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EE 202 Section 1
Lab 1

Quality Factor of an Inductor

Software Implementation

In this lab experiment we assumed that we are given a real inductor that its inner resistance and inductance is unknown. Since its essential values are unknown it is expected to find these values using the methods we have designed. To verify the methods LtSpice has been used in this experiment. To realize this part first of all I add a real inductor (Figure-1) with sample values close to real ones by assuming its values are unknown. Using the methods designed, the real values and calculation values are compared and then the quality factor (Q) is calculated. These steps are repeated for 4 different frequencies in the range 1MHZ-5MHZ.

To calculate Quality Factor the equation below will be used.

$$Q = \frac{\omega * L}{R}$$

Figure-1: A real inductor model

Methods:

For both of the method the given (Figure-2) model will be used. However, for one of the method load resister values will take one more value. In addition to that frequency will be changed to examine the quality factor at 4 different frequencies.

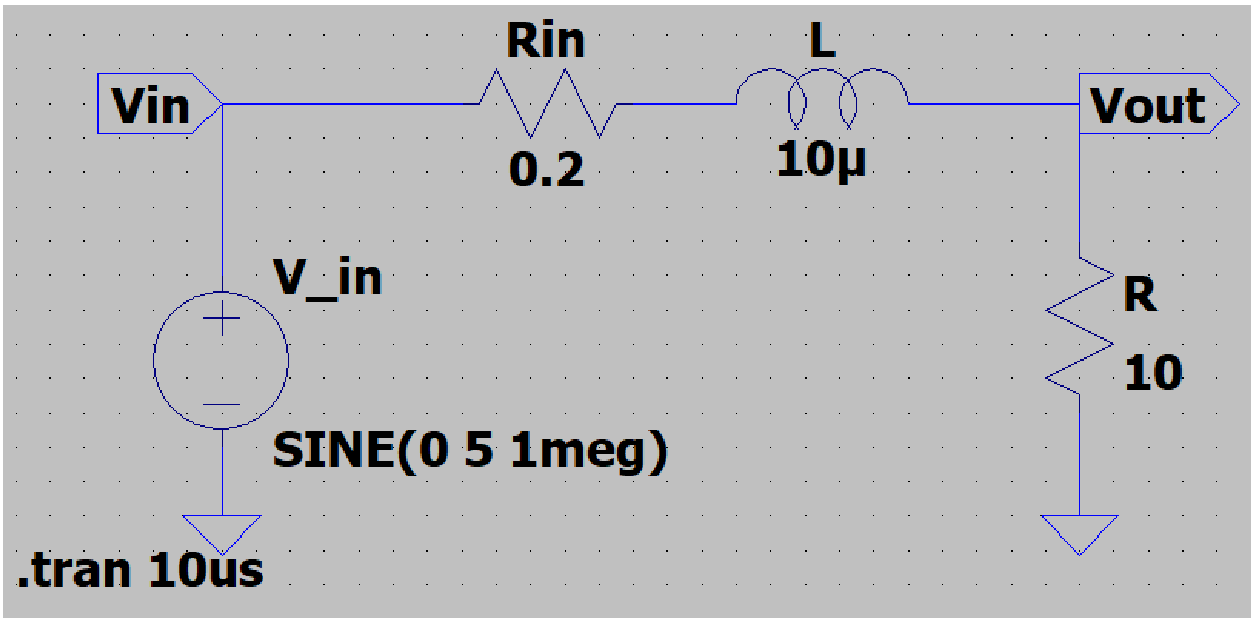

Figure-2: Circuit Model for Simulations

Method 1 (Phase Angle Method):

In this method phase angle between the input voltage and output voltage (shown in Figure-2) is used. To find the phase angle between input and output time difference between them is necessary since the following formula will be used. To find the time difference between signal cursors function is used.

$$\theta = \omega * \Delta t$$

$$\theta = 2 * \pi * f * \Delta t$$

θ: Phase difference between signals

f: natural frequency set by us

Δt: time difference between signals

In order to use phase angle, we need to work on phasor domain. To work on phasor domain, a simple voltage divider formula is used. Process to work on phasor domain is given below.

$$\frac{V_{out}}{V_{in}} = \frac{R_{load}}{R_{in} + R_{load} + j\omega L}$$

$$\frac{V_{in} * R_{load}}{V_{out}} = R_{in} + R_{load} + j\omega L$$

$$R_{load}\left(\frac{V_{in}}{V_{out}} - 1\right) = R_{in} + j\omega L$$

We have both complex and real values at right hand of the equation. To have complex and real values at left hand of the equation given formula is used.

$$\frac{V_{in}}{V_{out}} = \left|\frac{V_{in}}{V_{out}}\right| * e^{j*\theta}$$

By substitution of Euler's Formula, we have the following equation.

$$\frac{V_{in}}{V_{out}} = \left|\frac{V_{in}}{V_{out}}\right| * e^{j*\theta} = \left|\frac{V_{in}}{V_{out}}\right| (\cos(\theta) + j * \sin(\theta))$$

Using the findings this equation below is obtained.

$$R_{load}\left(\frac{V_{in}}{V_{out}}-1\right)=R_{in}+j\omega L=R_{load}\left(\left|\frac{V_{in}}{V_{out}}\right|(\cos(\theta)+j*\sin(\theta))-1\right)=R_{in}+j\omega L$$

$$R_{load}\left(\left|\frac{V_{in}}{V_{out}}\right|(\cos(\theta)+j*\sin(\theta))-1\right)=R_{in}+j\omega L$$

It is known that complex values at both of right and left hand must be equal to each other. Similarly real values of both sides will be equal as well. Using this information, we have the inner resistance and the inductance values of the inductor.

$$R_{load}*\left|\frac{V_{in}}{V_{out}}\right|\cos(\theta)-R_{load}=R_{in}$$

$$R_{load}*\left|\frac{V_{in}}{V_{out}}\right|\sin(\theta)=\omega L$$

$$R_{load}*\left|\frac{V_{in}}{V_{out}}\right|\sin(\theta)*\frac{1}{\omega}=L$$

As seen in the last two equation we have obtained the values of inner resistance and inductance.

