

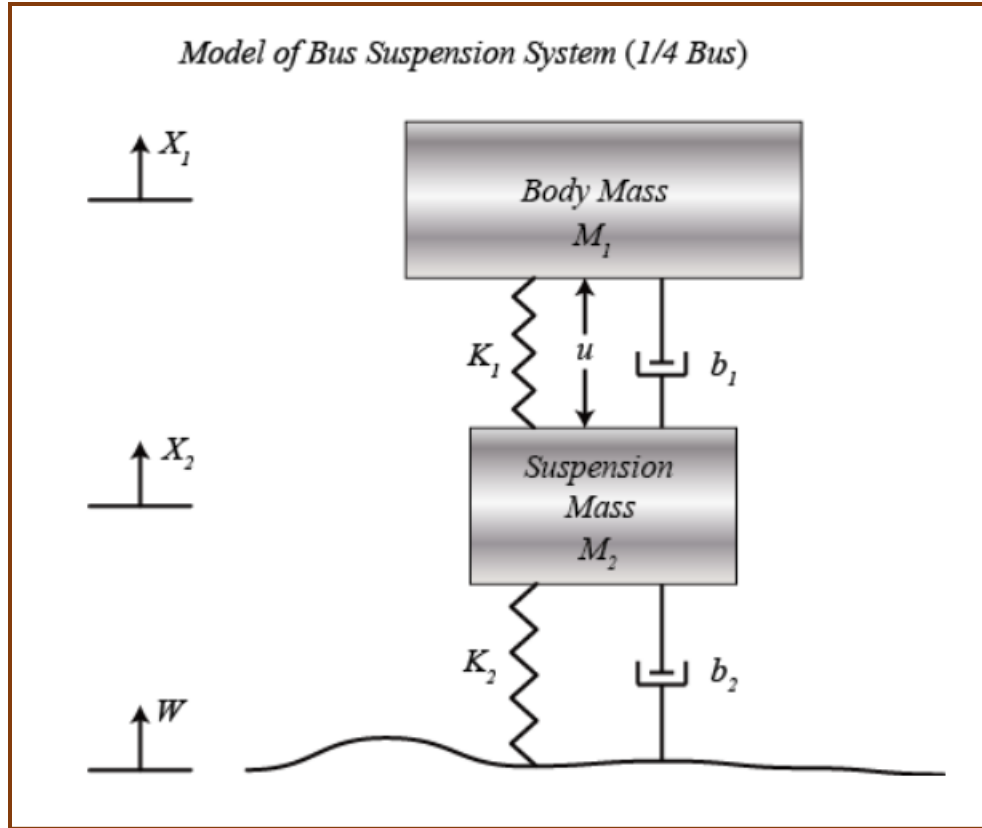
Yildiz Technical University  
Faculty of Mechanical Engineering  
Department of Mechatronics Engineering

MKT 3822 – Lab 3 (System Dynamics & Control)

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**Table A.1:** We will analyze and control of automotive suspension system in this project. This model is for an active suspension system which is used control force “U” in order to control the motion. W is the road disturbance.

We found equations of Motion in order to find Transfer Functions by using;

$$\sum \text{Inputs} - \sum \text{Outputs} = \sum f_{\text{external}}$$

$$(M_1 s^2 + b_1 s + K_1) X_1(s) - (b_1 s + K_1) X_2(s) = U(s)$$

$$-(b_1 s + K_1) X_1(s) + (M_2 s^2 + (b_1 + b_2)s + (K_1 + K_2)) X_2(s) = (b_2 s + K_2) W(s) - U(s)$$

We write these equation in Matrix Form  $[A] \begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$

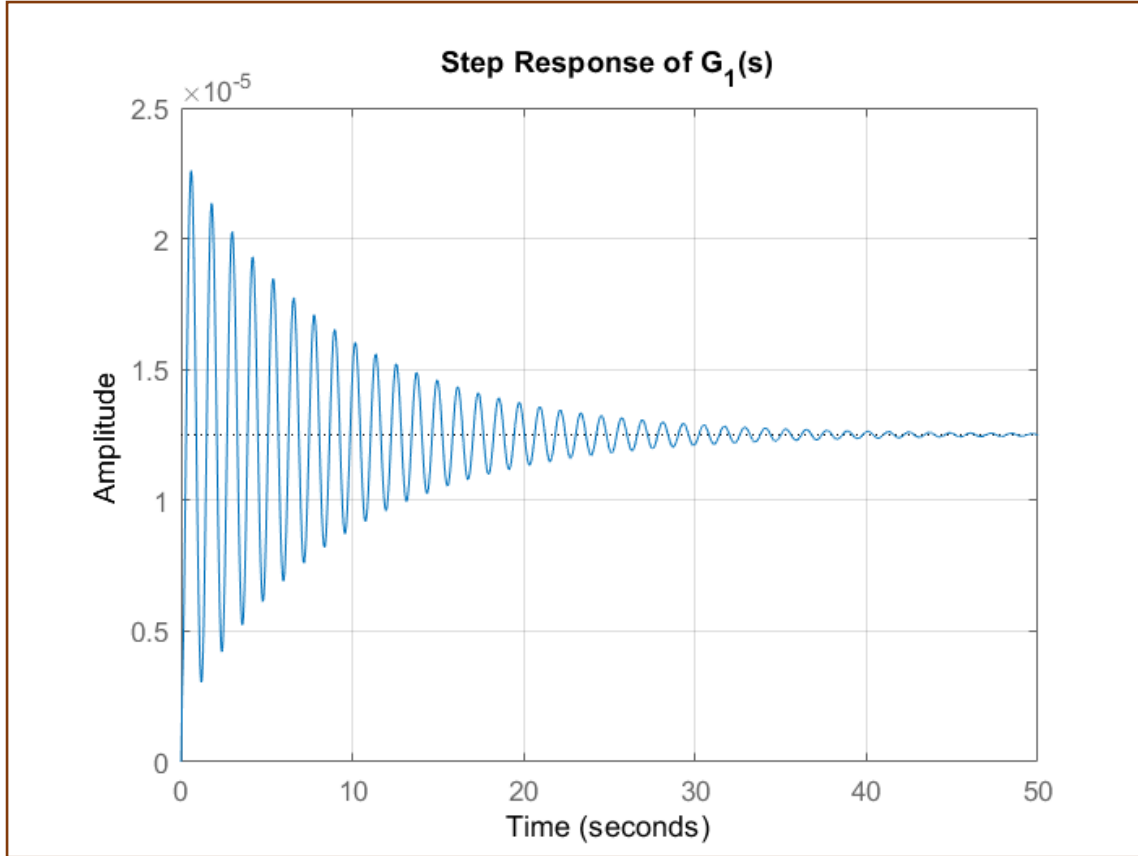
And, we found transfer function of  $G_1(s)$  by taking the only effect of  $U(s)$  and neglect the  $W(s) = 0$ .

$$G_1(s) = \frac{X_1(s) - X_2(s)}{U(s)} = \frac{(M_1 + M_2)s^2 + b_2 s + K_2}{\text{determinant of } A}$$

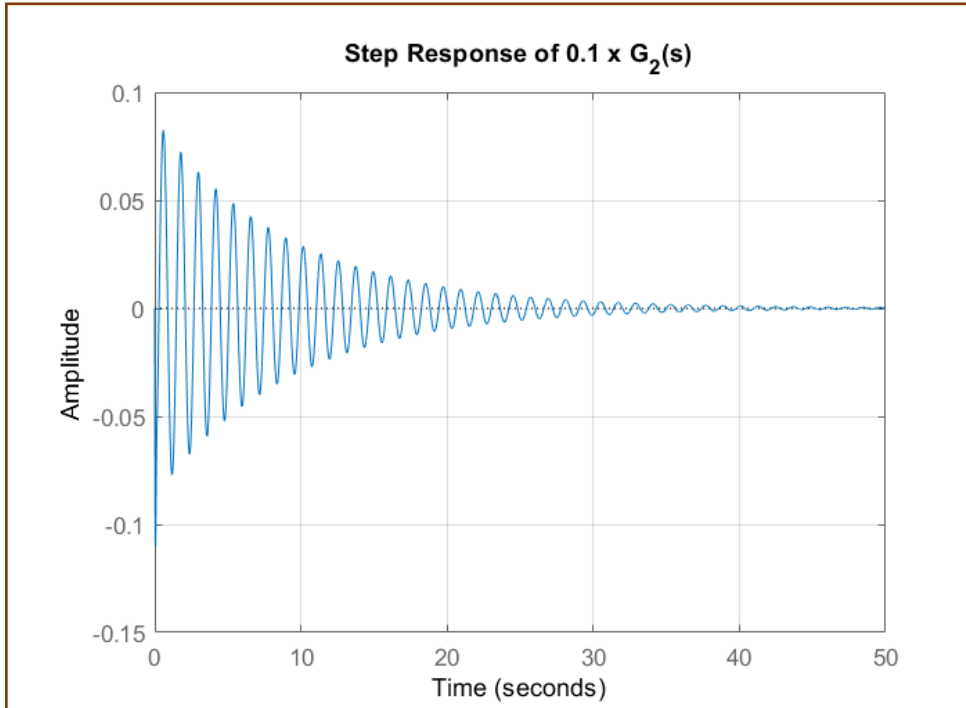
Then, we found transfer function of  $G_2(s)$  by taking the only effect of  $W(s)$  and neglect the  $U(s) = 0$ .

$$G_2(s) = \frac{X_1(s) - X_2(s)}{W(s)} = \frac{-M_1 b_2 s^3 - M_1 K_2 s^2}{\text{determinant of } A}$$

When we enter the system parameters, we obtained the step responses of  $G_1(s)$  and  $G_2(s)$ .



