EEE 321 LAB WORK 4

December 11, 2022

1 Introduction

In this lab experiment it is expected to work on Fourier Series Expansion concept using the given functions ($y_a(t)$). It is known by the course gains so far that for a continuous function to be stored in a digital environment it must be stored firstly. In this lab experiment the functions will be stored by using the sampling period $T_s = 1/8s$, in this way the functions will look more like continuous. After discretization of functions, their fourier series expansions will be found analytically, and fourier series coefficients will be plotted to show the spectrum of the given period signals. Then using the found analytical results, which are fourier series coefficients, discrete function $Z_N[n]$ will be calculated and plotted. In the last part of the lab experiment zeroth, first, second, and third harmonics of $y_a(t)$ will be plotted. Fourier series expansion of any periodic function (let $x(t)$) is given below, where T is its fundamental period.

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j\frac{2\pi}{T}kt}$$

2 Questions & Results

2.1 First Function

Given $y_a(t)$, which is a rectangular waveform defined as below:

$$y_a(t) = \begin{cases} 0 & t \in [0,6)s \\ 4 & t \in [6,10)s \\ 0 & t \in [10,16)s \end{cases}$$

It can be seen easily that $y_a(t)$ is periodic with a period of 16 seconds.

2.1.1 Part a

$y_a(t)$ is discretized using the sampling period $T_s = 1/8$s such that $y_[n] = y_a(nT_s)$ and $n \in [-30, 225]$. Corresponding plot of the function is shown below. Since y[n] is a discrete

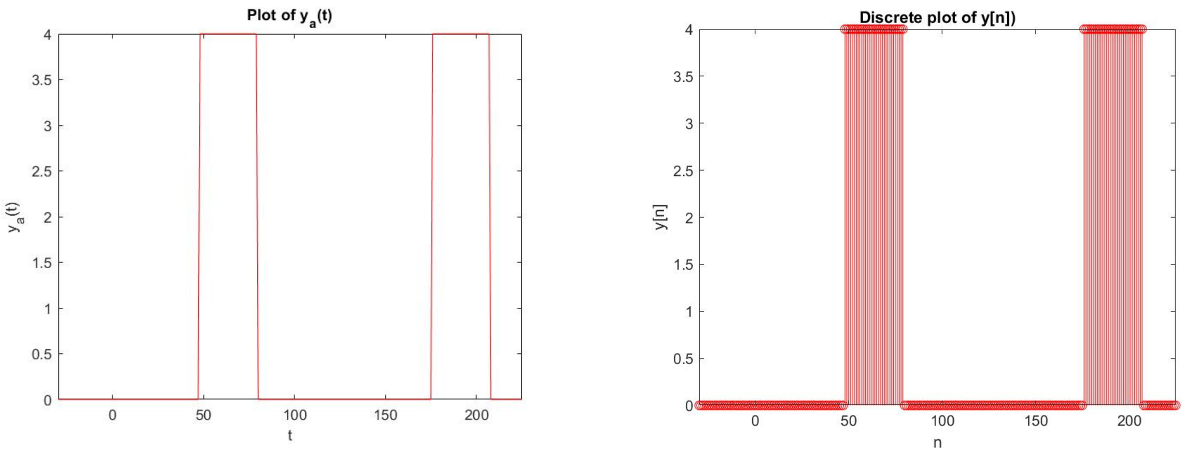

Figure 1: Continuous plot of $y_a(t)$ and Discrete plot of y[n]

signal its plot is drawn in discrete fashion but in lab manual it is indicated that all the functions should be thought as continuous so its continuous plot is drawn as well.

2.1.2 Part b

Fourier series expansion of any function (let $x(t)$) is defined as below, which T is the period of the function and a_k is weight of sinusoidal components.

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j\frac{2\pi}{T}kt}$$

In order to find the fourier series expansion, firstly fourier series coefficients (a_k's) must be found. The formula and the procedure to find them are given below. In this procedure Euler theorem has been used to convert sinusoidal expressions to exponential expressions.(o.o.p means over one period).

$$a_k = \frac{1}{T}\int_{o.o.p} y_a(t)e^{-j\frac{2\pi}{T}kt}\,dt = \frac{1}{16}\int_0^{16} y_a(t)e^{-j\frac{2\pi}{16}kt}\,dt$$
$$= \frac{1}{16}\int_6^{10} 4e^{-j\frac{\pi}{8}kt}\,dt = \frac{4}{16}\int_6^{10} e^{-j\frac{\pi}{8}kt}\,dt$$

For $k \neq 0$

$$= \frac{4}{16}\bigg|_6^{10} \frac{e^{-j\frac{\pi}{8}kt}}{-j\frac{\pi}{8}k} = \frac{2j}{\pi k}\bigg(e^{-j\frac{\pi}{8}(10)k} - e^{-j\frac{\pi}{8}(6)k}\bigg) = \frac{2j}{\pi k}e^{-j\frac{\pi}{8}8k}\bigg(e^{-j\frac{\pi}{8}2k} - e^{-j\frac{\pi}{8}2k}\bigg)$$

Using Euler theorem

$$= \frac{2j}{\pi k}e^{-j\pi}\bigg(-2jsin(\frac{\pi}{4}k)\bigg) = \frac{2j}{\pi k}\bigg(cos(\pi k) + jsin(\pi k)\bigg)\bigg(-2jsin(\frac{\pi}{4}k)\bigg)$$

Using $sin(\pi k) = 0$ for all integer k's.

$$a_k = \frac{4}{\pi k} cos(\pi k) sin(\frac{\pi}{4} k)$$

For $k = 0$

$$a_k = a_0 = \frac{1}{16} \int_0^{16} y_a(t) e^{-j\frac{2\pi}{16}kt} = \frac{1}{16} \int_6^{10} 4e^{-j\frac{2\pi}{16}0t} = \frac{4}{16} \int_6^{10} e^0 \, dt$$

$$= \frac{4}{16} \int_6^{10} 1 \, dt = \frac{4}{16} \Big|_6^{10} 1 = \frac{4}{16} \Big(10 - 6 \Big) = 1 = a_0$$

The fourier series coefficients of $y_a(t)$ are found. Now its fourier series expansion can be written as below.

$$y_a(t) = \sum_{k=-\infty}^{\infty} a_k e^{j\frac{2\pi}{T}kt}$$

$$= \left(\sum_{k=-\infty}^{-1} \frac{4}{\pi k} cos(\pi k) sin(\frac{\pi}{4} k) e^{j\frac{2\pi}{16}kt} \right) + a_k e^{j\frac{2\pi}{16}kt} \Big|_{k=0} + \left(\sum_{k=1}^{\infty} \frac{4}{\pi k} cos(\pi k) sin(\frac{\pi}{4} k) e^{j\frac{2\pi}{16}kt} \right)$$

$$= \left(\sum_{k=-\infty}^{-1} \frac{4}{\pi k} cos(\pi k) sin(\frac{\pi}{4} k) e^{j\frac{2\pi}{16}kt} \right) + 1 + \left(\sum_{k=1}^{\infty} \frac{4}{\pi k} cos(\pi k) sin(\frac{\pi}{4} k) e^{j\frac{2\pi}{16}kt} \right)$$

2.1.3 Part c

In this part of the lab experiment, it is expected to plot the spectrum of $y_a(t)$. Since the spectrum of a periodic signal is complex amplitudes of the complex sinusoidal components of a signal and the sinusoidal componenets are the exponential expressions in the fourier series expansion formula, a_k's are the magnitudes of the sinusoidal components. Therefore, it is needed to plot the fourier series coefficients, which are found in part b, with proper frequencies. Since $f_0 = 1/T_0$, we found the frequency as below.

$$\omega = \omega_0 k = \frac{2\pi}{T_0} k = \frac{2\pi}{16} k = 2\pi f_0 k = 2\pi f$$

$$f = f_0 k$$

Using this frequencies, plot of corresponding amplitudes of sinusoidal components can be drawn as below.

