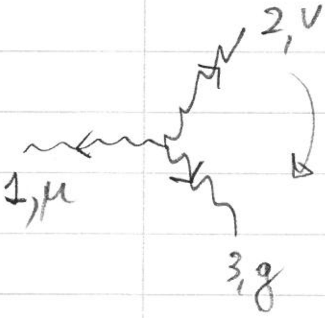
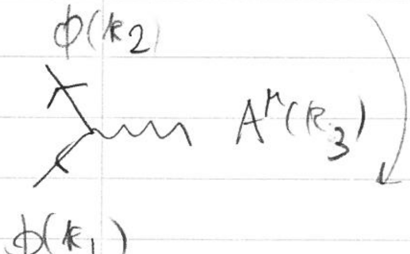
FEYNMAN RULES AND

ALL TREES WE NEED!

COLOUR-ORDERED FEYNMAN RULES



$$= i\frac{g}{\sqrt{2}}\left[(k_1-k_2)_\rho\,\eta_{\mu\nu}+(k_2-k_3)_\mu\,\eta_{\rho\nu}+(k_3-k_1)_\nu\,\eta_{\mu\rho}\right]$$



$$= +\frac{ig}{\sqrt{2}}\left(k_2^\mu-k_1^\mu\right) = -\frac{ig}{\sqrt{2}}\left(k_1-k_2\right)^\mu$$

From p. 11 of Dixon's report (for the first one).
and p. 2 of Bern De Freitas Wong 9912033

POLARISATIONS

$$\varepsilon^{(+)}(k) = \frac{\langle \xi|\mu|k]}{\sqrt{2}\,\langle \xi k\rangle}$$

$$\varepsilon^{(-)}(k) = -\frac{[\xi|\mu|k\rangle}{\sqrt{2}\,[\xi k]} = +\frac{\langle k|\mu|\xi]}{\sqrt{2}\,[k\xi]}$$

DERIVING GLUON 3-PT AMPLITUDES FROM $\underbrace{\text{FEYNMAN}}_{\text{COLOUR ORDERED}}$ RULES

$$\varepsilon^{(\pm)\mu}(k) = \pm \frac{\langle \xi^{\mp} | \gamma^\mu | k^{\mp} \rangle}{\sqrt{2} \langle \xi^{\mp} | k^{\pm} \rangle}$$

or

$$\boxed{\begin{aligned} \varepsilon^{(+)\mu}(k) &= + \frac{\langle \xi | \mu | k]}{\sqrt{2} \langle \xi k \rangle} \\ \varepsilon^{(-)\mu}(k) &= - \frac{[\xi | \mu | k\rangle}{\sqrt{2} [\xi k]} \end{aligned}}$$

with $\sigma^\mu_{\alpha\dot\alpha} \sigma_{\mu\beta\dot\beta} = 2 \varepsilon_{\alpha\beta} \varepsilon_{\dot\alpha\dot\beta}$.

$\Big($from $\sigma^\mu_{\alpha\dot\alpha} \bar\sigma_\mu^{\dot\beta\beta} = \delta_{\alpha\dot\alpha} \delta^{\dot\beta\beta} - (2 \delta_\alpha^\beta \delta_{\dot\alpha}^{\dot\beta} - \delta_{\alpha\dot\alpha} \delta^{\dot\beta\beta}) = 2 \delta_\alpha^\beta \delta_{\dot\alpha}^{\dot\beta}$

$\Rightarrow \sigma^\mu_{\alpha\dot\alpha} \sigma_{\mu\gamma\dot\gamma} = \sigma^\mu_{\alpha\dot\alpha} \bar\sigma_\mu^{\dot\beta\beta} \varepsilon_{\dot\gamma\dot\beta} \varepsilon_{\gamma\beta} = \varepsilon_{\dot\gamma\dot\beta} \varepsilon_{\gamma\beta} \cdot 2 \delta_\alpha^\beta \delta_{\dot\alpha}^{\dot\beta} =$

$= 2 \varepsilon_{\gamma\alpha} \varepsilon_{\dot\gamma\dot\alpha} \Big)$

with the overall sign changed to (+) to agree with Zh.

