Date: 26.12.2018
Group Number: 1

Yildiz Technical University
Faculty of Mechanical Engineering
Department of Mechatronics Engineering

MKT 3801 – System Dynamics

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Number: 1506A023

Date: 26.12.2018
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Layout

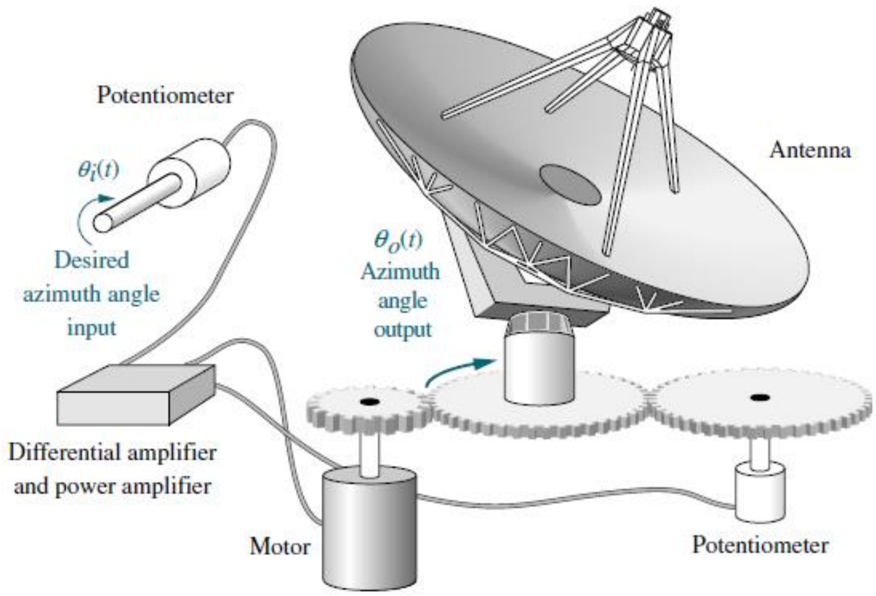


Figure 1: An Antenna Azimuth Position Control System

Schematic

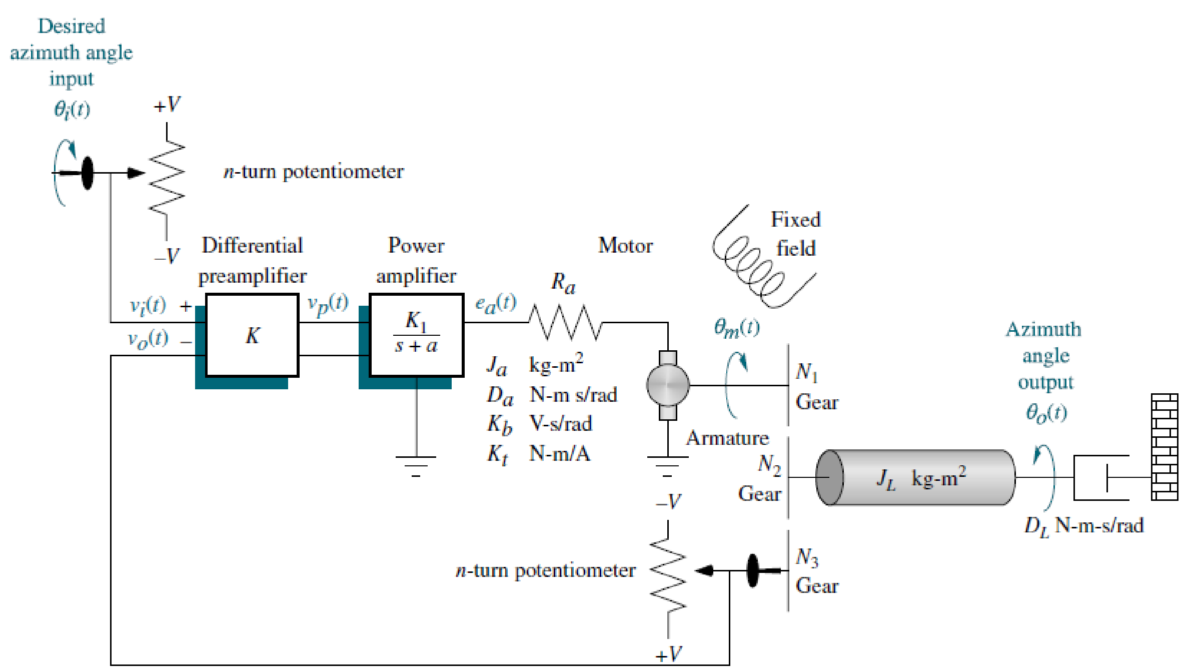


Figure 2: Schematic Diagram of Antenna Azimuth

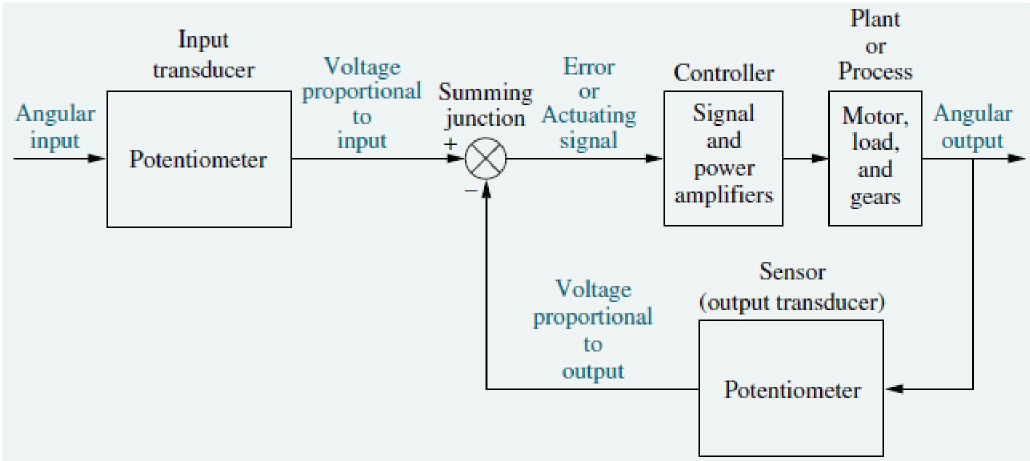


Figure 3: Functional Block Diagram

I. Project Name: Modeling and Analysis of Dish Antenna System

II. Project Purpose & Problem Definition:

The purpose of this system is to have the azimuth angle output of the antenna, $\theta_0(t)$, follow the input angle of the potentiometer, $\theta_i(t)$. The input command is an angular displacement. The potentiometer converts the angular displacement into a voltage. The signal and power amplifiers increases the difference between