Margin Analysis of a System

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1. Introduction

The aim of this laboratory assignment is to understand and get familiar with margins, which are gain phase and delay margin. We estimate these margins using mathematical models and verify them.

2. Laboratory Content

2.1 Margin Estimation

In the previous laboratory assignment, we had designed three PI controllers. The bode diagram of our open loop transfer function is:

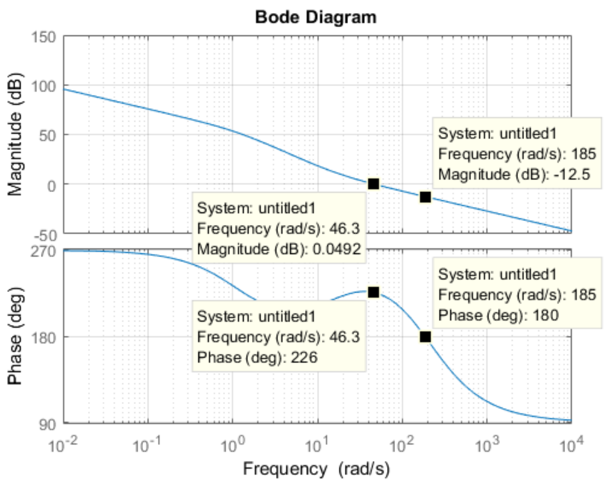

Figure 1: Bode Diagram with Margin Data Points

For phase margin, we look at the frequency where the magnitude is 1, i.e. 0 dB. This frequency is observed on the Bode plot as $\omega = 46.3\ \frac{rad}{sec}$. At this frequency, on the phase plot, the corresponding phase is $\phi = 226°$. Then the Phase Margin (PM) is:

$$\phi_{PM} = 226° - 180° = 46° = 0.8\, rad$$

To calculate the Gain Margin (GM), we find the frequency where the phase is $\phi = 180°$ and find the corresponding gain on this frequency. As it can be seen from the Bode phase plot, the phase becomes $\phi = 180°$ at the $\omega = 185\ \frac{rad}{sec}$ and the gain at that frequency is -12.5 dB. Then, our gain margin is

$$GM = 0 - (-12.5) = 12.5 \text{ dB}$$

This margin corresponds to the gain K:

$$20\log(K) = 12.5\, dB$$

$$K = 10^{\frac{12.5}{20}} = 4.22$$

Delay Margin (DM) is $DM = \frac{\phi_{PM}}{\omega_x}$ where ω_x is the frequency where the gain is 0 dB. In our case, ω_x is found as $46.3\frac{\text{rad}}{\text{sec}}$. Therefore, delay margin is:

$$DM = \frac{\phi_{PM}}{\omega_x} = \frac{0.8\, rad}{46.3\, rad/s} = 0.017\, s$$

2.2 Margin Verification

To verify these estimated margins, we use DC motor hardware. For input $r(t) = 40u(t)$, we configured our system and increased the gain K until we find the gain that makes the system unstable. We had found K1 = 29.4372 in the previous lab and now, we have several trials and error attempts to find that gain:

