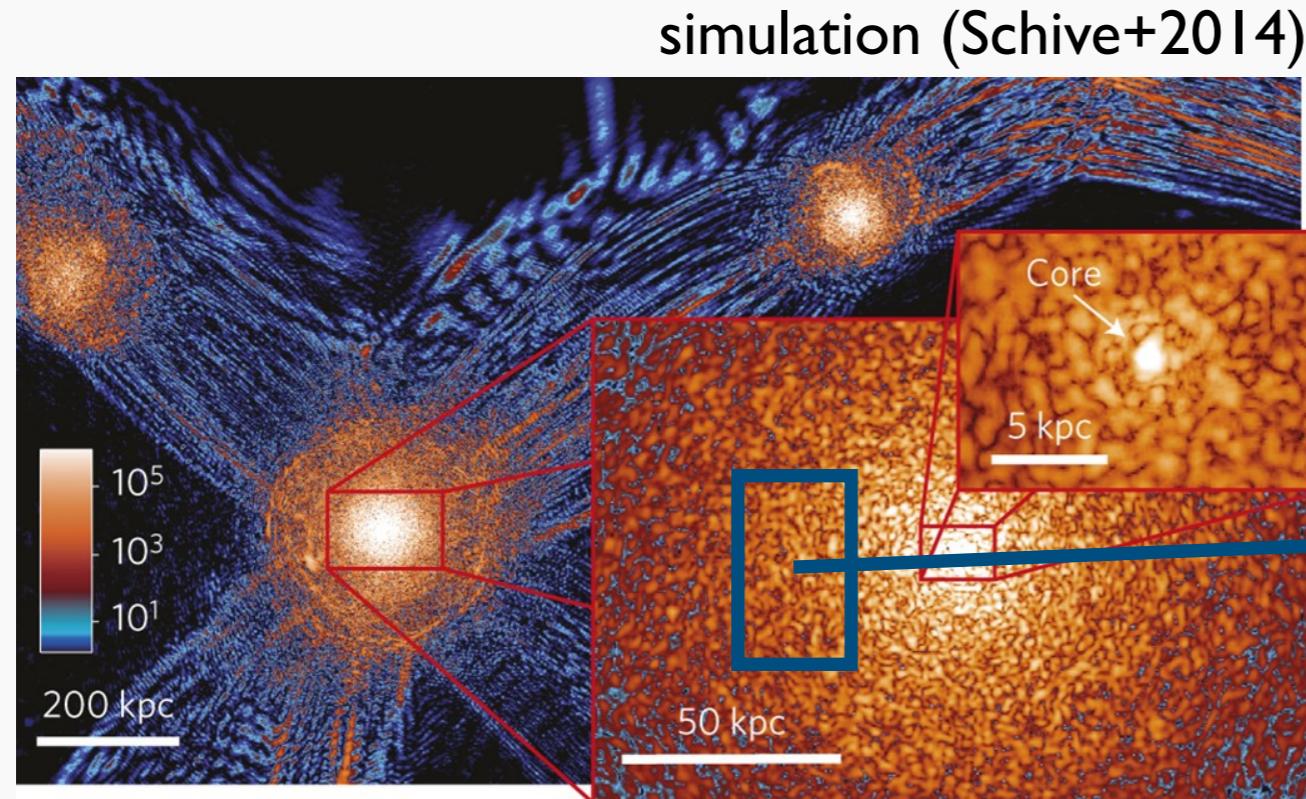
# $P(k)$ of FDM?

- wave effect below de Broglie wavelength
- dark matter halo consists of quantum clumps with their size  $\sim \lambda_{dB}$





# Analytic model of $P(k)$ in FDM



- derive  $P(k)$  assuming superposition of Gaussian clumps

$$P(k) = \left( \frac{\Sigma_h(x)}{\Sigma_h(x) + \Sigma_b(x)} \right)^2 \frac{4\pi \lambda_c^3}{3r_h(x)} \exp\left(-\frac{\lambda_c^2 k^2}{4}\right)$$

