# EEE 321 LAB WORK 2

October 14, 2022

## 1 Introduction

In this lab experiment it is expected to work on convolution concept, using the given impulse of given discrete linear and time(shift) invariant system (LTI). It is known by the course gains that for a system to use convolution, it must be both linear and time invariant. Therefore, in this lab experiment these properties will be introduced. Then, asked questions will be answered, and the output of the given system will be found for different input functions. This solution will be done analytically, then using the analytical results MATLAB program will be written. However, when it is written in MATLAB, meaningful non-infinite size will be used as indexes. Finally plots of the output functions will be drawn.
Given impulse response is shown below.

$$h[n] = \left(\frac{8}{9}\right)^n u[n-3] \tag{1}$$

### 1.1 Linearity

For any system T the linearity condition is given below,this condition must be satisfied for all a, b scalars and $x_1(t), x_2(t)$ functions.

The system equations $T[x_1(t)] = y_1(t)$ and $T[x_1(2)] = y_1(2)$ are given.For any a,b scalars we have the equation below (2) for linear systems.

$$T[ax_1(t) + bx_2(t)] = ay_1(t) + by_2(t) \tag{2}$$

### 1.2 Time Invariance

For any system T the shift invariance condition is given below,this condition must be satisfied for all $t_0$ scalar and $x1(t)$ functions.

The system equation $T[x(t)] = y(t)$ is given.For any a,b scalars we have the equation below (3) for time invariant systems.

$$T[x(t-t_0)] = y(t-t_0) \tag{3}$$

## 2 Questions & Results

As mentioned before given impulse response is shown below. We are expected to find system is casual or not, and system is stable or not.

$$h[n] = \left(\frac{8}{9}\right)^n u[n-3]$$

- Checking Casuality

$$T[x[n]] = y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=-\infty}^{\infty} x[k](8/9)^{n-k}u[n-k-3]$$

As seen from the equation when $n < k$, $y[n]$ becomes zero (0) because $u[n-k-3] = 0$ for $n < k$. When $n > k$ there is no future item to calculate $y[n]$ in the equation. Thus, it is seen that the output doesn't depend on the future inputs so this system is casual.

- Checking Stability

$$T[x[n]] = y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=-\infty}^{\infty} x[k](8/9)^{n-k}u[n-k-3]$$

Let $|x[n]| < B$ for a scalar B.Then we got the inequality below.

$$|y[n]| = |\sum_{k=-\infty}^{\infty} x[k](8/9)^{n-k}u[n-k-3]| < |B\sum_{k=-\infty}^{\infty}(8/9)^{n-k}u[n-k-3]|$$

$$= |B\sum_{k=n-3}^{\infty}(8/9)^{n-k}u[n-k-3]| = B|\sum_{k=n-3}^{\infty}(8/9)^{n-k}|$$

$$= B.(8/9)^3(\frac{1}{1-8/9}) = (512/81)B$$

It is seen that when $|x[n]|$ is bounded $|y[n]|$ is bounded as well. Thus the system is stable.

### 2.1 Input Function: a)

$$x_1[n] = \begin{cases} 2 & if \quad 0 \le n \le 9 \\ 0 & else \end{cases}$$

Using the given input function and impulse response, corresponding convolution and its numerical result is given below.

$$y_1[n] = x_1[n] * h[n] = \sum_{k=-\infty}^{\infty} x_1[k]h[n-k]$$

Using the input function's and impulse response's specifications

$$y_1[n] = \sum_{k=0}^{9} x_1[k]h[n-k] = 2\sum_{k=0}^{9}(8/9)^{n-k}u[n-k-3]$$

$$= \begin{cases} 0 & if \quad n-3<0 \\ 2\sum_{k=0}^{n-3}(8/9)^{n-k} & if \quad 0 \le n-3 < 9 \\ 2\sum_{k=0}^{9}(8/9)^{n-k} & else \end{cases}$$

$$= \begin{cases} 0 & if \quad n<3 \\ 18(8/9)^3(1-(8/9)^{n-2}) & if \quad 3 \le n < 12 \\ 18(8/9)^{n-9}(1-(8/9)^{10}) & else \end{cases} \tag{4}$$

As seen from the equation above, for $3 \le n$ $y_1[n] > 0$ and $y_1[n]$ converges to zero as n increases. This is because $(8/9)^{n-k}$ decreases and converges to zero. Therefore MATLAB code is written for only a small interval. However, the pattern of y function can be seen properly. The corresponding $y_1[n]$ figure is given below, which is in figure-1. Also in figure-1 another graph is also given which is created by using the proper convolution function that is written by me. In the figure below comparison of two graph is provided. As seen the results are same so that means numerical result is found correctly.