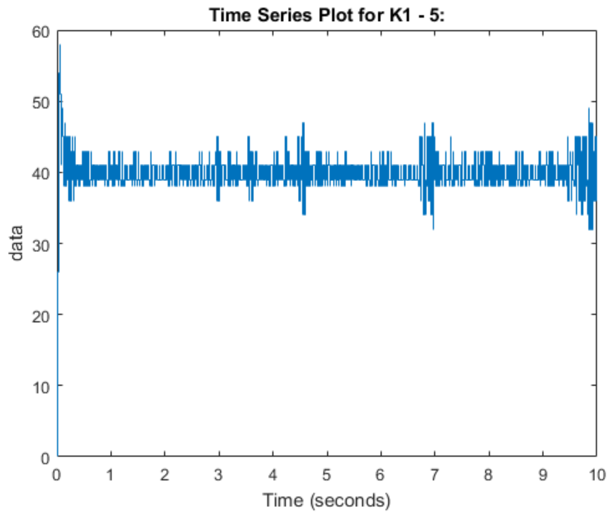

Figure 2: Response of the system with gain K1-5

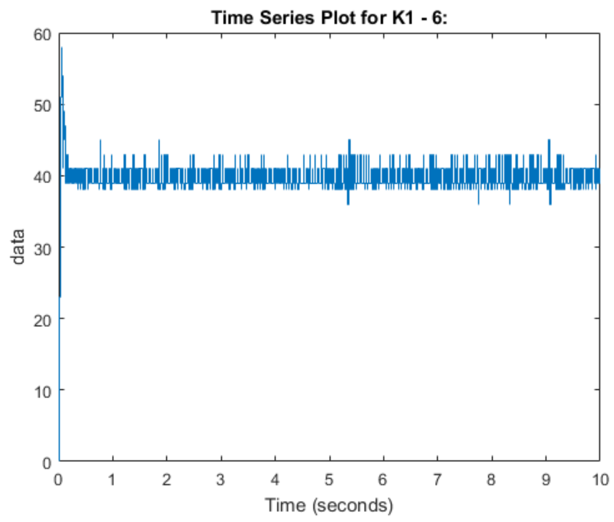

Figure 3: Response of the system with gain K1-6

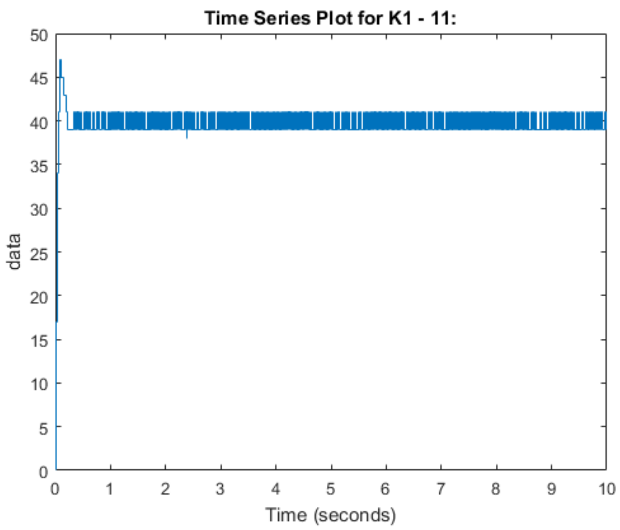

Figure 4: Response of the system with gain K1-11

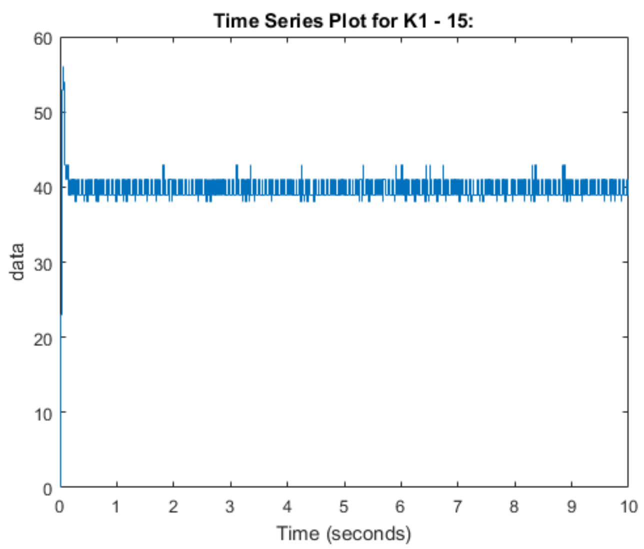

Figure 5: Response of the system with gain K1-15

According to these trial and errors, the gain K_f that makes the system unstable is about:

$$K_f = K1 - 10 = 29.4372 - 10 = \quad 19.4372$$

Therefore, experimentally, our gain margin is observed as 19.4.

The second part of the experiment is about delay margin and its verification using hardware. Again, as we did for gain margin, we find the delay that makes the system unstable by trial and error.