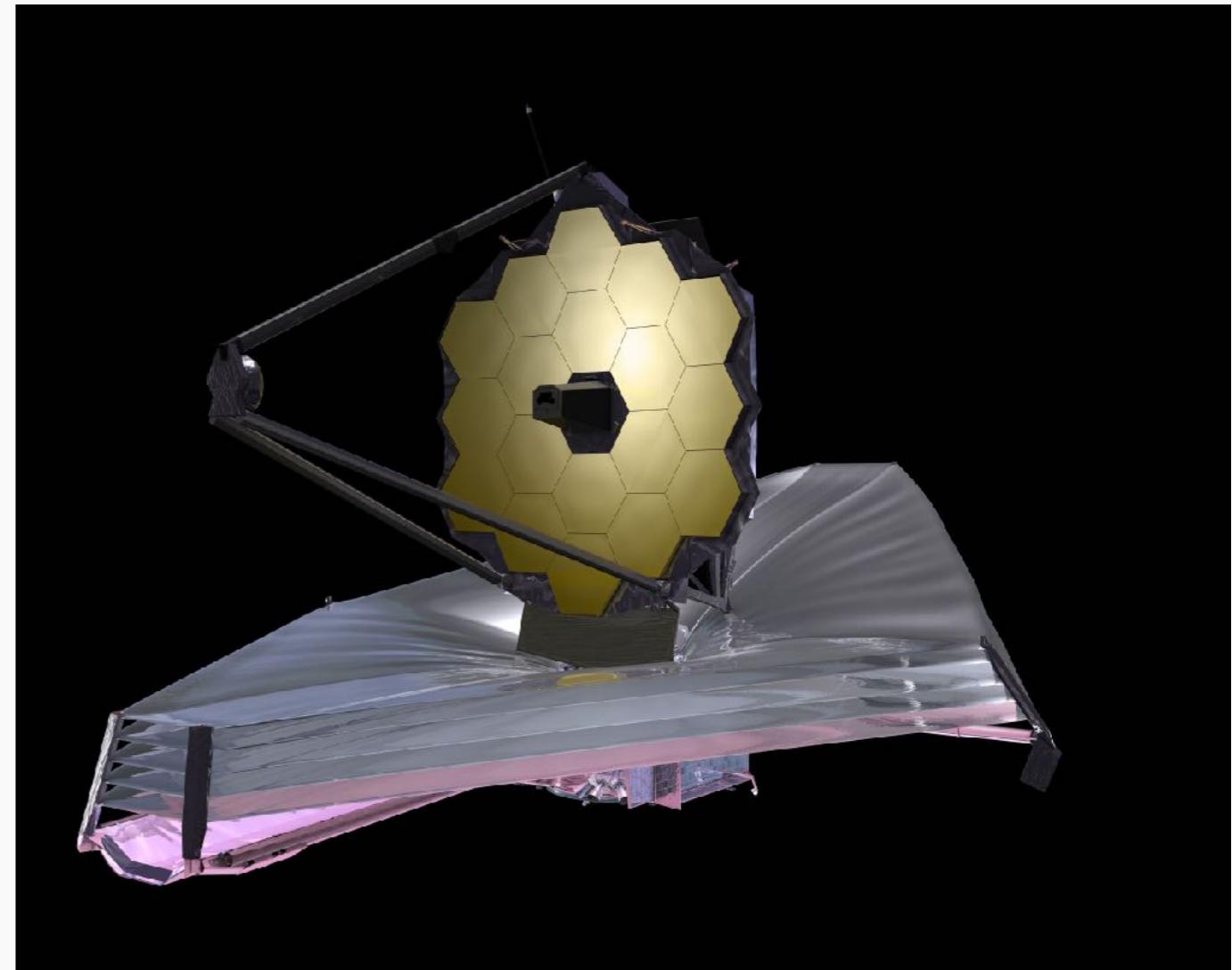
$$r_h(x) = \frac{\Sigma_h^2(x)}{\int_Z dz \rho_h^2(r)} = \frac{\left(\int_Z dz \rho_h(r)\right)^2}{\int_Z dz \rho_h^2(r)}$$

