RESTRİCTED 3 BODY PROBLEM

OĞUZ MÜNİB ESEN

2018405132

A) DERIVATION

We can convert the two second order ODE’s to a system of first order ODE’s with four elements.

Let where

$y_1'(t) = x_1$, $x_1(0) = 0$, $\mu = m_1$

$y_1(t) = x_2$, $x_2(0) = 0.994$, $\mu^* = m_2$

$y_2'(t) = x_3$, $x_3(0) = -2.0015851063$

$y_2(t) = 4$, $x_4(0) = 0$

Then,

$$\dot{x}_1 = x_2 + 2x_3 - m_2 \frac{(x_2 + m_1)}{D_1} - m_1 \frac{(x_2 - m_2)}{D_2}$$

$$\dot{x}_2 = x_1$$

$$\dot{x}_3 = x_4 - 2x_1 - m_2 \cdot \frac{x_4}{D_1} - m_1 \cdot \frac{x_4}{D_2}$$

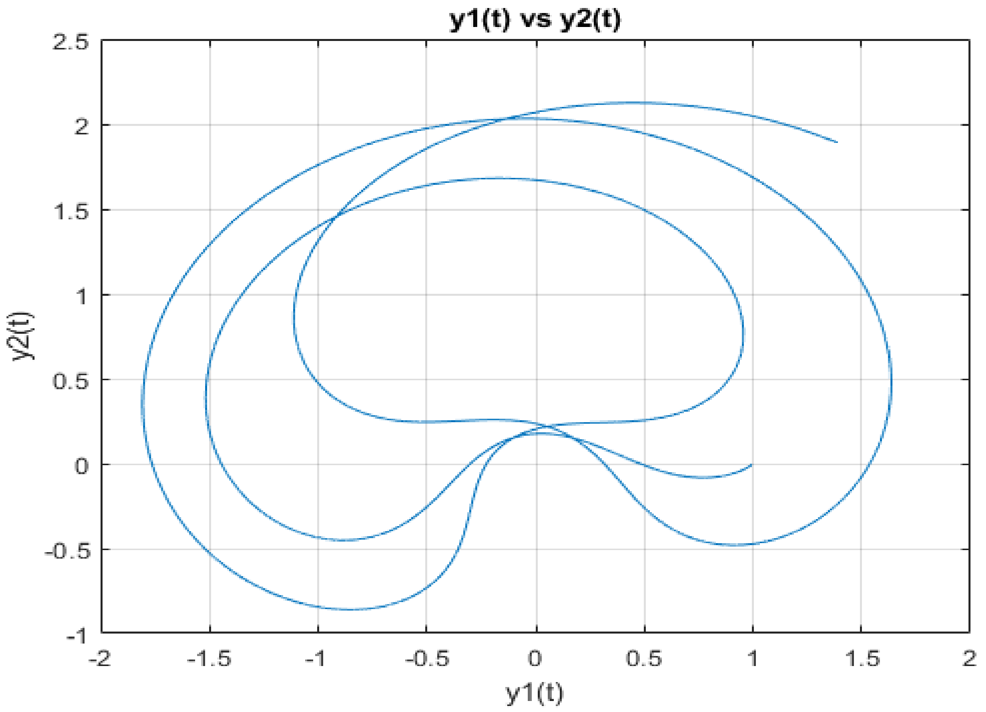
$$\dot{x}_4 = x_3$$

And

$$D_1 = \left[(x_2 + m_1)^2 + x_4^2\right]^{3/2}, \quad D_2 = \left[(x_2 - m_2)^2 + x_4^2\right]^{3/2}$$