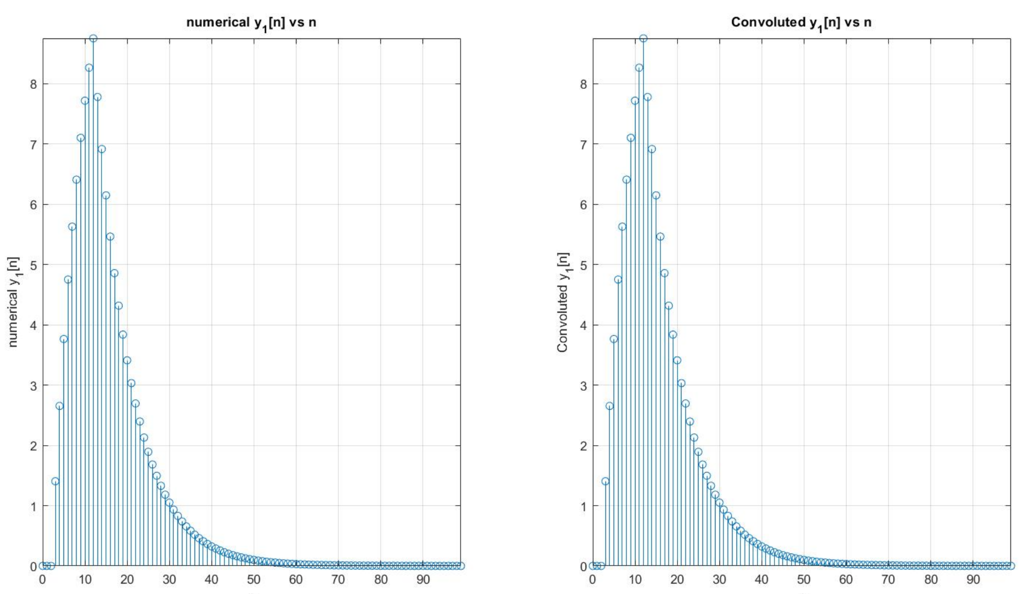


Figure 1: Comparison of Numerical $y_1[n]$ result vs Convolution $y_1[n]$ result

## 2.2 Input Function: b)

$$x_2[n] = \begin{cases} 2 & if \quad 0 \le n \le 5 \\ -2 & if \quad 6 \le n \le 9 \\ -4 & if \quad 10 \le n \le 15 \\ 0 & else \end{cases}$$

Using the given input function and impulse response, corresponding convolution and its numerical result is given below.

$$y_2[n] = x_2[n] * h[n] = \sum_{k=-\infty}^{\infty} x_2[k]h[n-k]$$

Using the input function's and impulse response's specifications

$$y_2[n] = \sum_{k=0}^{15} x_2[k]h[n-k]$$

It can be seen that $y_2[n] = y_1[n] - y_1[n-6]$. Since the system is LTI, this transformation can be used to calculate $y_2[n]$. To calculate $y_2[n]$ equation (4) is used. Firstly it is needed to calculate $y_1[n-6]$. It is a straightforward shifting operation since the system is time invariant. Also direct subtraction of $2y_1[n-6]$ from$y_1[n]$ is used without a problem since this system is linear. Using these facts the equations for both $y_1[n-6]$ and $y_2[n]$ are given below.

$$y_1[n-6] = \begin{cases} 0 & if \quad n < 9 \\ 18(8/9)^3(1-(8/9)^{n-8}) & if \quad 3 \le n < 12 \\ 18(8/9)^{n-15}(1-(8/9)^{10}) & else \end{cases}$$

Using equation (4) and above equation, $y_2[n]$ is given below.

$$y_2[n] = \begin{cases} 0 & if \quad n < 3 \\ 18(8/9)^3(1-(8/9)^{n-2}) & if \quad 3 \le n < 12 \\ 18(8/9)^3(1-(8/9)^{n-2}) - 36(8/9)^3(1-(8/9)^{n-8}) & if \quad 3 \le n < 12 \\ 18(8/9)^{n-9}(1-(8/9)^{10}) - 36(8/9)^3(1-(8/9)^{n-8}) & if \quad 3 \le n < 12 \\ 18(8/9)^{n-9}(1-(8/9)^{10}) - 36(8/9)^{n-15}(1-(8/9)^{10}) & else \end{cases} \tag{5}$$

As seen from the equation above they were not simplified to see the results better. In this way it can be realized easily that for $12 \le n$ $y_2[n] > 0$ and $y_2[n]$ converges to zero as n increases. This is because $(8/9)^{n-k}$ decreases and converges to zero. Therefore MATLAB code is written for only a small interval. However, the pattern of y function can be seen properly. The corresponding $y_2[n]$ figure is given below, which is in figure-2. Also in figure-2 another graph is also given which is created by using the proper convolution function that is written by me. In the figure below comparison of two graph is provided. As seen the results are same so that means numerical result is found correctly.