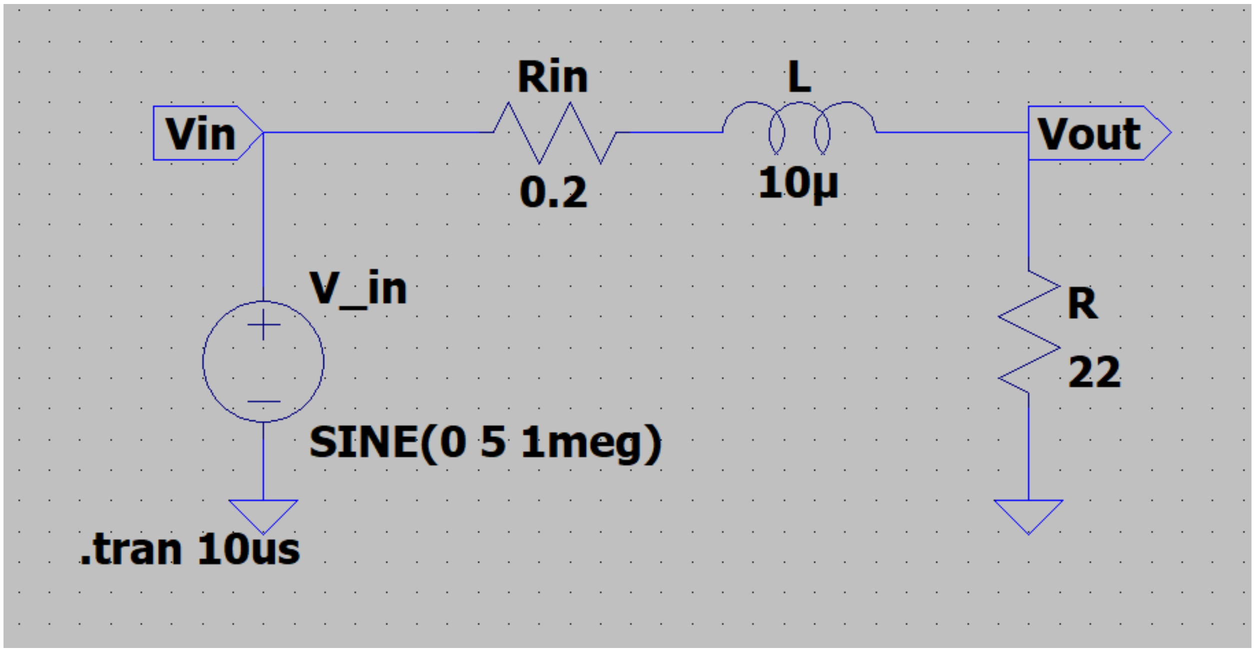

Figure-3: 5V, 1 MHZ voltage is applied, Load Resistor= 22 Ω

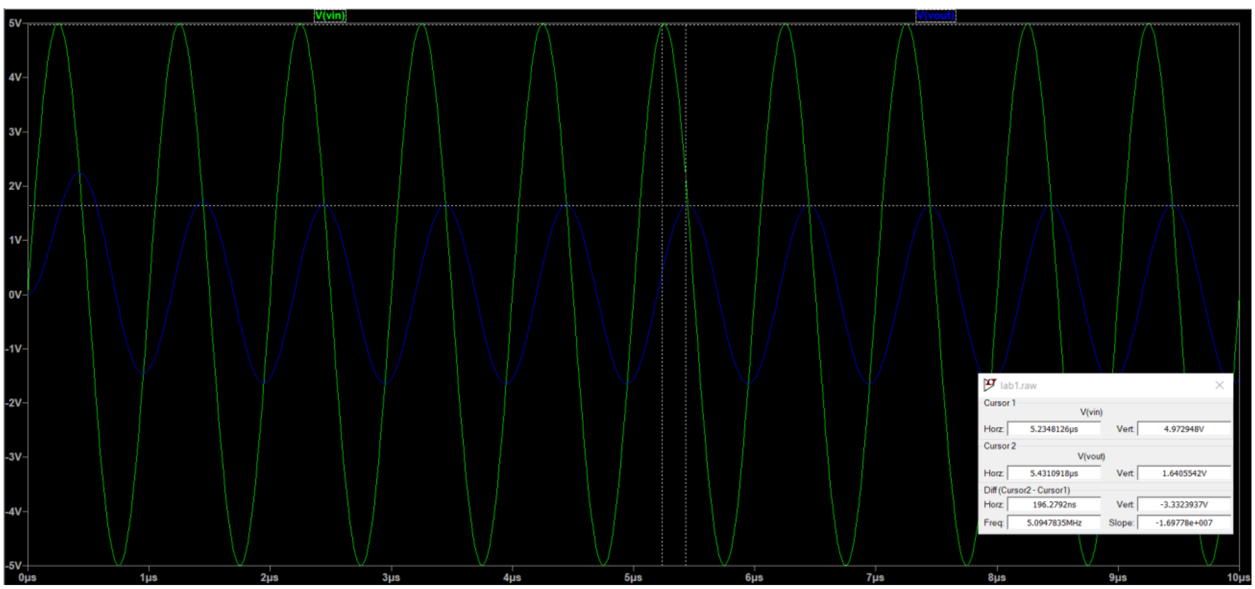

Figure-4: Simulation Result and Require Values to Calculate Q for the Circuit given in Figure-3