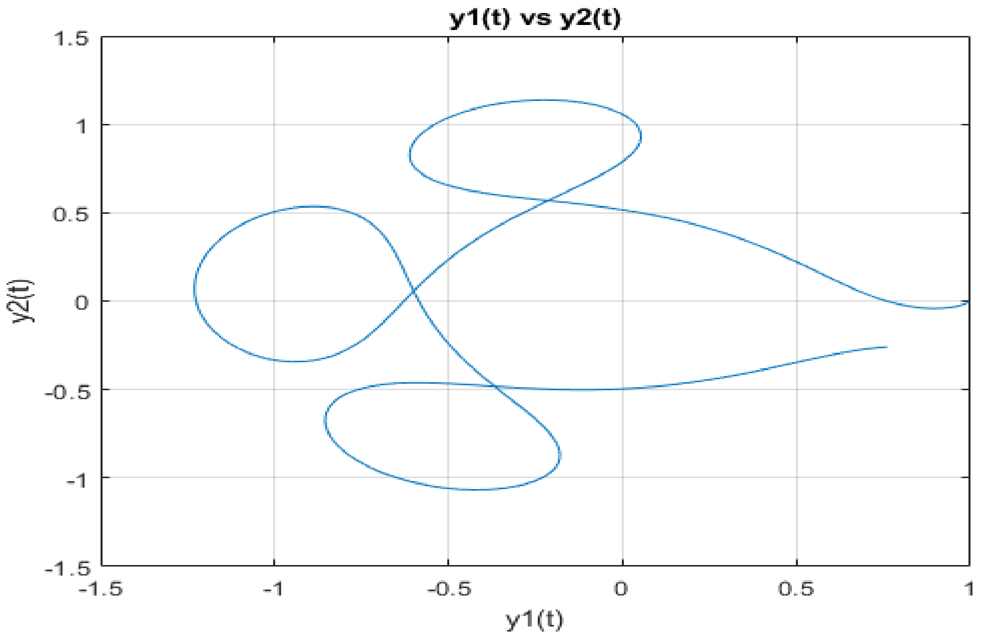
B) Euler Method

We will implement Euler's method with h=T/24000. It took 0.017624 seconds to run Euler's method.



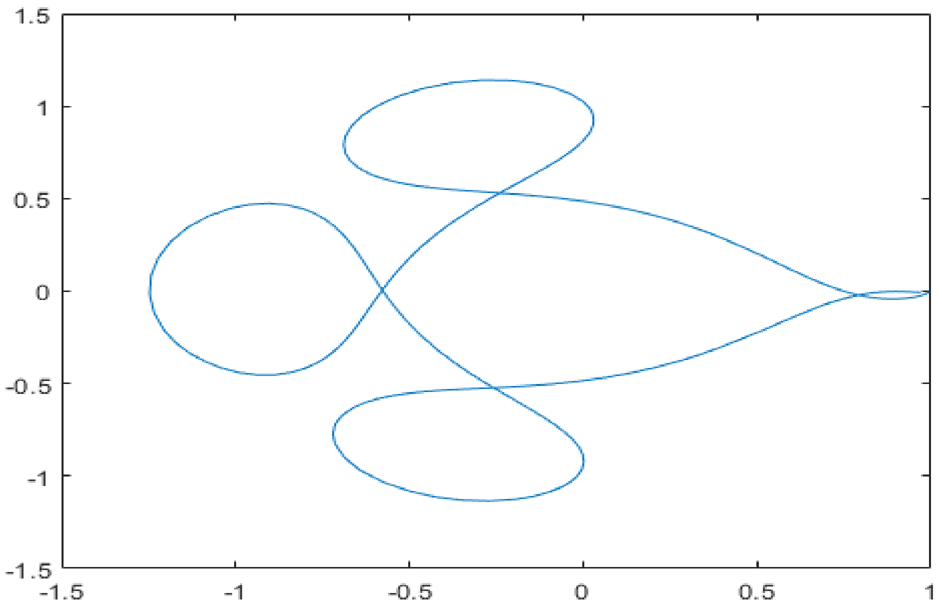
C) RK-4 Method

We will implement Range-Kutta(4) Method with h=T/6000. It took 0.041127 seconds to run rk4.



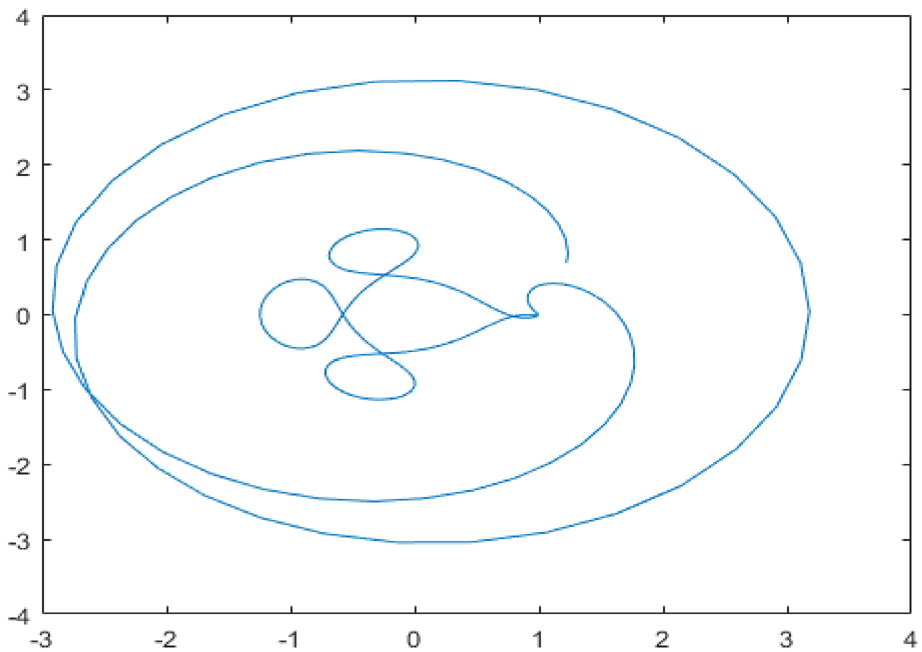
D) SOLUTION USING ODE45

Now we will use the ode45 function that is built in MATLAB. ode 45 is an efficient way of solving differential equations. It took 0.063376 seconds to run ode45.