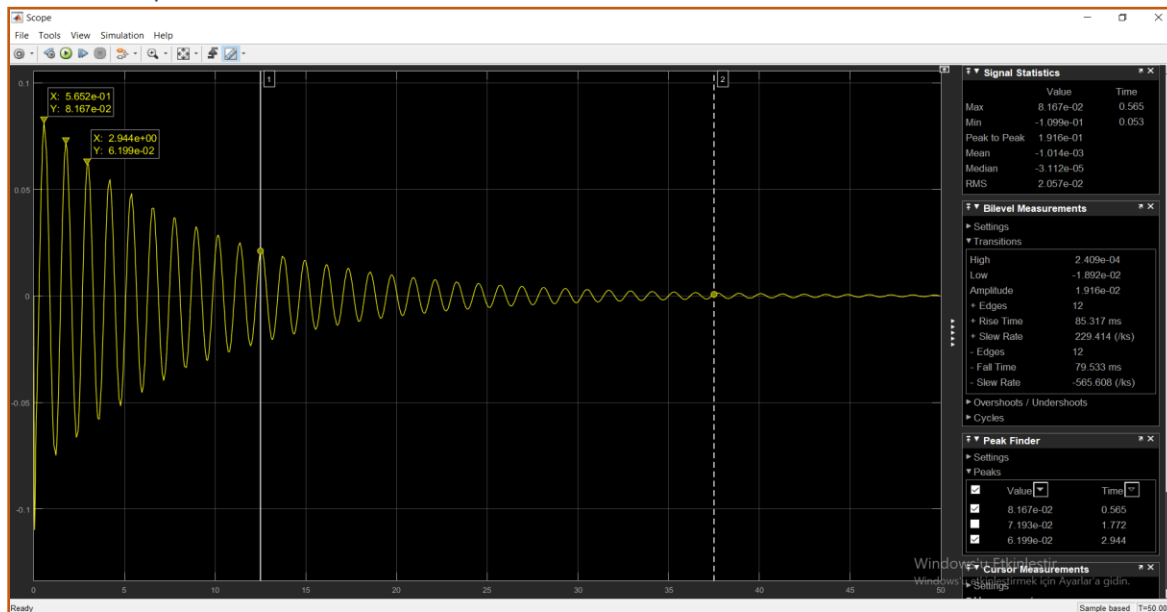
**Table A.2:** We obtained very small oscillation with respect to Open-Loop response for a unit step input but the system reached to Steady-State on the for a long time after so it is undesirable situation. And, the system is under-damped. It means that value of  $\zeta$  is between 0 and 1.



**Table A.3:** We gave 10 cm step disturbance. There is still very long oscillation. It means Settling Time is very long. And then, there is big oscillation initially. The people who in the bus are uncomfortable. Still, this system has not got feedback mechanism. We need to use Closed-Loop System. In the Open-Loop Transfer Functions the system cannot satisfy our request.

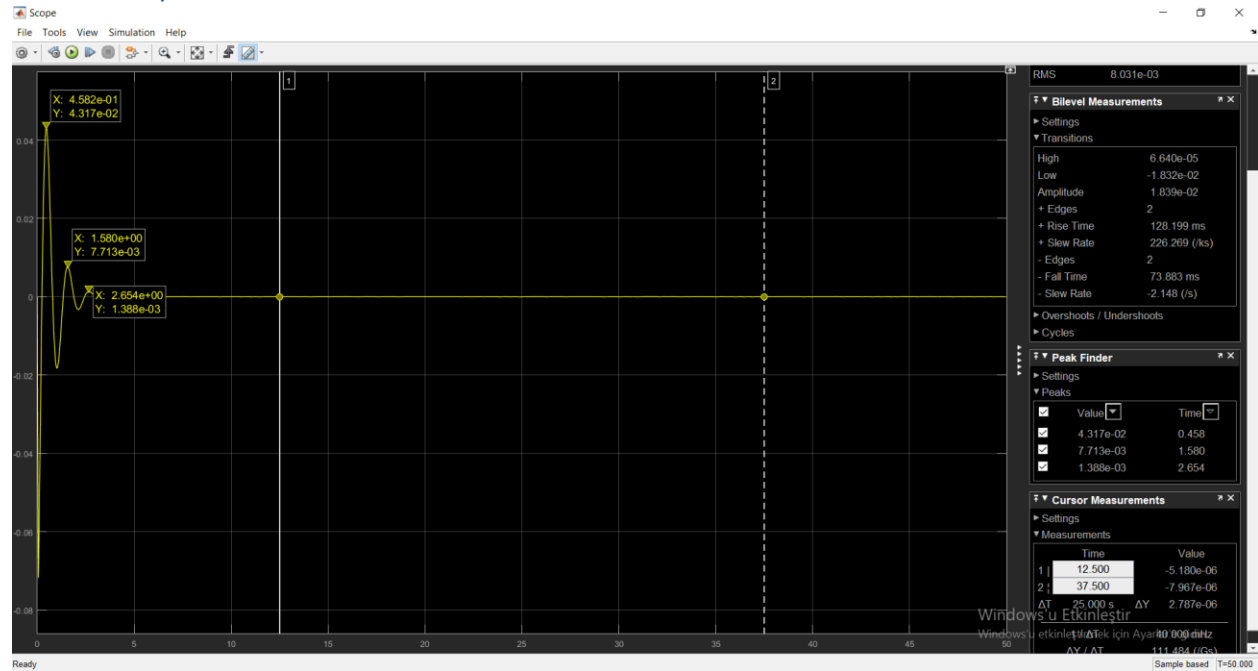
### PID Controller:

#### Effect of Proportional Control

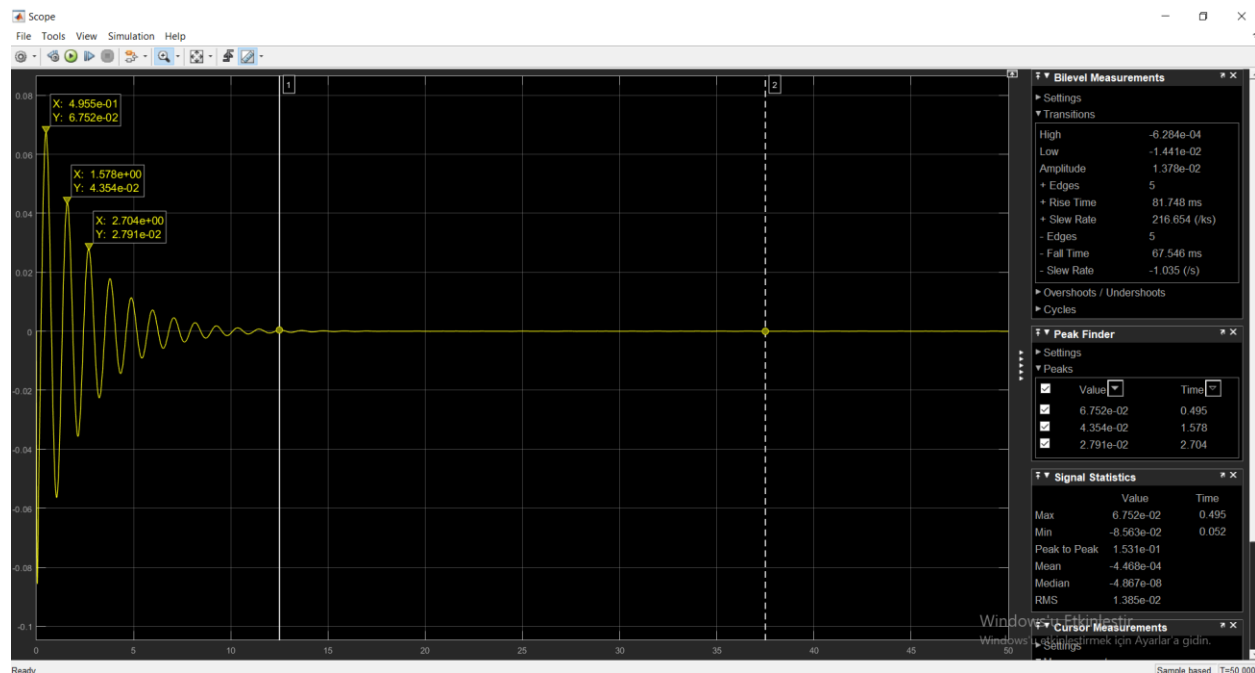


**Table A.4:** As you can see, there is no effect of proportional gain to the system because of Reference Input is equal to 0. Normally, Proportional Optimization have an impact to value that is sitting. Assume that we gave 3 value to the reference input. If the proportional gain will choose very high value, the result will be more accurate than the lower values. But reference input must zero because of we want the final value is zero. Proportional Controller is important due to adjust of system level.