the input and output voltages. In other words, the purpose of the project is analyzing a control system for the antenna azimuth position using MATLAB and Simulink.

We have to select the DC Motor by using the given parameters and, we have to calculate the required estimates with DC Motor that is chosen and electrical, rotational systems.

III. Analysis of Elements and Overall System:

a. Selection of DC Motor:

We have to select the DC Motor with respect to the given parameters and requirements are in Table 1.

• Motor Output Torque	• T = 5 mNm
• Motor speed:	• n = 5000 rpm
• Supply Voltage:	• U = 18.5V
• Current source, max:	• I = 1 A
• Space max:	• diameter=30mm, • length=60mm

Table 1: DC Motor Parameters

So, the power of the DC Motor is calculated.

$$P_{motor} = Torque \times AngularVelocity = T \times \omega \qquad \& \qquad \omega = 2 \times \pi \times n$$

$$P_{motor} = T \times 2 \times \pi \times n$$

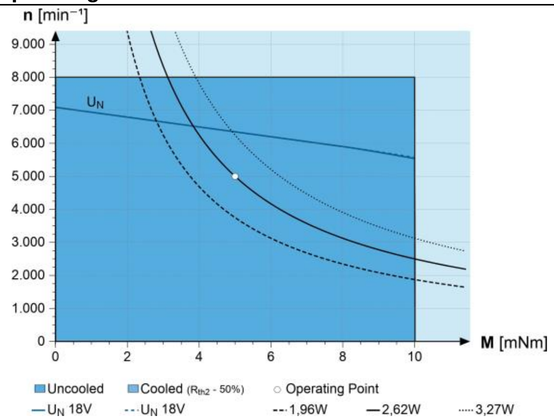
$$P_{motor} = 5\,mNm \times 2 \times \pi \times 5000\,rpm$$

