

Cs301 Assignment 4

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Recursive formulation

For finding recursive formulation of tourist problem first we need to define sub problem which is shortest path to source to destination. In example this was Istanbul to Bursa. In example there is a direct line between each other and this was shortest path but intermediate city path can be shorter in other cases. Lets imagine that Istanbul Bursa travel time takes 320 minute. So Istanbul-Eskisehir was shortest then $\min(\text{ist-bur}) = \min(\text{ist-esk}) + \min(\text{esk-bur})$. This will create overlapping sub problems. I added Ankara and selected as destination to problem to show overlaps better.

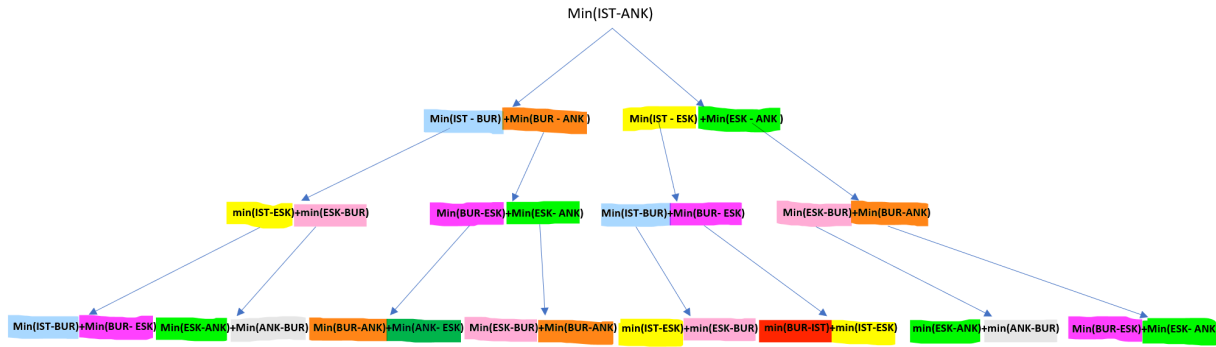


Figure 1: same color highlight shows overlap

For the problem have all the cities have both train and bus line between them this doubles. So recursive formulation happens for both train and bus:

Train or Bus $(k) (i,j) = 0$ if $i=j$

Train or Bus $(k) (i,j) = \text{Train or Bus}[i][j]$ if $k=0$

Train or Bus $(k) (i,j) = \min(\text{Train or Bus}(k-1)(i,l) + \text{Same choice}(k-1)(l,j), \text{Train or Bus}(k-1)(i,l) + \text{Other choice}(k-1)(i,l) + \text{transfer time})$ if $k \geq 1$

Pseudocode

Algorithm 1: DP with memoization

```
function Shortestpath();  
for every intermediate city do  
    for each starting city do  
        for each destination city do  
            Bus[i][j] = min(Bus[i][j], Bus[i][l]+Bus[l][j], Train[i][l]+Bus[l][j]+transfer time);  
            Train[i][j] = min(Train[i][j], Train[i][l]+Train[l][j], Bus[i][l]+Train[l][j]+transfer time);  
            Matrix[i][j] = min(Train[i][j], Bus[i][j]);  
        end  
    end  
end  
Print Matrix[0]
```

Asymptotic time and space complexity analysis

Asymptotic time of algorithm takes $O(n^3)$ because of 3 times nested loop. Space complexity takes up $O(n^2)$ space due to 2 nxn matrixes for shortest path and general path.

Experimental evaluations of your algorithm