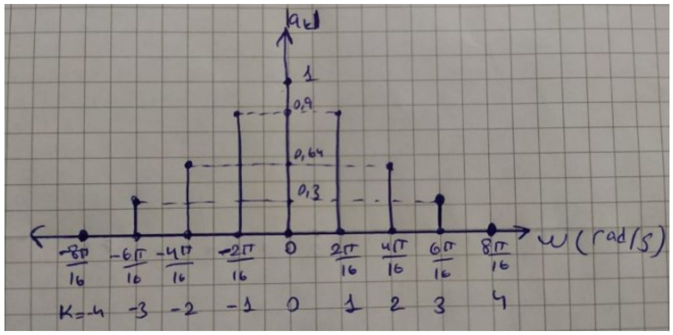

Figure 2: Spectrum of $y_a(t)$

2.1.4 Part d

In this part of the lab experiment, it is needed to compute a discrete $z_N[n]$ function, which is shown below. This function is actually sum of weighted sinusoidal components in a specified interval. That is to say, sum of fourier series expansions' elements in a finite interval.

$$Z_N[n] = \sum_{k=-\infty}^{\infty} a_k e^{j\omega_0 knT_s}$$

N=120
$Z_N[n]$ for N=120 is calculated using MATLAB, and using the result, plot of $Z_{120}[n]$ is drawn as below.

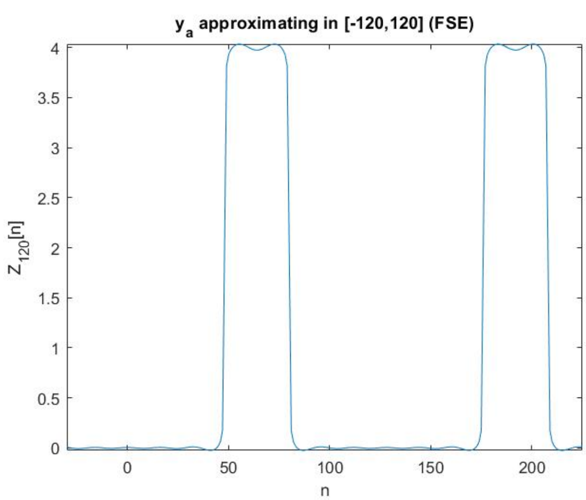

Figure 3: y_a approximation from -120 to 120

2.1.5 Part e.1

N=60
$Z_N[n]$ for N=60 is calculated using MATLAB, and using the result, plot of $Z_{60}[n]$ is drawn as below.

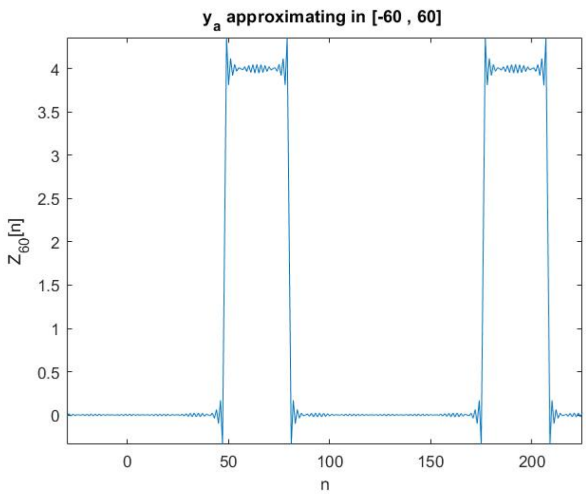

Figure 4: y_a approximation from -60 to 60

2.1.6 Part e.2

N=30
$Z_N[n]$ for N=30 is calculated using MATLAB, and using the result, plot of $Z_{30}[n]$ is drawn as below.

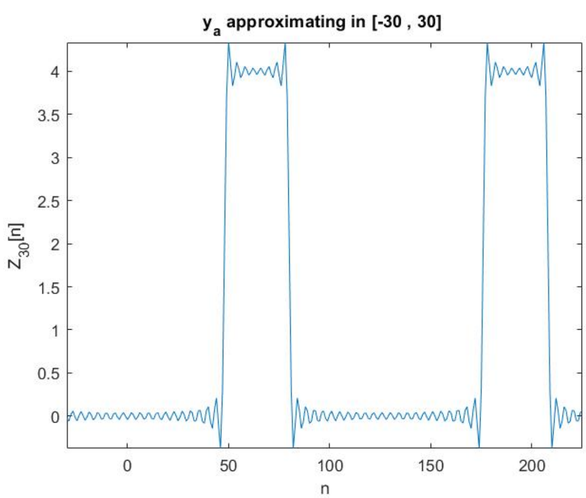

Figure 5: y_a approximation from -30 to 30

2.1.7 Part f

N=5
$Z_N[n]$ for N=5 is calculated using MATLAB, and using the result, plot of $Z_5[n]$ is drawn as below.

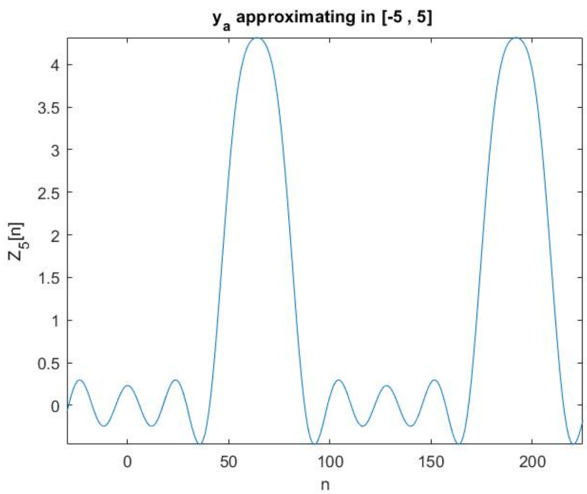

Figure 6: y_a approximation from -5 to 5

2.1.8 Part g

N=3
$Z_N[n]$ for N=3 is calculated using MATLAB, and using the result, plot of $Z_3[n]$ is drawn as below.

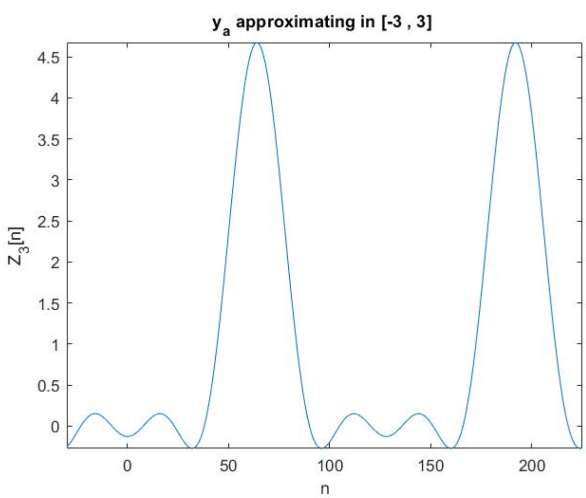

Figure 7: y_a approximation from -3 to 3

2.1.9 Part h

N=1
$Z_N[n]$ for N=1 is calculated using MATLAB, and using the result, plot of $Z_1[n]$ is drawn as below.

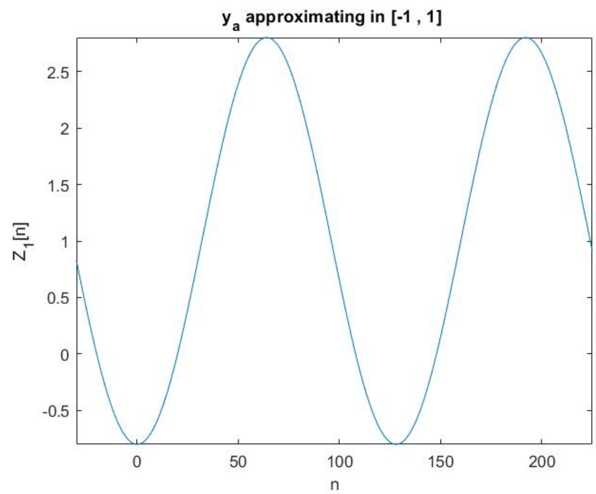

Figure 8: y_a approximation from -1 to 1

2.1.10 Comparison of Functions From d to h

To compare the plots N=120, N=30, N=5 and N=1 are chosen and they have drawn on same plot, also to compare those function with the original function it is drawn to the left of the figure, which are shown below.