$$P_2 = 2.6179W = 2.6180W$$

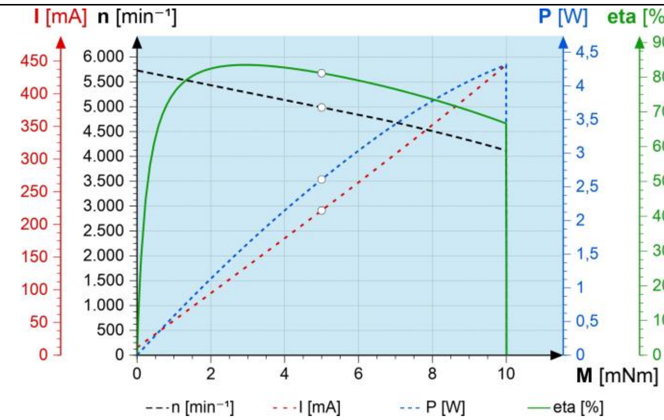
My Requirements

Motor and Gearhead Preselection (optional)		
Motor series	2232U018SR	
Gear series	All	
Global settings		
Ambient temperature	22	°C
Available diameter	30	mm
Available length	60	mm
Available supply voltage	18,5	V
Advanced settings		
Available current	1	A
Efficiency, min.	10	%
RTH2 Reduction		%
Your entries		
Load transmission	Direct rotational	
Required load speed	5.000	1/min
Required load torque	5	mNm
Results of the Load Calculation		
Load current	221,46	mA
Load voltage	14,56	V
Motor winding temperature	31,73	°C
Motor housing temperature	29,89	°C
Required motor torque	5	mNm
Required motor speed	5.000	1/min
Output power	2,62	W
Efficiency (over all)	81,18	%
Overall Dimensions		
Diameter	22	mm
Length	32,2	mm
Mass	62	g

Operating Area



Characteristic curves



Motor Characteristic Data			
Nominal voltage	U_N	18	V
Terminal resistance	R	9,04	Ω
Torque constant	k_M	24,1	mNm/A
No load speed	n_0	7.100	1/min
Stall torque	M_H	47,6	mNm
Speed constant	k_n	397	1/min/V
Rotor inductance	L	0,4	mH
Slope of n-M curve	Δn/ΔM	149	1/min/mNm
Rotor inertia	J	3,8	gcm²
Mechanical time constant	τ	6	ms
RTH2 Reduction		0	%
Efficiency max.	η_{max}	86	%

As you can see on this Table, we select that the DC Motor with respect to the given parameters.

Drive System: 2232 U 018 SR

i. <u>Optimizing the preselection:</u>

To optimize the DC Motor 's operation and life performance, the required speed n has to be higher than half the no load speed n_0 at nominal voltage, and the load torque T has to be less than half the stall torque M_H .

$$n \geq \frac{n_0}{2} \ so\ that, 10000\ min^{-1} \geq 7100\ min^{-1}$$

$$M \leq \frac{M_H}{2}\ so\ that, 10\ mNm\ \leq 46.7\ mNm$$

ii. <u>Performance characteristics at nominal voltage at 18V</u>

Stall Current:

$$I_H = \frac{U_N}{R} = \frac{18\ V}{9.04\ \Omega} = 1.991\ A$$

Torque at max. efficiency:

$$M_{opt} = \sqrt{M_H \times M_R} = \sqrt{47.6\ mNm\ \times 0.28 mNm} = 3.650\ mNm$$

iii. Calculation of the main parameters:

In this application the available supply voltage is lower than the nominal voltage of the selected motor. The calculation under load therefore is made of 18.5 V.

No-load speed (n_0) at 18.5V:

$$n_0 = \frac{U - (I_0 \times R)}{2 \times \pi \times k_M}$$

inserting the values;

Table 2.1

Supply Voltage	U	=	18.5	V
Terminal resistance	R	=	9.04	Ω
No-load current	I_0	=	0.0116	A
Torque constant	k_M	=	24.1	mNm/A

$$n_0 = \frac{18.5\,V - 0.0116\,A \times 9.04\Omega}{2 \times \pi \times 24.1 mNm/A} = 8208\,min^{-1}$$

(We divided by 60 in order to find min^{-1})

Stall current I_H:

$$I_H = \frac{U}{R} = \frac{18.5\,V}{9.04\,\Omega} = 2.046\,A$$

Stall torque M_H:

$$M_H = k_M \times \left(\frac{U}{R} - I_0\right) = (24.1\,mNm/A)\left(\frac{18.5\,V}{9.04\,\Omega} - 0.0116\,A\right) = 49.04\,mNm$$

Efficiency, max η_{max} :

$$\eta_{max} = \left(1 - \sqrt{\frac{I_0 \times R}{U}}\right)^2 = \left(1 - \sqrt{\frac{0.0116\,A \times 9.04\,\Omega}{18.5\,V}}\right)^2 = 85.510\%$$

At the point of max. efficiency, the torque delivered is:

$$M_{opt} = \sqrt{M_H \times M_R}$$

Inserting the values

Table 2.2

Friction Torque	M_R	=	0.28	mNm
Stall Torque with 18.5 V	M_H	=	49.04	mNm

So;

$$M_{opt} = \sqrt{49.04\,mNm \times 0.28\,mNm} = 3.705\,mNm$$

iv. Calculation of operation point at 18.5 V

When the torque $M = 5\ mNm$ at the working point is taken into consideration $I, n, P,$ and η can be calculated.