Results for Method-1:

Time Difference (Δt)	Frequency (f)	R_{load}	V_{in}	V_{out1}	Phase Angle (θ)	R_{in}	L	Q
196ns	1 MHZ	22 Ω	5V	1.64V	1.233°	0.204	10.07	310.25
110ns	2 MHZ	22 Ω	5V	0.843V	1.399°	0.224	10.23	573.04
78ns	3 MHZ	22 Ω	5V	0.572V	1.454°	0.255	10.13	748.12
60ns	4 MHZ	22 Ω	5V	0.425V	1.484°	0.284	10.26	906.89

Table-1: Results of Simulation Using Method-1

Method-2 (Voltage Divider Method) :

In this method voltage divider formula and transfer function are used again. However, rather than phase angle or time difference, two different load resistors are used to have 2 different equations with 2 unknowns.

Voltage divider and transfer functions formulas for different load resistors are given below.

For first load resistance:

$$\frac{V_{out1}}{V_{in}} = \frac{R_{load1}}{R_{in} + R_{load} + j\omega L}$$

$$(\frac{V_{out1}}{V_{in}})^2 = \frac{{R_{load1}}^2}{(R_{in} + R_{load1})^2 + (\omega L)^2}$$

$$(\frac{V_{in}}{V_{out1}})^2 * {R_{load1}}^2 = (R_{in} + R_{load1})^2 + (\omega L)^2$$

For second load resistance:

$$\frac{V_{out2}}{V_{in}} = \frac{R_{load2}}{R_{in} + R_{load} + j\omega L}$$

$$(\frac{V_{out2}}{V_{in}})^2 = \frac{{R_{load2}}^2}{(R_{in} + R_{load2})^2 + (\omega L)^2}$$

$$(\frac{V_{in}}{V_{out2}})^2 * {R_{load2}}^2 = (R_{in} + R_{load2})^2 + (\omega L)^2$$

Using the findings, the equation below is obtained:

$$(\frac{V_{in}}{V_{out1}})^2 * {R_{load1}}^2 - \left(\frac{V_{in}}{V_{out2}}\right)^2 * {R_{load2}}^2 = (R_{in} + R_{load1})^2 - (R_{in} + R_{load2})^2$$

$$(\frac{V_{in}}{V_{out1}})^2 * {R_{load1}}^2 - \left(\frac{V_{in}}{V_{out2}}\right)^2 * {R_{load2}}^2 = (R_{load1} - R_{load2}) * (R_{load1} + R_{load2} + 2 * R_{in})$$

$$\frac{(\frac{V_{in}}{V_{out1}})^2 * {R_{load1}}^2 - \left(\frac{V_{in}}{V_{out2}}\right)^2 * {R_{load2}}^2}{2(R_{load1} - R_{load2})} - \frac{(R_{load1} + R_{load2})}{2} = R_{in}$$

By Substitution what we obtained for inner resistance in one of the equations we can easily find the value of inductance. It is given below.

$$(\frac{V_{in}}{V_{out1}})^2 * {R_{load1}}^2 = (R_{in} + R_{load1})^2 + (\omega L)^2$$

$$(\frac{V_{in}}{V_{out1}})^2 * {R_{load1}}^2 - (R_{in} + R_{load1})^2 = (\omega L)^2$$

$$\sqrt{\frac{(\frac{V_{in}}{V_{out1}})^2 * {R_{load1}}^2 - (R_{in} + R_{load1})^2}{\omega^2}} = L$$

$$\sqrt{\frac{(\frac{V_{in}}{V_{out1}})^2 * {R_{load1}}^2 - ((\frac{(\frac{V_{in}}{V_{out1}})^2 * {R_{load1}}^2 - \left(\frac{V_{in}}{V_{out2}}\right)^2 * {R_{load2}}^2}{2(R_{load1} - R_{load2})} - \frac{(R_{load1} + R_{load2})}{2}) + R_{load1})^2}{\omega^2}}$$

$$= L$$

As seen at the end we have found the values of inductance of inner resistor.

In this step I decided to use 22 Ω and 10 Ω resistance as load resistors.

LtSpice sample signal form and sample required values for this method are given below.