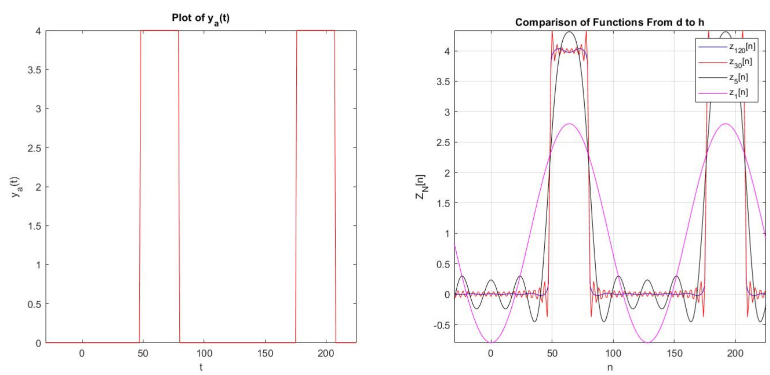

Figure 9: Comparison of Functions From d to h and y[n]

As seen from the figure $Z_{120}[n]$ is most similar plot to $y_a[n]$ whereas $Z_1[n]$ is quite different. This is because, to calculate $Z_N[n]$, elements from fourier series expansion of $y_a[n]$ are used. To calculate $Z_{120}[n]$ 241 elements at the center are used but to calculate $Z_1[n]$ only 3 elements are used, which makes $Z_{120}[n]$ more similar to $y_a[n]$ than $Z_1[n]$. Also paying attention to Gibbs phenomenon it can be seen that when Z=120 the passing looks like an instant jump because of more sinusoidal terms, when Z=1 the wave looks like sinusoidal because of less sinusoidal terms. Briefly it can be said that the higher N is the higher the quality of the $Z_N[n]$ approximation.

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2.1.11 Part I

In this part of the experiment zeroth, first, second, and third harmonics of $y_a(t)$, will computed and drawn on the same plot using same scale,which is shown below in figure. The equation of the k'th harmonic (for $k \neq 0$) is given below. Zeroth harmonic is equal to a_0 which is just a constant . Thus, Zeroth harmonic is DC component of $y_a(t)$.

$$k'th\ harmonic = a_{-k}e^{-j\omega_0 kt} + a_k e^{j\omega_0 kt}$$

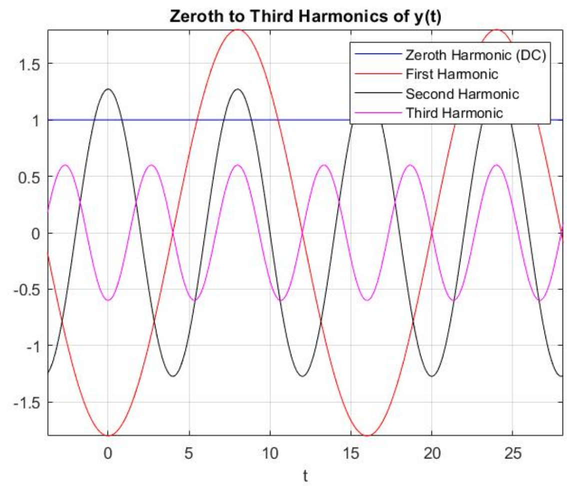


Figure 10: Zeroth to Third Harmonics of $y_a(t)$

2.2 Second Function

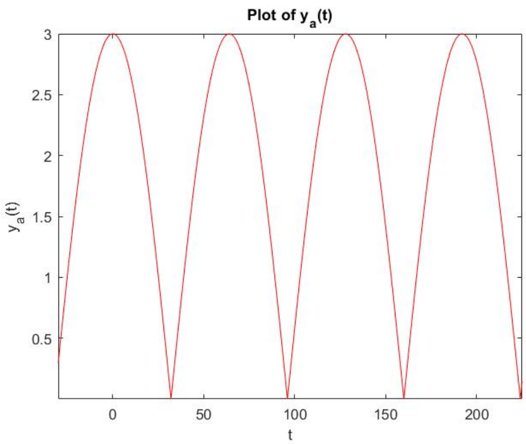
Given $y_a(t)$, which is a rectangular waveform defined as below:

$$y_a(t) = \left| 3cos\left(\frac{\pi}{8}t\right) \right|$$

It can be seen easily that $y_a(t)$ is an absolute value of a cosine function. Normally period of cosine function is 2π but this is an absolute value of a cosine function. Therefore the period of this function is π, using this fact its fundamental period (T) is found as $\frac{\pi}{\pi/8} = 8s$

2.2.1 Part a

$y_a(t)$ is discretized using the sampling period $T_s = 1/8$s such that $y_[n] = y_a(nT_s)$ and $n \in [-30, 225]$. Corresponding plot of the function is shown below. Since y[n] is a discrete signal its plot is drawn in discrete fashion but in lab manual it is indicated that all the functions should be thought as continuous so its continuous plot is drawn as well.



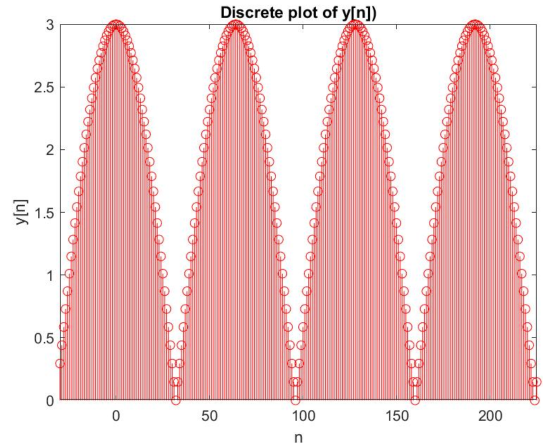


Figure 11: Continuous plot of $y_a(t)$ and Discrete plot of y[n]

2.2.2 Part b

In order to find the fourier series expansion, firstly fourier series coefficients (a_k's) must be found. The formula and the procedure to find them are given below. In this procedure Euler theorem has been used to convert sinusoidal expressions to exponential expressions.

$$a_k = \frac{1}{T}\int_{o.o.p} y_a(t)e^{-j\frac{2\pi}{T}kt}\,dt = \frac{1}{8}\int_{o.o.p}\left|3cos\left(\frac{\pi}{8}t\right)\right|e^{-j\frac{2\pi}{8}kt}\,dt$$

Using $cos\left(\frac{\pi}{8}t\right) \geq 0$ for $t \in [-4, 4]$

$$= \frac{1}{8}\int_{-4}^{4} 3cos\left(\frac{\pi}{8}t\right)e^{-j\frac{2\pi}{8}kt}\,dt$$

Using Euler theorem

$$= \frac{3}{8}\int_{-4}^{4}\left(\frac{e^{j\frac{\pi}{8}t}+e^{-j\frac{\pi}{8}t}}{2}\right)e^{-j\frac{2\pi}{8}kt}\,dt = \frac{3}{16}\int_{-4}^{4}\left(e^{j\frac{\pi}{8}t-j\frac{2\pi}{8}kt}+e^{-j\frac{\pi}{8}t-j\frac{2\pi}{8}kt}\right)dt$$

For $k \neq 1/2$ and $k \neq -1/2$, noting that no need to worry about 1/2 or -1/2 since k is integer.

$$= \frac{3}{16}\Bigg|_{-4}^{4}\left(\frac{e^{j\frac{\pi}{8}t-j\frac{2\pi}{8}kt}}{j\frac{\pi}{8}-j\frac{2\pi}{8}k}+\frac{e^{-j\frac{\pi}{8}t-j\frac{2\pi}{8}kt}}{-j\frac{\pi}{8}-j\frac{2\pi}{8}k}\right)$$

$$= \frac{3}{16}\left(\frac{e^{j\frac{\pi}{8}4}e^{-j\frac{2\pi}{8}4k}-e^{j\frac{\pi}{8}(-4)}e^{-j\frac{2\pi}{8}(-4)k}}{j\frac{\pi}{8}-j\frac{2\pi}{8}k}+\frac{e^{-j\frac{\pi}{8}(4)}e^{-j\frac{2\pi}{8}k(4)}-e^{-j\frac{\pi}{8}(-4)}e^{-j\frac{2\pi}{8}k(-4)}}{-j\frac{\pi}{8}-j\frac{2\pi}{8}k}\right)$$

Using $e^{j\pi/2} = j$ and $e^{-j\pi/2} = -j$

$$= \frac{3}{16}\left(\frac{je^{-j\pi k}+je^{j\pi k}}{j\frac{\pi}{8}-j\frac{2\pi}{8}k}+\frac{-je^{-j\pi k}-je^{j\pi k}}{-j\frac{\pi}{8}-j\frac{2\pi}{8}k}\right) = \frac{3}{16}\left(e^{-j\pi k}+e^{j\pi k}\right)\left(\frac{1}{\frac{\pi}{8}-\frac{2\pi}{8}k}+\frac{1}{\frac{\pi}{8}+\frac{2\pi}{8}k}\right)$$

Using Euler's theorem

$$= \frac{3}{\pi}\big(2cos(\pi k)\big)\left(\frac{1}{1-4k^2}\right) = \frac{6cos(\pi k)}{\pi(1-4k^2)} = a_k$$

The fourier series coefficients of $y_a(t)$ are found. Now its fourier series expansion can be written as below.

$$y_a(t) = \sum_{k=-\infty}^{\infty} a_k e^{j\frac{2\pi}{T}kt} = \sum_{k=-\infty}^{\infty} \frac{6cos(\pi k)}{\pi(1-4k^2)} e^{j\frac{2\pi}{8}kt}$$

2.2.3 Part c

In this part of the lab experiment, it is expected to plot the spectrum of $y_a(t)$. Since the spectrum of a periodic signal is complex amplitudes of the complex sinusoidal components of a signal and the sinusoidal componenets are the exponential expressions in the fourier series expansion formula, a_k's are the magnitudes of the sinusoidal components. Therefore, it is needed to plot the fourier series coefficients, which are found in part b, with proper frequencies. Since $f_0 = 1/T_0$, we found the frequency as below.

$$\omega = \omega_0 k = \frac{2\pi}{T_0}k = \frac{2\pi}{8}k = 2\pi f_0 k = 2\pi f$$

$$f = f_0 k$$

Using this frequencies, plot of corresponding amplitudes of sinusoidal components can be drawn as below.

