EEE 444 Robust Feedback Theory HW2 Report

Oğuz Altan – 21600966

Part 1)

In this assignment, we are given a standart feedback control system with the plant

$$P_\delta(s) = \frac{e^{-hs}}{\tau s - 1}$$

where $\tau \in [0.2, 0.25]$ and h $\in [0, 0.05]$ and we have plant

$$P(s) = \frac{1}{0.2s - 1}$$

The uncertainty is defined as $\delta = P_\delta - P$

We try to find an uncertainty weight in the form $W_a(s) = \frac{a_1 s}{(a_2 s+1)^2}$, $a_1, a_2 > 0$ i.e we try to find values a_1 and a_2, so that $|W_a(j\omega)| > ||\delta(j\omega)||$ for all ω and $|W_a(j\omega)|$ is as small as possible for each ω.

To do these, as we are given intervals for τ and h, we select 10 values of each of them in their intervals. After, We construct $P_\delta(s)$ transfer functions for each relevant τ and h. Thus, we get, in total $10.10 = 100$ $P_\delta(s)$ transfer functions. Using these transfer functions, we calculate and plot the uncertainities $\delta = P_\delta - P$ and plot them all, on top of each other. To ensure the condition $|W_a(j\omega)| > ||\delta(j\omega)||$, we select $a_1$and a_2, accordingly. I chose values $a_1 = 0.05$ and $a_2 = 0.7$. The plots of uncertainities and uncertainty weight is given as following:

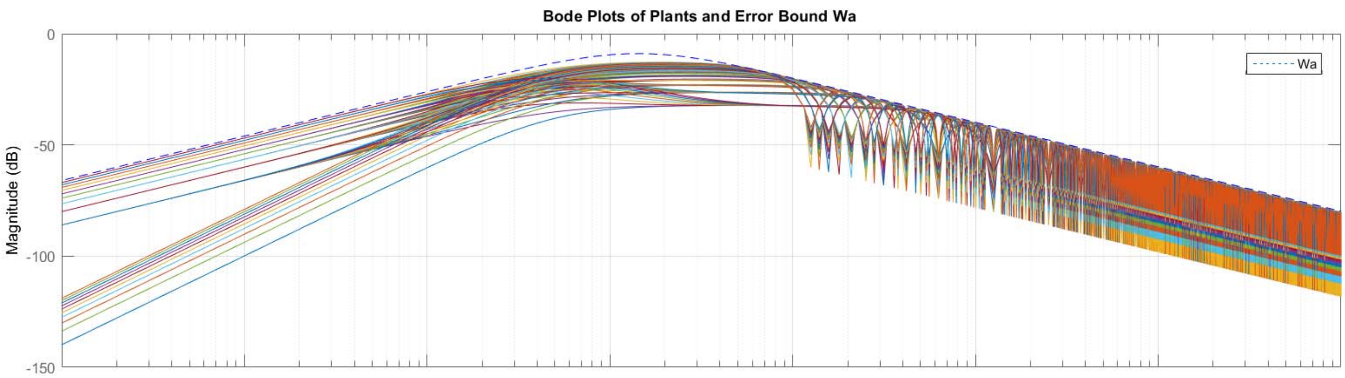


Figure 1: Bode Plots of Uncertainities and Error Bound Wa

As it can be seen, the error bound Wa is bigger than the largest possible error, hence the condition $|W_a(j\omega)| > ||\delta(j\omega)||$ is satisfied.

Part 2)

In this part, we find the allowable interval for $\beta > 0$ so that the controller

$$C(s) = \frac{(15s + 1)}{\beta s}$$

is robustly stabilizing (C, P) for all $P_\Delta \in$ P. For robustly stabilizing check, for each fixed value of β, we look at the inequaility

$$\left|\left|W_a C(1 + PC)^{-1}\right|\right|_\infty < 1$$

If this equation holds, then the system is robustly stable. We find the range of β that stabilizes the system. The figure below shows the value of $\left|\left|W_a C(1 + PC)^{-1}\right|\right|_\infty$ for the interval β that makes this term less than 1.