Use now the 3-pt vertex to get

$$\boxed{\begin{aligned} A^{\text{tree}}_{\substack{\text{COLOUR}\\\text{ORDERED}}}(123) = \frac{i g}{\sqrt{2}} \Big[&(k_1 - k_2) \cdot \varepsilon^{(3)} (\varepsilon^{(1)} \cdot \varepsilon^{(2)}) + \\ &+ (k_2 - k_3) \cdot \varepsilon^{(1)} (\varepsilon^{(2)} \cdot \varepsilon^{(3)}) + (k_3 - k_1) \cdot \varepsilon^{(2)} (\varepsilon^{(3)} \cdot \varepsilon^{(1)}) \Big] \end{aligned}}$$

agrees with (2,12) Zh's review.

Note: no f^{abc} on RHS since this is colour-ordered.

- For MHV, $1^- 2^- 3^+$; all terms with $\varepsilon^{(1)} \cdot \varepsilon^{(2)}$ can be made to vanish by choosing $\xi_2 = \xi_1$.

Further choose $\xi_3 = k_1 \Rightarrow$ we get

$$(\varepsilon^{(3)} \cdot \varepsilon^{(1)}) \sim \langle \xi_3 | \mu | k_3] \, [\xi_1 | \mu | k_1 \rangle = \langle k_1 | \mu | k_3] [\xi_1 | \mu | k_1 \rangle$$

$= 0$. Then

$$k_2 \cdot \varepsilon^{(1)} = k_2 \cdot (-) \frac{[\xi_1 | \mu | k_1 \rangle}{\sqrt{2} [\xi_1 k_1]} = -\frac{1}{\sqrt{2}} \frac{[\xi_1 2] \langle 21 \rangle}{[\xi_1 1]}$$

$$k_3 \cdot \varepsilon^{(1)} = (-k_1 - k_2) \cdot \varepsilon^{(1)} = -k_2 \cdot \varepsilon^{(1)} = \text{above} \Rightarrow$$

$$\boxed{(k_2 - k_3) \cdot \varepsilon^{(1)} = -\sqrt{2} \frac{[\xi_1 2] \langle 21 \rangle}{[\xi_1 1]}}$$ and

$$\varepsilon^{(2)} \cdot \varepsilon^{(3)} = -\frac{1}{2} \frac{[\xi_2 | \mu | 2 \rangle \langle \xi_3 | \mu | k_3]}{[\xi_2 2] \langle \xi_3 3 \rangle}$$

Use now $[a|\mu|b\rangle [c|\mu|d\rangle = 2 [ac] \langle db \rangle \Rightarrow$

$$\boxed{\varepsilon^{(2)} \cdot \varepsilon^{(3)} = -\frac{[\xi_2 3] \langle \xi_3 2 \rangle}{[\xi_2 2] \langle \xi_3 3 \rangle} = -\frac{[\xi_2 3] \langle 12 \rangle}{[\xi_2 2] \langle 13 \rangle}} \Rightarrow$$

$$A(1^- 2^- 3^+) = \frac{ig}{\sqrt{2}} (-\sqrt{2})(-) \frac{[\xi_1 2] \langle 21 \rangle}{[\xi_1 1]} \frac{[\xi_2 3] \langle 12 \rangle}{[\xi_2 2] \langle 13 \rangle}$$

- choose $\xi_1 = \xi_2 \Rightarrow \; = i g f^{abc} \cdot \frac{\langle 21 \rangle \cdot \langle 12 \rangle}{\langle 13 \rangle} \frac{[\xi 3]}{[\xi 1]}$