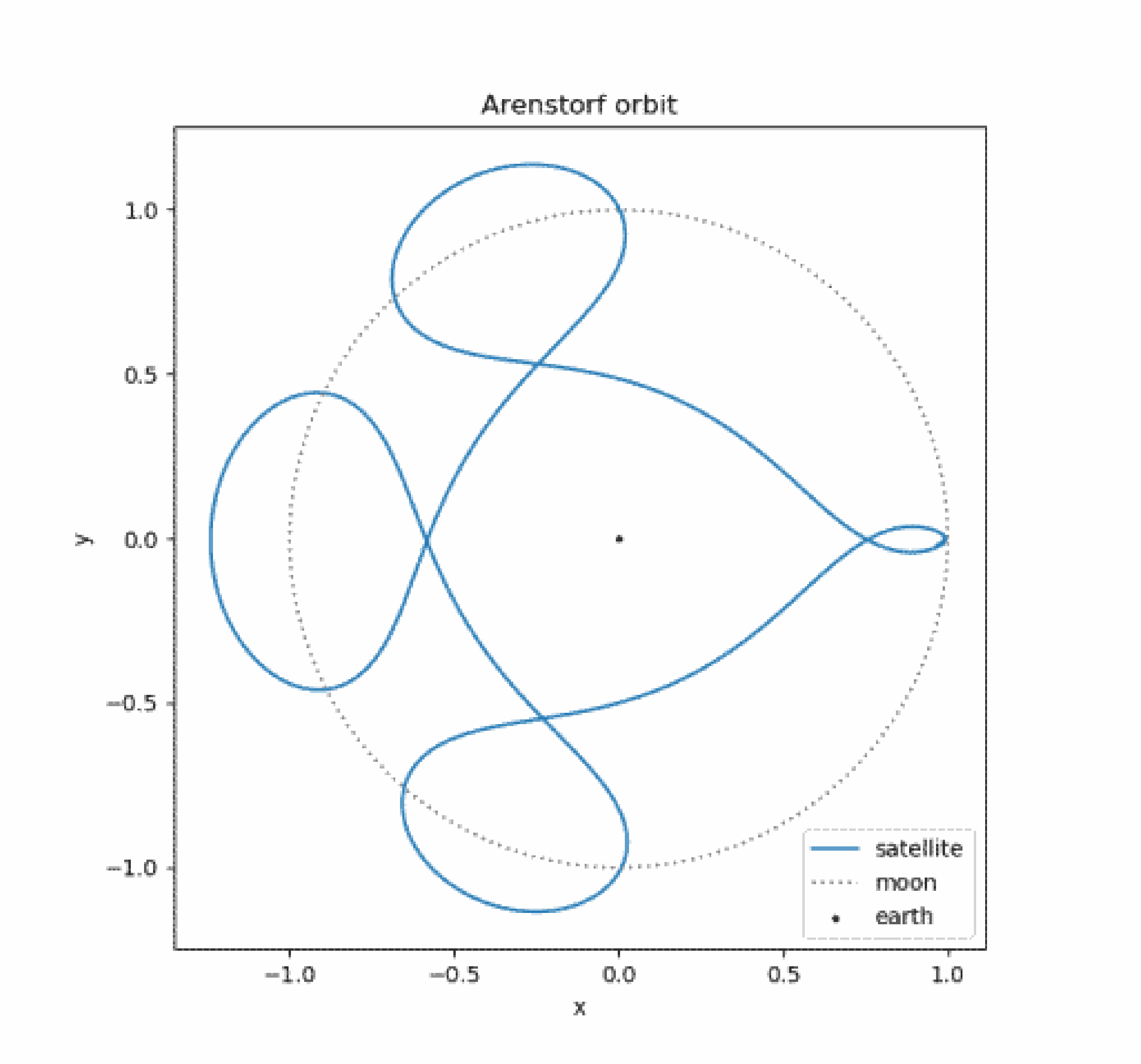
E) PROPERTIES OF TRAJECTORY

If we run the code for 2 periods (2T) using ode45, we see that the body escapes, and starts rotating in an almost circular way.



F) DISCUSSION OF PART B, C AND D

The model poses the “Arenstrorf Orbit”. We can see that Euler’s method does not come close to the answer even with the quarter step size of Range-Kutta(4). Range-Kutta(4) has a close answer but ode45 has no doubt the best answer.



Euler’s Method is much faster than the other methods. Than we have Range-Kutta(4), and ode45 is the slowest. But still, we are doing a computation that involves spacecrafts. Because they are very expensive, hard to replace and hard to relaunch, we want as much precision as we can have with speed being a secondary concern. This is why we should prefer Range-Kutta(4) to Euler’s Method and ode45 to all of the methods.