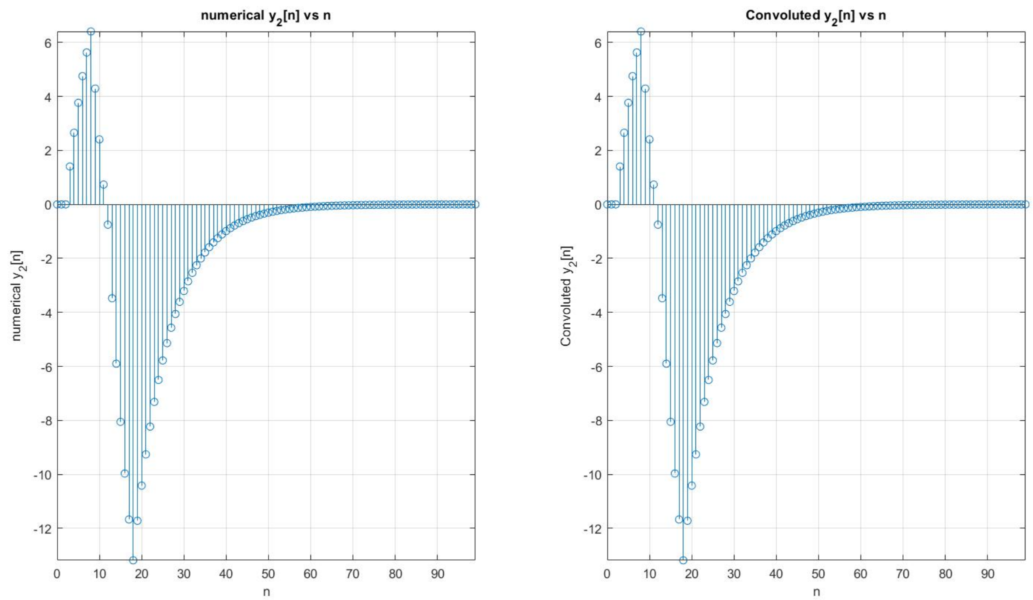


Figure 2: Comparison of Numerical $y_2[n]$ result vs Convolution $y_2[n]$ result

## 2.3 Input Function: c)

$$x_3[n] = \begin{cases} e^{j(1/4)n} & if \quad 3 \leq n \leq 23 \\ 0 & else \end{cases}$$

Using the given input function and impulse response, corresponding convolution and its numerical result is given below.

$$y_3[n] = x_3[n] * h[n] = \sum_{k=-\infty}^{\infty} x_3[k]h[n-k]$$

Using the input function's and impulse response's specifications

$$y_3[n] = \sum_{k=3}^{23} x_3[k]h[n-k] = \sum_{k=3}^{23} e^{j(1/4)k}(8/9)^{n-k}u[n-k-3]$$

$$= \begin{cases} 0 & if \quad n-6<0 \\ \sum_{k=3}^{n-3} e^{j(1/4)k}(8/9)^{n-k} & if \quad 3 \leq n-3 < 23 \\ \sum_{k=3}^{23} e^{j(1/4)k}(8/9)^{n-k} & else \end{cases}$$

$$= \begin{cases} 0 & if \quad n<6 \\ e^{j(3/4)}(\frac{8}{9})^{n-3}\left(\frac{(1-(\frac{9e^{j/4}}{8})^{n-5}}{1-(\frac{9e^{j/4}}{8})}\right) & if \quad 6 \leq n < 26 \\ e^{j(3/4)}(\frac{8}{9})^{n-3}\left(\frac{(1-(\frac{9e^{j/4}}{8})^{21}}{1-(\frac{9e^{j/4}}{8})}\right) & else \end{cases} \tag{6}$$

As seen from the equation above they were not simplified to see the results better. In this way it can be realized easily that for $26 \leq n$ $y_3[n] > 0$ and $y_3[n]$ converges to zero as n increases. This is because $(8/9)^{n-k}$ decreases and converges to zero. Therefore MATLAB code is written for only a small interval. However, the pattern of y function can be seen properly. The corresponding $y_3[n]$ figure is given below, which is in figure-3. Also in figure-3 another graph is also given which is created by using the proper convolution function that is written by me. In the figure below comparison of two graph is provided. As seen the results are same so that means numerical result is found correctly.

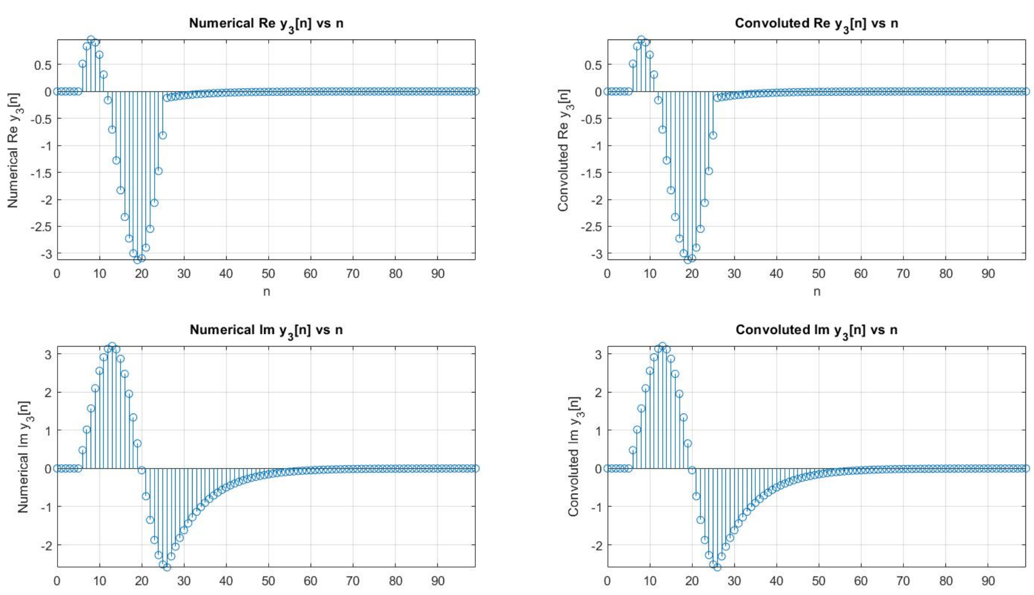


Figure 3: Comparison of Numerical $y_3[n]$ result vs Convolution $y_3[n]$ result

## 2.4 Input Function: d)

$$x_4[n] = \begin{cases} -2sin[(1/4)n] & if \quad 3 \leq n \leq 23 \\ 0 & else \end{cases}$$

