# Determining The Mass of the Z-Boson

Oguz Tanatar

## Introduction

In High Energy Physics, one of the interesting and most fundamental quantities of a particle is its mass. In this report, we will use data from the ATLAS (A Toroidal Lhc ApparatuS) experiment at CERN to measure the mass of the $Z^0$ boson, one of the three particles which mediate the weak nuclear force.

When two protons collide between the center-of-mass energies of 8 TeV and 13 TeV, they can produce a $Z^0$ boson, which can, in 10% of cases, decay into a lepton-antilepton pair. Other possible products of decay are a neutrino-antineutrino pair, or a quark-antiquark pair. Due to the very weak interactions of neutrinos, and the instability of quarks, muons, and tauons, the most feasible option is to measure the properties of the produce electron-positron pairs and deduce the $Z^0$ boson's invariant mass by applying relevant symmetries. The $Z^0$ boson's mass will be first measured using the curve_fit capabilities of Python's scipy.optimize library and will later be checked using a chi-square map.

## Invariant Mass Distribution

In ATLAS, we can measure the produced leptons' total energies $(E)$ , their transverse momenta $(p_T)$, their pseudorapidities $(\eta)$, and their azimuthal angles $(\phi)$. These quantities can be brought together in the following way to find their momenta:

$$p_x = p_T \cos(\phi), \quad p_y = p_T \cos(\phi), \quad p_z = p_T \sinh(\eta) \tag{1}$$

Then, we can derive the individual masses for each data point using Einstein's Energy relation,

$$M = \sqrt{E^2 - \left(p_x^2 + p_y^2 + p_z^2\right)} \tag{2}$$

After formatting this data into a histogram, we can fit it using the Breit-Wigner function,

$$D(m, m_o, \Gamma) = \frac{1}{\pi} \frac{\frac{\Gamma}{2}}{(m - m_o)^2 + \left(\frac{\Gamma}{2}\right)^2} \tag{3}$$

where $\Gamma$ is the width parameter ($GeV$), and $m_o$ is the true rest mass ($\frac{GeV}{c^2}$). Using the curve_fit capabilities of scipy.optimize, a fitting range from 87 GeV to 93 GeV and a normalization of $\frac{5000}{2}$, we can find best fit parameters and their uncertainties to be $m_o = 90.3 \pm 0.1$ GeV and $\Gamma_{\mathrm{exp}} = 6.4 \pm 0.2$ GeV. Our fit, data, and residuals between the fit and data are shown in Figure 1. The $\chi^2$, n.d.o.f, and p-values were found to be 10, 10, and 0.4, respectively.

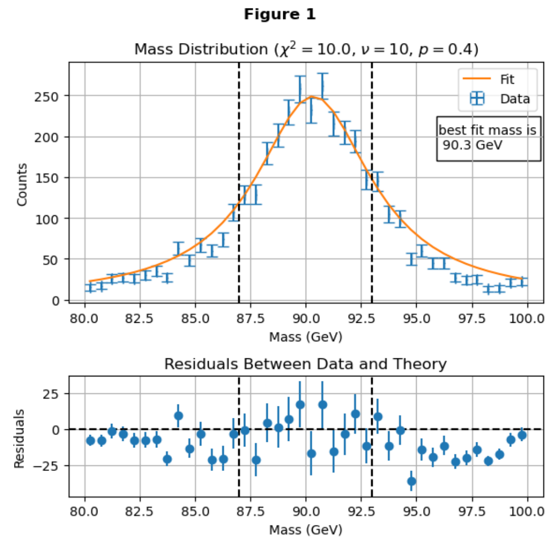


*Figure 1: The vertical dashed lines represent the fitting range. (a) The mass histogram of the $Z^0$ boson, overlayed with the Breit-Wigner function fitted over the mass bin centers of 87 to 93 GeV. The statistically relevant quantities are listed in the title, and the best-fit mass is 90.3 GeV. (b) Residuals between the fit and data. The horizontal dashed line represents perfect agreement.*

To check the validity of our fit, we can run some statistical tests, namely the reduced-$\chi^2$ test and the chidist test. Looking at the reduced-$\chi^2$ first, we see that $\frac{\chi^2}{n.d.o.f} = 1$ exactly. Generally, it should be roughly equal to one, so at this level of precision, we see that our fit passes the test. Shifting to the p-value, we know that for acceptable agreement between theory and data, the p-value must be between 0.05 and 0.95. 0.4 lies comfortably in the middle of this range, so our fit passes this test too. Since the fit passes both aforementioned tests, we know that our fit is valid.