Next write $\frac{[\xi 3]}{[\xi 1]} = \frac{[\xi 3] \langle 32 \rangle}{[\xi 1] \langle 32 \rangle} = -\frac{[\xi 1] \langle 12 \rangle}{[\xi 1] \langle 32 \rangle} = \frac{\langle 12 \rangle}{\langle 23 \rangle} \Rightarrow$

$$A(1^-_a 2^-_b 3^+_c) = ig \frac{(-)\langle 12\rangle^2 \langle 12\rangle}{\langle 13\rangle\langle 23\rangle} \quad \text{or}$$

$$\boxed{A(1^-_a 2^-_b 3^+_c) = ig \frac{\langle 12\rangle^3}{\langle 23\rangle\langle 31\rangle}}$$

For $\overline{\text{MHV}}$ $1^+2^+3^-$:

Choose $\xi_1 = \xi_2$ and $\xi_3 = k_1$ again which makes $\varepsilon^{(3)}\cdot\varepsilon^{(1)} = 0$

$\Rightarrow$ Need

$$\varepsilon^{(2)}\cdot\varepsilon^{(3)} = -\frac{1}{(\sqrt{2})^2}\frac{\langle\xi_2|\mu|2]}{\langle\xi_2 2\rangle}\frac{[\xi_3|\mu|3\rangle}{[\xi_3 3]} = -\frac{2}{2}\frac{\langle\xi_2 3\rangle[\xi_3 2]}{\langle\xi_2 2\rangle[\xi_3 3]}$$

And $(k_2 - k_3)\cdot\varepsilon^{(1)} = 2k_2\cdot\varepsilon^{(1)} + 0 =$

$$= 2\cdot\frac{1}{\sqrt{2}}\frac{\langle\xi_1|2|1]}{\langle\xi_1 1\rangle} = \sqrt{2}\frac{\langle\xi_1|2|1]}{\langle\xi_1 1\rangle} \Rightarrow$$

$$A^h(1^+_a 2^+_b 3^-_c) = \frac{igf^{abc}}{\sqrt{2}}\cdot(-)\sqrt{2}\frac{\langle\xi_2 3\rangle[\xi_3 2]}{\langle\xi_2 2\rangle[\xi_3 3]}\frac{\langle\xi_1 2\rangle[21]}{\langle\xi_1 1\rangle}$$

Using $\xi_1 = \xi_2$ and $\xi_3 = k_1$ $\Rightarrow$

$$A(1^+_a 2^+_b 3^-_c) = -ig\frac{\langle\xi_2 3\rangle[12]}{\cancel{\langle\xi_2 2\rangle}[13]}\frac{\cancel{\langle\xi_2 2\rangle}[21]}{\langle\xi_2 1\rangle}$$

and

$$\frac{\langle\xi_2 3\rangle}{\langle\xi_2 1\rangle} = \frac{\langle\xi_2 3\rangle[32]}{\langle\xi_2 1\rangle[32]} = -\frac{\cancel{\langle\xi_2 1\rangle}[12]}{\cancel{\langle\xi_2 1\rangle}[32]} = +\frac{[12]}{[23]}$$

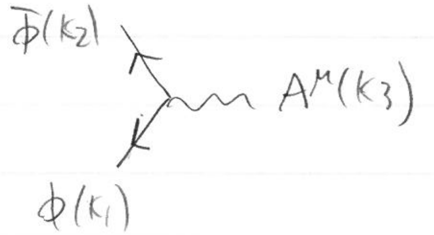
$$A(1^+_a 2^+_b 3^-_c) = -ig \cdot \frac{[12][21]}{[13]} \cdot \frac{[12]}{[23]} \Rightarrow$$

$$\boxed{A(1^+_a 2^+_b 3^-_c) = -ig \frac{[12]^3}{[12][23][31]}}$$

In agreement with Bern et al.