## Effect of Proportional Control and Derivative Control:



**Table A.5:** In this graph, we can deduce to effect of PD Controller. In this graph, Reference Input is equal to zero again because we want the value must be zero. And then, Gain of Proportional Controller is very high value and Gain of Derivative Controller has also high value. As you can see, Derivative Controller effects to oscillation optimization and it reduces to Settling Time, therefore Oscillation will decrease. Derivative Controller is important due to access to exponential terms and these exponential goes to zero in process of time.



**Table A.6:** We see how to effect signal when using Integral Controller in PID Controller. Integral Controller is used in order to get rid of Steady-State Error. It is important due to not stay in the Steady-State.

**Note:** We assume that the vehicle runs onto a 10-cm step. Therefore, Disturbance value, it means  $W$ , entered by 0.1. We used this arbitrary values to show effect of  $(K_p, K_d, K_i)$ .

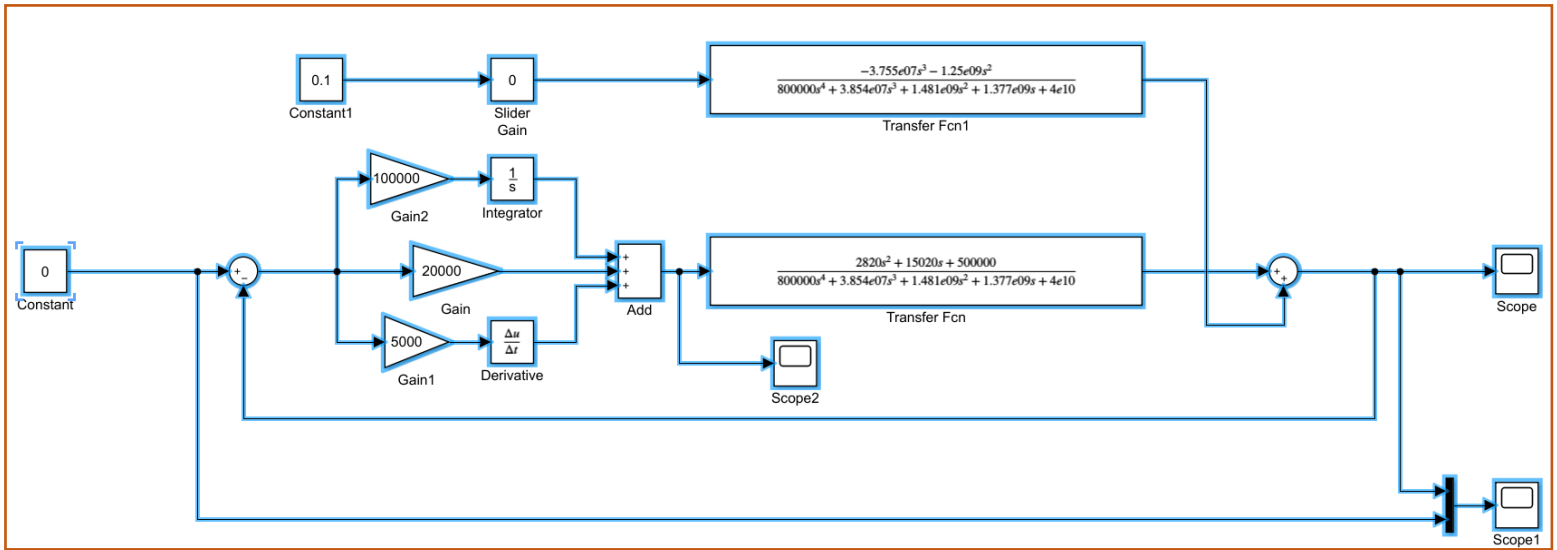
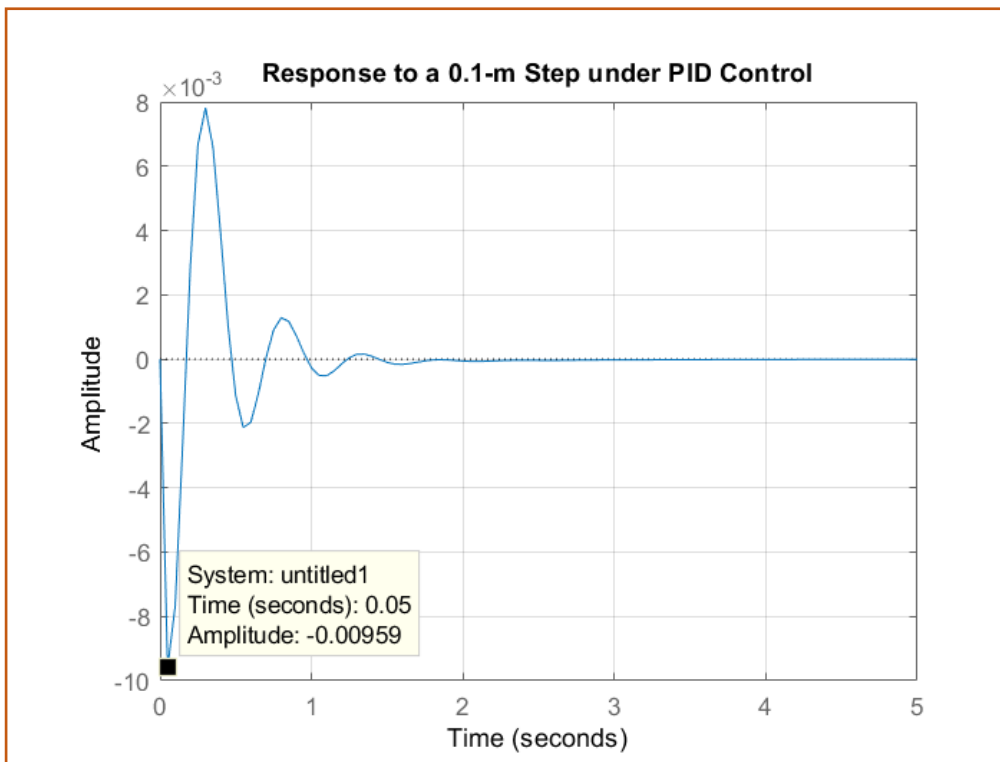
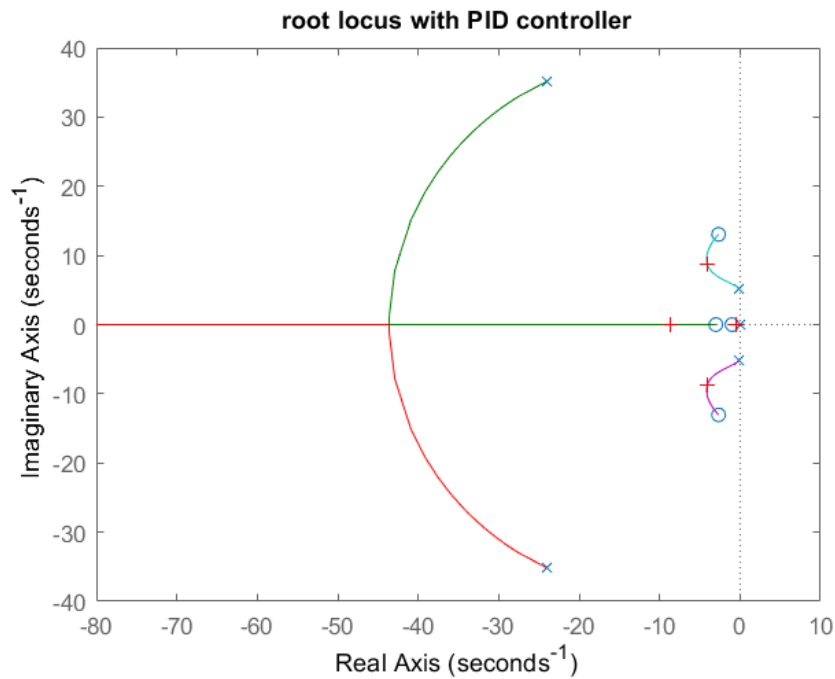