Using the given input function and impulse response, corresponding convolution and its numerical result is given below.

$$y_4[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x_4[k]h[n-k]$$

Using the input function's and impulse response's specifications

$$y_4[n] = \sum_{k=3}^{23} x_4[k]h[n-k] = \sum_{k=3}^{23} -2sin[(1/4)k](8/9)^{n-k}u[n-k-3]$$

Since it is known that $(sin(k) = (e^{jk} - e^{-jk})/2i)$, we can write this equation in terms of e.

$$y_4[n] = \sum_{k=3}^{23} -2\frac{e^{jk/4} - e^{-jk/4}}{2j}(8/9)^{n-k}u[n-k-3]$$

$$= \begin{cases} 0 & if \quad n-6<0 \\ \sum_{k=3}^{n-3} \frac{e^{jk/4}-e^{-jk/4}}{-j}(8/9)^{n-k} & if \quad 3 \leq n-3 < 23 \\ \sum_{k=3}^{23} \frac{e^{jk/4}-e^{-jk/4}}{-j}(8/9)^{n-k} & else \end{cases}$$

$$= \begin{cases} 0 & if \quad n<6 \\ j\left[e^{j(3/4)}(\frac{8}{9})^{n-3}\left(\frac{(1-(\frac{9e^{j/4}}{8})^{n-5}}{1-(\frac{9e^{j/4}}{8})}\right) - e^{-j(3/4)}(\frac{8}{9})^{n-3}\left(\frac{(1-(\frac{9e^{-j/4}}{8})^{n-5}}{1-(\frac{9e^{-j/4}}{8})}\right)\right] & if \quad 6 \leq n < 26 \\ j\left[e^{j(3/4)}(\frac{8}{9})^{n-3}\left(\frac{(1-(\frac{9e^{j/4}}{8})^{21}}{1-(\frac{9e^{j/4}}{8})}\right) - e^{-j(3/4)}(\frac{8}{9})^{n-3}\left(\frac{(1-(\frac{9e^{-j/4}}{8})^{21}}{1-(\frac{9e^{-j/4}}{8})}\right)\right] & else \end{cases} \tag{7}$$

As seen from the equation above they were not simplified to see the results better. In this way it can be realized easily that for $26 \leq n$ $y_4[n] > 0$ and $y_4[n]$ converges to zero as n increases. This is because $(8/9)^{n-k}$ decreases and converges to zero. Therefore MATLAB code is written for only a small interval. However, the pattern of y function can be seen properly. The corresponding $y_4[n]$ figure is given below, which is in figure-4. Also in figure-4 another graph is also given which is created by using the proper convolution function that is written by me. In the figure below comparison of two graph is provided. As seen the results are same so that means numerical result is found correctly.

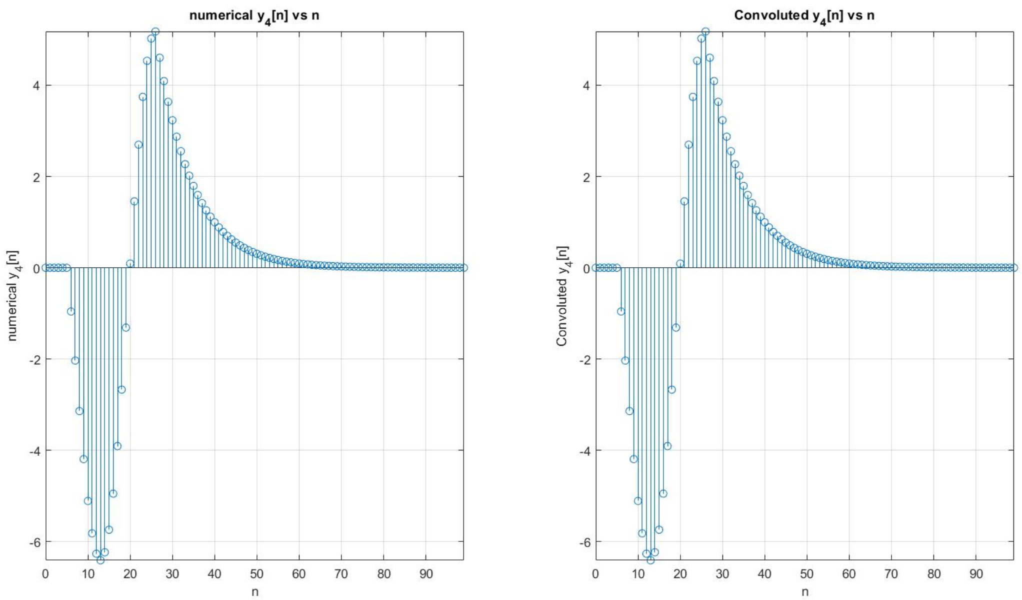


Figure 4: Comparison of Numerical $y_4[n]$ result vs Convolution $y_4[n]$ result

## 2.5 Input Function: e)

$$x_5[n] = \begin{cases} cos[(1/4)n] & if \quad 3 \leq n \leq 23 \\ 0 & else \end{cases}$$