SCALAR – SCALAR – GLUON AMPLITUDES FROM COLOUR-ORDERED FEYNMAN RULES

Use the colour-ordered Feynman rules of Zeng – de Freitas – Wong.



$$= -\frac{ig}{\sqrt{2}}(k_1-k_2)^\mu$$

Then use $\varepsilon^{(+)}(k_3) = \frac{\langle \xi|\mu|3]}{\sqrt{2}\langle \xi 3\rangle}$, $\varepsilon^{(-)}(k_3) = -\frac{[\xi|\mu|3\rangle}{\sqrt{2}[\xi 3]}$ $\Rightarrow$

$$A(1_\phi 2_{\bar\phi} 3_{g^+}) = -\frac{ig}{\sqrt{2}}\frac{\langle \xi|1-2|3]}{\sqrt{2}\langle \xi 3\rangle} = \frac{2ig}{\sqrt{2}\sqrt{2}}\frac{\langle \xi|2|3]}{\langle \xi 3\rangle}$$

$$A(1_\phi 2_{\bar\phi} 3_{g^-}) = -\frac{ig}{\sqrt{2}}(-)\frac{[\xi|1-2|3\rangle}{[\xi 3]} = \frac{-2ig}{\sqrt{2}\sqrt{2}}\frac{[\xi|2|3\rangle}{[\xi 3]}$$

or

$$\boxed{\begin{aligned} A(1_\phi 2_{\bar\phi} 3_{g^+}) &= ig\frac{\langle \xi|2|3]}{\langle \xi 3\rangle} \\ A(1_\phi 2_{\bar\phi} 3_{g^-}) &= ig\frac{\langle 3|2|\xi]}{[3\xi]} \end{aligned}}$$

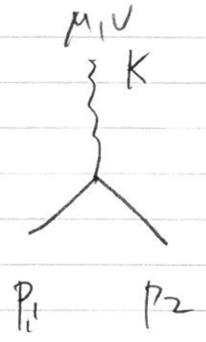
$1+2+3=0$

We will also use

$$\boxed{\begin{aligned} A(1_\phi 2_{\bar\phi} 3_{h^{++}}) &= \left[A(1_\phi 2_{\bar\phi} 3_{g^+})\right]^2 \\ A(1_\phi 2_{\bar\phi} 3_{h^{--}}) &= \left[A(1_\phi 2_{\bar\phi} 3_{g^-})\right]^2 \end{aligned}}$$

Note resulting MINUS SIGNS from $i^2=-1$!

THREE-POINT $h^{\pm} S \bar{S}$ AMPLITUDE.



$$= -i\varkappa \left[\frac{1}{2}\eta^{\mu\nu}\left(p_1\cdot p_2 - m_\varphi^2\right) - \frac{1}{2}\left(p_1^\mu p_2^\nu + p_2^\mu p_1^\nu\right)\right]$$

from Theodor's Schuster thesis

Use the polarisation vectors for gluons

$$\varepsilon^{(+)}(k) = \frac{\langle \xi|\mu|k]}{\sqrt{2}\langle \xi k\rangle} \qquad \varepsilon^{(-)}(k) = \frac{\langle k|\mu|\xi]}{\sqrt{2}[k\xi]} \quad \Rightarrow$$

For $\varepsilon^{(+)} \otimes \varepsilon^{(+)} = \varepsilon^{(++)}_{\mu\nu}$: the $\eta^{\mu\nu}$ gives 0 and we get

$$(-i\varkappa)\left(-\frac{1}{2}\right)\cdot 2\left(\frac{1}{\sqrt{2}}\right)^2 \frac{\langle \xi|p_1|k]}{\langle \xi k\rangle}\frac{\langle \xi|p_2|k]}{\langle \xi k\rangle} =$$