## 2D Parameter Scan

To further investigate how the fit behaves as we vary the fitting parameters, we can make a two-dimensional $\chi^2$-map, using the fitting parameters as independent variables. To calculate this, a 2D meshgrid was made in numpy over the plotting domains ($m_o \in [89, 93]$ GeV and $\Gamma \in [5, 8]$ GeV), and the difference between the $\chi^2$ associated with each combination of fitting parameter and the minimum $\chi^2$ was calculated. The plot of this map is shown in Figure 2.

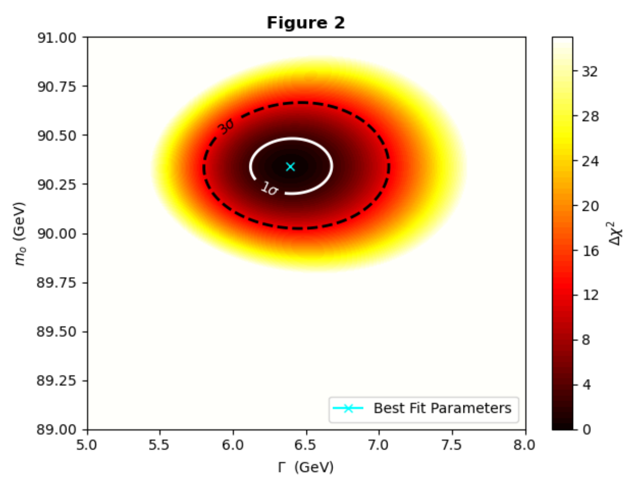


*Figure 2: $\Delta\chi^2$ contour map for the width and invariant mass fitting parameters. The minimum $\chi^2$ is denoted by the cyan x mark. The black region corresponds to smaller differences and the brighter regions correspond to larger differences. For convenience, the $3\sigma$ contour has been marked in a black dashed line, and the $1\sigma$ has been marked in a white solid line. The $\Delta\chi^2$ was also clipped at 35 units for ease of reading.*

To determine the levels at which to place the $1\sigma$ and $3\sigma$ marks, we can use the well-known fact that in a map such as this one, A spread of one standard deviation corresponds to a $\chi^2$ value of approximately 2.3, and three standard deviations corresponds to $\chi^2 \approx 11.83$. Here, we switch out the $\chi^2$ for $\Delta\chi^2$, as the minimum $\chi^2$ corresponds to zero.

## Discussion and Future Work

The mass and width parameters we determined for the $Z^0$ boson was $90.3 \pm 0.1$ GeV and $6.4 \pm 0.2$ GeV, respectively. With this fit, we found a $\chi^2$ of 10.0, with 10 degrees of freedom and a p-value of 0.4.

Given our findings, we can test for agreement in mass with the accepted values of $91.2 \pm 0.0$ GeV, for invariant mass. We can see if there is agreement between these two values by running a difference-over-sigma test, for which we get a result of $\sim 9$, which confirms significant disagreement between the experimental and accepted value. To fix this disagreement, several improvements to the experiment can be made.

Firstly, we can look at systematic uncertainties. As we accelerate protons and collide them, we get a significant amount of noise that we can filter out through Fourier analysis. However, in doing so, we also lose some of the signal, which creates systematic uncertainty. Keeping this, along with other sources of systematic uncertainty, we can generate better agreement between the accepted value for the $Z^0$ boson's mass, and the one we found.

Another component of error, which ought to be highlighted, can be attributed to the finite energy resolution of the detector. The limited capabilities of the detector should be included in the uncertainties of our data and in our data processing.

Finally, another step we can take to enhance realism is to also propagate the uncertainty in our fitting parameters. In Fig. 1, it is apparent that although uncertainty was applied to the data, the fit did not have uncertainty plotted, although the uncertainty in the fitting parameters can have more of an impact on our results.