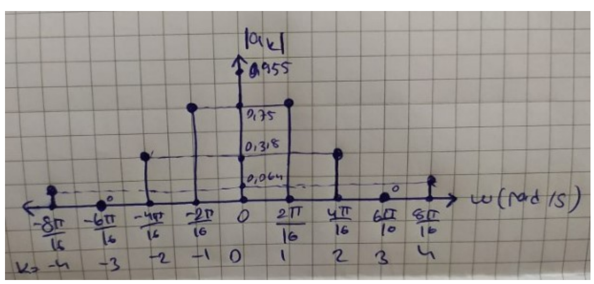


Figure 12: Spectrum of $y_a(t)$

2.2.4 Part d

In this part of the lab experiment, it is needed to compute a discrete $z_N[n]$ function, which is shown below. This function is actually sum of weighted sinusoidal components in a specified interval. That is to say, sum of fourier series expansions' elements in a finite interval.

$$Z_N[n] = \sum_{k=-\infty}^{\infty} a_k e^{j\omega_0 knT_s}$$

N=120
$Z_N[n]$ for N=120 is calculated using MATLAB, and using the result, plot of $Z_{120}[n]$ is drawn as below.

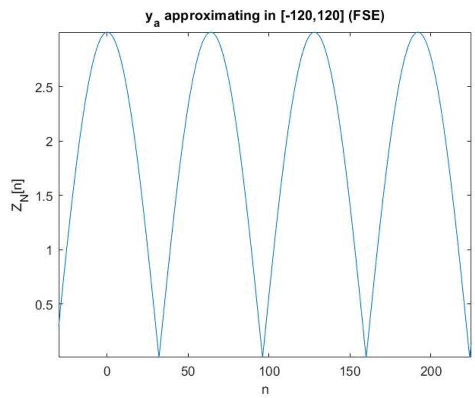


Figure 13: y_a approximation from -120 to 120

2.2.5 Part e.1

N=60
$Z_N[n]$ for N=60 is calculated using MATLAB, and using the result, plot of $Z_{60}[n]$ is drawn as below.

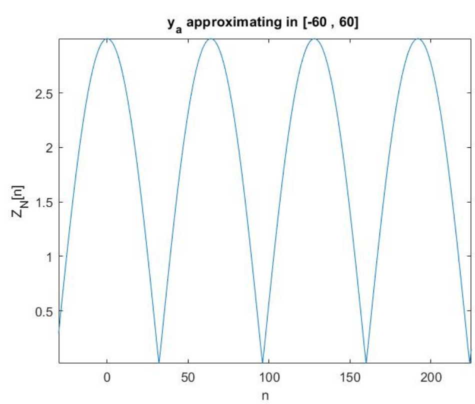


Figure 14: y_a approximation from -60 to 60

2.2.6 Part e.2

N=30
$Z_N[n]$ for N=30 is calculated using MATLAB, and using the result, plot of $Z_{30}[n]$ is drawn as below.

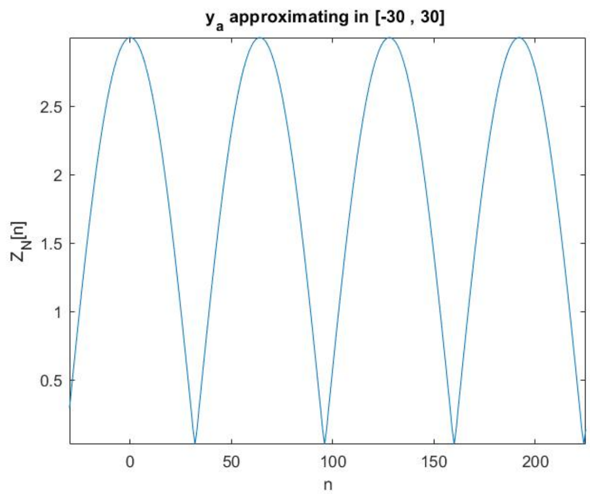


Figure 15: y_a approximation from -30 to 30

2.2.7 Part f

N=5
$Z_N[n]$ for N=5 is calculated using MATLAB, and using the result, plot of $Z_5[n]$ is drawn as below.

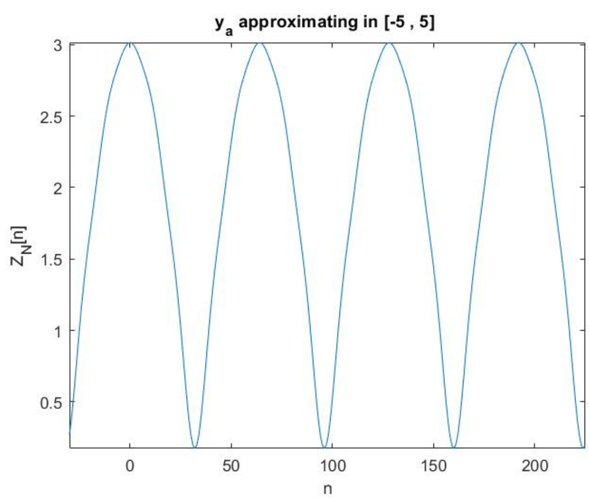


Figure 16: y_a approximation from -5 to 5

2.2.8 Part g

N=3
$Z_N[n]$ for N=3 is calculated using MATLAB, and using the result, plot of $Z_3[n]$ is drawn as below.

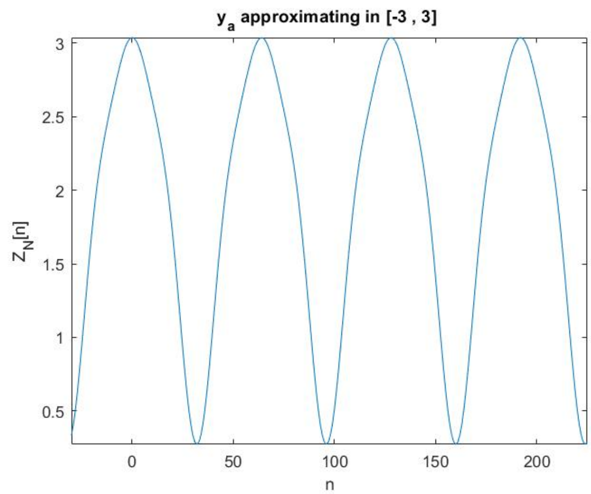


Figure 17: y_a approximation from -3 to 3

2.2.9 Part h

N=1
$Z_N[n]$ for N=1 is calculated using MATLAB, and using the result, plot of $Z_1[n]$ is drawn as below.

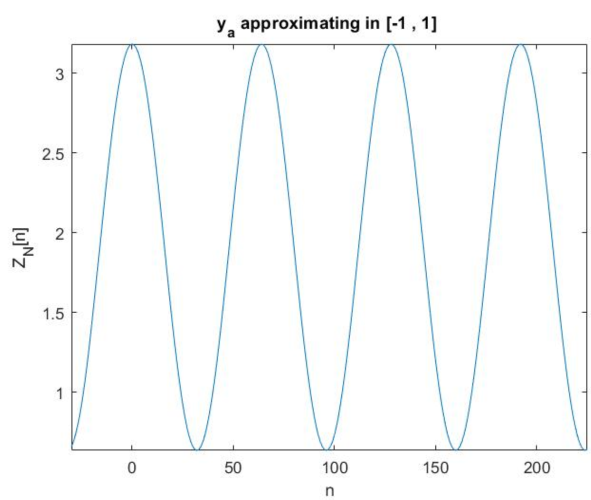


Figure 18: y_a approximation from -1 to 1

2.2.10 Comparison of Functions From d to h

To compare the plots N=120, N=30, N=5 and N=1 are chosen and they have drawn on same plot, also to compare those function with the original function it is drawn to the left of the figure, which are shown below.

