ANKARA BİLİM ÜNİVERSİTESİ

MATH 224-LAPLACE-EXERCISES

24-25 Spring Semester

Student No : **Date :**

Name & Surname : **Signature :**

Q1) Find the Laplace and the inverse Laplace transform of the following functions.

(a) $\mathcal{L}\Big\{e^{-7t} + 3\sin(2t) - 5\cos(4t)\Big\}$

(b) $\mathcal{L}\Big\{t^2e^{3t} - \sin(3t)e^{2t} + 4\cos(t)e^{-6t}\Big\}$

(c) $\mathcal{L}^{-1}\Big\{\dfrac{6}{s^4} + \dfrac{7}{s^2+49} + \dfrac{5}{s+3}\Big\}$

(d) $\mathcal{L}^{-1}\Big\{\dfrac{7s^3+18s-15}{(s^2-1)(s^2+4)}\Big\}$

(e) $\mathcal{L}^{-1}\Big\{\dfrac{2s}{s^2+8s+17}\Big\}$

(f) $\mathcal{L}^{-1}\Big\{\dfrac{7s\ e^{3s}}{s^2+4}\Big\}$

(g) $\mathcal{L}^{-1}\Big\{e^{-3s}\dfrac{1}{s^2+4} + e^{4s}\dfrac{s}{s^2+1}\Big\}$

Q2) Use Laplace transform to solve the initial value problem

$$-2y' + y = 0; \qquad y(0) = 1$$

Q3) Use Laplace transform to solve the initial value problem

$$y'' - 4y' + 4y = 8t; \qquad y(0) = 5, y'(0) = 0$$

Q4) Use Laplace transform to solve the initial value problem

$$y'' + y' = 6e^{2t}; \quad y(0) = 0,\ y'(0) = 3$$

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Q5) Use the Laplace transform to solve the initial value problem

$$y'' + y = \cos 2t; \quad y(0) = 0, y'(0) = 1$$

Q6) Use the Laplace transform to solve the initial value problem

$$y'' - 6y' + 9y = 50\cos t; \quad y(0) = 2, y'(0) = 0$$

Q7) Use the Laplace transform to solve the initial value problem

$$y'' - 2y' + y = te^{2t}; \quad y(0) = 2, y'(0) = 6$$

Q8) Find $\mathcal{L}\Big\{(t^2 - t + 5)H(t-3)\Big\}$

Q9) Find Laplace transform of the following function:

$$f(t) = \begin{cases} 0 & \text{if } t < 2 \\ \\ t^2 - t & \text{if } t \geq 2 \end{cases}$$

Q10) Use Laplace transform to solve the initial value problem:
$y'' + 4y = f(t),\ y(0) = 0, y'(0) = 0$, where

$$f(t) = \begin{cases} 0 & \text{if } t < 3 \\ \\ 8t - 20 & \text{if } t \geq 3 \end{cases}$$

f(t)	$\mathcal{L}[f](s)$
e^{at}	$\dfrac{1}{s-a}$
t^n	$\dfrac{n!}{s^{n+1}}$
$\sin bt$	$\dfrac{b}{s^2+b^2}$
$\cos bt$	$\dfrac{s}{s^2+b^2}$

$\mathcal{L}[y'] = sY(s) - y(0)$

$\mathcal{L}[y''] = s^2Y(s) - sy(0) - y'(0)$

Shifting Theorems:
$\mathcal{L}[e^{ct}f(t)](s) = F(s-c)$

$\mathcal{L}[H(t-a)f(t-a)](s) = e^{-as}\mathcal{L}[f(t)](s) = e^{-as}F(s)$

$H(t-a)f(t-a) = \mathcal{L}^{-1}[e^{-as}F(s)]$