$\Rightarrow$ using $p_1+p_2+k=0$ $\Rightarrow$

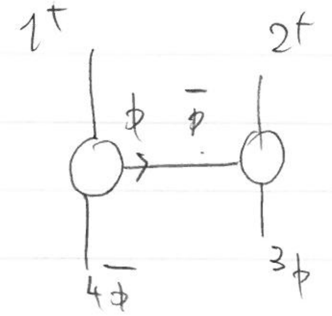
$$\boxed{A\left(K^{++}\, p_1^{\phi}\, p_2^{\bar\phi}\right) = -\frac{i\varkappa}{2}\left[\frac{\langle \xi|p_2|k]}{\langle \xi k\rangle}\right]^2}$$

For $-$ helicity: $\varepsilon^{(-)}(k) = \dfrac{\langle k|\mu|\xi]}{\sqrt{2}[k\xi]}$ and we get

$$\boxed{A\left(K^{--}\, p_1^{\phi}\, p_2^{\bar\phi}\right) = -\frac{i\varkappa}{2}\left[\frac{\langle k|p_2|\xi]}{[k\xi]}\right]^2}$$

These results are $(\text{ours}) \times \left[-\frac{i\varkappa}{2}\right]$

$$\boxed{A_{YM}(1^+ 2^+ 3_\phi 4_{\bar\phi})}$$



u $\quad \hat\lambda_1 = \lambda_1 + z\lambda_2$, $\quad \hat{\tilde\lambda}_2 = \tilde\lambda_2 - z\tilde\lambda_1$ $\quad \boxed{\begin{matrix}\langle 12] \\ \text{SHIFT}\end{matrix}}$

Use $\quad A(1^+ 2_\phi 3_{\bar\phi}) = i\,\dfrac{\langle \xi|3|1]}{\langle \xi 1\rangle}$

Then get $\qquad + \hat P = 2+3$

$$\text{Diagram} = i\,\frac{\langle q_1|4|1]}{\langle q_1 1\rangle}\cdot\frac{i}{(p_1+p_4)^2-\mu^2}\cdot i\,\frac{\langle q_2|-\hat P|\hat 2]}{\langle q_2 2\rangle} =$$

$$= -i^2\,\frac{\langle q_1|4|1]\,\langle q_2|3|\hat 2]}{\langle 21\rangle\langle 12\rangle}\;\frac{i}{(p_1+p_4)^2-\mu^2}$$

Choose $\quad q_1 = 2 \quad q_2 = \hat 1 \quad \Rightarrow$

$$= +\underbrace{\langle 2|4|1]\langle \hat 1|3|\hat 2]}\;\frac{i}{(p_1+p_4)^2-\mu^2}\;\frac{1}{\langle 21\rangle\langle 12\rangle}$$

$$\downarrow$$

$$\mathrm{Tr}(\hat 2 4 \hat 1 3) = -\mathrm{Tr}(\hat 2 3 \hat 1 3) = -2\Big[\,2\overbrace{(\hat 2 3)}^{=0}(\hat 1 3) - (\hat 2\hat 1)3^2\Big]$$

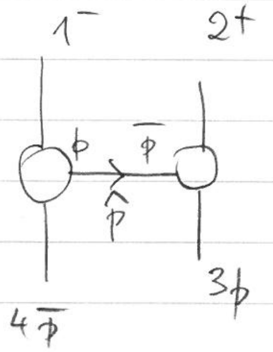
$$= +2(\hat 1\hat 2)3^2 = +\langle 21\rangle[12]\cdot\mu^2 \quad \Rightarrow$$

$$\text{Diagram} = \mu^2\,\frac{\cancel{\langle 21\rangle}[12]}{\cancel{\langle 21\rangle}\langle 12\rangle}\;\frac{i}{(p_1+p_4)^2-\mu^2} \quad \Rightarrow$$

$$\boxed{A(1^+ 2^+ 3_\phi 4_{\bar\phi}) = \mu^2\,\frac{[12]}{\langle 12\rangle}\;\frac{i}{(p_1+p_4)^2-\mu^2}}$$