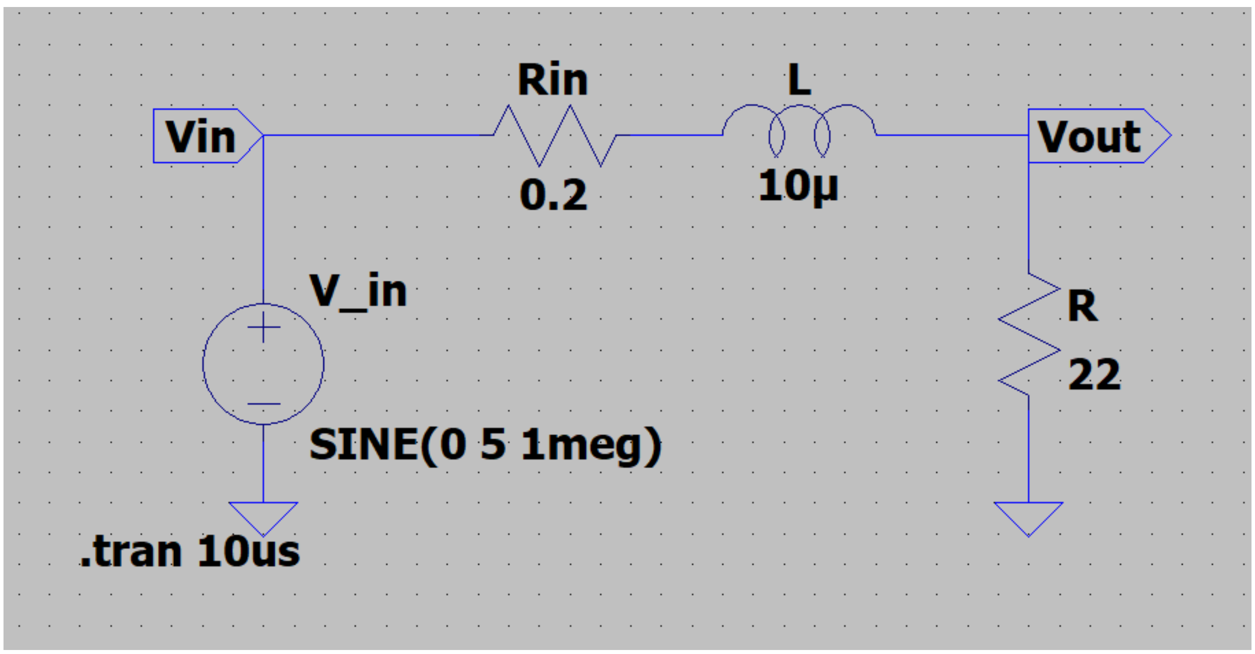

Figure-5: 5V, 1 MHZ voltage is applied, Load Resistor= 22 Ω

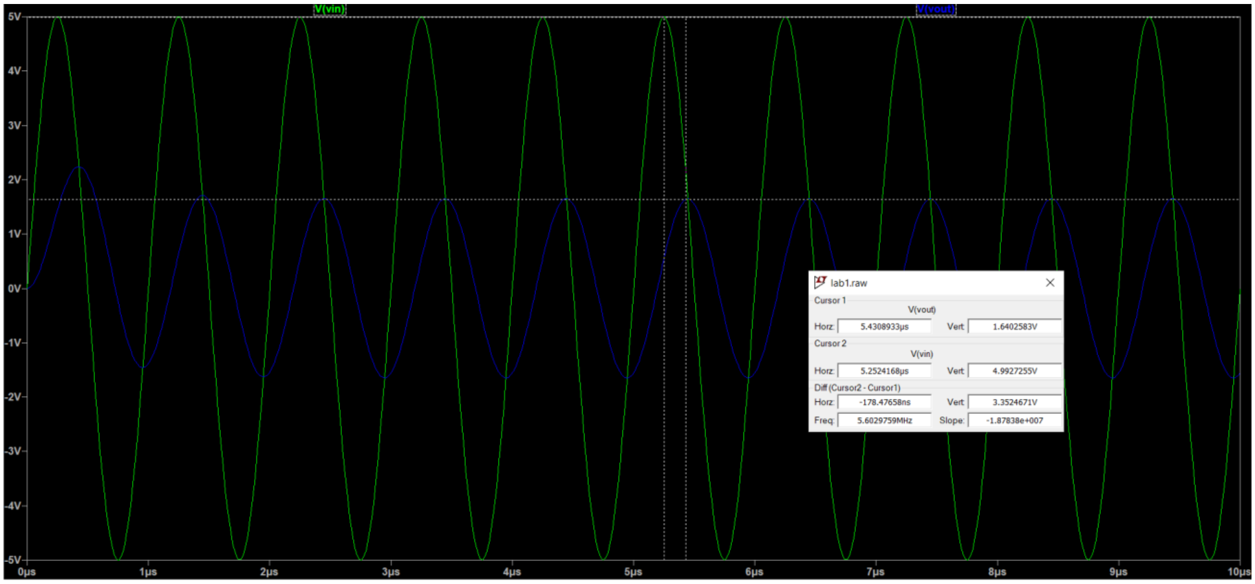

Figure-6: Simulation Result and Require Values to Calculate Q for the Circuit given in Figure-5

Results for Method-2:

V_{in}	V_{out1}	V_{out2}	R_{load1}	R_{load2}	Frequency (f)	R_{in}	L	Q
5 V	1.64 V	0.78 V	22 Ω	10 Ω	1 Mhz	0.236 Ω	10.07 µF	267.78
5 V	0.84V	0.39 V	22 Ω	10 Ω	2 Mhz	1.510 Ω	10.21 µF	84.98
5 V	0.57 V	0.26 V	22 Ω	10 Ω	3 Mhz	3.954 Ω	10.11 µF	48.18
5 V	0.423V	0.19 V	22 Ω	10 Ω	4 Mhz	4.638 Ω	10.24µF	55.5

Table-2: Results of Simulation Using Method-2

Hardware implementation

In this step we are required to create our sample models from our models in real life. To realize that we have used 2 different inductors. The first inductor is and core inductor and the other one is obtained from lab. To have a 10 μF core inductor T38-8 is used since its A_L is 21 ${}^{nH}/_{N^2}$ the number of turns is arranged to 20 then by making them closer its value is increased. The values for core inductor are given below.

$$L = A_L * n^2$$

$$8.82\ \mu\text{F} = 20 * 21^2$$

n=number of turns

According to formula inductance is 8.82; however, to increase its value winds are squeezed, the new inductance value is given below.

