PI Controller Design with Least Settling Time

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1. Introduction

The aim of this laboratory assignment is to design a PI controller such that the step response of the closed velocity loop has least settling time, so that the system gives a fast step response. For an LTI system, the settling time is minimum when the maximum value of the real parts of poles of the closed loop transfer function is minimized. This can be seen using the settling time approximation for second order systems:

$$T_s = \frac{4}{\zeta w_n}$$

Using the s-plot, we can see that the term ζw_n in the denominator of the equation above, is the real part of a pole in the second order systems. As we aim to find the least settling time, picking the poles, which are furthest away from the imaginary axis, is an important factor.
The given system consists of a plant with 2 poles and a controller with 1 pole, making the transfer function third order. Using the dominance of the poles which are the closest pole pair, we need to put this pair of poles away from imaginary axis to minimize the settling time in our system.

2. Laboratory Content

2.1 Analytical Part

In the first lab, we designed and found parameters of DC motor. The two fundamental parameters in our system are K and τ. The transfer function of plant is:

$$G_p(s) = \frac{K}{(\tau s + 1)} e^{-hs}$$

In this transfer function, K and τ are calculated as the average of the K and τ that are already found in first lab.

In addition, in the first lab, a delay was effective in the response of the system. This delay is first pade approximated and modeled as the following:

$$e^{-hs} \cong \frac{-\frac{hs}{2} + 1}{\frac{hs}{2} + 1}$$

Time delay h is assumed to be exactly 10 milliseconds (h = 0.01).

With the delay model, our transfer function of plant becomes:

$$G_p(s) \cong \frac{K}{(\tau s + 1)} \frac{-\frac{hs}{2} + 1}{\frac{hs}{2} + 1}$$

We design a PI (proportional–integral) controller, the transfer function of this controller is:

$$G_C(s) = K_c \frac{-\frac{s}{z} + 1}{s}$$

As a conclusion of this section, to make the closest pair of poles away from the imaginary axis, the key part is to find and put an appropriate zero in our system's transfer function and find the related gain K.

2.2 Software and Calculations Part

In this code block, K and tau values found in first lab are averaged and the corresponding plant transfer function is created. Also, as we find the minimum of maximum real parts of the closed loop poles, I find them in this code block and store them in a vector, with the related gain K values.

```
clc; close all;
K = (41.86 + 32.47 + 12.041) / 3;
tau = (0.031 + 0.044 + 0.097) / 3;
h = 0.01;
plant = tf([-K*h 2*K],[tau*h (2*tau)+h 2]);
maks = zeros(991, 65);
mins = zeros(991, 1);
Ks = zeros(991,1);
count = 1;
z = -100:0.1:-1;
for i = -100:0.1:-1
    g_c = tf([-1 i], [i 0]);
    [R,L] = rlocus(g_c * plant);
    for j = 1:65
        maks(count, j) = max(real(R(:,j)));
    end
    count = count + 1;
end
[mins, Ks] = min(maks, [], 2);
```

For the 991 linearly separated samples of z in the specified range (-100:0.1:-1), we obtain the d vs z plot and select the global minimum from that plot, namely $z1$.

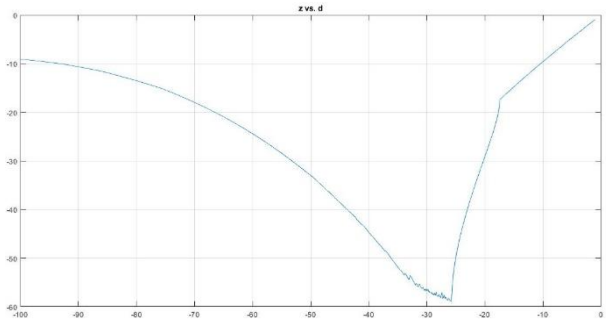

Figure 1 – d vs. z plot

For this z1 value, we plot the root locus as below:

```
[~, i] = min(mins);
figure;
z1 = z(i);
g_c = tf([-1 z1], [z1 0]);
rlocus(g_c * plant); % Gain 2.0657
title('Root Locus for z_{1}');
```

