Homework #3 Solution

$$P(s) = \frac{4(s-2)}{(s^2-2s+2)} \qquad z=2 \qquad p_{1,2} = 1 \pm j$$

let $D(s) = \left(\frac{s^2-2s+2}{s^2+2s+2}\right)$ $\quad N(s) = \frac{4(s-2)}{s^2+2s+2}$

$X(p_{1,2}) = 1/N(p_{1,2})$ let $X(s) = \frac{x_1 s + x_0}{(s+1)}$ $\qquad X(1+j) = \left(\frac{x_1(1+j)+x_0}{1+j+1}\right) = \frac{(1+j)^2+2(1+j)+2}{4(1+j-2)}$

$$\begin{bmatrix} 1+j & 1 \\ 1-j & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_0 \end{bmatrix} = \begin{bmatrix} (2+j)/N(1+j) \\ (2-j)/N(1-j) \end{bmatrix} \Rightarrow \begin{matrix} x_1 = -2 \\ x_0 = +3 \end{matrix}$$

$X(s) = \frac{-2s+3}{s+1}$; $Y(s) = \left(\frac{s+13}{s+1}\right)$

(a) $C = \frac{X+DQ}{Y-NQ}$: $Q \in \mathcal{H}_\infty$, where X, Y, N, D are above.

(b) $\left.\begin{matrix} C(0) = \infty \\ C(\pm j3) = \infty \end{matrix}\right\} \Rightarrow \begin{matrix} Q(0) = Y(0)/N(0) \\ Q(\pm j3) = Y(\pm j3)/N(\pm j3) \end{matrix}$ three interpolation conditions

a 2nd Order $Q(s)$ solves this problem : $Q(s) = \frac{q_2 s^2 + q_1 s + q_0}{(s+2)(s+3)}$

poles of $Q(s)$: $\{-2,-3\}$ are selected arbitrarily

$$\begin{bmatrix} 0 & 0 & 1 \\ (j3)^2 & j3 & 1 \\ (-j3)^2 & (-j3) & 1 \end{bmatrix} \begin{bmatrix} q_2 \\ q_1 \\ q_0 \end{bmatrix} = \begin{bmatrix} d_q(0)\, Y(0)/N(0) \\ d_q(j3)\, Y(j3)/N(j3) \\ d_q(-j3)\, Y(-j3)/N(-j3) \end{bmatrix}$$

$d_q(s) = (s+2)(s+3)$

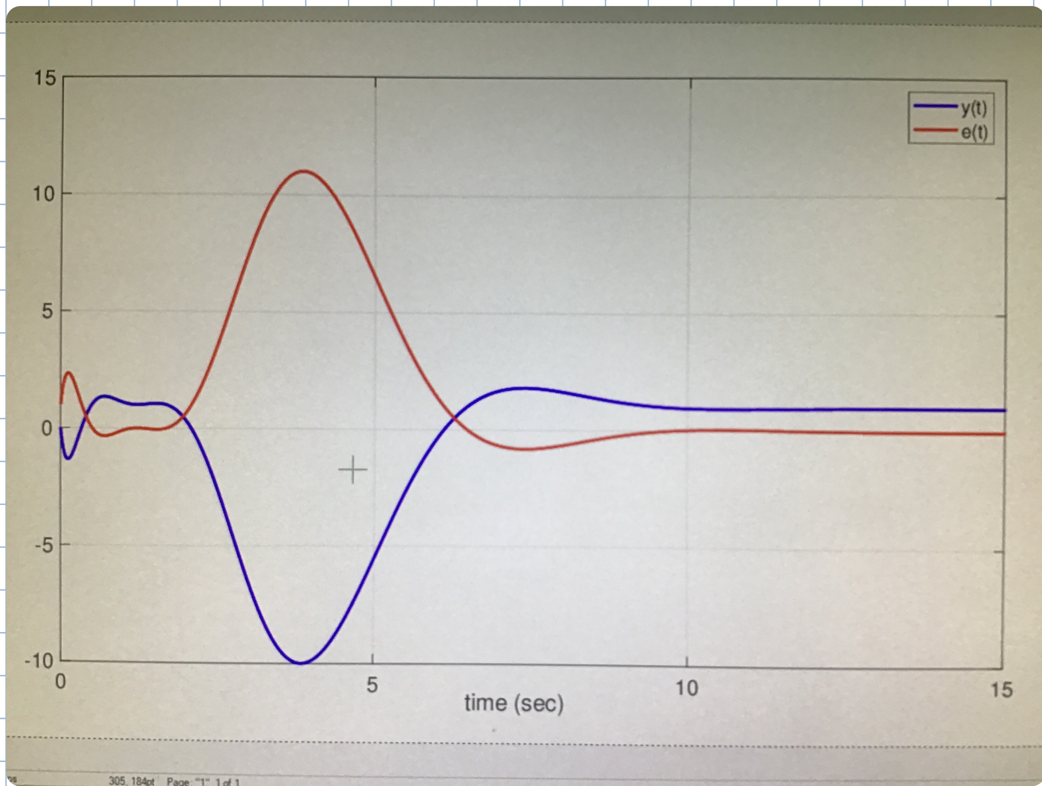
$$Q(s) = \frac{-4.565s^2 + 11.72s - 19.5}{(s+2)(s+3)}$$