Figure-7: Resistance Value for Winded Inductor

Figure-8: Inductance Value for Winded Inductor

Rather than winded core inductor we build, an axial inductor is used as well. Its values are given below.

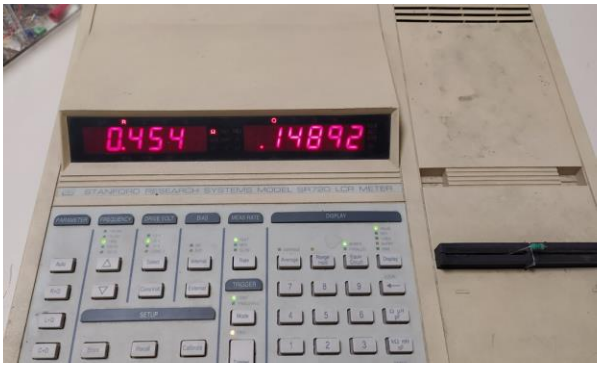

Figure-9: Resistance Value for Axial Inductor

Figure-10: Inductance Value for Axial Inductor

After measurements methods are applied for both of inductors using the circuit below.

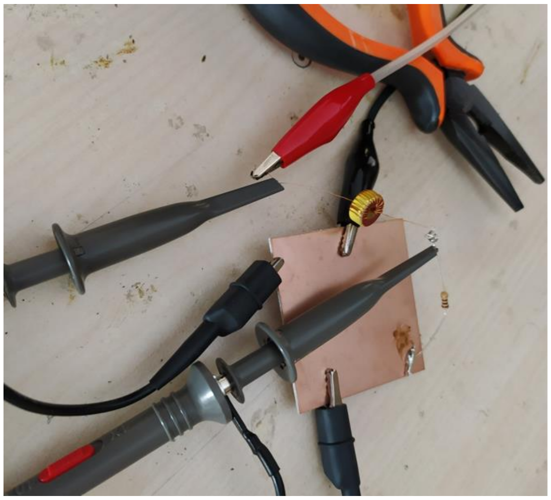

Figure-11: Real Life Implementation of the Sample Circuit

Results:

Method-2, Axial Inductor:

V_{in}	V_{out1}	V_{out2}	R_{load1}	R_{load2}	Frequency (f)	R_{in}	L	Q
4.6V	0.72V	0.34V	10 Ω	4.7 Ω	1 MHZ	0.206 Ω	10.037 µF	305.3227
4.8V	0.68V	0.32V	10 Ω	4.7 Ω	1.2 MHZ	0.209 Ω	9.263 µF	334.7101
5V	0.64V	0.30V	10 Ω	4.7 Ω	1.4 MHZ	0.224 Ω	8.805 µF	346.1458
5.2V	0.56V	0.26V	10 Ω	4.7 Ω	1.6 MHZ	0.256 Ω	9.180 µF	359.7645