Using the given input function and impulse response, corresponding convolution and its numerical result is given below.

$$y_5[n] = x_5[n] * h[n] = \sum_{k=-\infty}^{\infty} x_5[k]h[n-k]$$

Using the input function's and impulse response's specifications

$$y_5[n] = \sum_{k=3}^{23} x_5[k]h[n-k] = \sum_{k=3}^{23} cos[(1/4)k](8/9)^{n-k}u[n-k-3]$$

Since it is known that $(cos(k) = (e^{jk} + e^{-jk})/2)$, we can write this equation in terms of e.

$$y_4[n] = \sum_{k=3}^{23} \frac{e^{jk/4} + e^{-jk/4}}{2}(8/9)^{n-k}u[n-k-3]$$

$$
= \begin{cases} 0 & if \quad n-6<0 \\ \sum_{k=3}^{n-3} \frac{e^{jk/4}+e^{-jk/4}}{2}(8/9)^{n-k} & if \quad 3 \leq n-3 < 23 \\ \sum_{k=3}^{23} \frac{e^{jk/4}+e^{-jk/4}}{2}(8/9)^{n-k} & else \end{cases}
$$

$$
= \begin{cases} 0 & if \quad n<6 \\ \left[e^{j(3/4)}(\frac{8}{9})^{n-3}\left(\frac{(1-(\frac{9e^{j/4}}{8})^{n-5}}{1-(\frac{9e^{j/4}}{8})}\right) + e^{j(-3/4)}(\frac{8}{9})^{n-3}\left(\frac{(1-(\frac{9e^{-j/4}}{8})^{n-5}}{1-(\frac{9e^{-j/4}}{8})}\right)\right]/2 & if \quad 6 \leq n < 26 \\ \left[e^{j(3/4)}(\frac{8}{9})^{n-3}\left(\frac{(1-(\frac{9e^{j/4}}{8})^{21}}{1-(\frac{9e^{j/4}}{8})}\right) + e^{j(-3/4)}(\frac{8}{9})^{n-3}\left(\frac{(1-(\frac{9e^{-j/4}}{8})^{21}}{1-(\frac{9e^{-j/4}}{8})}\right)\right]/2 & else \end{cases} \tag{8}
$$

As seen from the equation above they were not simplified to see the results better. In this way it can be realized easily that for $26 \leq n$ $y_5[n] > 0$ and $y_5[n]$ converges to zero as n increases. This is because $(8/9)^{n-k}$ decreases and converges to zero. Therefore MATLAB code is written for only a small interval. However, the pattern of y function can be seen properly. The corresponding $y_5[n]$ figure is given below, which is in figure-5. Also in figure-5 another graph is also given which is created by using the proper convolution function that is written by me. In the figure below comparison of two graph is provided. As seen the results are same so that means numerical result is found correctly.

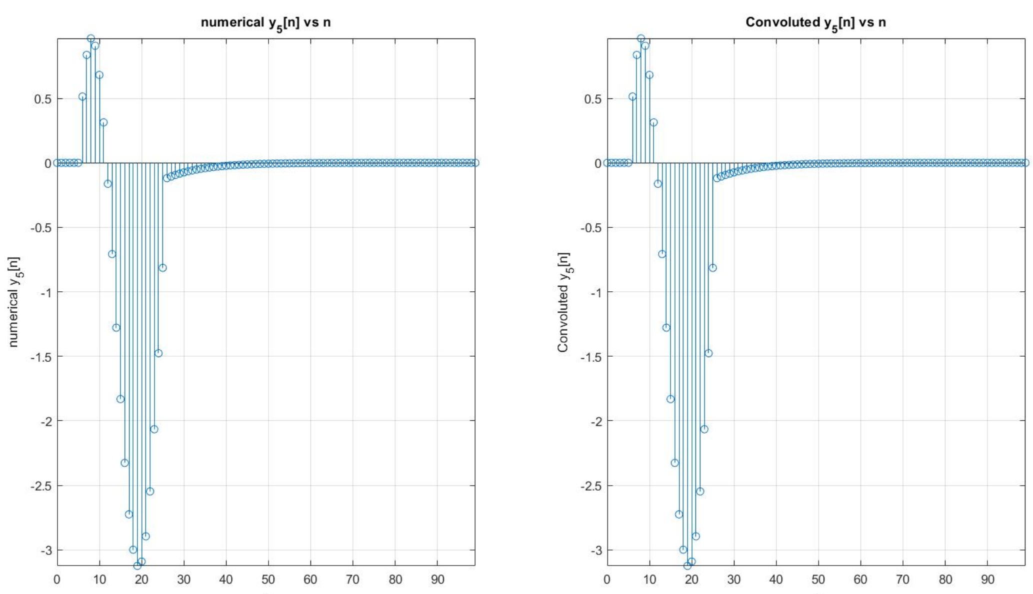


Figure 5: Comparison of Numerical $y_5[n]$ result vs Convolution $y_5[n]$ result

## 2.6 Input Function: f)

$$x_6[n] = x_1[n] - 3jx_2[n]$$

Using the given input function and impulse response, corresponding convolution and its numerical result is given below.