Current at the operating point:

$$I_{Last} = \frac{M + M_R}{k_M} = \frac{5\ mNm + 0.28\ mNm}{24.1\ mNm/A} = 0.219\ A$$

Speed at the operating point:

$$n = \frac{U - R \times I_{Last}}{2 \times \pi \times k_m} = \frac{18.5\ V - \ (9.04\ \Omega)(0.219\ A)}{2 \times \pi \times (24.1\ mNm/A)} = 6545.920\ min^{-1}$$

Output power at the operating point:

$$P_2 = M \times 2 \times \pi \times n = \ (5\ mNm) \times 2\ \times \pi \times (6545.920\ min^{-1}) = 3.427\ W$$

Efficiency at the operating point:

$$\eta = \frac{P_2}{U \times I} = \frac{3.427\ W}{(18.5\ V) \times (0.219\ A)} = 84.585\ \%$$

Supply voltage at the operating point:

The exact supply voltage at the operating point can now be obtained with the following equation:

$$U = I_{load} \times R\ + 2 \times \pi \times n \times k_m$$

$$= 0.219\ A\ \times 9.04\ \Omega + \ 2 \times \pi\ \times 5000\ min^{-1} \times 24.1 \frac{mNm}{A} = \ 14.598\ V$$

Series 2232 … SR

Values at 22°C and nominal voltage		2232 U	018 SR		
1	Nominal voltage	U_N	18		V
2	Terminal resistance	R	9,04		Ω
3	Efficiency, max.	$\eta_{max.}$	86		%
4	No-load speed	n_0	7 100		min^{-1}
5	No-load current, typ. (with shaft ø 2 mm)	I_0	0,0116		A
6	Stall torque	M_H	47,6		mNm
7	Friction torque	M_R	0,28		mNm
8	Speed constant	k_n	397		min^{-1}/V
9	Back-EMF constant	k_E	2,52		mV/min^{-1}
10	Torque constant	k_M	24,1		mNm/A
11	Current constant	k_I	0,042		A/mNm
12	Slope of n-M curve	$\Delta n/\Delta M$	149		min^{-1}/mNm
13	Rotor inductance	L	400		µH
14	Mechanical time constant	τ_m	6		ms
15	Rotor inertia	J	3,8		gcm^2
16	Angular acceleration	$\alpha_{max.}$	120		$\cdot 10^3 rad/s^2$
17	Thermal resistance	R_{th1} / R_{th2}	4 / 13		K/W
18	Thermal time constant	τ_{w1} / τ_{w2}	7 / 340		s
19	Operating temperature range:				
	– motor		-30 … +85 (optional version	-55 … +125)	°C
	– winding, max. permissible		+125		°C
20	Shaft bearings		sintered bearings	ball bearings, preloaded	
21	Shaft load max.:		(standard)	(optional version)	
	– with shaft diameter		2	2	mm
	– radial at 3 000 min^{-1} (3 mm from bearing)		1,5	8	N
	– axial at 3 000 min^{-1}		0,2	0,8	N
	– axial at standstill		20	10	N

IV. Model of Each Component and Overall System

Subsystem	Input	Output
Input Potentiometer	Angular Rotation from User, $\theta_i(t)$	Voltage to preamp, $V_i(t)$
Preamp	Voltage from Potentiometers, $V_e(t) = V_i(t) - V_0(t)$	Voltage to power amp, $V_p(t)$
Power amp	Voltage from preamp, $V_p(t)$	Voltage to motor, $E_a(t)$
Motor	Voltage from power amp, $E_a(t)$	Angular rotation to load, , $\theta_0(t)$
Output potentiometer	Angular rotation from load, $\theta_0(t)$	Voltage to preamp, , $V_0(t)$

Table 3.1: Subsystems of the antenna azimuth position control

Input Potentiometer; Output Potentiometer:

Transfer Function of Input and Output Potentiometers must be same since there are configured in the same way. We can neglect the dynamics for the potentiometers and simply find the relationship between the output voltage and the input angular displacement. In the center position the output voltage is zero. Assume that the five turns toward either the positive 18.5 volts or the negative 18.5 volts yields a voltage change of 18.5 volts. Thus, the transfer function, $V_i(s)/\theta_i(s)$ is equal to:

$$\frac{V_i(s)}{\theta_i(s)} = \frac{10}{10\pi} = \frac{1}{\pi} = 0.318$$

Pre Amplifier; Power Amplifier:

The transfer functions of both amplifiers are the ratio of the Laplace transforms of the output voltage divided by the input voltage.