Table-3: Results of Hardware Implementation Using Axial Inductor and Method-2

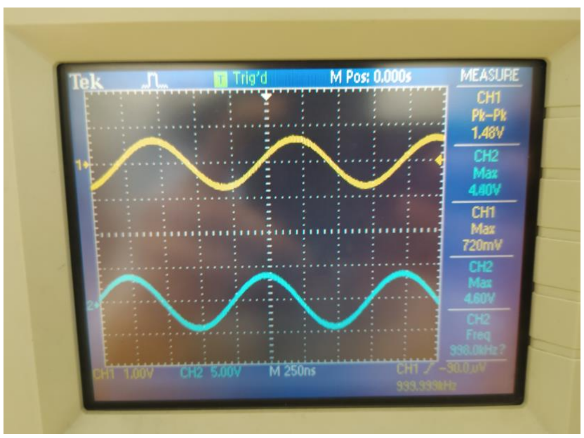

Figure-12: Input and Output Signal for axial inductor at 1 MHZ

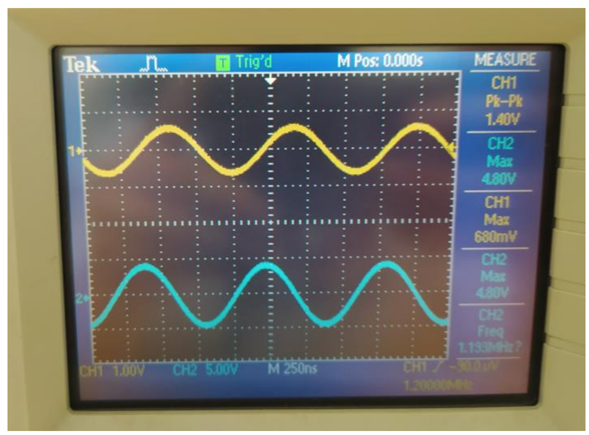

Figure-13: Input and Output Signal for axial inductor at 1.2 MHZ

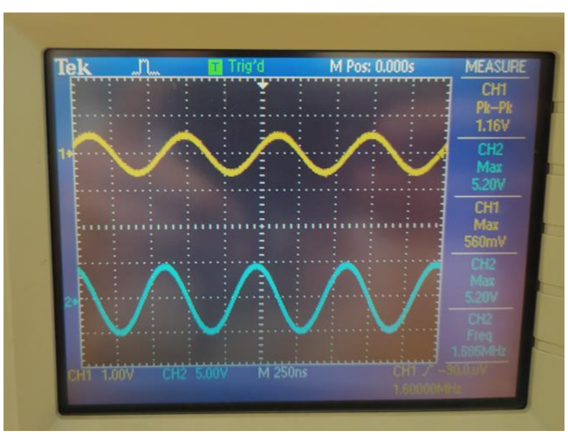

Figure-15: Input and Output Signal for axial inductor at 1.6 MHZ

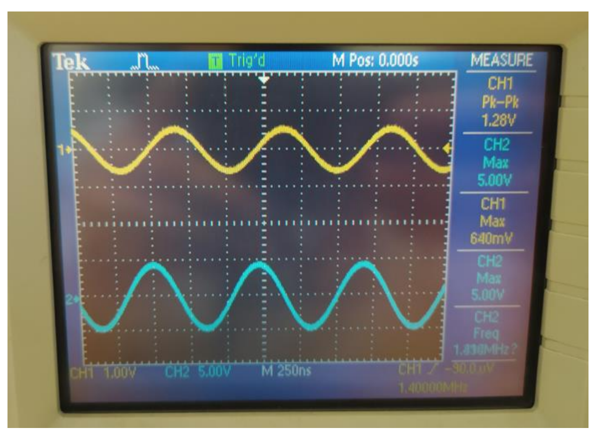

Figure-14: Input and Output Signal for axial inductor at 1.4 MHZ

Method 2 winded core inductor

V_{in}	V_{out1}	V_{out2}	R_{load1}	R_{load2}	Frequency (f)	R_{in}	L	Q
5.8V	0.92V	0.44V	10 Ω	4.7 Ω	1 MHZ	0.451 Ω	9.89μF	137.62
6.2V	0.88V	0.42V	10 Ω	4.7 Ω	1.2 MHZ	0.465 Ω	9.24 μF	149.82
6.6V	0.76V	0.36V	10 Ω	4.7 Ω	1.4 MHZ	0.488 Ω	9.80 μF	176.63
7V	0.72V	0.34V	10 Ω	4.7 Ω	1.6 MHZ	0.529 Ω	9.61 μF	182.52

