

# Estimating Parameters of a DC motor and Simulation

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## 1. Introduction

The aim of this laboratory assignment is to estimate the parameters of a DC motor using the input signal, output data and the given closed loop system. In addition, the design of the DC motor plant and calculation of its transfer function accordingly and simulation of the system via Simulink within MATLAB are done. Finally, the simulation system results are compared with the given data and error and differences are explained.

## 2. Laboratory Content

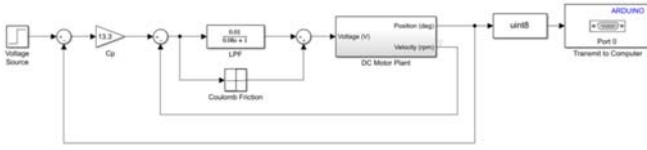


Fig. 1: Test setup provided by company

With the given information in the assignment sheet, the test setup can be simplified to the figure as below:

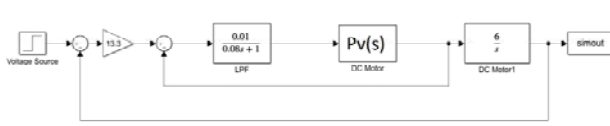


Fig. 2: Simplified and Equivalent Setup

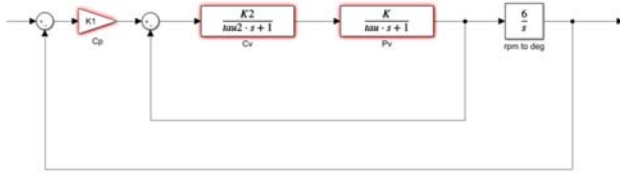


Fig. 3: Simplified and Equivalent Setup with transfer function names

Our aim is to estimate the parameters of DC motor in the form  $K/(\tau s + 1)$ . The required work are divided into subparts and explained in subsections as follows.

**2.1)** First, we find the closed loop transfer function of the position loop  $T_p(s)$ , in terms of  $K$  and  $\tau$ .

The simplified transfer function of this block diagram in Figure 3 is calculated as

$$P_p(s) = \frac{P_p(s)C_v(s)}{1 + P_p(s)C_v(s)}$$

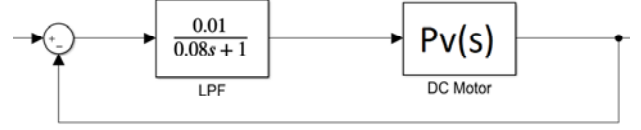


Fig. 4: Feedback Block Diagram Simplification

Writing the value of transfer function of LPF in the equation, we get

$$P_p(s) = \frac{\frac{K}{\tau s + 1} \frac{0.01}{0.08s + 1}}{1 + \frac{K}{\tau s + 1} \frac{0.01}{0.08s + 1}}$$

$$= \frac{0.01K}{0.01K + (0.08s + 1)(\tau s + 1)}$$

We found the transfer function of  $P_p(s)$ , now we can proceed with the considering the other block of our model.

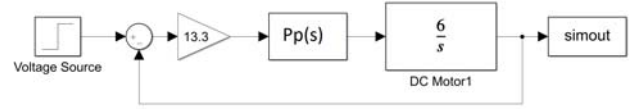


Fig. 5: Whole Block Diagram with  $P_p(s)$

The transfer function of the closed position loop is

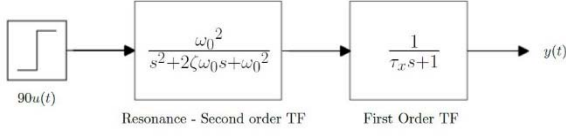
$$T_p(s) = \frac{\frac{6(13.3)P_p(s)}{s}}{1 + \frac{6(13.3)P_p(s)}{s}}$$

$$T_p(s) = \frac{\frac{6(0.133)K}{s(0.08s + 1)(\tau s + 1) + 0.01Ks}}{1 + \frac{6(0.133)K}{s(0.08s + 1)(\tau s + 1) + 0.01Ks}}$$

$$T_p(s) = \frac{6(0.133)K}{s(0.08s + 1)(\tau s + 1) + 0.01Ks + 6(0.133)K}$$

We have obtained closed position loop.

2.2) We now separate  $T_p(s)$  into 2 parts.



**Fig. 6:** Simplified form of closed loop transfer function

The transfer function of the block diagram at Figure 6 is

$$T(s) = \frac{\omega_0^2}{(s^2 + 2\zeta\omega_0s + \omega_0^2)(\tau_x s + 1)}$$

We have 2 transfer function equations of third degree. Equating the parameters with proper functions of parameters of  $T(s)$ , we have:

$$\omega_0^2 = 6(0.133)K\alpha$$

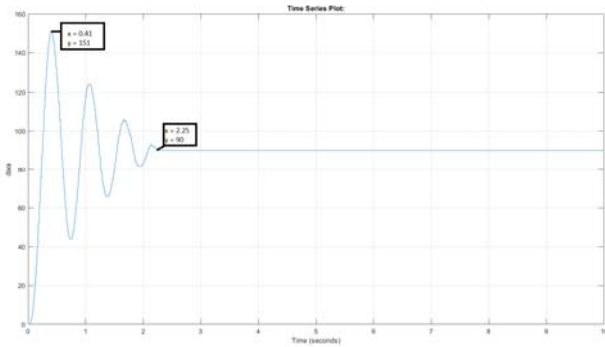
$$\tau_x = 0.08\tau\alpha$$

$$2\zeta\omega_0\tau_x + 1 = (0.08 + \tau)\alpha$$

$$2\zeta\omega_0 + \omega_0^2\tau_x = (0.01K + 1)\alpha$$

As numerators of two transfer functions may not be equal,

2.3) Using the approximations on the data plot in MATLAB, we can estimate natural frequency  $\omega_0$  and damping constant  $\zeta$ . For the  $90u(t)$  step input, we have following data plot:



**Fig. 7:** Plot of the given position data

In the lab assignment, it is advised to assume the dominance of second order transfer function to be able to do approximations. Therefore, we can use settling time and peak time approximations. Using the plot of the given data, the peak time and settle time can be observed.