$$C = \frac{X+DQ}{Y-NQ} \Rightarrow C(s) = \frac{-6.5654(s-1.333)(s-0.04527)(s^2-0.36s+8.1)}{s(s^2+9)(s+37.33)}$$

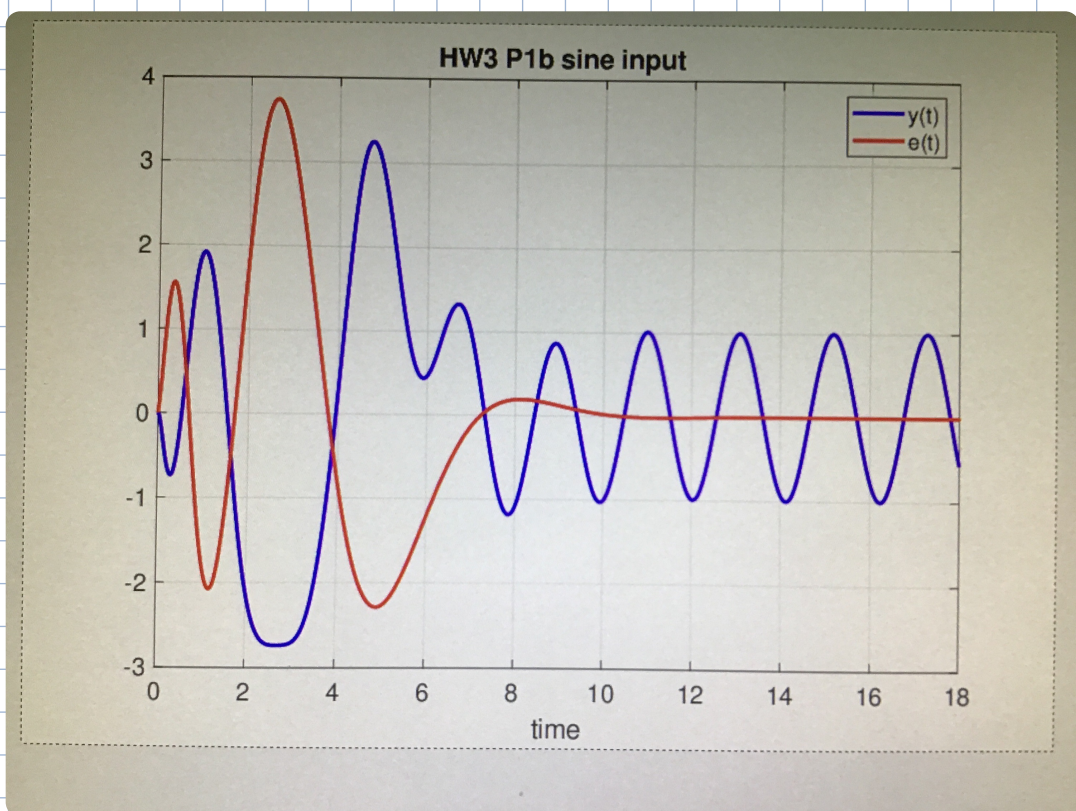
Implement the system in Simulink : final time $\simeq$ 15–18 sec fixed step size of 0.01sec.

Verify that the steady state error is zero for $r(t)$ = unit step and $r(t) = \sin(3t)$

Closed loop system poles are in $\mathbb{C}_-$: for this design poles are at $\{-2.9, -2.19, -1.08 \pm j1.11, -0.89 \pm j0.955\}$



Step response



Sinusoidal response