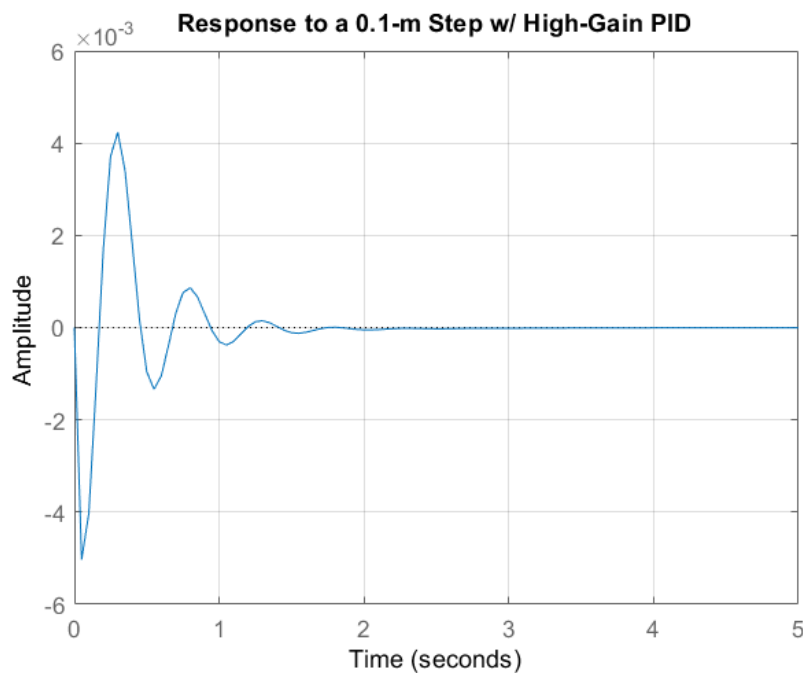
Table A.7: PID Controller in Simulink



**Table A.8:** Maximum Percent Overshoot of System Response with PID controller is  $9 \times 10^{-3}$  m (9 mm). It does not satisfy our expectation even if Settling Time of Response is less than 5 seconds. We have to find Root-Locus of this system to adjust the Suitable Overshoot to find valid gain. When we increase the gains, system will be more accurate.

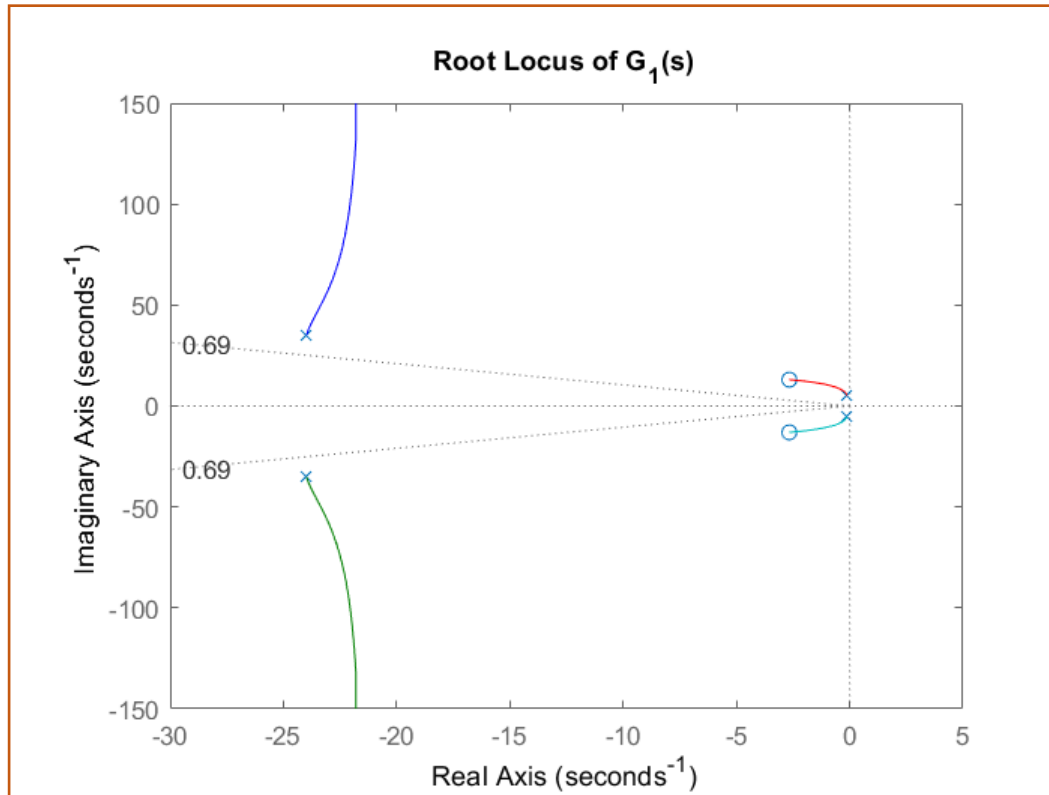


**Table A.9:** Graph of Root Locus with PID Controller seems to us position of roots and zeros number of asymptotes, number of branches, real axis portion of locus.

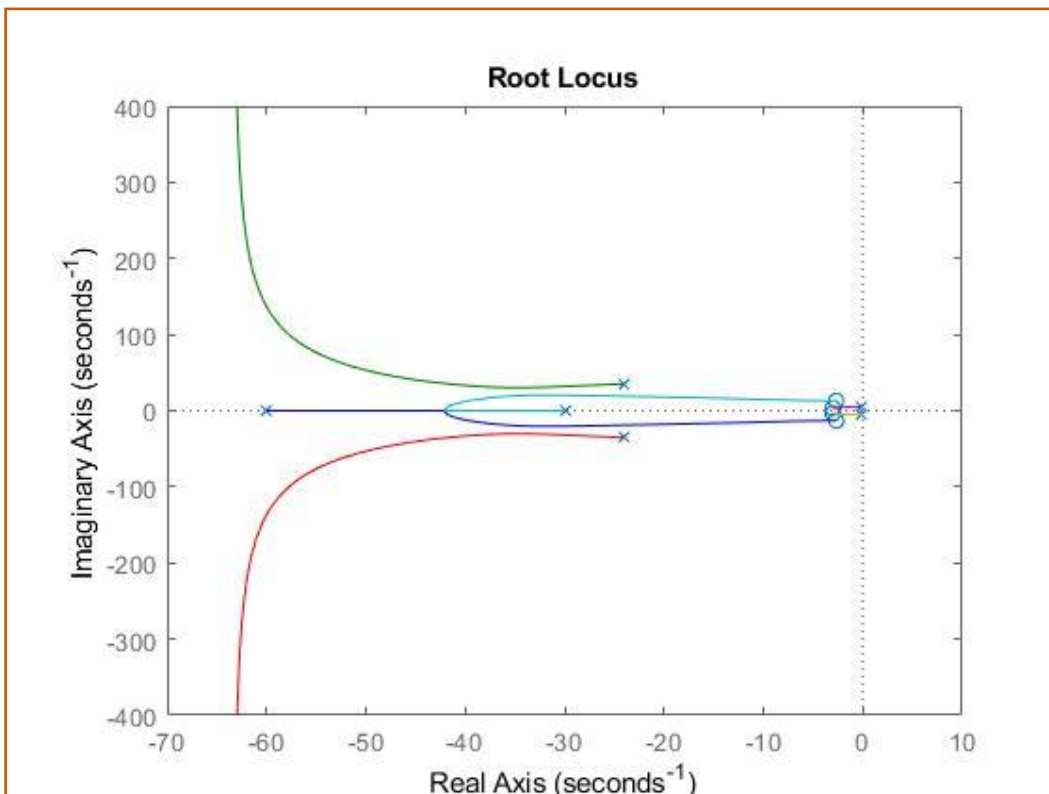


**Table A.10:** When we increase the gains ( $K_p$ ,  $K_d$ ,  $K_i$ ), we find the figure on the left side. When we multiply by 10 each gain, amplitude decreases. When each gain was divided by 10, amplitude increases. As we can see, Maximum Percent Overshoot and Settling Time are adjusted with respect to requirements.