# Progress with JWST

NASA



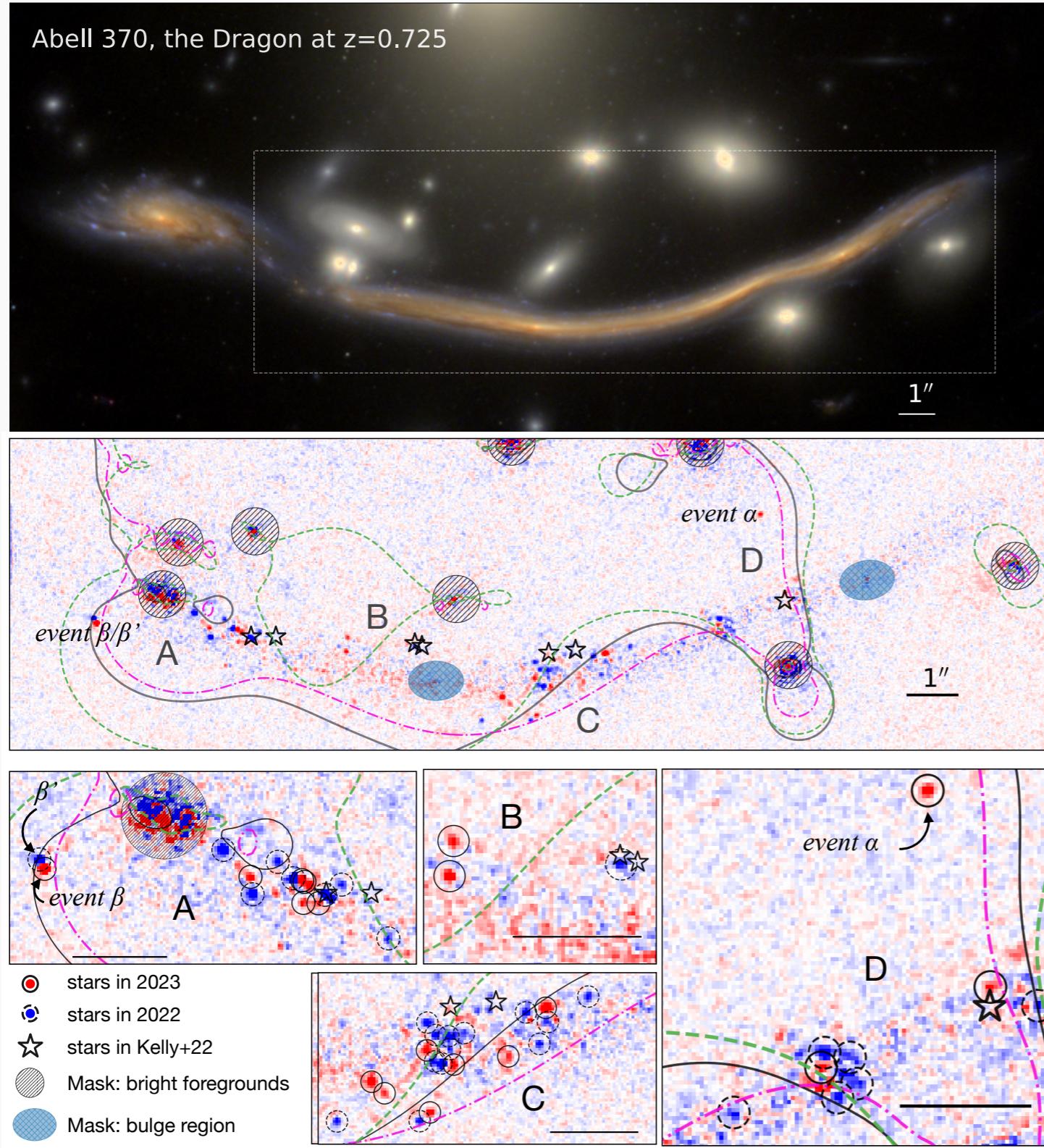
NASA



- more caustic crossings needed to study DM
- **JWST** is the solution!