Table-4: Results of Hardware Implementation Using Winded Inductor and Method-2

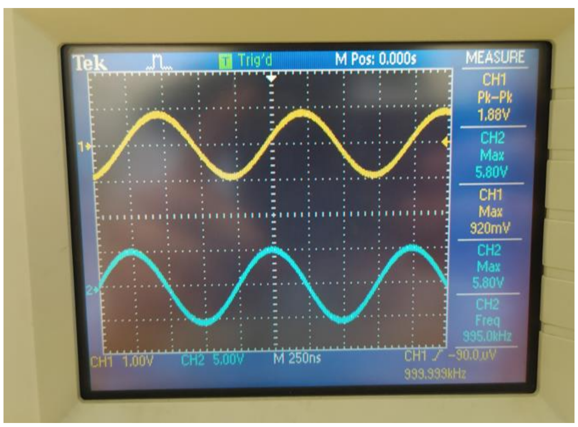

Figure-16: Input and Output Signal for Winded inductor at 1 MHZ

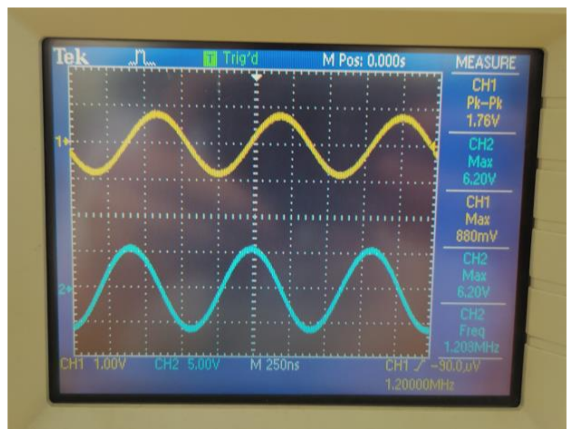

Figure-17: Input and Output Signal for Winded inductor at 1.2 MHZ

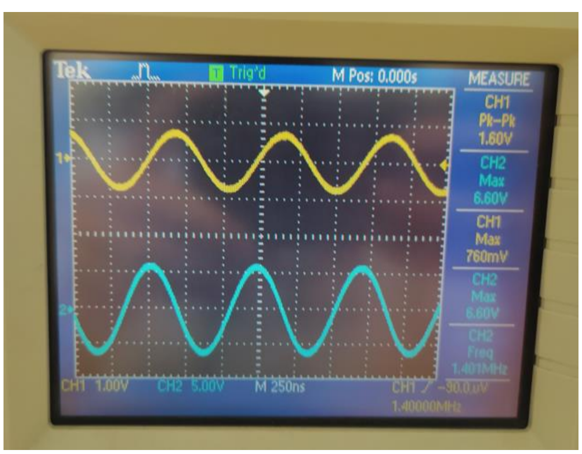

Figure-18: Input and Output Signal for Winded inductor at 1.4 MHZ

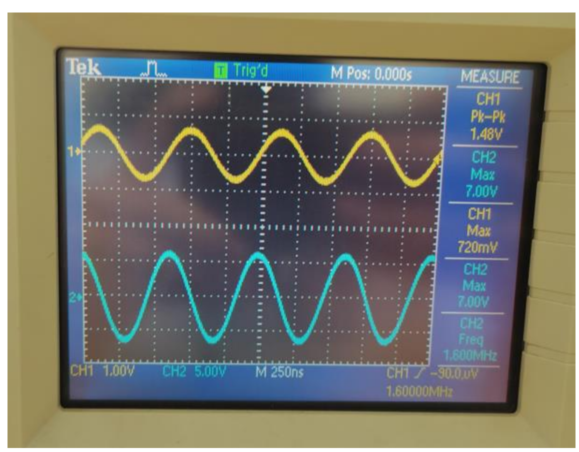

Figure-19: Input and Output Signal for Winded inductor at 1.6 MHZ

Method 1 axial inductor

Time Difference (Δt)	Frequency (f)	R_{load}	V_{in}	V_{out1}	Phase Angle (θ)	R_{in}	L	Q
188ns	1.2MHZ	10Ω	4V	0.56V	1.424°	0.44Ω	9.37 µF	158.94
164ns	1.4MHZ	10Ω	4V	0.52V	1.434°	0.46Ω	8.66 µF	166.18
144ns	1.6MHZ	10Ω	4.4V	0.5V	1.451°	0.50Ω	8.69 µF	175.40
128ns	1.8MHZ	10Ω	4.4V	0.48V	1.456°	0.53Ω	8.05 µF	171.99

Table-5: Results of Hardware Implementation Using Axial Inductor and Method-1

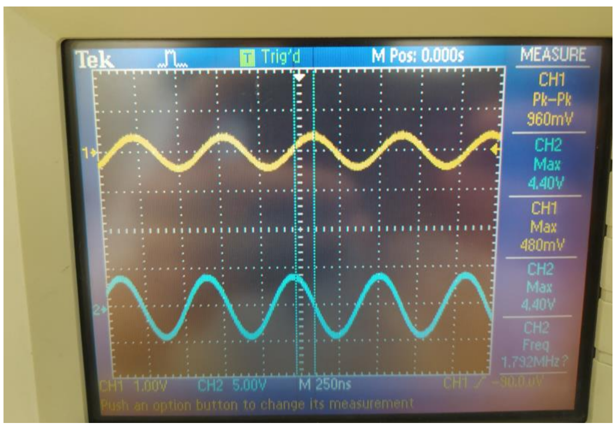

Figure-20: Input and Output Signal for Axial inductor at 1.8 MHZ

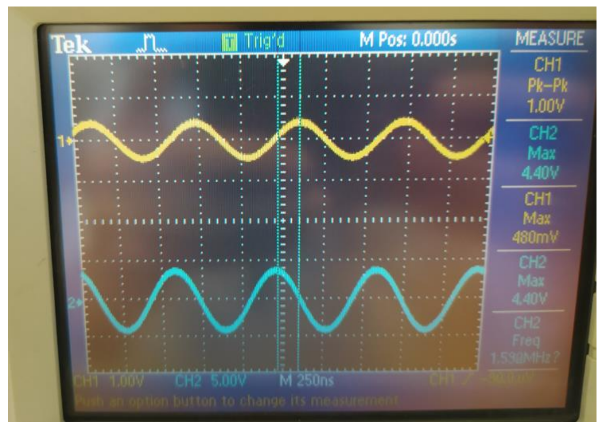

Figure-21: Input and Output Signal for Axial inductor at 1.6 MHZ

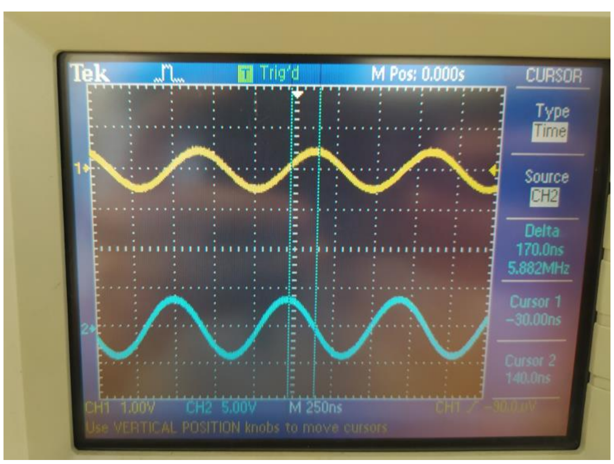

Figure-22: Input and Output Signal for Axial inductor at 1.4 MHZ

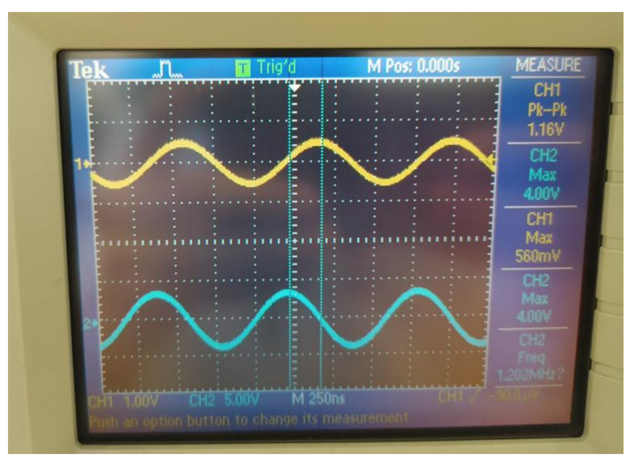

Figure-23: Input and Output Signal for Axial inductor at 1.2 MHZ

Method 1 winded core inductor

Time Difference (Δt)	Frequency (f)	R_{load}	V_{in}	V_{out1}	Phase Angle (θ)	R_{in}	L	Q
228ns	1MHZ	10 Ω	7.6V	1.19V	1.430°	0.206 Ω	11.52 µF	351.77
192ns	1.2MHZ	10 Ω	7.6V	1.04V	1.453°	0.213 Ω	11.49 µF	406.77
166ns	1.4MHZ	10 Ω	8.2V	0.94V	1.453°	0.245 Ω	9.85 µF	353.14
146ns	1.6MHZ	10 Ω	8.4V	0.88V	1.463°	0.258 Ω	9.44 µF	368.38