## Root Locus Controller Design

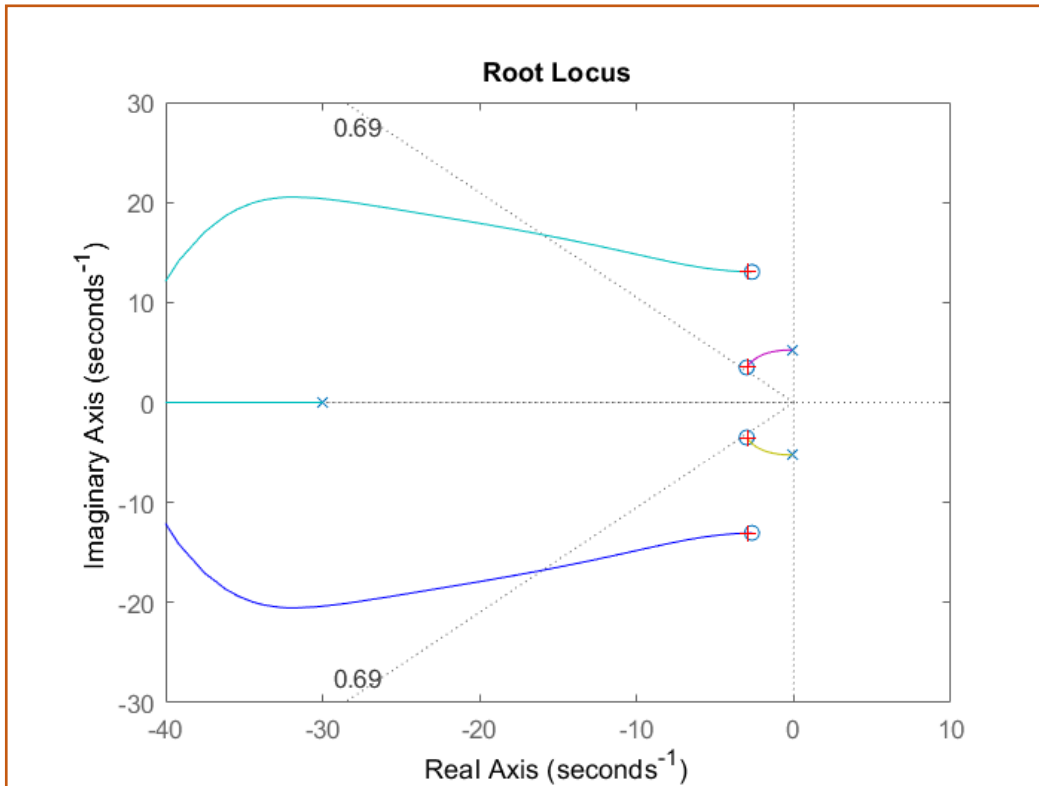


**Table A.11:** The purpose of Root-Locus is to forecast the Closed-Loop Response by using Open-Loop Root-Locus Plot. Root locus of the  $G_1(s)$  is on the left side. There are 2 zeros and 2 poles that are very close to Imaginary Axis and they can make the system by Marginally Stable. It is undesirable situation if we want to control the systems. The system must be Stable in order to control. That's why, to get faster reason we put 2 poles which is in the Real Axis to the Left-Half Plane.

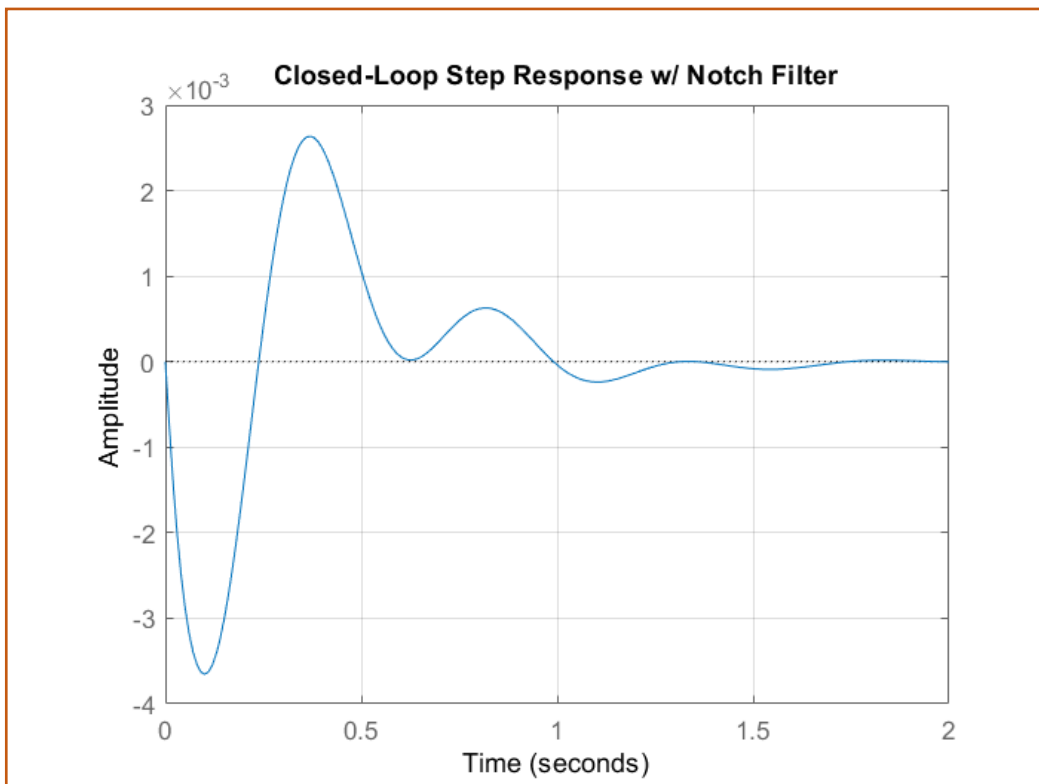


**Table A.12:** We used Notch Filter in order to change locus by adding 2 poles on the Left-Half plane. Zeros were added at  $3 \pm 3.5i$  and poles were added at 30 and 60. Root-Locus will be like is on the Left Side.