For Preamplifier,

$$\frac{V_p(s)}{V_e(s)} = K$$

For Power Amplifier,

$$\frac{E_a(s)}{V_p(s)} = \frac{10^2}{s + 10^2} = \frac{100}{s + 100}$$

-Motor and Load:

Inertia	J_a	3.8	gcm^2
Inductance	L_a	400	μH
Resistance	R_a	9.04	Ω
Back-EMF Constant	K_b	2.52	mV/min^{-1}
Torque Constant	K_t	24.1	mNm/A

Table 3.2: Motor Parameters

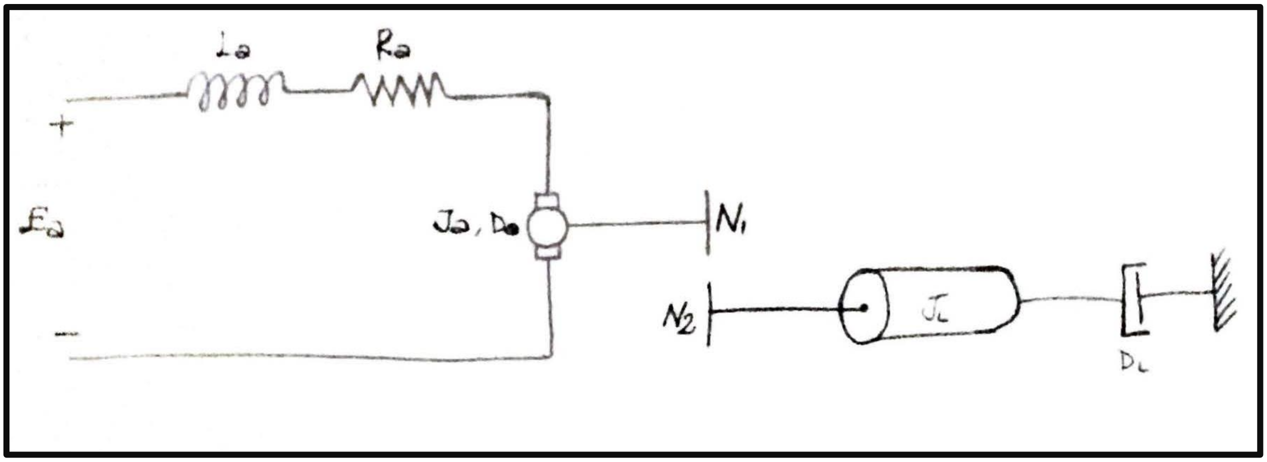
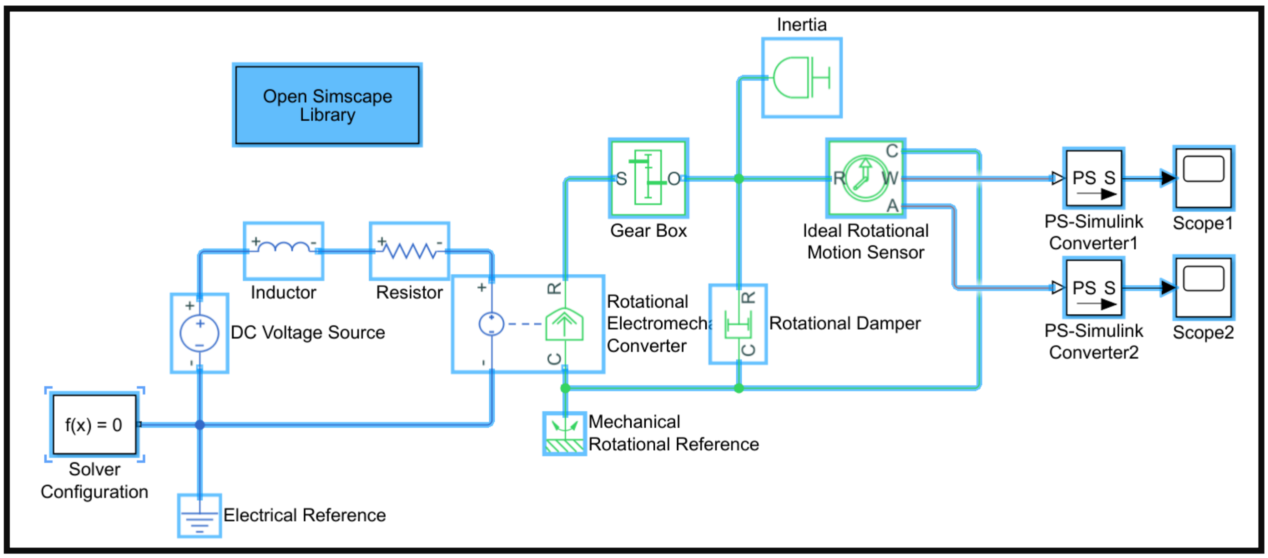


Table 3.3.a: System Model



→This diagram was created by using ssc_new command.