Table-6: Results of Hardware Implementation Using Winded Inductor and Method-1

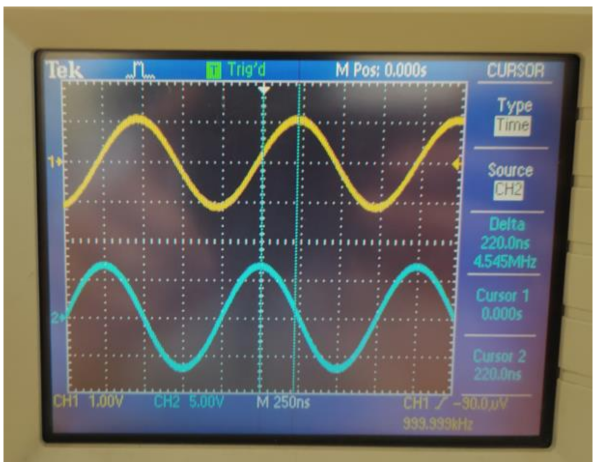

Figure-24: Input and Output Signal for Winded inductor at 1 MHZ

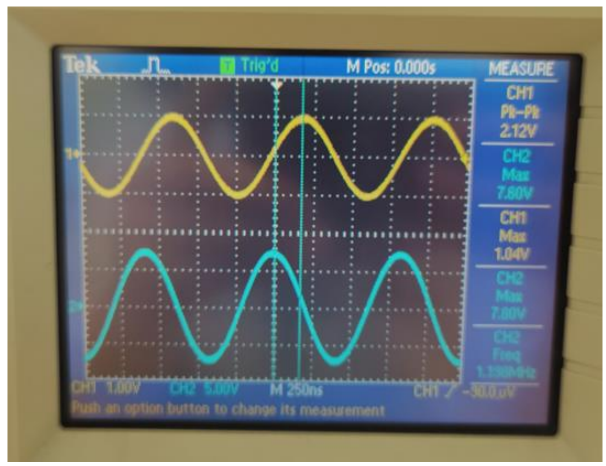

Figure-25: Input and Output Signal for Winded inductor at 1.2 MHZ

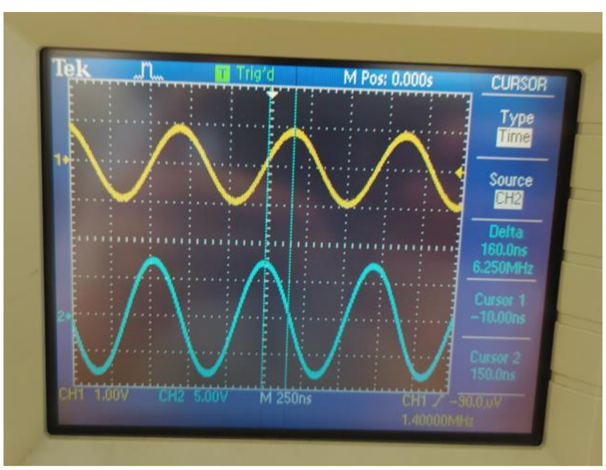

Figure-26: Input and Output Signal for Winded inductor at 1.4 MHZ

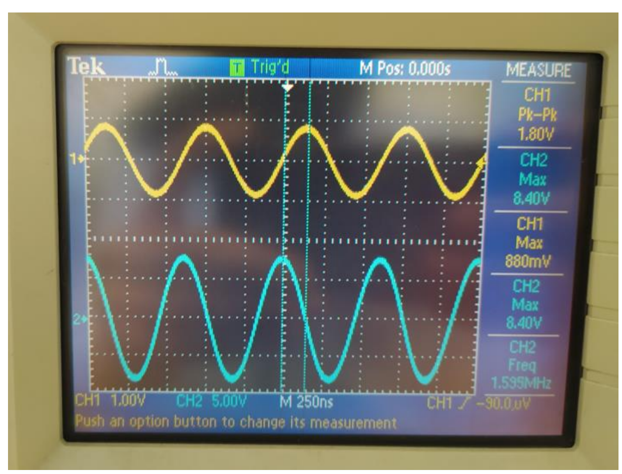

Figure-27: Input and Output Signal for Winded inductor at 1.6 MHZ

Conclusion

The lab consists of 2 parts. In the software lab we designed a method to calculate quality factor of a real inductor and simulate them. In the hardware part the methods are applied in real life for different inductors that one of them is winded by us and the other one is axial inductor taken from lab. In the software part it is observed that when the frequency is increased, the quality factor increases as expected since angular frequency increases with frequency and quality factor is right proportional with angular frequency. However, at high frequencies error percentage was high and calculation of quality factor was becoming wrong. The reason for this error is that when the frequency is high measuring the needed data is more difficult and getting wrong data is more possible since the signals becomes more sensitive. Because of this in the hardware implementation part I decided to choose frequencies in the range 1-2MHZ. As expected in the hardware implementation quality factor is increased at higher frequencies. However, there were still some errors. Fortunately using 1-2MHZ range helped to get more closer results since error percentages were less than 1-5MHZ range. However, the reason for errors in this part were generally related to measurement hardness since oscilloscopes were not enough sensitive to measure the important decimals. All in all, in this laboratory experiment it is understood that inner resistance and inductance value of an inductor can be found using phase angle or voltage divider methods, also it is revealed that quality factor increases at high frequencies.