Queen Mary
University of London

Scattering Amplitudes

Samuel Overington | ID 170431121

11-03-2020

- Scattering amplitudes
- Feynman diagrams
- difference between On-shell and Off-shell
- Current methods using Gauge theories:
    - BCFW recursion / Yang Mills
    - loop level / tree level recursion

- Aim: To find and produce generators using the 'On-Shell' methods.

- A definition of Scattering amplitudes (what they do/ what they are for)

- How have they been calculated using Feynman diagrams

- Problem with current method of computation (using Feynman diagrams)

- difference between On-shell and Off-shell

- How do they relate to Gauge theory

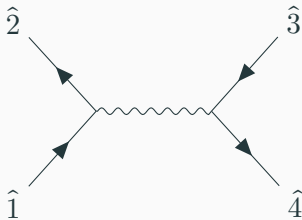- Gauge theories and spinor matrices for Gluons and massless particles

Scattering Amplitudes

- Predicting and measuring fundamental particle interactions.
- Map Energy and momentum.
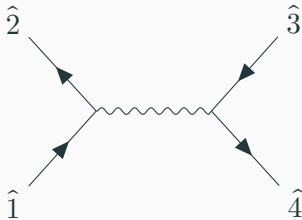- Preserve Einstein Energy momentum relation.

Einsteins Energy momentum relationship.

$$E^2 = (\vec{p}c)^2 + (mc^2)^2$$

A Feynman Diagram

Scattering Amplitudes

└─Scattering Amplitudes

In Quantum Field Theory (QFT), a scattering amplitude is the mechanism for predicting and measuring fundamental particle interactions by mapping the energy and momentum associated with each entering particle to exiting particles while preserving Einsteins energy-momentum relationship

Feynman Diagrams:

- Set boundary conditions.
- Energy Momentum Conservation law.
- Virtual Particles (Off Shell)

Feynman Diagrams:
- Set boundary conditions.
- Energy Momentum Conservation law.
- Virtual Particles (Off Shell)

Traditionally, they have been computed using Feynman diagrams, which set the boundary conditions to the summed values of all entering particles and those exiting within the same interaction, such that they conserve this E-M relation between the entering and exiting particles at the macro level.

The Feynman method makes use of virtual particles, so called because they are gauge variant or Off-Shell (or off the mass shell) to keep tally of the energy and momenta of all the paths the particles and interactions make, so that an individual interaction may temporarily break the energy momentum relations creating such a virtual particle with negative mass, or energy @Hodges:2013aa, @Plefka:2014aa. Although it allows for broken symmetries, each virtual particle must cancel out by the final exiting interaction, ensuring that the system as a whole is conservative and invariant and we are only left with On-Shell particles at the boundaries.

$$E^2 = (\vec{p}c)^2 + (mc^2)^2 \quad ,\text{"Shell" radius} \quad = \sqrt{mE}$$
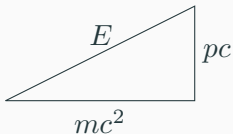

Figure 1: Einstein Energy momentum relationship

2020-03-10

$E^2 = (\vec{p}c)^2 + (mc^2)^2$, "Shell" radius $= \sqrt{mE}$

**Figure 5:** Einstein Energy momentum relationship

To explain more about on and off shell: Particles are said to be "on the mass shell," or simply "on-shell," if their behaviour satisfies the relationship between energy and momentum given by Einstein in his theory of special relativity. Virtual particles are those that don't satisfy this relationship, and are said to be "off-shell.". The diagram is to illustrate the relationship. In momentum space, this is precisely the equation of a spherical "shell" with radius $\sqrt{mE}$

- Massless particles produce parabolic surface
- Massive particles produce a cone shape
- Real particles momentum vectors along "Shell surface"
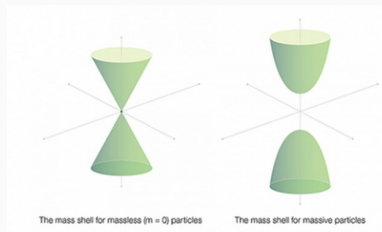
(a) Parabolic surface for massive particles
Conic surface for massless particles
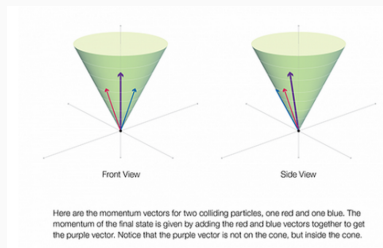
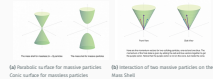(b) Interaction of two massive particles on the
Mass Shell

Figure 2: Source: Perimeter Institute

If you graph it, you obtain a parabolic surface for massive particles, and a cone for massless particles, like a photon.

This is known as the mass shell, it is quite literally a shell or surface. The momentum of a real particle can be represented as a vector on the surface, hence the expression on shell. Virtual particles do not have these on the surface, hence they are off shell. The point is that real particles have momentum vectors that are on the shell – not inside it, but on it. So when two particles (or any other objects) collide, their total momentum has to be preserved. To calculate the momentum in the final state, the momentum vectors from the initial state should be added together. (with natural units this is $E^2 = p^2 + m^2$)

When two massless on-shell particles interact and produce a third particle, The momentum vector of the final particle won't lie along the surface of the cones.

