I Introduction

As is taught in most introductory physics classes, the force between two masses is,

$$F = -G \frac{M * m}{r^2} \hat{r}$$

Where r is the distance between the two masses (in meters), M and m (in kilograms), $\hat{r}$ is the direction between the two masses (unitless) pointing from M to m, and G is an experimentally determined constant ($6.67 * 10^{-11} \frac{m^3}{kg\ s^2}$.

Because the force is a vector, we can solve an associated equation to find the gravitational potential, which is given by,

$$V = -\frac{GM}{r}$$

To find the necessary parameters for the rocket to reach the moon, we must understand the forces that we must overcome, and how the forces on the rocket change as the rocket follows its trajectory.

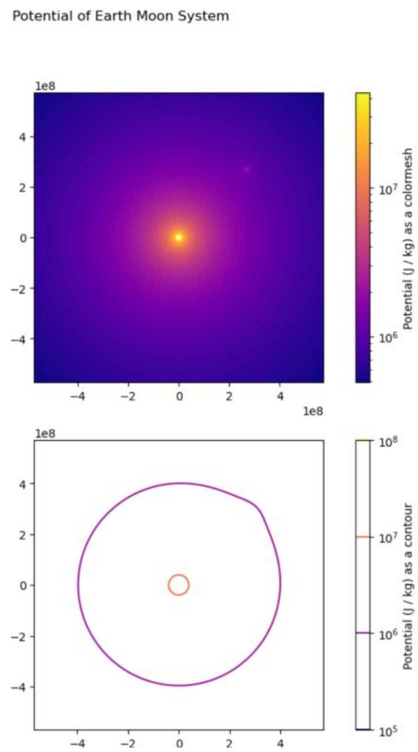
To create visual representations of the potential and forces at play, we have used matplotlib.pyplot, a python library for effectively plotting numerical data. To compute this data, we used numpy, a python library to process large groups of numerical data.

II The Gravitational Potential of the Earth Moon System

The gravitational potential of the earth moon system is the sum of the individual potentials of the earth and the moon. We took the position of earth to be at the origin, and the position of the moon to be at the lunar distance, at 45 degrees from the positive x-axis. We plugged these into a generic potential function defined by Equation 2.

We added a command into the potential function, np.where(r < R, 0, V). This command checks if the position being plugged into the potential is within the mass. If so, it returns "not a number, which essentially avoids the potential blowing up at the position of the mass. Since we do not care about gravity inside of a stellar mass, this is a valid measure to take.

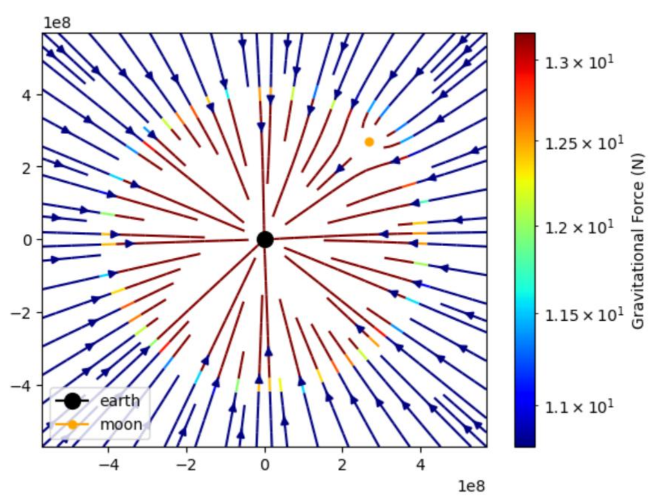
We plotted potential out to a radius of 1.5 lunar distances, the contours and color mesh plots of which are shown below.



It should be noted that the values of potential are scalars, and so we only plot a magnitude. We do not need to concern ourselves over the direction of anything. Further, it should also be noted that the slight bump in the contour and the corresponding yellow spot in the colormesh are represented by the positions of the moon.

III The Gravitational Force of the Earth Moon System

The force of the Earth Moon system, however, are vectors, in that we must plot their magnitude and direction. This is plotted according to the equation defining force as listed in the introduction, and the plot is shown below.



The legend in the plot shows the positions of the earth and moon. To get the Saturn V rocket to the moon, we must overcome the forces that pull the rocket to the earth. Again, note that an "np.where" command was used to avoid the forces blowing up at the positions of the masses of the earth and moon.

IV Projected Performance of Saturn V Stage 1

To overcome these forces, we use a rocket that propels fuel in the direction opposite its velocity, propelling it forward. The change in speed of the velocity is given by the equation,

$$\Delta v = v_e \ln\left(\frac{m_o}{m(t)}\right) - gt$$

Where $m(t) = m_o - \dot{m}t$ in kilograms, $v_e = 2.4 * 10^3 \frac{m}{s}$, and $g = 9.8\frac{m}{s^2}$ (acceleration due to gravity on the surface of the earth. For the Saturn V rocket, $m_o = 2.8 * 10^6 kg$, $\dot{m} = 1.3 * 10^4 \frac{kg}{s}$, and t is time in seconds. To get burn time T, we must equate $m(t)$ to $m_f = 7.5 * 10^5\ kg$. To find the altitude that the rocket can travel to with burning, we must find

$$h = \int_0^T \Delta v\, dt$$

Solving the above equation for burn time, we get $T = 157.69\ s$, $h = 74093.98\ m$. The distance from the surface of the earth to the surface of the moon is about 371,885 meters, meaning that the rocket must be able to burn fuel for a significantly larger amount of time before it can reach the moon. It should be noted that the command "np.where(m < mf, 0, fxn)" was used to ensure that the change in velocity becomes zero after all the fuel was expended.

V Discussion and Future Work

In experiment, the rocket was found to burn fuel for a time of 160 seconds, reaching an altitude of 70,000 meters. The likely reason for our time overestimate is likely due to our assumption of uniform burning rate. This isn't what really happens when launching a rocket but was used for computational efficiency. On top of this, the reason for an overestimate of height could be due to the assumption of uniform gravity, on top of the uniform fuel burning assumption. We also neglected drag, which is proportional to the square of the speed of the rocket at these velocities. This neglected drag force slows the rocket down considerably in practice, resulting in a lower experimental altitude.

These overestimates should be kept in mind in future tests, and consequent rocket designs should be adjusted accordingly.