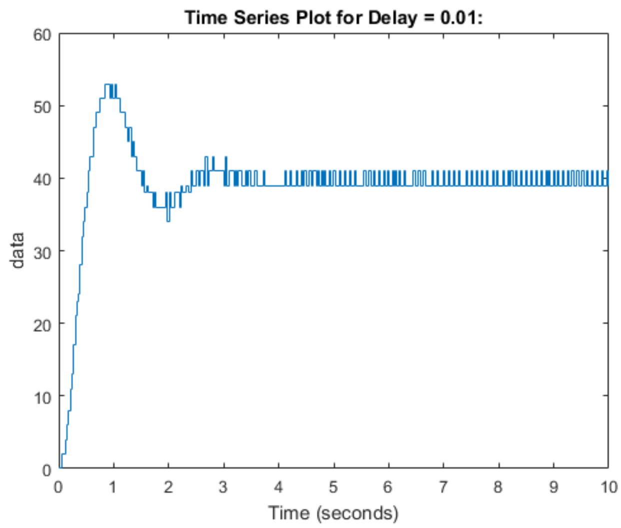

Figure 6: Response of the system with delay = 0.01 s

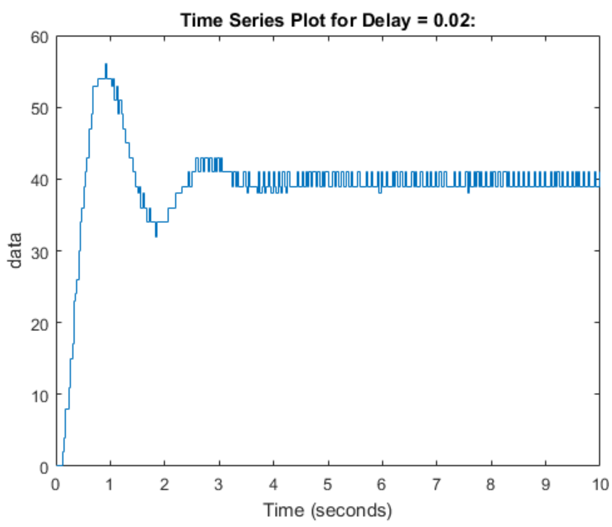

Figure 7: Response of the system with delay = 0.02 s

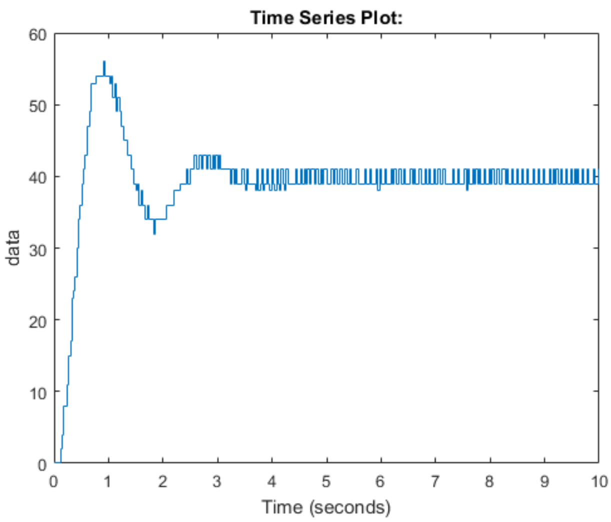

Figure 8: Response of the system with delay = 0.035 s

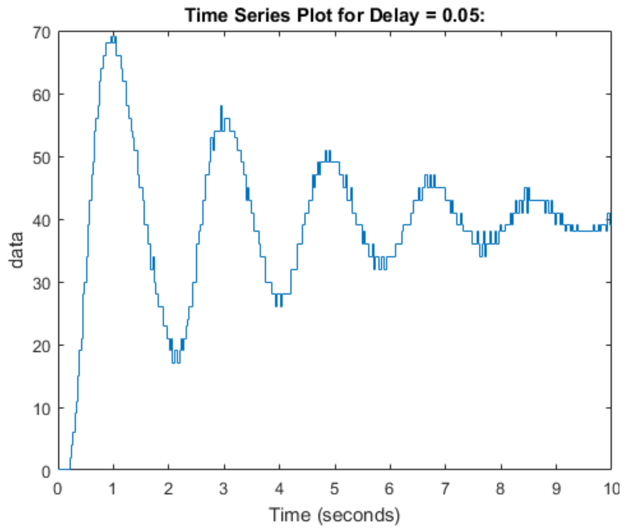

Figure 9: Response of the system with delay = 0.05 s

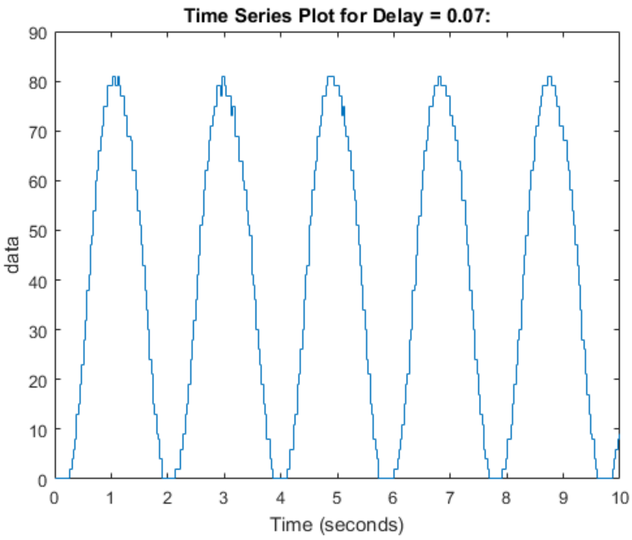

Figure 10: Response of the system with delay = 0.07 s

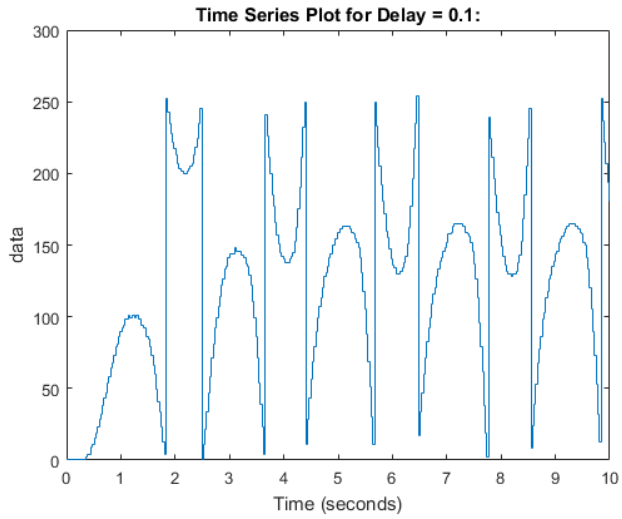

Figure 11: Response of the system with delay = 0.1 s

Looking at the plots, we can say that system becomes unstable for about 0.06 s delay, thus according to these trial and errors, the delay margin h_f that makes the system unstable is:

$$h_f = 0.06\, s = 60\, ms$$

3. Conclusion

In this laboratory work, we mathematically analyzed the system that we had designed in the previous lab. We calculated gain, phase and delay margin using the Bode plot of open loop system. In the practical part, we used DC motor and tried to found the gain, phase and delay margin with trial and error procedure. Comparing the margins that are found theoretically and with trial and error in the second part of the experiment, we observe differences between them. This is due to linear first order approximation that we used. As our system is not linear but rather nonlinear, we expect to obtain error in our results. Also, DC motor plant that we use in the laboratory is not ideal, it may occur physical and mechanical errors and inaccuracies.