picture text: Momentum vectors for two colliding particles, (red blue). Momentum in final state given by adding v1 + v2 = v purple.
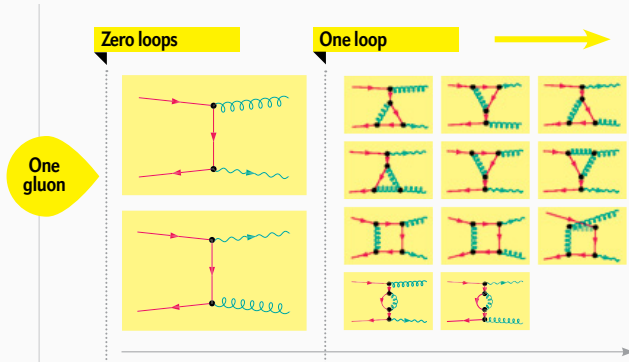
Figure 3: Source: Scientific American

Problem with current method of computation (using
Feynman diagrams)
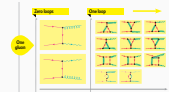
Each Feynman diagram provides an intuitive way to visualize one possible way that
particles might interact. The trouble is that there are countless other ways, too. A
quark-quark interaction might produce more than one gluon or involve more than
one virtual-particle loop, or both. The calculations quickly become unmanageable.

Feynman diagrams are classified by the number of external lines and the number
of closed loops they have. Loops represent one of the quintessential features of
quantum theory: virtual particles.

Though not directly observable, virtual particles have a measurable effect on the
strength of forces. They obey all the usual laws of nature, such as the conservation
of energy and of momentum, with one caveat: their mass can differ from that of
the corresponding "real" (that is, directly observed) particles. Loops represent their
ephemeral life cycle: they pop into existence, move a short distance, then vanish
again. Their mass determines their life expectancy: the heavier they are, the shorter
they live

So here is an feynman diagram with 2 particles, and the complexity of loops within

Feynman diagrams help visualise one possible way particles interact.
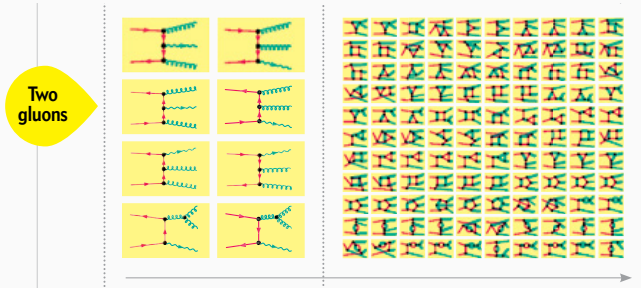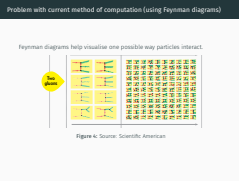

Figure 4: Source: Scientific American

Figure 4: Source: Scientific American

└─Problem with current method of computation (using Feynman diagrams)

At very short distances and for very simple collisions, including the distances relevant for collisions at the LHC, we can get away with considering only uncomplicated Feynman diagrams
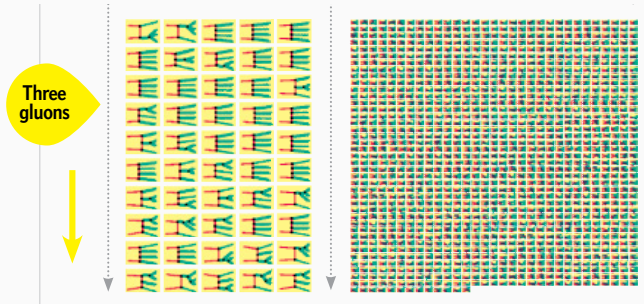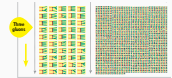
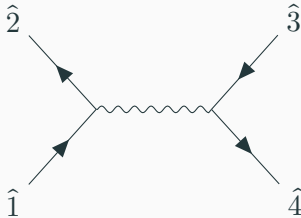**Figure 5:** Source: Scientific American

Figure 5: Source: Scientific American

For messy collisions, though, the full complexity of the Feynman technique comes rushing in.

Britto-Cachazo-Feng-Witten (BCFW) approach:

- Allows us to calculate with ease any tree level amplitude.
- two gluon into n = 8 gluon, 10,525,9000 contributing Feynman diagrams.
- Particles have complex momentum.

└─Gauge theories and Spinor matrices for Gluons and
    massless particles were

Feynman rules allow one to write down any tree-level amplitude as a sum of all Feynman diagrams contributing. In practice, this straightforward approach can quickly become cumbersome, as the number of Feynman diagrams typically grows factorially with the number of external legs.

To give an example: for the scattering of two gluons into n gluons the number of contributing Feynman diagrams grows from 4 (n = 2), to 220 (n = 4), to 34,300 (n = 6) to an enormous 10,525,900 for n = 8 [1]
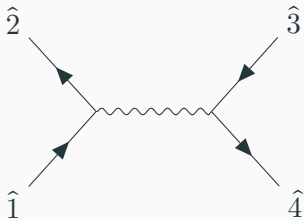
$$\lambda_1 \to \hat{\lambda}_1(z) = \lambda_1 - z\lambda_n$$
$$\tilde{\lambda}_1 \to \hat{\tilde{\lambda}}_1(z) = \tilde{\lambda}_1 - z\tilde{\lambda}_n$$

└─ BCFW

The new approach from Cachazo et al is different. Two real on-shell particles come enter. Each one produces two more on-shell particles – but these particles have complex momentum, meaning that the number representing their momentum comprises both real and imaginary numbers.

If drawn on a Feynman diagram, the complex state would be coming out of the plane because we are thinking of the plane as being a real space, not a complex space. Below, we see two on-shell particles coming in and producing two complex on-shell particles each, which then merge and produce two more on- shell particles with real momentum.

The key to the BCFW recursion is to study how the amplitude behaves under a complex deformation of the particle momenta preserving the on-shell conditions. For this we perform the following complex shift of the helicity spinors for two neighboring legs 1 and n