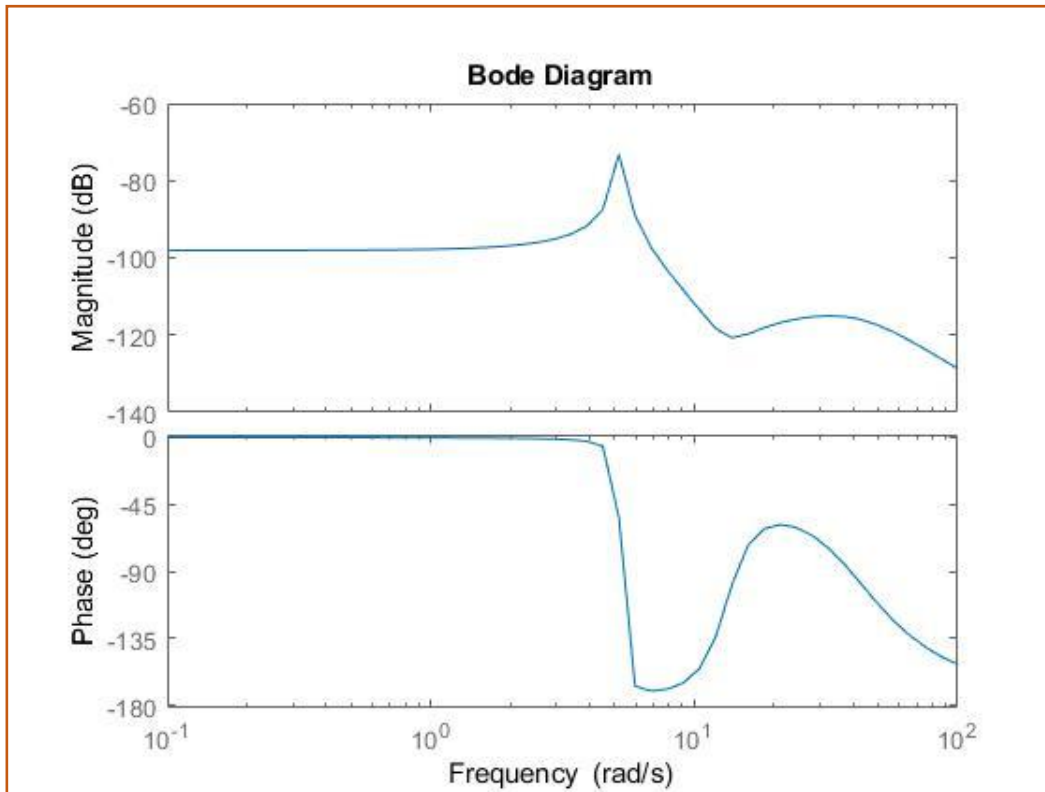


**Table A.13:** Particular Region of the Root Locus

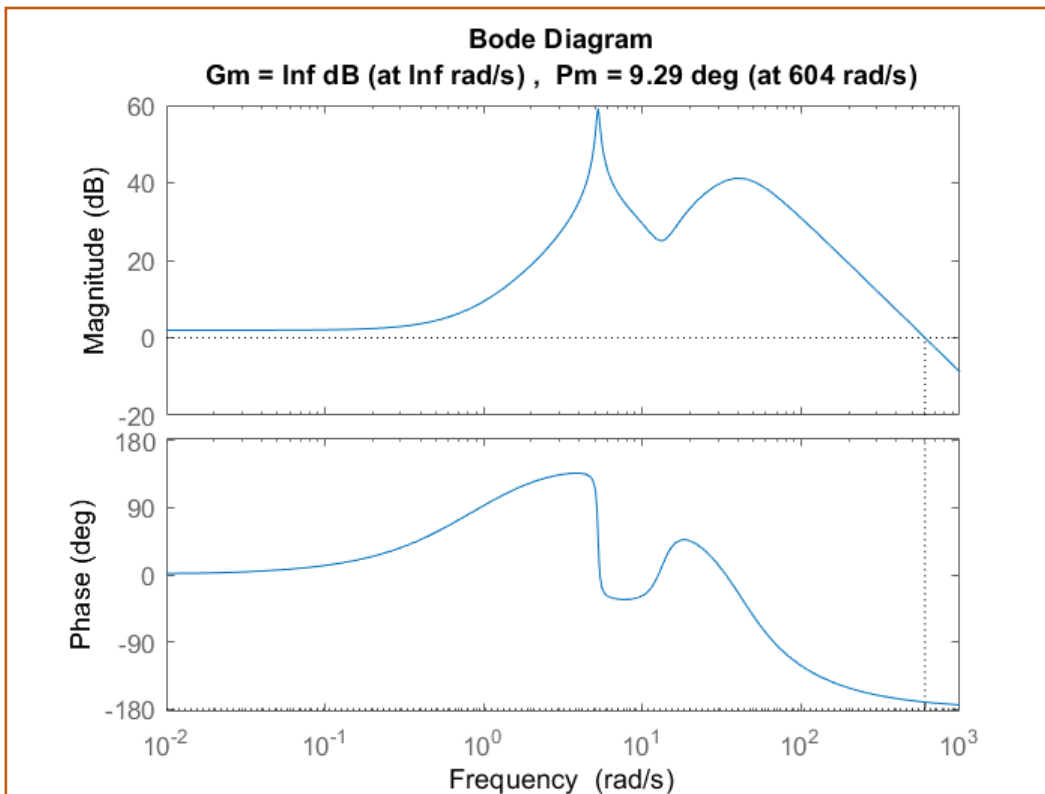


**Table A.14:** This plot demonstrates the Maximum Percent Overshoot is approximately 3.75 mm and Settling time is equal to 2 seconds. Therefore, the required points are gratified. Even if we choose the disturbance Settling Time will be the same but Maximum Overshoot will be changed.

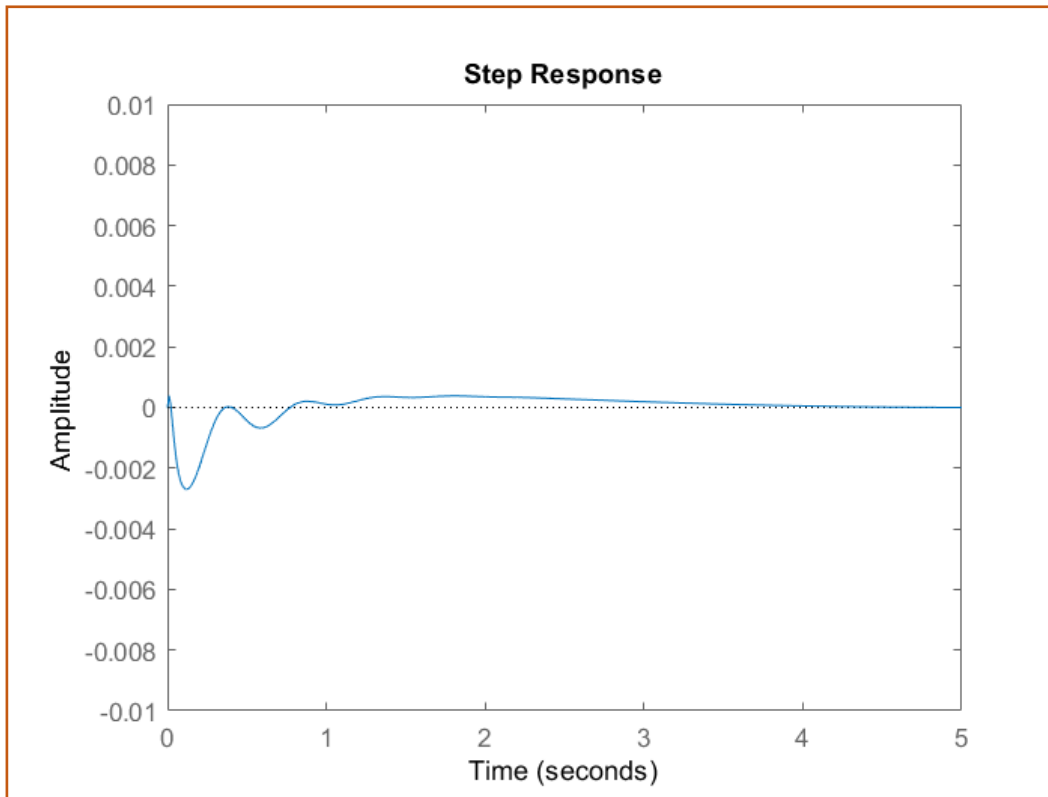
## Frequency Response Controller Design



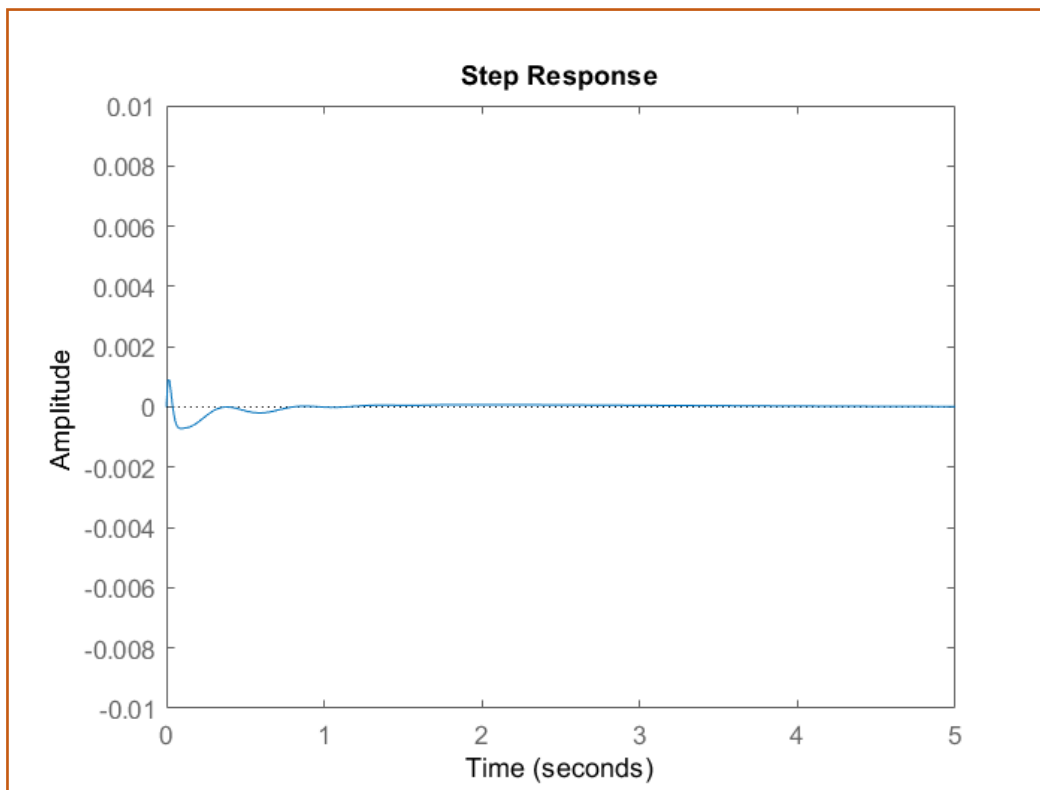
**Table A.15:** If we want to obtain the dynamic properties of the system and again if we will do it statically, we have to use Frequency Domain Analysis. What is up with that? It means, we can obtain the values which is observing dynamically in the Time Domain, statically in the Frequency Domain. If frequency of this system increases, amplification rate will decrease with respect to the equation. When the Phase angle is minimum, the overshoot will be minimal level. But we can do it with compensator. It means we can obtain minimal overshoot level by using compensator



**Table A.16:** In this plot, Open-Loop Bode Plot was changed by adding Controller. As you can see, the phase curve is concave at about 5 rad/sec. Lead controller can affect +90 degrees maximally thus we have to add with respect to expectation. It can be much more one. When we increase the 'W', the magnitude will decrease but also new plot resembles previous plot. When we decrease the 'W', the magnitude will increase but also new plot resembles previous plot and phase diagram will be more ovoid.