$$y_6[n] = x_6[n] * h[n] = \sum_{k=-\infty}^{\infty} x_6[k]h[n-k]$$

Using the input function's and impulse response's specifications

$$y_6[n] = \sum_{k=0}^{15} x_6[k]h[n-k] = \sum_{k=0}^{15}(x_1[n] - 3jx_2[n])h[n-k] =$$

$$= \sum_{k=0}^{9} x_1[n]h[n-k] - 3j\sum_{k=0}^{15} x_2[n]h[n-k] = y_1[n] - 3jy_2[n]$$

It can be seen that we need equation (4) and equation (5) to calculate $y_6[n]$. It can be done directly thanks to linearity property.

$$y_6[n] = y_1[n] - 3jy_2[n]$$

$$y_6[n] = \begin{cases} 0 & if \quad n < 3 \\ 18(\frac{8}{9})^3(1-(\frac{8}{9})^{n-2}) - 54j(\frac{8}{9})^3(1-(\frac{8}{9})^{n-2}) & if \quad 3 \leq n < 9 \\ 18(\frac{8}{9})^3(1-(\frac{8}{9})^{n-2}) - 3j(18(\frac{8}{9})^3(1-(\frac{8}{9})^{n-2}) - 36(\frac{8}{9})^3(1-(\frac{8}{9})^{n-8})) & if \quad 9 \leq n < 12 \\ 18(\frac{8}{9})^{n-9}(1-(\frac{8}{9})^{10}) - 3j(18(\frac{8}{9})^{n-9}(1-(\frac{8}{9})^{10}) - 36(\frac{8}{9})^3(1-(\frac{8}{9})^{n-8})) & if \quad 12 \leq n < 18 \\ 18(\frac{8}{9})^{n-9}(1-(\frac{8}{9})^{10}) - 3j(18(\frac{8}{9})^{n-9}(1-(\frac{8}{9})^{10}) - 36(\frac{8}{9})^{n-15}(1-(\frac{8}{9})^{10})) & else \end{cases} \tag{9}$$

As seen from the equation above they were not simplified to see the results better. In this way it can be realized easily that for $18 \leq n$ $y_6[n] > 0$ and $y_6[n]$ converges to zero as n increases. This is because $(8/9)^{n-k}$ decreases and converges to zero. Therefore MATLAB code is written for only a small interval. However, the pattern of y function can be seen properly. The corresponding $y_6[n]$ figure is given below, which is in figure-6. Also in figure-6 another graph is also given which is created by using the proper convolution function that is written by me. In the figure below comparison of two graph is provided. As seen the results are same so that means numerical result is found correctly.