As seen from the figure $Z_{120}[n]$ is most similar plot to $y_a[n]$ whereas $Z_1[n]$ is different. This is because, to calculate $Z_N[n]$, elements from fourier series expansion of $y_a[n]$ are used.

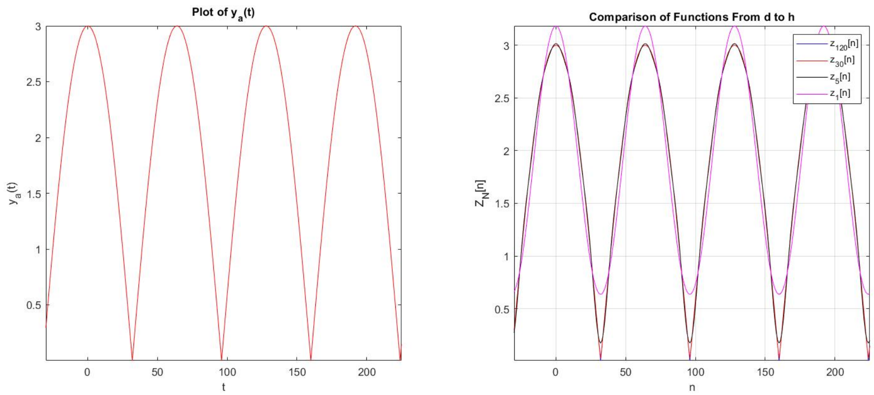


Figure 19: Comparison of Functions From d to h and y_a

To calculate $Z_{120}[n]$ 241 elements at the center are used but to calculate $Z_1[n]$ only 3 elements are used, which makes $Z_{120}[n]$ more similar to $y_a[n]$ than $Z_1[n]$. Also paying attention to Gibbs phenomenon it can be seen that when Z=120 the zero points looks like an instant turn and the function more looks like Full-wave rectifier because of more sinusoidal terms, when Z=1 the wave looks like sinusoidal because of less sinusoidal terms. Briefly it can be said that the higher N is the higher the quality of the $Z_N[n]$ approximation.

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2.2.11 Part I

In this part of the experiment zeroth, first, second, and third harmonics of $y_a(t)$, will computed and drawn on the same plot using same scale,which is shown below in figure. The equation of the k'th harmonic (for $k \neq 0$) is given below. Zeroth harmonic is equal to a_0 which is just a constant . Thus, Zeroth harmonic is DC component of $y_a(t)$.

$$k'th\ harmonic = a_{-k}e^{-j\omega_0 kt} + a_k e^{j\omega_0 kt}$$

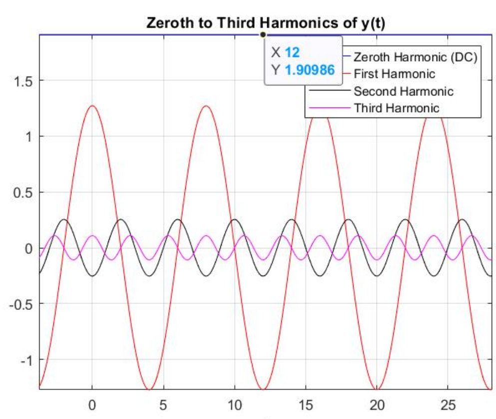

Figure 20: Zeroth to Third harmonics of $y_a(t)$

2.3 Third Function

Given $y_a(t)$, which is a rectangular waveform defined as below:

$$y_a(t) = \begin{cases} \left|3cos(\frac{\pi}{8}t)\right| & t \in [-4,4)s \\ 0 & t \in [4,12)s \end{cases}$$

It can be seen easily that $y_a(t)$ is periodic with a period of 16 seconds.

2.3.1 Part a

$y_a(t)$ is discretized using the sampling period $T_s = 1/8$s such that $y[n] = y_a(nT_s)$ and $n \in [-30, 225]$. Corresponding plot of the function is shown below. Since y[n] is a discrete signal its plot is drawn in discrete fashion but in lab manual it is indicated that all the functions should be thought as continuous so its continuous plot is drawn as well.

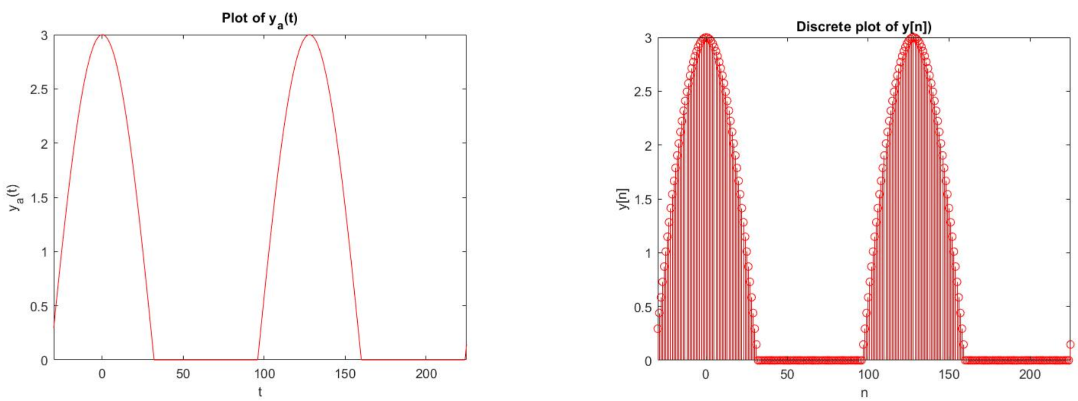

Figure 21: Continuous plot of $y_a(t)$ and Discrete plot of y[n]

2.3.2 Part b

In order to find the fourier series expansion, firstly fourier series coefficients (a_k's) must be found. The formula and the procedure to find them are given below. In this procedure Euler theorem has been used to convert sinusoidal expressions to exponential expressions.

$$a_k = \frac{1}{T}\int_{o.o.p} y_a(t)e^{-j\frac{2\pi}{T}kt}\,dt = \frac{1}{16}\int_{o.o.p}\left|3cos\left(\frac{\pi}{8}t\right)\right|e^{-j\frac{2\pi}{16}kt}\,dt$$

Using $cos\left(\frac{\pi}{8}t\right) \geq 0$ for $t \in [-4,4]$, and $y_a(t) = 0$ for $t \in [4,12)$

$$= \frac{1}{16}\int_{-4}^{4} 3cos\left(\frac{\pi}{8}t\right)e^{-j\frac{2\pi}{16}kt}\,dt$$

Using Euler theorem

$$= \frac{3}{16}\int_{-4}^{4}\left(\frac{e^{j\frac{\pi}{8}t} + e^{-j\frac{\pi}{8}t}}{2}\right)e^{-j\frac{2\pi}{16}kt}\,dt = \frac{3}{32}\int_{-4}^{4}\left(e^{j\frac{\pi}{8}t - j\frac{2\pi}{16}kt} + e^{-j\frac{\pi}{8}t - j\frac{2\pi}{16}kt}\right)dt$$

For $k \neq 1$ and $k \neq -1$

$$= \frac{3}{32}\Bigg|_{-4}^{4}\left(\frac{e^{j\frac{\pi}{8}t - j\frac{\pi}{8}kt}}{j\frac{\pi}{8} - j\frac{\pi}{8}k} + \frac{e^{-j\frac{\pi}{8}t - j\frac{\pi}{8}kt}}{-j\frac{\pi}{8} - j\frac{\pi}{8}k}\right)$$

$$= \frac{3}{32}\left(\frac{e^{j\frac{\pi}{8}4}e^{-j\frac{\pi}{8}4k} - e^{j\frac{\pi}{8}(-4)}e^{-j\frac{\pi}{8}(-4)k}}{j\frac{\pi}{8} - j\frac{\pi}{8}k} + \frac{e^{-j\frac{\pi}{8}(4)}e^{-j\frac{\pi}{8}k(4)} - e^{-j\frac{\pi}{8}(-4)}e^{-j\frac{\pi}{8}k(-4)}}{-j\frac{\pi}{8} - j\frac{\pi}{8}k}\right)$$