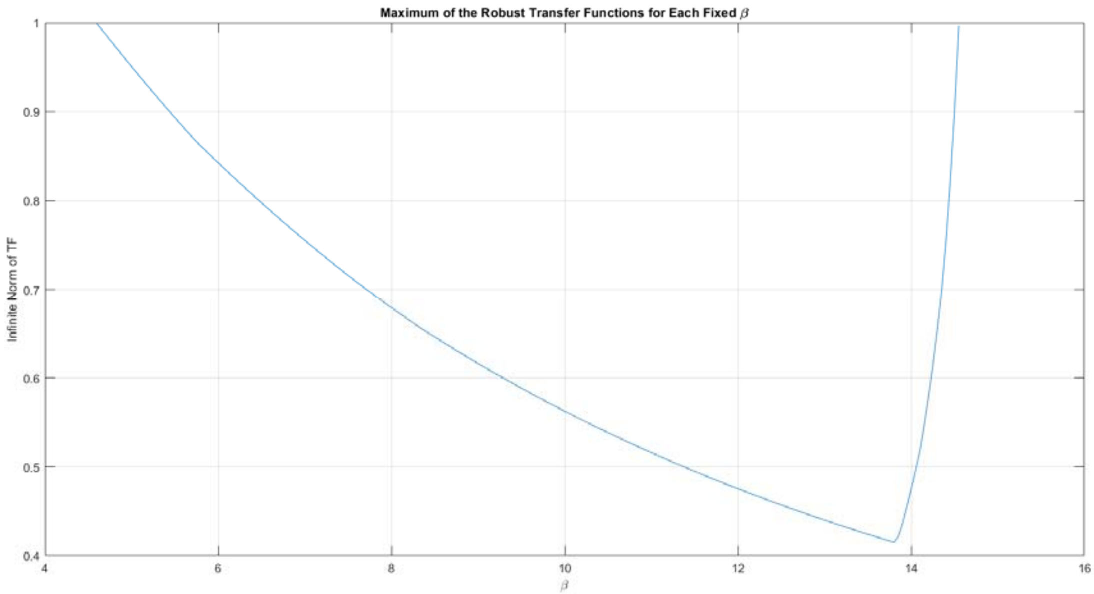


Figure 2: Maximum of the Robust Transfer Function for Each Fixed β

The range of β values making the term $\left|\left|W_a C(1 + PC)^{-1}\right|\right|_\infty$ less than 1 is calculated by MATLAB as $\beta \in [4.6, 14.55]$.

Part 3)

Now, we pick a value of β in the interval determined above and verify that the feedback system formed by the controller C and the plant $P_1(s) = \frac{e^{-0.05s}}{0.25s-1}$ is stable. I pick $\beta = 10$ and look at the delay margin of the system (C, P_1). The delay margin (DM) of the system calculated by MATLAB is $DM = 0.1326$, which is bigger than 0.05, the delay added to the system by the plant $P_1(s)$. Therefore, the system is stable for the chosen β value.

Another way to check the stability is to analyse the Nyquist Diagram of the open loop transfer function of the system CP_1. The Nyquist Diagram is given as following:

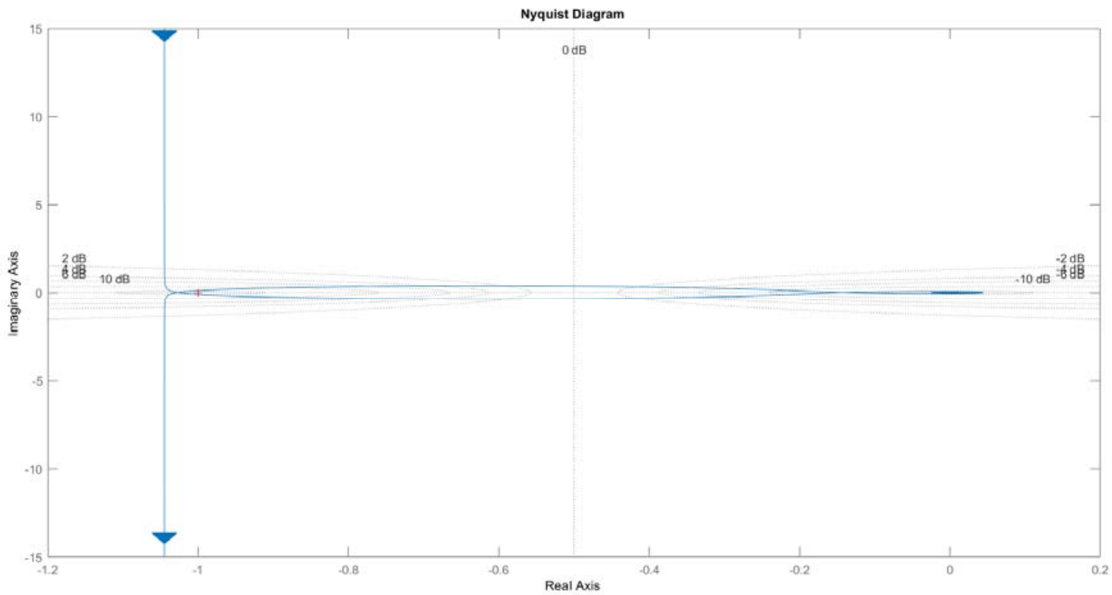


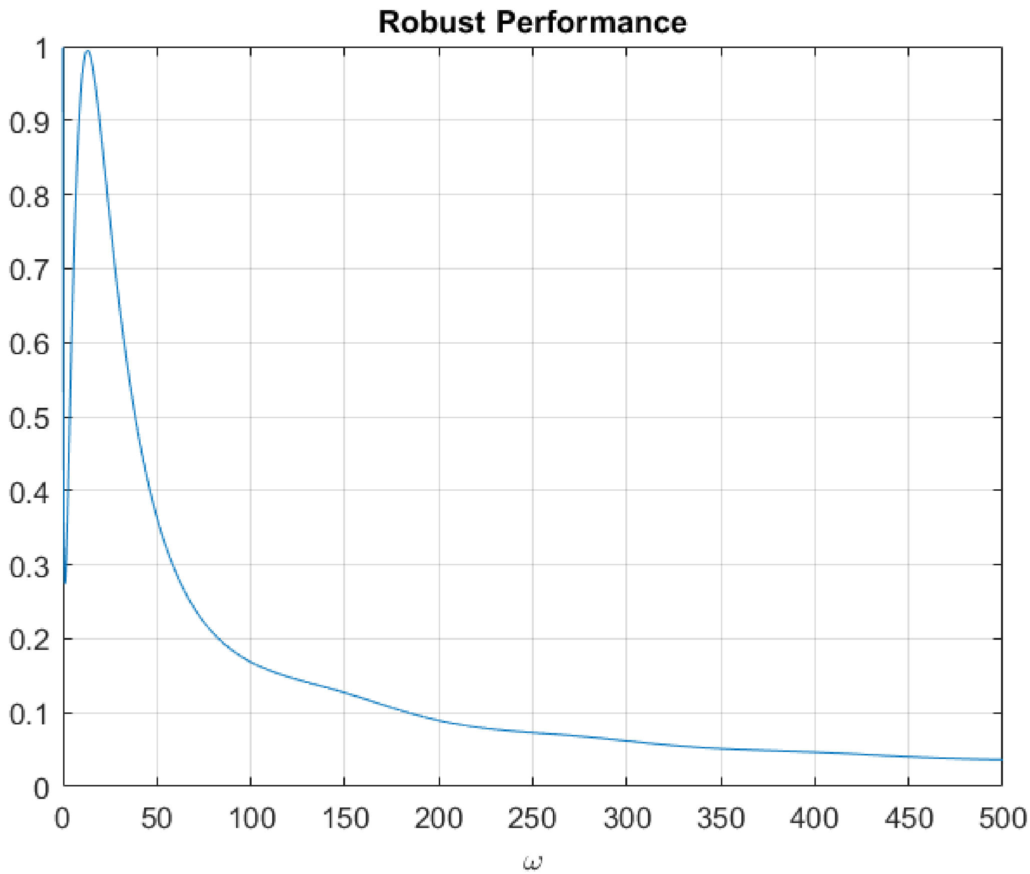
Figure 3: Nyquist Plot of the Open Loop Transfer Function CP_1

As it can be seen on the plot, -1 is encircled once, therefore the system is stable.