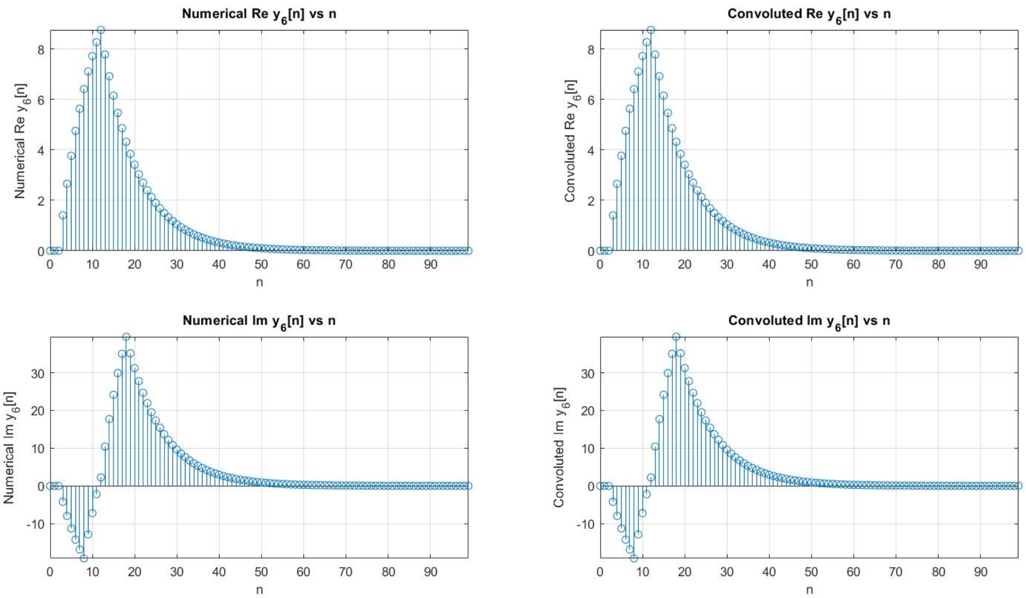


Figure 6: Comparison of Numerical $y_6[n]$ result vs Convolution $y_6[n]$ result

# 3 Appendix-Matlab Code

```
% y_1
y_1 = calc(@x_1,60);
figure("Position",[100 50 1350 700])
subplot(1,2,2);
stem([0:99],y_1(1:100));
xlabel('n');
ylabel('y_1[n]');
grid on
axis tight
title('y_1[n] vs n');
subplot(1,2,1);
stem([0:99],a_1(99));
xlabel('n');
ylabel('numerical y_1[n]');
grid on
axis tight
title('numerical y_1[n] vs n');
disp(y_1);


% y_2
y_2 = calc(@x_2,60);
```

```
figure("Position",[100 50 1350 700])
subplot(1,2,2);
stem([0:99],y_2(1:100));
xlabel('n');
ylabel(' y_2[n]');
grid on
axis tight
title(' y_2[n] vs n');
subplot(1,2,1);
stem([0:99],b_1(99));
xlabel('n');
ylabel('numerical y_2[n]');
grid on
axis tight
title('numerical y_2[n] vs n');
disp(y_2);

% y_3
y_3 = calc(@x_3,60);
figure("Position",[100 50 1350 700])
subplot(2,2,2);
stem([0:99],real(y_3(1:100)));
xlabel('n');
ylabel('Re {y_3[n]}');
grid on
axis tight
title('Re {y_3[n]} vs n');
subplot(2,2,4);
stem([0:99],imag(y_3(1:100)));
xlabel('n');
ylabel('Im {y_3[n]}');
grid on
axis tight
title('Im {y_3[n]} vs n');

subplot(2,2,1);
stem([0:99],real(c_1(99)));
xlabel('n');
ylabel('Numerical Re {y_3[n]}');
grid on
axis tight
title('Numerical Re {y_3[n]} vs n');
subplot(2,2,3);
stem([0:99],imag(c_1(99)));
xlabel('n');
```

```
ylabel('Numerical Im {y_3[n]}');
grid on
axis tight
title('Numerical Im {y_3[n]} vs n');
disp(y_3);

% y_4
y_4 = calc(@x_4,60);
figure("Position",[100 50 1350 700])
subplot(1,2,2);
stem([0:99],y_4(1:100));
xlabel('n');
ylabel('y_4[n]');
grid on
axis tight
title('y_4[n] vs n');

subplot(1,2,1);
stem([0:99],d_1(99));
xlabel('n');
ylabel('numerical y_4[n]');
grid on
axis tight
title('numerical y_4[n] vs n');
disp(y_4);

% y_5
y_5 = calc(@x_5,60);
figure("Position",[100 50 1350 700])
subplot(1,2,2);
stem([0:99],y_5(1:100));
xlabel('n');
ylabel('y_5[n]');
grid on
axis tight
title('y_5[n] vs n');

subplot(1,2,1);
stem([0:99],e_1(99));
xlabel('n');
ylabel('numerical y_5[n]');
grid on
axis tight
title('numerical y_5[n] vs n');
disp(y_5);
```

```
% y_6
y_6 = calc(@x_6,60);
figure("Position",[100 50 1350 700])

subplot(2,2,2);
stem([0:99],y_6(1:100));
xlabel('n');
ylabel('Re {y_6[n]}');
grid on
axis tight
title('Re {y_6[n]} vs n');
subplot(2,2,4);
stem([0:99],imag(y_6(1:100)));
xlabel('n');
ylabel('Im {y_6[n]}');
grid on
axis tight
title('Im {y_6[n]} vs n');

subplot(2,2,1);
stem([0:99],f_1(99));
xlabel('n');
ylabel('Numerical Re {y_6[n]}');
grid on
axis tight
title('Numerical Re {y_6[n]} vs n');
subplot(2,2,3);
stem([0:99],imag(f_1(99)));
xlabel('n');
ylabel('Numerical Im {y_6[n]}');
grid on
axis tight
title('Numerical Im {y_6[n]} vs n');

disp(y_6);
%
% CONVOLUTION FUNCTION
function resu = calc(inp,inter)
    resu = zeros(1,121);
    for t= 0:120
        temp=zeros(1,121);
        for i= inter*(-1):inter
            temp(i+61)=inp(i).*(uf(t-i-3).*(8/9).^(t-i));
        end
```

```
        resu(t+1)=sum(temp);
    end
end
%
% UNIT FUNCTION
function  o_uf= uf(i)
     if(i>=0)
          o_uf = 1;
     else
          o_uf = 0;
     end
end

%INPUT FUNCTIONS
% X_1





function  x_1 = x_1(n)
     if(n<=9 && n>=0)
          x_1 = 2;
     else
          x_1 = 0;
     end
end

%
% X_2

function  x_2 = x_2(n)
     if(n<=5 && n>=0)
          x_2 = 2;
     elseif(n<=9 && n>=6)
          x_2 = -2;
     elseif(n<=15 && n>=10)
          x_2 = -4;
     else
          x_2 = 0;
     end
end
%
% X_3
```

```
function x_3 = x_3(n)
    if(n<=23 && n>=3)
        x_3 = exp((n)*1i/4);
    else
        x_3 = 0;
    end
end
%
%X_4

function x_4 = x_4(n)
    x_4=(-2)*imag(x_3(n));
end
%
%X_5
function x_5 = x_5(n)
    x_5=real(x_3(n));
end
%
%X_6





function x_6 = x_6(n)
    x_6 = x_1(n)-1i*3*x_2(n);
end



%OUTPUTS
%Y_1
function resu=a_1(n)
resu = zeros(1,100);
for t = 0:n
    if t<3 && t>-1
        resu(t+1)=0;
    elseif (t<=11 && t>=3)
        resu(t+1)= 18*(8/9)^3.*(1-(8/9)^(t-2));
    else
        resu(t+1)= 18*(8/9).^(t-9).*(1-(8/9)^(10));
    end
end
end
```