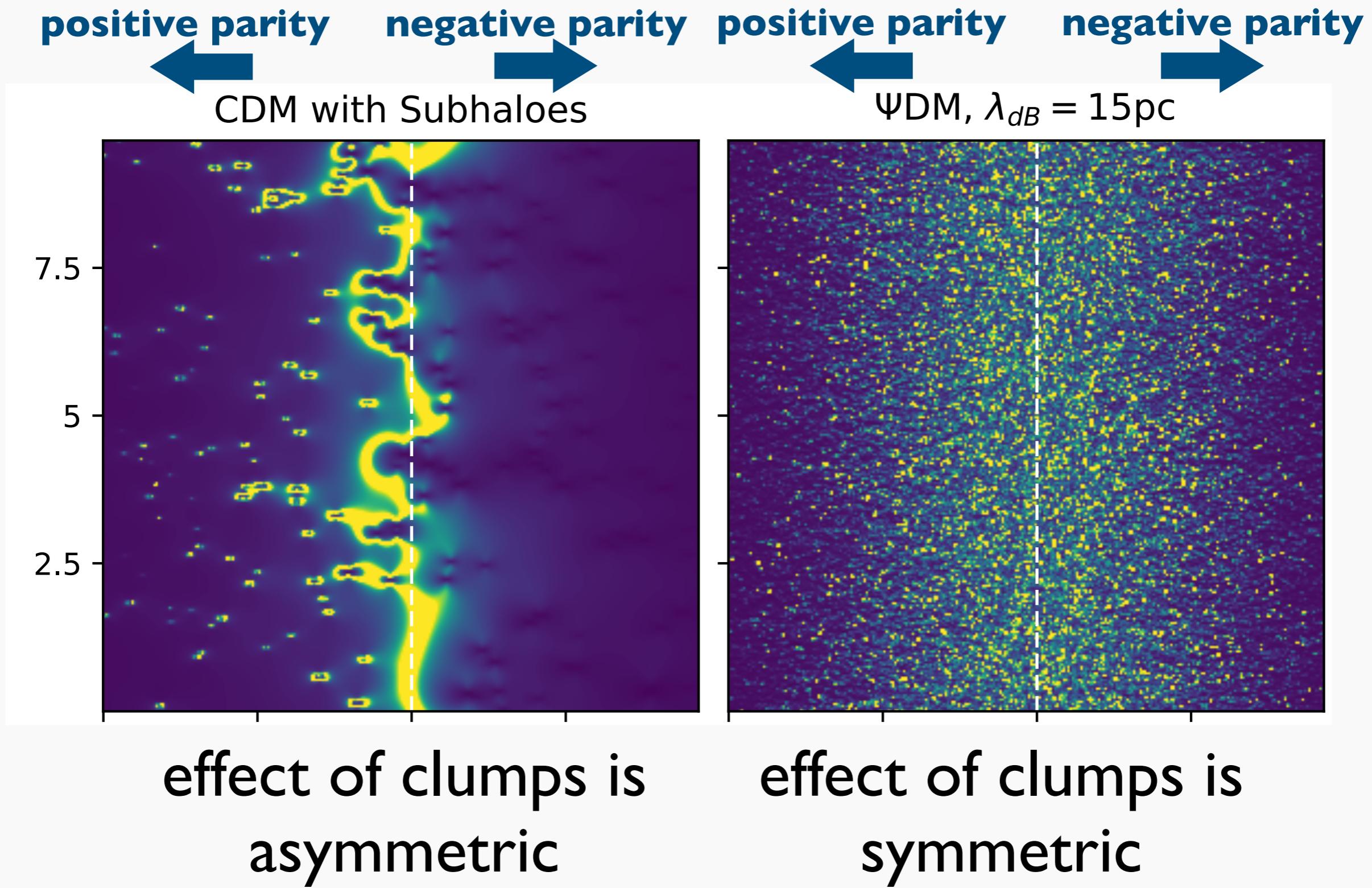


# >40 lensed stars in “Dragon”



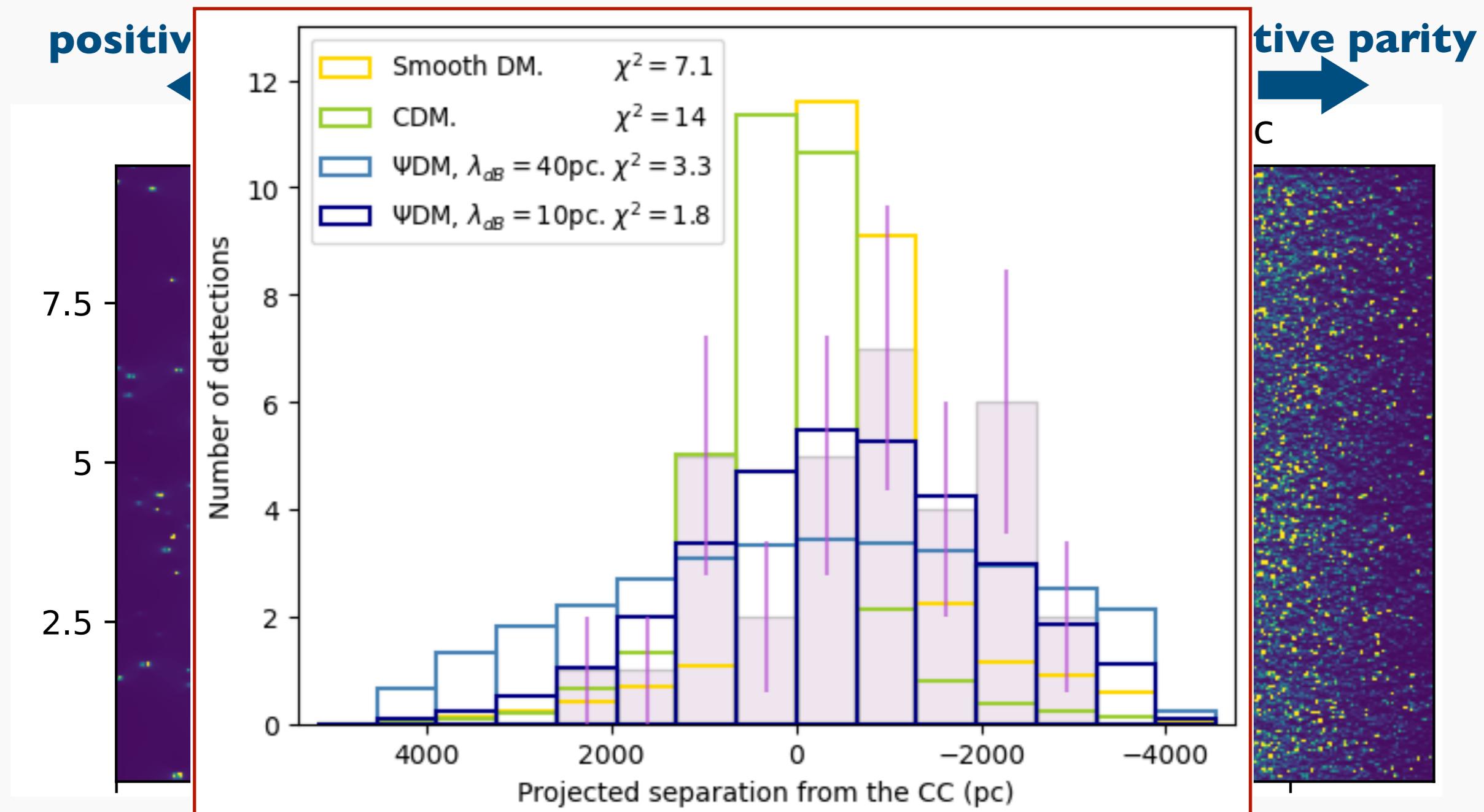


# Constraint from skewness





# Constraint from skewness



Dragon Arc observation favors FDM (?)

asymmetric symmetric

# Summary

- caustic crossings are new phenomena reported for the first time in 2018
- highly magnified (~thousands) individual stars
- interpretation rather complicated, but their basic properties now understood thanks to the progress of theoretical studies
- they offer a new route to probe the nature of dark matter
  - sensitive to the PBH abundance
  - probe DM small scale density fluctuations