Table 3.3.b: System Model by using MATLAB Simulink

→The equivalent inertia, J_m, is

$$J_m = J_a + \left(\frac{N_1}{N_2}\right)^2 \cdot J_L = 3.8\times 10^{-7} + 1 \cdot \left(\frac{20}{100}\right)^2 = 0.04$$

where J_L = 1 is the load inertia at θ_0. When we are doing from g.cm^2 to $kg.m^2$, we multiply by $10^{-7}$and we assume that the $N_1 = 20$, $N_2 = 100$.

→The equivalent viscous damping, D_m, at the armature is

$$D_m = D_a + \left(\frac{N_1}{N_2}\right)^2 \cdot D_L$$

To calculate D_a value we have to use this equation:

$$i_a = \frac{D_a V_a + K_b T_L}{D_a R_a + K_b K_T}$$

The corresponding no-load current required can be found by setting $T_L = 0$, so:

$$i_a = \frac{D_a V_a}{D_a R_a + K_b K_T}$$

$$i_a D_a R_a + i_a K_b K_T = D_a V_a$$

$$i_a D_a R_a - D_a V_a = -i_a K_b K_T$$

$$D_a[i_a R_a - V_a] = -i_a K_b K_T$$

$$D_a = \frac{-i_a K_b K_T}{i_a R_a - V_a}$$

Selected motor which U = 18.5 V and $i_a = 0.0116\ A$ that's way D_a will be very small considered to damper of antenna D_L. Therefore D_a can be neglected. Therefore;

$$D_m = \left(\frac{N_1}{N_2}\right)^2 \cdot D_L = 0.04$$

Finally,

$$\frac{\theta_m(s)}{E_a(s)} = \frac{K_t/(R_a J_m)}{s^2 + s \times \frac{1}{J_m}(D_m + \frac{K_t K_b}{R_a})} = \frac{0.067}{s^2 + s}$$

To complete the transfer function of the motor, we multiply by the gear ratio to arrive at the transfer function relating load displacement to armature voltage:

$$\frac{\theta_0(s)}{E_a(s)} = 0.2 \times \frac{\theta_m(s)}{E_a(s)} = \frac{0.0134}{s^2 + s}$$

Power amplifier:

$$G(s) = \frac{E_a(s)}{V_p(s)} = \frac{10^2}{s + 10^2} = \frac{100}{s + 100}$$

So, $(s + 100)E_a(s) = 100V_p(s)$

And then, $\frac{dE_a}{dt} = -100E_a + 100V_p(t)$

Since the output of the power amplifier is $E_a(t)$, the output equation is y = E_a

Motor and Load:

$$E_a(t) = i_a(t)R_a + K_b \frac{d\theta_m}{dt}$$

And, $T_m(t) = K_t i(t) = J_m \frac{d^2\theta_m}{dt^2} + \left(\frac{D_m R_a}{K_t} + K_b \right) \frac{d\theta_m}{dt}$

Defining the state variables x_1 and x_2 as

$$x_1 = \theta_m$$

$$x_2 = \frac{d\theta_m}{dt}$$

$$E_a(t) = \left(\frac{R_a J_m}{K_t} \right) \frac{dx_2}{dt} + \left(\frac{D_m R_a}{K_t} + K_b \right) x_2$$

$$\frac{dx_2}{dt} = -\frac{1}{J_m} \left(\frac{K_b K_t}{R_a} + D_m \right) x_2 + \left(\frac{K_t}{R_a J_m} \right) E_a(t)$$

So, the state equations are written,

$$\frac{dx_1}{dt} = x_2$$

$$\frac{dx_2}{dt} = -\frac{1}{J_m} \left(\frac{K_b K_t}{R_a} + D_m \right) x_2 + \left(\frac{K_t}{R_a J_m} \right) E_a(t)$$

The output $\theta_0(t) = \frac{1}{10}$ the displacement of the armature, which is x_1 . So, $y = 0.1x_1$