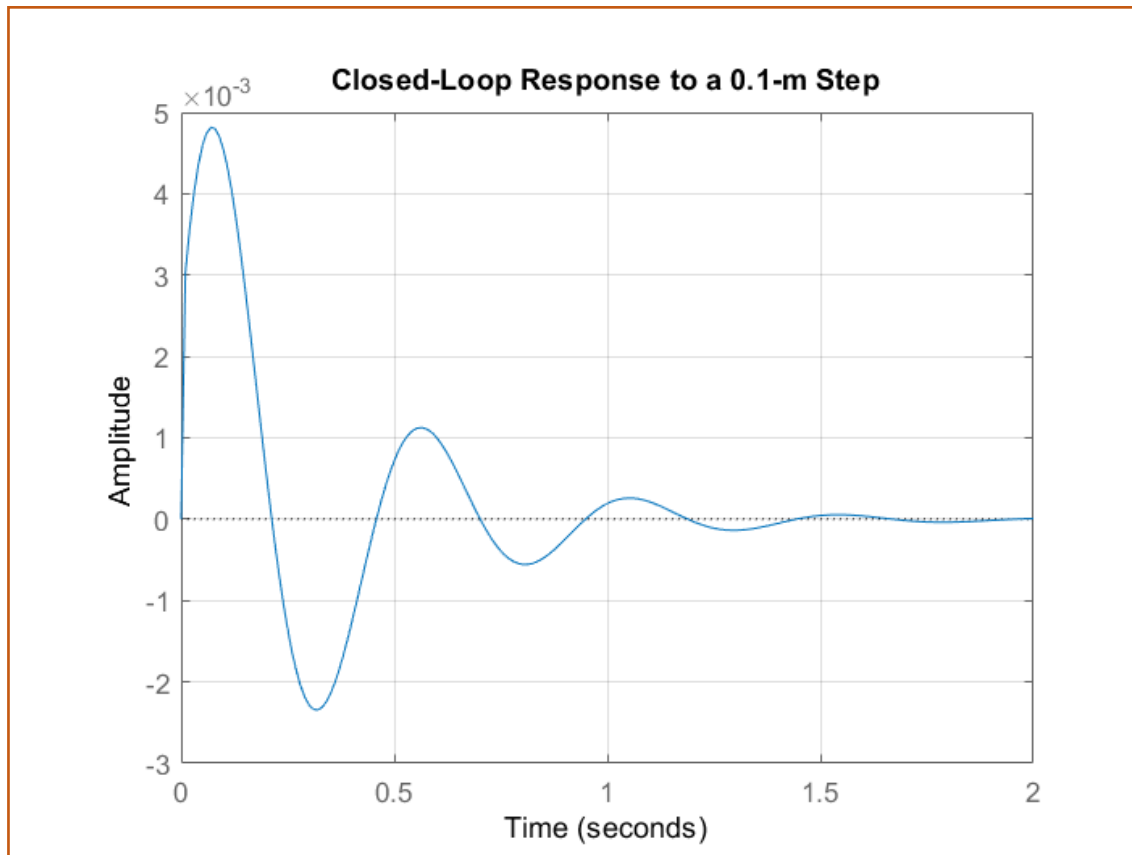


**Table A.17:** We multiplied by 0.1 because of the disturbance 10 cm. When we increase the 'W', maximum overshoot increases but settling time decreases. When we decrease the 'W', maximum overshoot and settling time decreases.



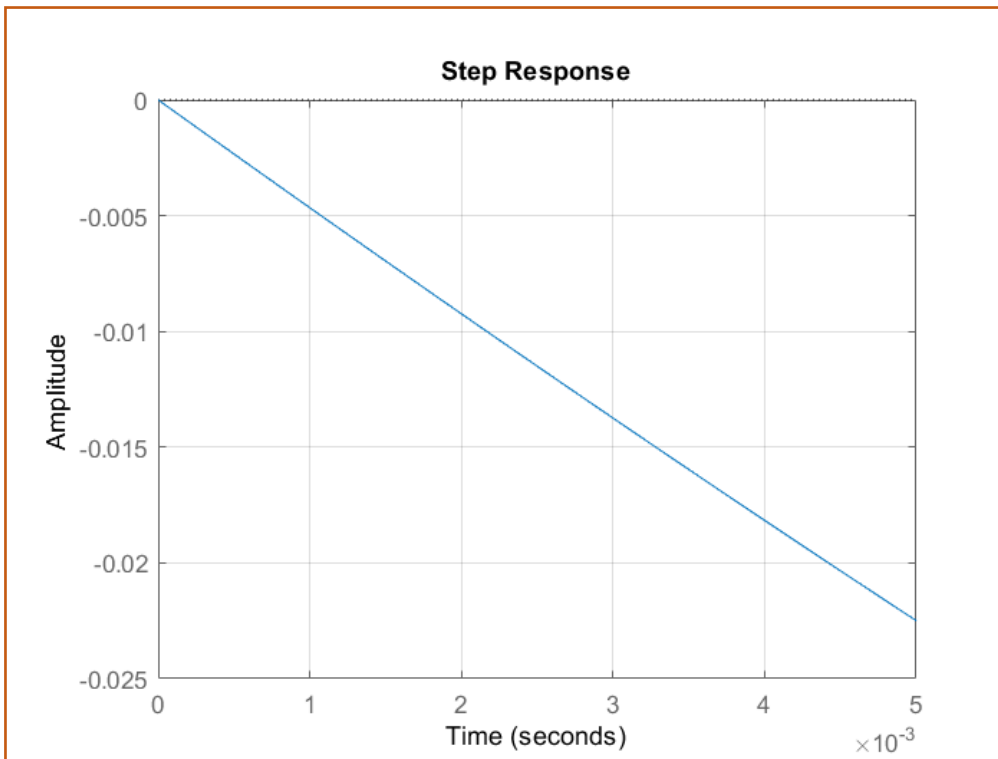
**Table A.18:** When we increase the 'W', maximum overshoot increases but, settling time increases. When we decrease the 'W', maximum overshoot increases but there are less oscillation, settling time decreases.

## State-Space Controller Design

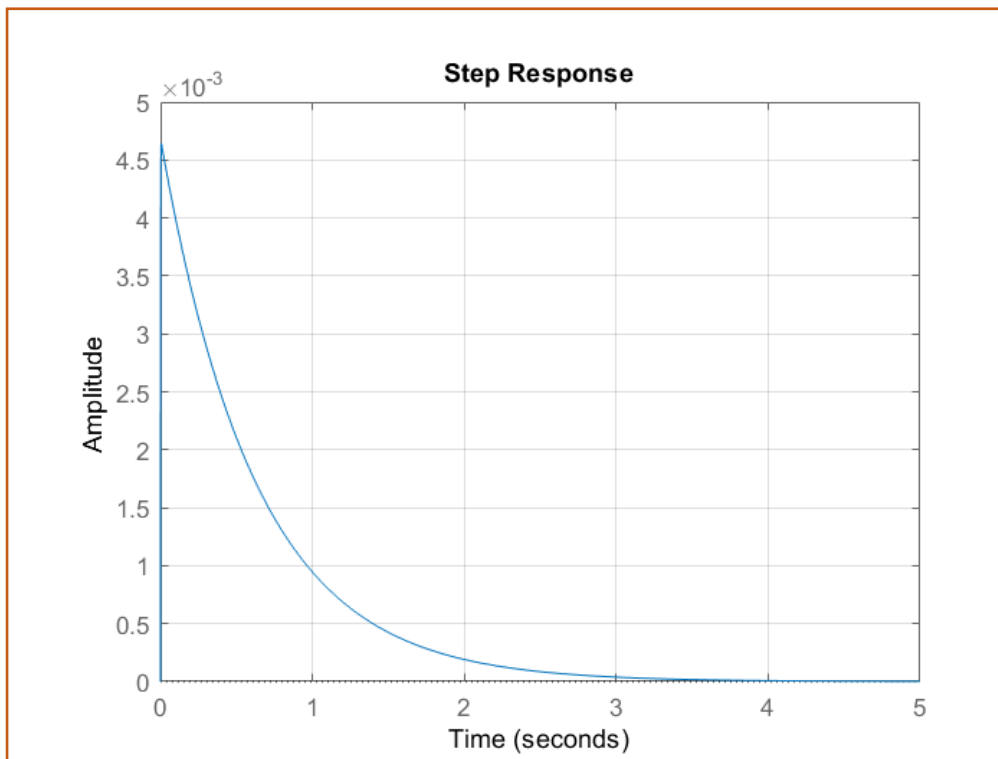


**Table A.19:** We can solve 'Reference Tracking Problem' and 'Stability Problem' with State-Space. We can adjust only second order response in Frequency Domain but by using State-Space, we can put any poles or states to the place wherever we want. On the other hand, we cannot set more than second pole in Frequency Domain. But there is zero location problem in closed loop in State-Space. The purpose on the full-state feedback is we can access to all poles and change values on the characteristic equation by using values of  $U$ . The coefficients of  $K$  select where the roots of the characteristic equation are in the  $s$ -plane. We can select coefficient of  $K$  arbitrary. If values of  $K$  is bigger than old coefficient, the system will be faster than previous system and vice versa.