Using $e^{j\pi/2} = j$ and $e^{-j\pi/2} = -j$

$$= \frac{3}{32}\left(\frac{je^{-j\pi k/2} + je^{j\pi k/2}}{j\frac{\pi}{8} - j\frac{\pi}{8}k} + \frac{-je^{-j\pi k/2} - je^{j\pi k/2}}{-j\frac{\pi}{8} - j\frac{\pi}{8}k}\right) = \frac{3}{32}\big(e^{-j\pi k/2} + e^{j\pi k/2}\big)\left(\frac{1}{\frac{\pi}{8} - \frac{\pi}{8}k} + \frac{1}{\frac{\pi}{8} + \frac{\pi}{8}k}\right)$$

Using Euler's theorem

$$= \frac{3}{2\pi}\big(2cos(\frac{\pi k}{2})\big)\left(\frac{1}{1-k^2}\right) = \frac{3cos(\frac{\pi k}{2})}{\pi(1-k^2)} = a_k$$

For $k = 1$

$$a_k = a_1 = \frac{1}{16}\int_{o.o.p}\left|3cos\left(\frac{\pi}{8}t\right)\right|e^{-j\frac{2\pi}{16}kt}\,dt$$

Using $cos\big(\frac{\pi}{8}t\big) \geq 0$ for $t \in [-4, 4]$

$$= \frac{1}{16}\int_{-4}^{4} 3cos\left(\frac{\pi}{8}t\right)e^{-j\frac{\pi}{8}t}\,dt$$

Using Euler theorem

$$= \frac{3}{16}\int_{-4}^{4}\left(\frac{e^{j\frac{\pi}{8}t} + e^{-j\frac{\pi}{8}t}}{2}\right)e^{-j\frac{\pi}{8}t}\,dt = \frac{3}{32}\int_{-4}^{4}\left(1 + e^{-2j\frac{\pi}{8}t}\right)dt$$

$$= \frac{3}{32}\Bigg|_{-4}^{4}\left(t + \frac{e^{-j\frac{\pi}{4}t}}{-j\frac{\pi}{4}}\right) = \frac{3}{32}\big(4 - (-4)\big) + \frac{3}{32}\left(\frac{e^{-\pi j} - e^{\pi j}}{-j\frac{\pi}{4}}\right)$$

$$= \frac{3}{4} + \frac{3}{32}\left(\frac{-1 - (-1)}{-j\frac{\pi}{4}}\right) = \frac{3}{4} + \frac{3}{16}\Big(0\Big) = \frac{3}{4} = a_1$$

For $k = -1$

$$a_k = a_{-1} = \frac{1}{16}\int_{o.o.p}\left|3cos\left(\frac{2\pi}{16}t\right)\right|e^{-j\frac{2\pi}{16}kt}\,dt$$

Using $cos\big(\frac{\pi}{8}t\big) \geq 0$ for $t \in [-4, 4]$

$$= \frac{1}{16}\int_{-4}^{4} 3cos\left(\frac{\pi}{8}t\right)e^{j\frac{\pi}{8}t}\,dt$$

Using Euler theorem

$$= \frac{3}{16}\int_{-4}^{4}\left(\frac{e^{j\frac{\pi}{8}t}+e^{-j\frac{\pi}{8}t}}{2}\right)e^{j\frac{\pi}{8}t}\,dt = \frac{3}{32}\int_{-4}^{4}\left(1+e^{2j\frac{\pi}{8}t}\right)dt$$

$$= \frac{3}{32}\Bigg|_{-4}^{4}\left(t+\frac{e^{j\frac{\pi}{4}t}}{j\frac{\pi}{4}}\right) = \frac{3}{32}\big(4-(-4)\big)+\frac{3}{32}\left(\frac{e^{\pi j}-e^{-\pi j}}{j\frac{\pi}{4}}\right)$$

$$= \frac{3}{4}+\frac{3}{32}\left(\frac{-1-(-1)}{j\frac{\pi}{4}}\right) = \frac{3}{32}+\frac{3}{32}\Big(0\Big) = \frac{3}{4} = a_{-1}$$

The fourier series coefficients of $y_a(t)$ are found. Now its fourier series expansion can be written as below.

$$y_a(t) = \sum_{k=-\infty}^{\infty} a_k e^{j\frac{2\pi}{T}kt} = \left(\sum_{k=-\infty,k\neq -1 or 1}^{\infty}\frac{3cos(\frac{\pi k}{2})}{\pi(1-k^2)}e^{j\frac{2\pi}{16}kt}\right)+\left(a_k e^{j\frac{2\pi}{16}kt}\right)\Bigg|_{k=1}+\left(a_k e^{j\frac{2\pi}{16}kt}\right)\Bigg|_{k=-1}$$

$$= \left(\sum_{k=-\infty,k\neq -1 or 1}^{\infty}\frac{3cos(\frac{\pi k}{2})}{\pi(1-k^2)}e^{j\frac{2\pi}{16}kt}\right)+\frac{3}{4}\left(e^{j\frac{2\pi}{16}t}+e^{-j\frac{2\pi}{16}t}\right)$$

2.3.3 Part c

In this part of the lab experiment, it is expected to plot the spectrum of $y_a(t)$. Since the spectrum of a periodic signal is complex amplitudes of the complex sinusoidal components of a signal and the sinusoidal componets are the exponential expressions in the fourier series expansion formula, a_k's are the magnitudes of the sinusoidal components. Therefore, it is needed to plot the fourier series coefficients, which are found in part b, with proper frequencies. Since $f_0 = 1/T_0$, we found the frequency as below.

$$\omega = \omega_0 k = \frac{2\pi}{T_0}k = \frac{2\pi}{16}k = 2\pi f_0 k = 2\pi f$$

$$f = f_0 k$$

Using this frequencies, plot of corresponding amplitudes of sinusoidal components can be drawn as below.

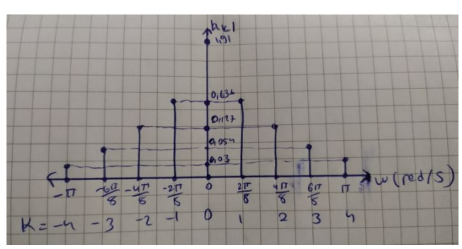

Figure 22: Spectrum of $y_a(t)$

2.3.4 Part d

In this part of the lab experiment, it is needed to compute a discrete $z_N[n]$ function, which is shown below. This function is actually sum of weighted sinusoidal components in a specified interval. That is to say, sum of fourier series expansions' elements in a finite interval.

$$Z_N[n] = \sum_{k=-\infty}^{\infty} a_k e^{j\omega_0 knT_s}$$

N=120
$Z_N[n]$ for N=120 is calculated using MATLAB, and using the result, plot of $Z_{120}[n]$ is drawn as below.

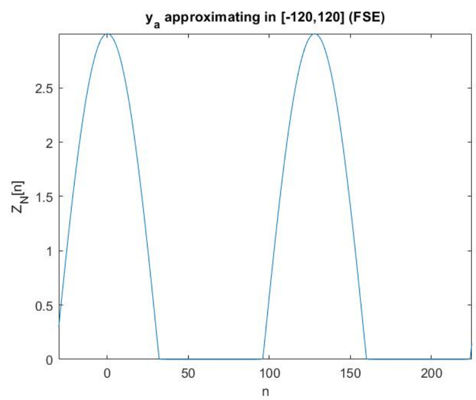

Figure 23: y_a approximation from -120 to 120

2.3.5 Part e.1

N=60
$Z_N[n]$ for N=60 is calculated using MATLAB, and using the result, plot of $Z_{60}[n]$ is drawn as below.

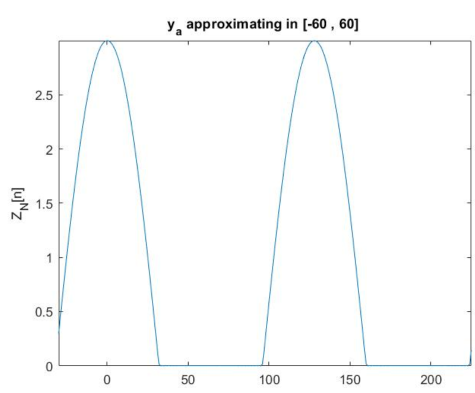

Figure 24: y_a approximation from -60 to 60

2.3.6 Part e.2

N=30
$Z_N[n]$ for N=30 is calculated using MATLAB, and using the result, plot of $Z_{30}[n]$ is drawn as below.