In vector-matrix form, $$\dot{x} = \begin{bmatrix} 0 & 1 \\ 1 & -\frac{1}{Jm}\left(\frac{K_b K_t}{R_a} + D_m\right) \end{bmatrix} x + \begin{bmatrix} 0 \\ \frac{K_t}{R_a Jm} \end{bmatrix} E_a(t)$$

$$y = [\,0.1 \quad 0\,]x$$

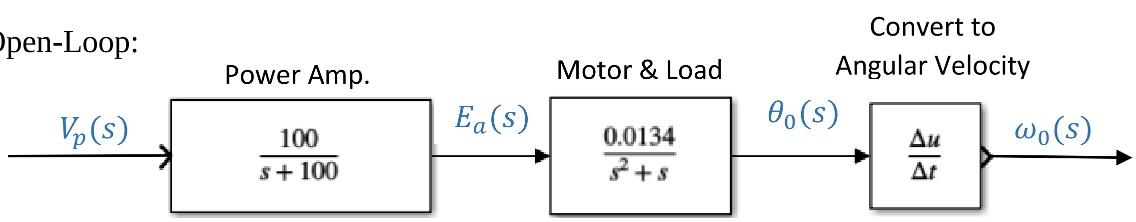
And we use the given parameters and values

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 1 & -\frac{1}{0.4}\left(\frac{24.1\times 10^{-3} \times 2.52 \times 10^{-3}}{9.04} + 0.4\right) \end{bmatrix} x + \begin{bmatrix} 0 \\ \frac{24.1 \times 10^{-3}}{9.04 \times 0.04} \end{bmatrix} E_a(t)$$

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0.067 \end{bmatrix} E_a(t)$$

$$y = [\,0.1 \quad 0\,]x$$

→Open-Loop:



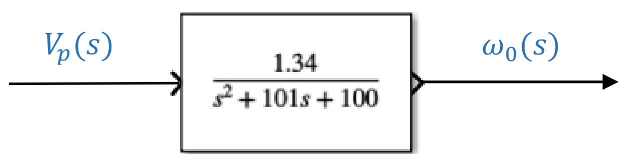


Table 4.1

The transfer function is:

$$G(s) = \frac{1.34}{s^2 + 101s + 100}$$

So; $\omega_n = \sqrt{100} = 10\ and\ \zeta$ = 5.05 (overdamped)

→In order to derive the angular velocity response to a step input, we multiply the transfer function by a step input, $\frac{1}{s}$ and we obtain;

$$\omega_0(s) = \frac{1.34}{s^3 + 101s^2 + 100s}$$

Expanding into partial fractions, we get

$$\omega_0(s) = \frac{0.0134}{s} + \frac{1.353 \times 10^{-4}}{s + 324} - \frac{0.0135}{s + 1}$$

from s-domain to time domain;

$$\omega_0(t) = 0.0134 + 1.353 \times 10^{-4} e^{-324t} - 0.0135e^{-t}$$

→First convert the transfer function into the state-space representation.

$$\frac{\omega_0(s)}{V_p(s)} = \frac{1.34}{s^2 + 101s + 100}$$

Cross-multiplying and taking the inverse Laplace transform with zero initial Conditions. We have

$$\dot{\omega}_0 + 101\dot{\omega}_0 + 100\omega_0 = 1.34V_p$$

Defining the phase variables as

$$x_1 = \omega_0$$
$$x_2 = \dot{\omega}_0 = \dot{x}_1$$

And,

$$x_2 = \dot{x}_1$$
$$\dot{x}_2 = -100x_1 - 101x_2 + 1.34V_p$$

where $V_p = 1$, a unit step. Since $x_1 = \omega_0$ is the output, the output equation is $y = x_1$