Part 4)

In this part, we determine if there exists β values in the interval determined in part 2, satisfying the the robust performance condition $\left|\frac{W_r(j\omega)}{\gamma_r} S(j\omega)\right| + |W(j\omega)C(j\omega)S(j\omega)| \leq 1\ \forall\omega$ where the sensitivity function S is $S = (1 + PC)^{-1}$.

For the value of $\gamma_r = 10$, we try to find the interval of β satisfying the robust performance condition, the interval is computed as $\beta \in [8.40\ 9.90]$. In the final part, we try to find the smallest value of γ_r possible so that we only have a single β that satisfies the robust performance condition. After MATLAB calculations, this value is computed as $\beta = 8.40$. The plot of robust performance for optimal value $\beta = 8.40$ can be seen below:



APPENDIX

```
clear all
clc

% q1

tau = [0.205 0.210 0.215 0.220 0.225 0.230 0.235 0.240 0.245 0.250];
h = [0.005 0.010 0.015 0.020, 0.025 0.030 0.035 0.040 0.045 0.050];

P = tf(1,[0.2 -1]);

for i = 1:10
    for j = 1:10
        P_unc = tf(1,[tau(i) -1],'InputDelay',h(j));
        [mag, phase] = bode(P_unc - P);
        bode(P_unc-P);
        hold on;
        disp(i);
    end
end

a1 = 0.05;
a2 = 0.07;

Wa = tf([a1 0],[a2*a2 2*a2 1]);

grid on;
bode(Wa,'b--');
title("Bode Plots of Plants and Error Bound Wa");
legend('Wa');
grid on
hold off

% q2

beta = 4.6:0.05:14.55;
isRobust = zeros(size(beta));
max_mag_vec = [];

for be = 1:length(beta)
    C = tf([15 1],[beta(be) 0]);
    robust_tf = (Wa*C/(1+P*C));
    [mag2,phase2] = bode(robust_tf);
    max_mag = max(mag2);
    max_mag_vec = [max_mag_vec max_mag];
end

figure;
```

```
plot(beta,max_mag_vec);
grid on;
title('Maximum of the Robust Transfer Functions for Each Fixed \beta');
xlabel('\beta')
ylabel('Infinite Norm of TF');

% q3

P1 = tf(1,[0.25 -1],'InputDelay',0.05);
beta_best = 10;
C_best = tf([15 1],[beta_best 0]);
tf_openloop = C*P1;
nyquist(tf_openloop)
grid on
margins = allmargin(tf_openloop);

% q4

Wr = tf(1,[1 0]);
gamma = 8.4;
beta_new = zeros(1,201);
beta_index = 1;
beta_cn = 0;
w = 0:0.01:500;

for n = 4.6:0.05:14.55
  Cnew = tf([15 1],[n 0]);
  Pnew = tf(1,[0.25 -1],'inputDelay',0.05);
  Snew = 1/(1+Pnew*Cnew);

  [mag_r1,phase] = bode(Wr*Snew/gamma,w);
  [mag_r2,phase] = bode(Wa*Cnew*Snew,w);

  mag_rmax = max(squeeze(mag_r1) + squeeze(mag_r2));

  if(mag_rmax <= 1)
    beta_new(beta_index) = n;
    beta_cn = beta_cn + 1;
  end
  beta_index = beta_index + 1;
end

Cnew2 = tf([15 1],[8.40 0]);
Pnew2 = tf(1,[0.25 -1],'inputDelay',0.05);
Snew2 = 1/(1+Pnew2*Cnew2);

[mag_r12,phase] = bode(Wr*Snew2/gamma,w);
[mag_r22,phase] = bode(Wa*Cnew2*Snew2,w);
```

```
mag_r2 = squeeze(mag_r12) + squeeze(mag_r22);
figure;
plot(w,mag_r2);
grid on
title('Robust Performance');
xlabel('\omega');
```