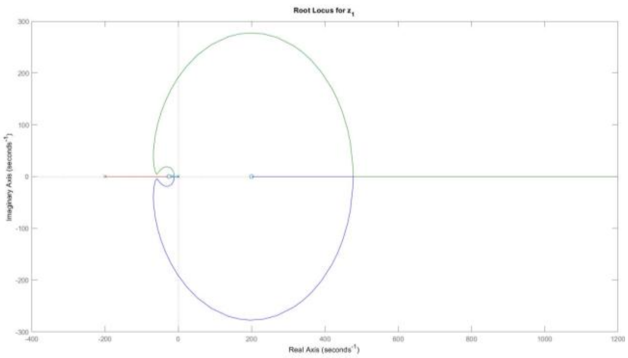

Figure 2 – z1 Root Locus

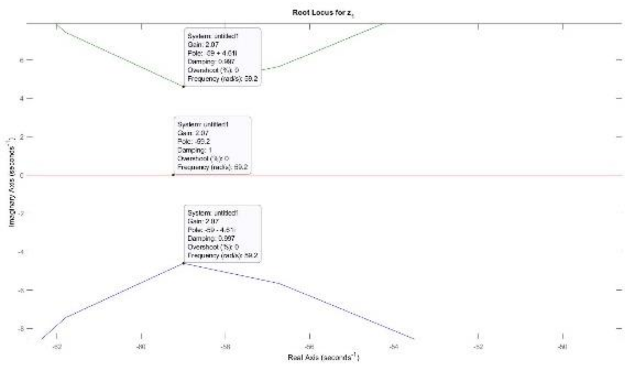

Figure 3 – z1 Root Locus with data points

This Figure 3 above shows the root locus of z1 with closest poles to imaginary axis having minimum real parts, including the corresponding gain Kc.

Gain is found as $K = 2.07$ and $\text{Re(poles)} < -59$ for z1.

We choose $z2 = \frac{z1}{2}$ and $z3 = \frac{z1}{3}$

```
z2 = z1 / 2;
g_c = tf([-1 z2], [z2 0]);
rlocus(g_c * plant); % Gain 3.9767
title('Root Locus for z_{2}');
figure;
z3 = z1 / 3;
g_c = tf([-1 z3], [z3 0]);
rlocus(g_c * plant); % Gain 3.0608
title('Root Locus for z_{3}');
hold off;
```

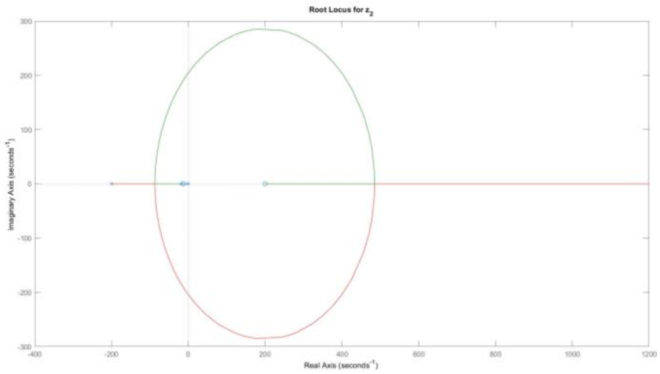

Figure 4 – z2 Root Locus

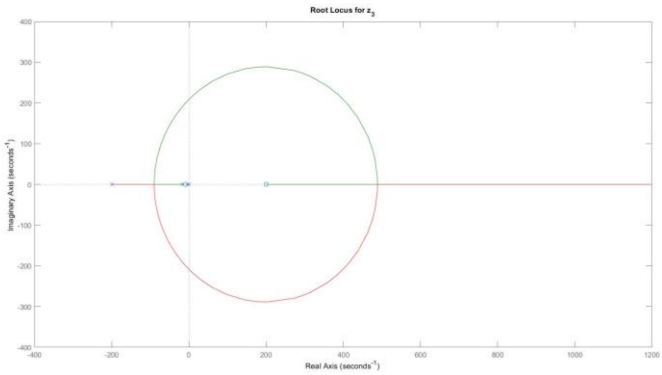

Figure 5 – z3 Root Locus

2.3 Simulink Simulations

In the Simulink simulation results shown below, the step input is $10u(t)$ and the step size is set to 0.01. The gain that we find from root locus is the gain of third order closed loop transfer function: KK_c