%Y_2

```
function resu=b_1(n)
resu = zeros(1,100);
for t = 0:n
    if t<3 && t>-1
        resu(t+1)=0;
    elseif (t<9 && t>=3)
        resu(t+1)= 18*(8/9)^3.*(1-(8/9).^(t-2));
    elseif (t<12 && t>=9)
        resu(t+1)= 18*(8/9)^3.*(1-(8/9).^(t-2))-36*(8/9)^3.*
        (1-(8/9).^(t-8));
    elseif (t<18 && t>=12)
        resu(t+1)= 18*(8/9)^(t-9).*(1-(8/9)^10)-36*(8/9)^3.*
        (1-(8/9).^(t-8));
    else
        resu(t+1)= 18*(8/9)^(t-9).*(1-(8/9)^10)-36*(8/9)^(t-15).*
        (1-(8/9)^10);
    end
end
end
```

%Y_3

```
function resu=c_1(n)
resu = zeros(1,100);
for t = 0:n
    z=(9/8).*exp(1i/4);
    if t<6 && t>-1
        resu(t+1)=0;
    elseif (t<26 && t>=6)
        resu(t+1)= (8/9).^(t-3).*exp(3*1i/4).*((1-z.^(t-5))/(1-(z)));
    else
        resu(t+1)= (8/9).^(t-3).*exp(3*1i/4).*((1-(z).^(21))/(1-(z)));
    end
end
end
```

%Y_4

```
function resu=d_1(n)
resu = zeros(1,100);
for t = 0:n
    z=(9/8).*exp(1i/4);
    p=(9/8).*exp(-1i/4);
    if t<6 && t>-1
        resu(t+1)=0;
    elseif (t<26 && t>=6)
        resu(t+1)= -((8/9).^(t-3).*exp(3*1i/4).*((1-z.^(t-5))/(1-(z)))
-     (8/9).^(t-3).*exp(-3i/4).*((1-p.^(t-5))/(1-(p))))./1i;
    else
        resu(t+1)= -((8/9).^(t-3).*exp(3*1i/4).*((1-(z).^(21))/(1-(z)))
- (8/9).^(t-3).*exp(-3i/4).*((1-(p).^(21))/(1-(p))))./1i
;
    end
end
end
```

```
%Y_5
function resu=e_1(n)
resu = zeros(1,100);
for t = 0:n
    z=(9/8).*exp(1i/4);
    p=(9/8).*exp(-1i/4);
    if t<6 && t>-1
        resu(t+1)=0;
    elseif (t<26 && t>=6)
        resu(t+1)= ((8/9).^(t-3).*exp(3*1i/4).*((1-z.^(t-5))/(1-(z)))+
(8/9).^(t-3).*exp(-3*1i/4).*((1-p.^(t-5))/(1-(p))))./2;
    else
        resu(t+1)= ((8/9).^(t-3).*exp(3*1i/4).*((1-(z).^(21))/(1-(z)))
+ (8/9).^(t-3).*exp(-3*1i/4).*((1-(p).^(21))/(1-(p))))./2
;
    end
end
end
```

```
%Y_6
function resu=f_1(n)
resu = zeros(1,100);
for t = 0:n
    if t<3 && t>-1
        resu(t+1)=0;
    elseif (t<9 && t>=3)
        resu(t+1)= 18*(8/9)^3.*(1-(8/9)^(t-2))-(54*(1i)*(8/9)^3.*
        (1-(8/9).^(t-2)));
    elseif (t<12 && t>=9)
        resu(t+1)= 18*(8/9)^3.*(1-(8/9)^(t-2))-(3i).*(18*(8/9)^3.*
        (1-(8/9).^(t-2))-36*(8/9)^3.*(1-(8/9).^(t-8)));
    elseif (t<18 && t>=12)
        resu(t+1)= 18*(8/9).^(t-9).*(1-(8/9)^(10))-(3i).*
        (18*(8/9)^(t-9).*(1-(8/9)^10)-36*(8/9)^3.*(1-(8/9).^(t-8)));
    else
        resu(t+1)= 18*(8/9).^(t-9).*(1-(8/9)^(10))-(3i).*
        (18*(8/9)^(t-9).*(1-(8/9)^10)-36*(8/9)^(t-15).*(1-(8/9)^10));
    end
end
end
```