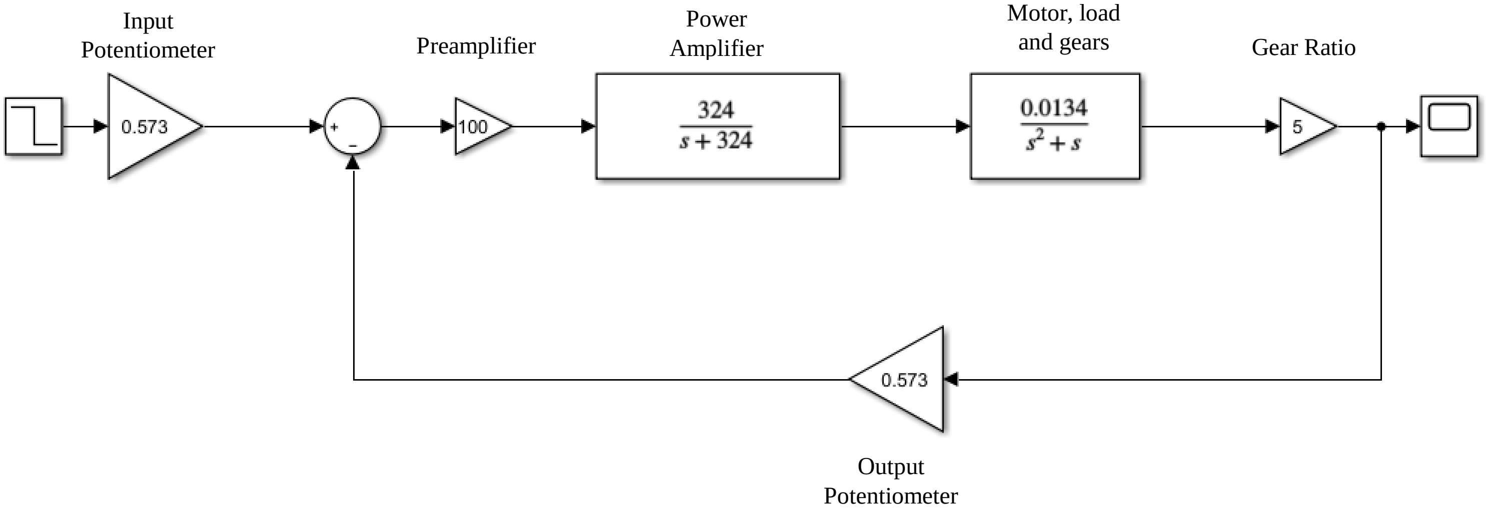


Table 4.2: Original Block Diagram reduction for the antenna azimuth position control system

```
>> sys = tf(4.3416, [1, 325, 324]);
>> stepinfo(sys)

ans =

  struct with fields:

        RiseTime: 2.1970
    SettlingTime: 3.9151
     SettlingMin: 0.0121
     SettlingMax: 0.0134
       Overshoot: 0
      Undershoot: 0
            Peak: 0.0134
        PeakTime: 10.5458
```

Table 4.3: Code of the Transfer Function of the Antenna Azimuth

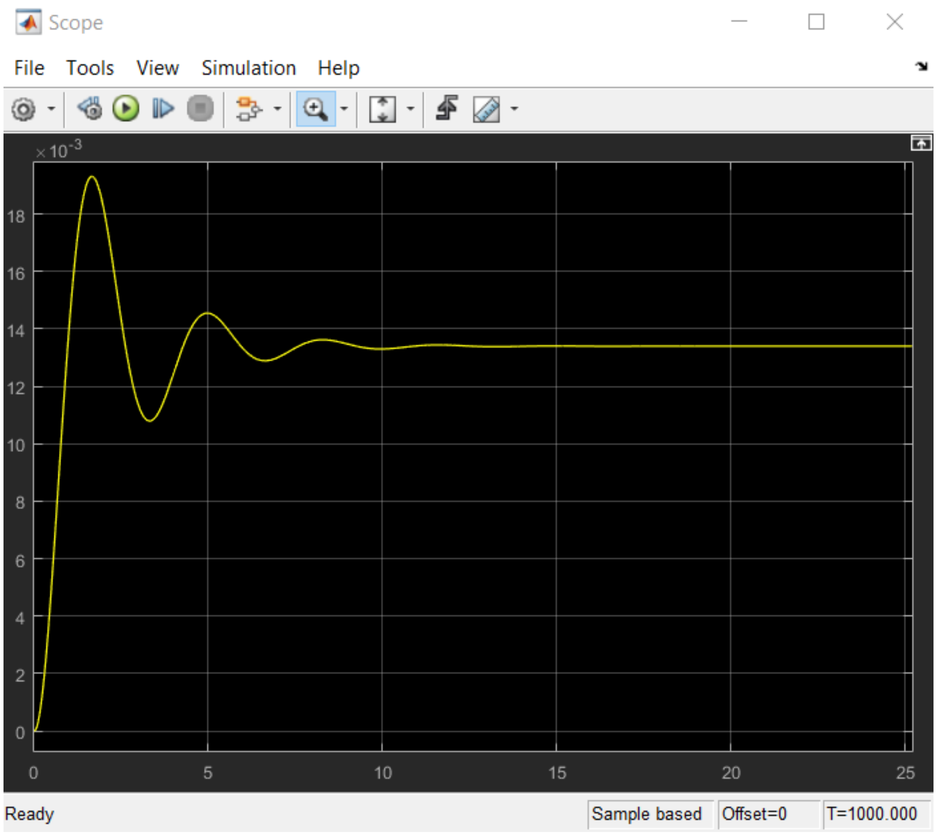


Table 4.4: Scope of Original Block Diagram

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