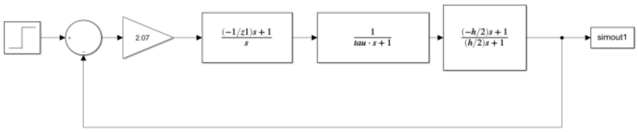

Figure 6 – Simulink Block diagram for z1

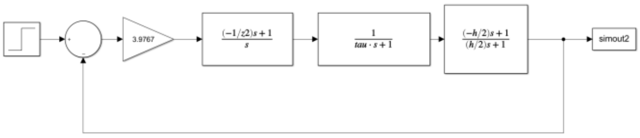

Figure 7 – Simulink Block diagram for z2

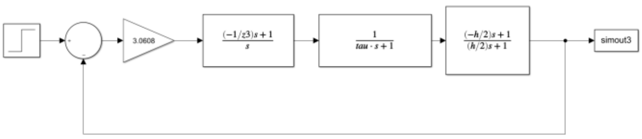

Figure 8 – Simulink Block diagram for z3

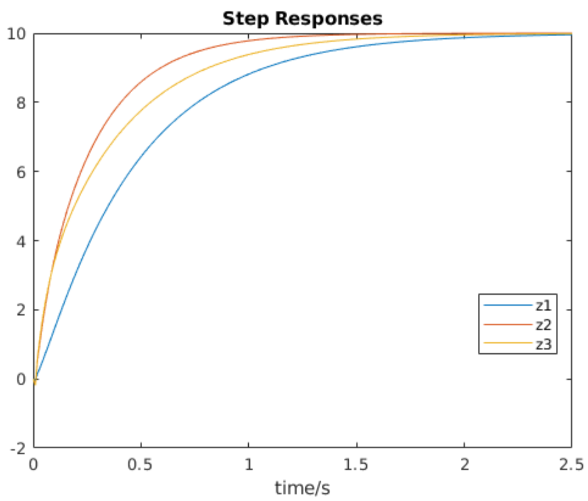

Figure 9 – Step Response of three closed-loop systems

Remark: In the laboratory assignment, it is stated that the gain found from the root locus is the controller's gain K_c. This statement causes unclarity, considering that the overall gain is K_c, as the locus plot has two zeros and three poles, which is in fact the transfer function of the entire closed loop i.e. this root locus can provide us the gain for the entire system KK_c, instead of only controller gain K_c.

Nevertheless, considering the controller gain K_c case, the resulting plot is shown below for convenience:

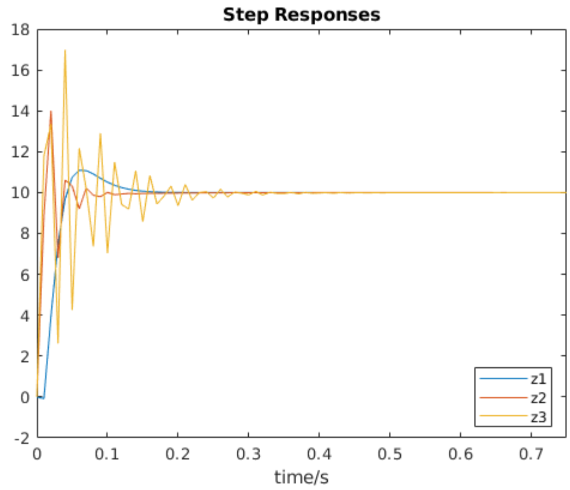

Figure 10 - Step Response of three closed-loop systems for K_c case

3. Conclusion

In this laboratory assignment, we aimed to design a PI controller for a second order system, so that to minimize the settling time, we used the way of forcing the poles to away from the imaginary axis. The relation between the settling time of the closed loop system and the poles' distance to imaginary axis in the complex plane is demonstrated. The best zero z1 in the overall transfer function is found and the root locus for z1, z2 and z3 are shown. Founding the corresponding z and K values, the Simulink Simulations are done and the results of the systems to step input are held on the same figure. It is expected, in theory, that the settling time is mostly related to z, the system with lower z value would have lower settling time such that the settling times of three z values would be related to the order

$$z1 > z2 > z3$$

In fact, in the simulatons, we observed that the actual order is:

$$z2 > z3 > z1$$

This order is also same for the gains of the systems. Therefore, we can comment that the system with the highest gain has the lowest settling time. In conclusion, the settling time of the systems, such as the analyzed one in this report, is related to zero z and gain, together.

REFERENCES

1. Lab-2 Preliminary Work Manual