The approximations are as follows:

$$T_s \cong \frac{4}{\zeta\omega_0} = 2.25 \text{ (Settling Time)}$$

$$T_p \cong \frac{\pi}{\sqrt{1-\zeta^2}} = 0.41 \text{ (Peak Time)}$$

Solving these equations together, we obtain:

$$\zeta \cong 7.87$$

$$\omega_0 = 0.23$$

Now, as we found natural frequency and damping constant, we can find other unknowns.

-For the two equations  $\tau_x = 0.08\tau\alpha$  and

$$2\zeta\omega_0\tau_x + 1 = (0.08 + \tau)\alpha$$

We can derive

$$0.715\alpha\tau = 1 - 0.08\alpha$$

-Also, we have

$$\omega_0^2 = 6(0.133)K\alpha$$

Which becomes

$$77.618 = K\alpha$$

-From the equation

$$2\zeta\omega_0\tau_x + 1 = (0.08 + \tau)\alpha$$

We have

$$4.95\tau\alpha = 0.01K\alpha + \alpha - 3.56$$

$$\tau\alpha = \frac{\alpha - 2.78}{4.95}$$

We had found

$$0.715\alpha\tau = 1 - 0.08\alpha$$

above, so using the equality of  $\alpha\tau$ 's, we obtain

$$\frac{1 - 0.08\alpha}{0.715} = \frac{\alpha - 2.78}{4.95}$$

The scalar is calculated as  $\alpha = 6.25$

Using this scalar  $\alpha$ , other unknowns can be calculated as

$$K = 12.41$$

$$\tau = 0.112$$

$$\tau_x = 0.056$$

2.5) The Simulink implementation with found parameters is as follows:



Fig. 8: Simulink Block Diagram

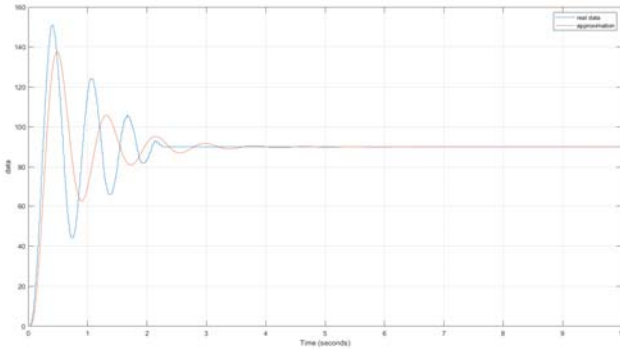


Fig. 9: Plot of the simulation result and given position data

The MATLAB code to plot the graph in the Figure 9, above, is as follows:

```
load('prelab1_posData.mat');
plot(position)
grid

samplepoints =
getdatasamples(position,[1:1000]);

[maxvalue, indexofmax] =
max(samplepoints);
disp(maxvalue);
disp(indexofmax);

plot(position)
hold on
plot(simout)
xlabel("Time (seconds)")
ylabel("data")
legend("real data", "approximation")
grid on
```

The MATLAB code used to calculate parameters  $K$  and  $\tau$  is as follows:

```
clc; clear all; close all;

load('prelab1_posData.mat');
% plot(position)

samples = getdatasamples(position, [1:
1001]);
[maxSample, indexMax] = max(samples);
disp(maxSample); disp(indexMax);
```

```
steady = samples(length(samples));
s2d = steady * 0.98;
s2u = steady * 1.02;
for k = 1:1001
    if samples(k) < s2u && samples(k) >
s2d
        settling = k / 100;
        break
    end
end
```

```
disp(settling);
inter1 = 4 / settling; % z_times_w
```

```
w = sqrt((inter1 ^ 2) + ((pi / (indexMax /
100)) ^ 2));
z = inter1 / w;
```

```
K1 = 13.3;
K2 = 0.01;
tau2 = 0.08;
```

```
K_alpha = (w^2) / (6 * K1 * K2);
first = K2 * K_alpha;
second = 2 * z * w;
third = 2 * z * w * tau2 * first;
fourth = 4 * (z^2) * (w^2) * tau2;
fifth = (w^2) * tau2;
denominator = 1 + ((w^2) * (tau2^2)) - (2
* z * w * tau2);
```

```
alpha = (-first + second + third - fourth
+ fifth) / denominator;
K = K_alpha / alpha;
tau = (1 - (tau2 * alpha)) / (1 - (2 * z *
w * tau2)) / alpha;
tau_x = tau2 * alpha * tau;
```

### 3. Conclusion

In this laboratory assignment, our goal was to work on a DC motor plant, find its transfer function, estimate its parameters, work on transfer function of the whole block diagram and simulate the estimated motor parameters via Simulink on MATLAB. The parameters  $K$ ,  $\tau$  and  $\tau_x$  are found first analytically and then using MATLAB. The transfer function of the closed position loop is found to be degree of 3, whereas using the approximation, the second order dominance is assumed. Therefore, there happened small deviations and errors comparing the original given data plot and simulation result. However, the calculation results are very close to exact values.

### REFERENCES

1. Laboratory Assignment 01 Preliminary Report Manual