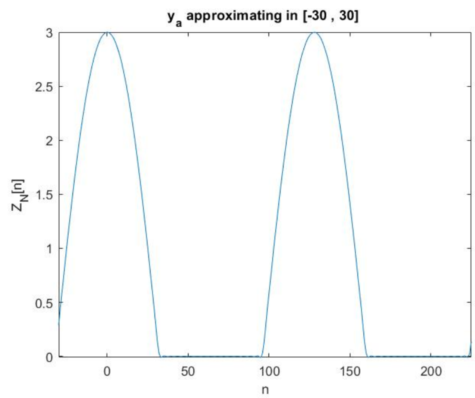

Figure 25: y_a approximation from -30 to 30

2.3.7 Part f

N=5
$Z_N[n]$ for N=5 is calculated using MATLAB, and using the result, plot of $Z_5[n]$ is drawn as below.

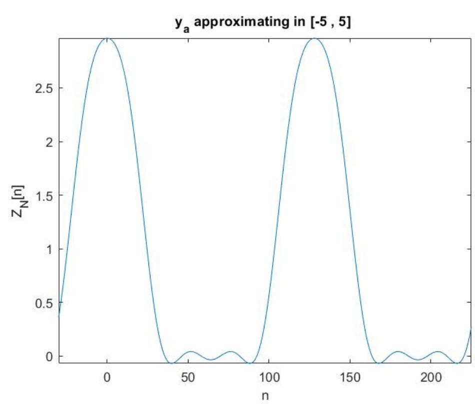

Figure 26: y_a approximation from -5 to 5

2.3.8 Part g

N=3
$Z_N[n]$ for N=3 is calculated using MATLAB, and using the result, plot of $Z_3[n]$ is drawn as below.

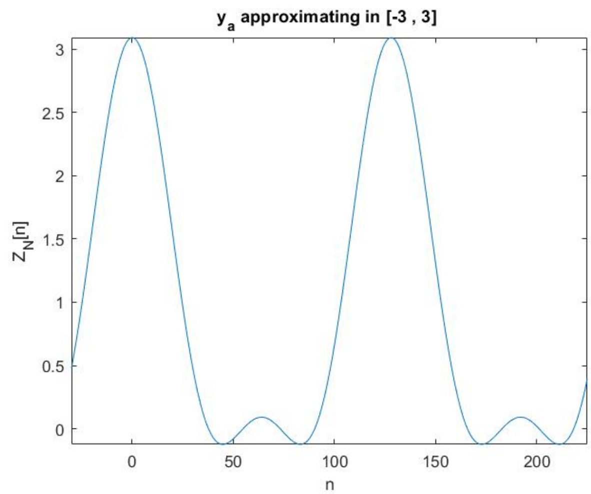

Figure 27: y_a approximation from -3 to 3

2.3.9 Part h

N=1
$Z_N[n]$ for N=1 is calculated using MATLAB, and using the result, plot of $Z_1[n]$ is drawn as below.

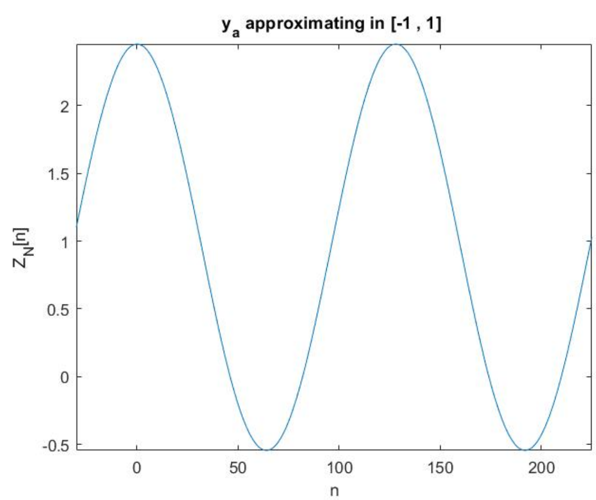

Figure 28: y_a approximation from -1 to 1

2.3.10 Comparison of Functions From d to h

To compare the plots N=120, N=30, N=5 and N=1 are chosen and they have drawn on same plot, also to compare those function with the original function it is drawn to the left of the figure, which are shown below.

As seen from the figure $Z_{120}[n]$ is most similar plot to $y_a[n]$ whereas $Z_1[n]$ is different. This is because, to calculate $Z_N[n]$, elements from fourier series expansion of $y_a[n]$ are used.

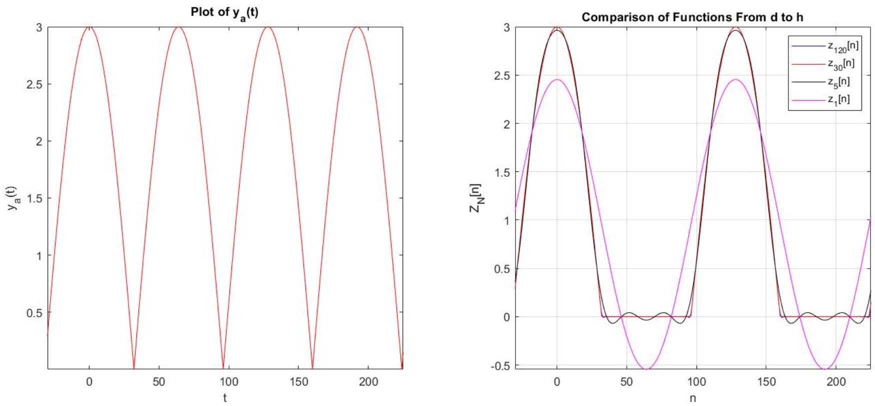

Figure 29: Comparison of Functions From d to h and $y_a(t)$

To calculate $Z_{120}[n]$ 241 elements at the center are used but to calculate $Z_1[n]$ only 3 elements are used, which makes $Z_{120}[n]$ more similar to $y_a[n]$ than $Z_1[n]$. Also paying attention to Gibbs phenomenon it can be seen that when Z=120 the zero points looks almost constant at zero and the function more looks like Half-wave rectifier because of more sinusoidal terms, when Z=1 the wave looks like sinusoidal because of less sinusoidal terms. Briefly it can be said that the higher N is the higher the quality of the $Z_N[n]$ approximation.

.

2.3.11 Part I

In this part of the experiment zeroth, first, second, and third harmonics of $y_a(t)$, will computed and drawn on the same plot using same scale,which is shown below in figure. The equation of the k'th harmonic (for $k \neq 0$) is given below. Zeroth harmonic is equal to a_0 which is just a constant . Thus, Zeroth harmonic is DC component of $y_a(t)$.