Margins are important concepts in Feedback Control Systems engineering, therefore understanding these and be able to verify them are necessary skills for engineering. Therefore, this laboratory work is very educational.

REFERENCES

1. Dorf, Richard C., and Robert H. Bishop. *Modern Control Systems*. 13th ed., Pearson, 201

APPENDIX

```
clc; close all;
% K = (41.86 + 32.47 + 12.041) / 3;
% tau = (0.031 + 0.044 + 0.097) / 3;
h = 0.01;
plant = tf([-K*h 2*K],[tau*h (2*tau)+h
2]);
maks = zeros(991, 65);
mins = zeros(991, 1);
Ks = zeros(991,1);
count = 1;
z = -100:0.1:-1;
for i = -100:0.1:-1
    g_c = tf([-1 i], [i 0]);
    [R,L] = rlocus(g_c * plant);
    for j = 1:65
        maks(count, j) =
max(real(R(:,j)));
    end
    count = count + 1;
end
[mins, Ks] = min(maks, [], 2);
plot(-100:0.1:-1, mins);
grid; title('z vs. d');
[~, i] = min(mins);
figure;
z1 = z(i);
g_c = tf([-1 z1], [z1 0]);
rlocus(g_c * plant);
title('Root Locus for z_{1}');
figure;
z2 = z1 / 2;
g_c = tf([-1 z2], [z2 0]);
rlocus(g_c * planttitle('Root Locus
for z_{2}');
figure;
z3 = z1 / 3;
g_c = tf([-1 z3], [z3 0]);
rlocus(g_c * plant);
title('Root Locus for z_{3}');
hold off;

zs = [z1 z2 z3];
for i = 1:3
  disp('------------- ');
  z = zs(i);
  g_c = tf([-1 z], [z 0]);
  [R, L] = rlocus(g_c * plant);
  a = zeros(1, 68);
  for m = 1:68
      a(m) = max(real(R(:,m)));
  end
  [T,I] = min(a);
  disp(T);
  disp(L(I));
  disp(R(:,I));
end
```