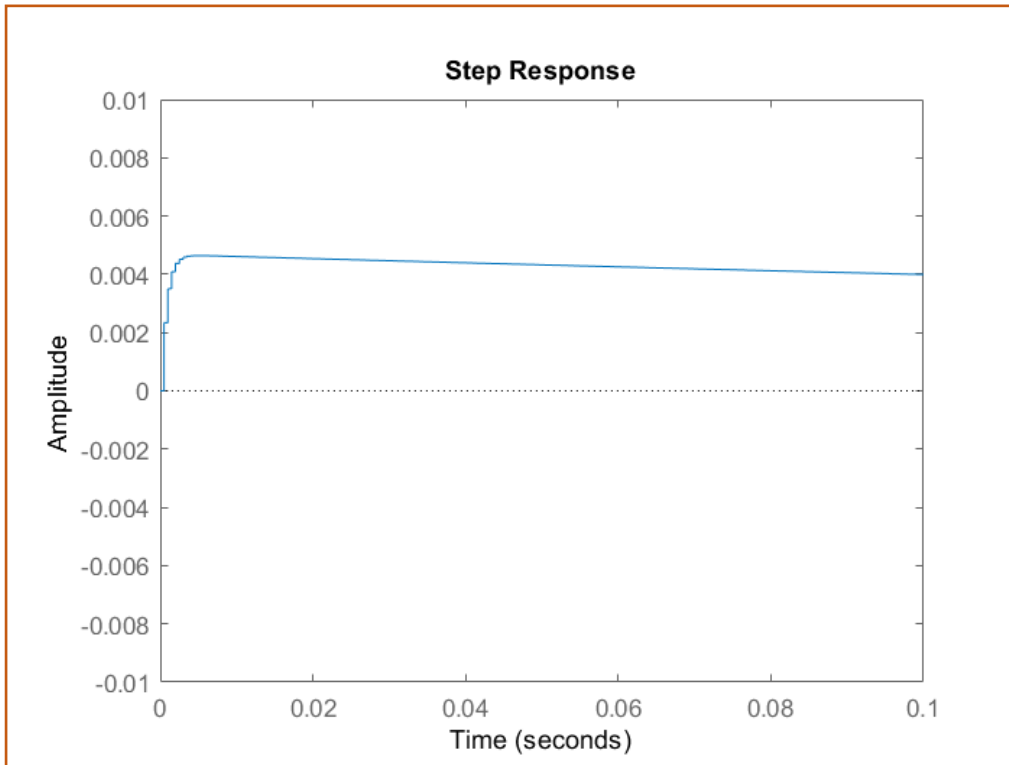
## Digital Controller Design



**Table A.20:** Even if there is a changing in the p4 and p5, this plot will be the same. The plot can be changed by changing the system parameters like coefficients of spring, mass and damper. It depends on these parameters. In the Digital Controller Design, we are interested in instantaneous change like sliding. Very small affects effect to the system very quickly. It changes our system parameters.

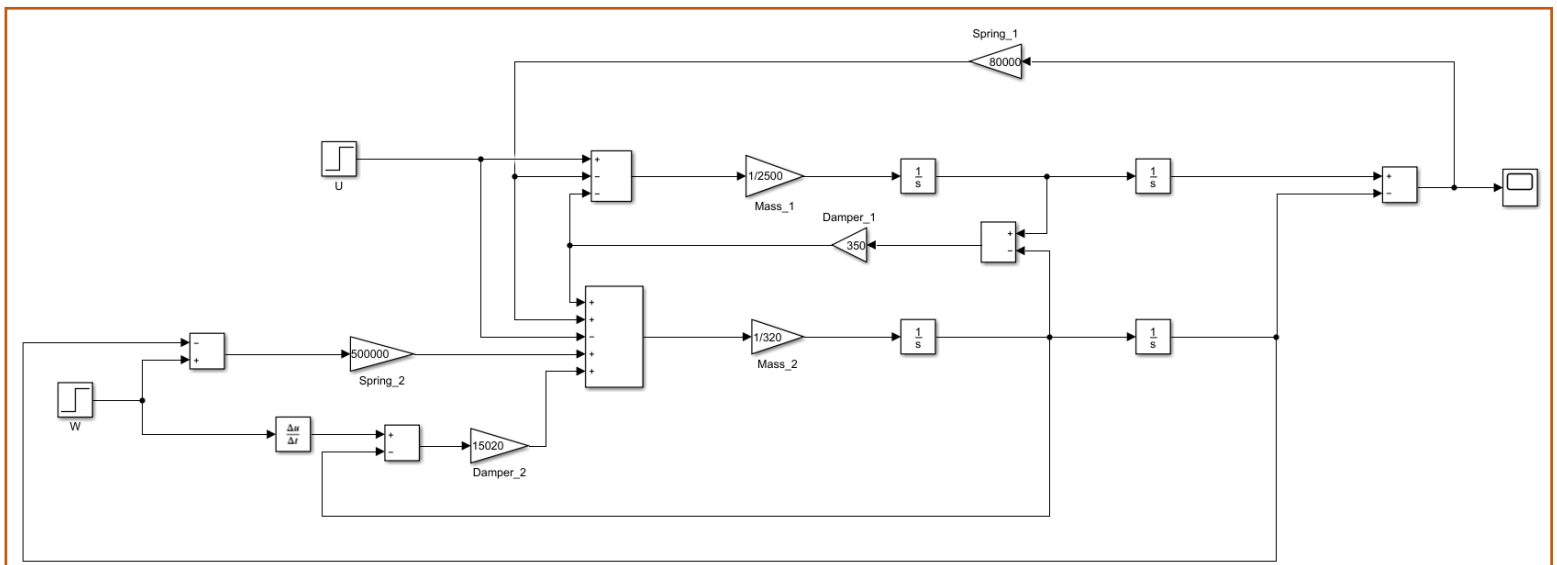


**Table A.21:** When we select the value of p5 '0.8', it means even if there is very small step-up, it will be effect to system huge impact (nearly 2.2 times much more).

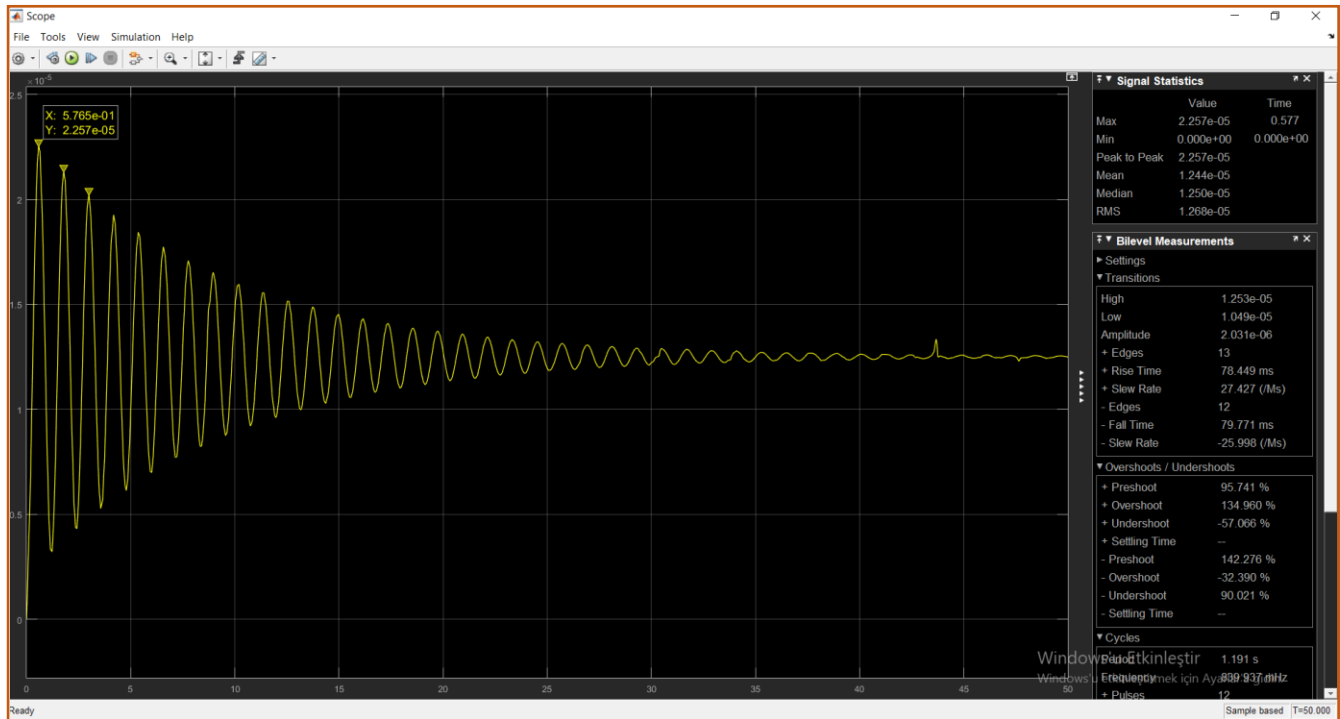


**Table A.22:** When we select the value of p5 '0.2', it means even if there is very small step-down, it will be effect to system huge impact (nearly 2 times less). When we select the value of p4 '1', it means approximately %0.8 step-up, the system will be peak value in the very small time and reach the steady-state.

## Modelling



**Table A.23:** This Modelling was created in the Suspension System. Required settings were done and the modelling was run.



**Table A.24:** The Scope of the Modelling ( Step Response of  $G_1(s)$  )

As you can see in the Simulink with Modelling. We obtained small overshoot with respect to Open-Loop response for a unit step input but the settling time does not satisfy. There is no any controller and we have to use it to reach the desired points.