$$k'th\ harmonic = a_{-k}e^{-j\omega_0 kt} + a_k e^{j\omega_0 kt}$$

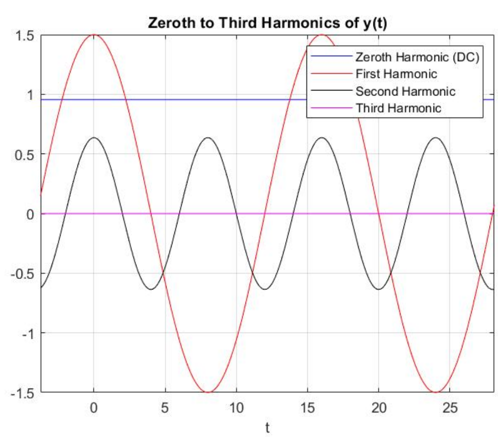

Figure 30: Zero to Third Harmonics of $y_a(t)$

3 Appendix-Matlab Code

```
%% Q1
T_s = 1/8;
n = -30:225;
disc = n.*T_s ;
T = 16;
y_t = @(disc) 0.*(0<=mod(disc,T) & mod(disc,T)<6)...
    + 4.*(6<=mod(disc,T) & mod(disc,T)<10)...
    + 0.*(10<=mod(disc,T) & mod(disc,T)<16);
y_n = y_t(disc);

% PART A
figure();
plot(n,y_n,'r');
title('Plot of y_a(t)');
xlabel('t');
ylabel('y_a(t)');
axis tight

figure();
stem(n,y_n,'r');
title('Discrete plot of y[n])');
xlabel('n');
ylabel('y[n]');
axis tight


% Part D


arraylen=120;
akarr1=a_k1(arraylen);
figure();
plot(n, fourier(disc,T,akarr1));
titl= sprintf('y_a approximating in [-120,120] (FSE)');
title(titl);
xlabel('n');
ylabel('Z_{120}[n]');
axis tight

%Part E1,E2,F,G,H
```

```
NS=[60,30,5,3,1];
for N=NS
    arraylen=N;
    akarr=a_k1(arraylen);
    figure();
    plot(n, fourier(disc,T,akarr));
    titl= sprintf('y_a approximating in [-%d , %d]  ',N,N);
    title(titl);
    lbe=sprintf("Z_{%d}[n]",N);
    xlabel('n');
    ylabel(lbe);
    axis tight
end

figure();
subplot(1,2,1);
plot(n,y_n,'r');
title('Plot of y_a(t)');
xlabel('t');
ylabel('y_a(t)');
axis tight
subplot(1,2,2);
plot(n, fourier(disc,T,a_k1(120)),'b');
hold on
plot(n, fourier(disc,T,a_k1(30)),'r');
hold on
plot(n, fourier(disc,T,a_k1(5)),'k');
hold on
plot(n, fourier(disc,T,a_k1(1)),'m');
hold off
title('Comparison of Functions From d to h')
legend('z_{120}[n]','z_{30}[n]','z_{5}[n]','z_{1}[n]');
xlabel('n');
ylabel('Z_N[n]');
axis tight
grid on




% PART I
% Plot the harmonics
spec_len=100;
magnitudes_1=a_k1(spec_len);
figure();
```

```
plot(disc, harmonik(disc, 0, magnitudes_1, T),'b');
hold on
plot(disc, harmonik(disc, 1, magnitudes_1, T),'r');
hold on
plot(disc, harmonik(disc, 2, magnitudes_1, T),'k');
hold on
plot(disc, harmonik(disc, 3, magnitudes_1, T),'m');
title('Zeroth to Third Harmonics of y(t)');
xlabel('t');
axis tight
grid on
legend('Zeroth Harmonic (DC)','First Harmonic' ...
    ,'Second Harmonic','Third Harmonic');
hold off

%% Q2
T_2 = 8;
y_t2 = @(disc) abs(3*cos(pi*disc/8));
y_n2 = y_t2(disc);

% PART A
figure();
plot(n,y_n2,'r');
title('Plot of y_a(t)');
xlabel('t');
ylabel('y_a(t)');
axis tight

figure();
stem(n,y_n2,'r');
title('Discrete plot of y[n])');
xlabel('n');
ylabel('y[n]');
axis tight


% Part D


arraylen=120;
akarr2=a_k2(arraylen);
figure();
plot(n, fourier(disc,T_2,akarr2));
titl= sprintf('y_a approximating in [-120,120] (FSE)');
title(titl);
```

```
xlabel('n');
ylabel('Z_N[n]');
axis tight

%Part E1,E2,F,G,H

NS=[60,30,5,3,1];
for N=NS
    arraylen=N;
    akarr_2=a_k2(arraylen);
    figure();
    plot(n, fourier(disc,T_2,akarr_2));
    titl= sprintf('y_a approximating in [-%d , %d]  ',N,N);
    title(titl);
    xlabel('n');
    ylabel('Z_N[n]');
    axis tight
end

figure();
subplot(1,2,1);
plot(n,y_n2,'r');
title('Plot of y_a(t)');
xlabel('t');
ylabel('y_a(t)');
axis tight
subplot(1,2,2);
plot(n, fourier(disc,T_2,a_k2(120)),'b');
hold on
plot(n, fourier(disc,T_2,a_k2(30)),'r');
hold on
plot(n, fourier(disc,T_2,a_k2(5)),'k');
hold on
plot(n, fourier(disc,T_2,a_k2(1)),'m');
hold off
title('Comparison of Functions From d to h')
legend('z_{120}[n]','z_{30}[n]','z_{5}[n]','z_{1}[n]');
xlabel('n');
ylabel('Z_N[n]');
axis tight
grid on
```

```
% PART I
% Plot the harmonics
magnitudes_2=a_k2(spec_len);
figure();
plot(disc, harmonik(disc, 0, magnitudes_2, T_2),'b');
hold on
plot(disc, harmonik(disc, 1, magnitudes_2, T_2),'r');
hold on
plot(disc, harmonik(disc, 2, magnitudes_2, T_2),'k');
hold on
plot(disc, harmonik(disc, 3, magnitudes_2, T_2),'m');
title('Zeroth to Third Harmonics of y(t)');
xlabel('t');
axis tight
grid on
legend('Zeroth Harmonic (DC)','First Harmonic', ...
    'Second Harmonic','Third Harmonic');
hold off


%% Q3

T_3 = 16;

y_t3 = @(disc) abs(3*cos(pi*disc/8)).*(0<=mod(disc,T) ...
    & mod(disc,T)<4)...
    + 0.*(4<=mod(disc,T) & mod(disc,T)<12)...
    + abs(3*cos(pi*disc/8)).*(12<=mod(disc,T) & mod(disc,T)<16);
y_n3 = y_t3(disc);

% PART A
figure();
plot(n,y_n3,'r');
title('Plot of y_a(t)');
xlabel('t');
ylabel('y_a(t)');
axis tight

figure();
stem(n,y_n3,'r');
title('Discrete plot of y[n])');
xlabel('n');
ylabel('y[n]');
axis tight
```

```
% Part D

arraylen=120;
akarr3=a_k3(arraylen);
figure();
plot(n, fourier(disc,T_3,akarr3));
titl= sprintf('y_a approximating in [-120,120] (FSE)');
title(titl);
xlabel('n');
ylabel('Z_N[n]');
axis tight

%Part E1,E2,F,G,H

NS=[60,30,5,3,1];
for N=NS
    arraylen=N;
    akarr_3=a_k3(arraylen);
    figure();
    plot(n, fourier(disc,T_3,akarr_3));
    titl= sprintf('y_a approximating in [-%d , %d] ',N,N);
    title(titl);
    xlabel('n');
    ylabel('Z_N[n]');
    axis tight
end

figure();
subplot(1,2,1);
plot(n,y_n2,'r');
title('Plot of y_a(t)');
xlabel('t');
ylabel('y_a(t)');
axis tight
subplot(1,2,2);
plot(n, fourier(disc,T_3,a_k3(120)),'b');
hold on
plot(n, fourier(disc,T_3,a_k3(30)),'r');
hold on
plot(n, fourier(disc,T_3,a_k3(5)),'k');
hold on
plot(n, fourier(disc,T_3,a_k3(1)),'m');
```

```
hold off
title('Comparison of Functions From d to h')
legend('z_{120}[n]','z_{30}[n]','z_{5}[n]','z_{1}[n]');
xlabel('n');
ylabel('Z_N[n]');
axis tight
grid on



% PART I
% Plot the harmonics
magnitudes_3=a_k3(spec_len);
figure();
plot(disc, harmonik(disc, 0, magnitudes_3, T_3),'b');
hold on
plot(disc, harmonik(disc, 1, magnitudes_3, T_3),'r');
hold on
plot(disc, harmonik(disc, 2, magnitudes_3, T_3),'k');
hold on
plot(disc, harmonik(disc, 3, magnitudes_3, T_3),'m');
title('Zeroth to Third Harmonics of y(t)');
xlabel('t');
axis tight
grid on
legend('Zeroth Harmonic (DC)','First Harmonic', ...
    'Second Harmonic','Third Harmonic');
hold off



%%
function result = fourier(n,T, ak)
result = 0;
l=(length(ak)-1)/2;
for j = -l:l
    result = result + ak(j+l+1).* exp(1i*(2*pi/T).*j.*n);
end
end
```

```
function result = a_k1(len)
result = zeros(0,len);
borders = -1*len:len;
for k = borders
    if(k==0)
        temp = 1;
    else
        temp = ((4)/(pi*k)) * cos(pi*k).*sin(pi*k/4) ;
        %temp = ((4i)/(2*pi*k)) * (exp(-10i*k*pi/8) - exp(-6i*pi*k/8));
    end
    result(k+len+1) = temp;
end
end


function result = a_k2(len)
result = zeros(0,len);
borders = -1*len:len;
for k = borders
    if (k==-1/2)  || (k==1/2)
        temp = 3/2;
    else
        temp = (6)*cos(pi*k) /  (pi*(1-4*k^2))    ;
    end
    result(k+len+1) = temp;
end
end


function result = a_k3(len)
result = zeros(0,len);
borders = -1*len:len;
for k = borders
    if (k==-1)  || (k==1)
        temp = 3/4;
    else
        temp = (3)*cos(pi*k/2) /  (pi*(1-k^2))    ;
    end
    result(k+len+1) = temp;
end
end
```

```
function result = harmonik(disc, k, ak, T)
len=(length(ak)-1)/2;
if k == 0
    result = ak(len+1).* exp((2i*pi/T).*k.*disc);
else
    result = ak(len+k+1).* exp((2i*pi/T).*k.*disc) ...
        + ak(len-k+1).* exp(-(2i*pi/T).*k.*disc);
end
end
```