in agreement with Bern and Morgan (3.3) of 9511336

$$\boxed{A_{YM}(1^- 2^+ 3_\phi 4_{\bar\phi})}$$



$\boxed{\text{Shift } \langle 21]}$

as usual for good large-z behavior

$$\hat\lambda_2 = \lambda_2 + z\lambda_1$$
$$\hat{\tilde\lambda}_1 = \tilde\lambda_1 - z\tilde\lambda_2$$

$$A(\hat 1^-, \hat P_\phi, 4_{\bar\phi}) = i\,\frac{\langle 1|4|q_1]}{[1 q_1]}$$

$$A(\hat 2^+, 3_\phi, -\hat P_{\bar\phi}) = i\,\frac{\langle q_2|-\hat P|2]}{\langle q_2 2\rangle}$$

$$\text{Diagram} = i\,\frac{\langle 1|4|q_1]}{[1 q_1]}\,\frac{i}{(p_1+p_4)^2-\mu^2}\; i\,\frac{\langle q_2|-\hat P|2]}{\langle q_2 2\rangle} \quad \text{with } \hat P = \hat 2 + 3$$

Choose $q_2 = 1$, $q_1 = 2$ and

$$i^2 \langle 1|4|2]\langle 1|-\hat P|2] = (-)(-)\langle 1|4|2]\langle 1|3|2] =$$

$$= -\langle 1|4|2]^2 \quad \Rightarrow$$

$$\text{Diagram} = -\frac{\langle 1|4|2]^2}{[12]\langle 12\rangle}\,\frac{i}{(p_1+p_4)^2-\mu^2} \quad \Rightarrow$$

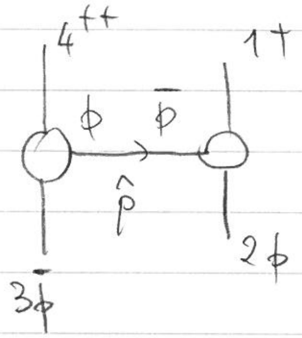
$$\boxed{A(1^- 2^+ 3_\phi 4_{\bar\phi}) = +\frac{\langle 1|4|2]^2}{s_{12}}\,\frac{i}{(p_1+p_4)^2-\mu^2}}$$

$$\boxed{4^{++}\, 1^{+}\, 2_{\phi}\, 3_{\bar\phi}}$$

$\hat\lambda_4 = \lambda_4 + z\lambda_1$ $\qquad\langle 4 1]$ shift.

$\hat{\tilde\lambda}_1 = \tilde\lambda_1 - z\tilde\lambda_4$

DIAGRAM 1



$$A_1 = -\frac{\langle q_1|3|4]^2}{\langle q_1 \hat 4\rangle^2}\,\frac{i}{(p_3+p_4)^2-\mu^2}\; i\,\frac{\langle q_2|-\hat P|\hat 1]}{\langle q_2 1\rangle}$$

Choose $q_2 = \hat 4 \quad q_1 = 1 \qquad [\hat P = \hat 1 + 2]$

$$\langle q_1|3|4]^2\langle q_2|-\hat P|\hat 1] = \langle 1|3|4]^2\langle \hat 4|-\hat P|\hat 1] =$$

$$= -\langle 1|3|4]^2\langle \hat 4|2|\hat 1] = -\langle 1|3|4]\,\mathrm{Tr}(\hat 1 3 \hat 4 2)$$

Use $\mathrm{Tr}(\hat 1 3 \hat 4 2) = -\mathrm{Tr}(\hat 1 2 \hat 4 2) = -2\left[2(\hat 1 2)(\hat 4 2) - (\hat 1 \hat 4)\, 2^2\right]$

But $2(\hat 1 2) = (\hat 1 + 2)^2 - 2^2 = \mu^2 - \mu^2 = 0 \quad \Rightarrow$

$\mathrm{Tr}(\hat 1 3 \hat 4 2) = +\mu^2 \cdot 2(\hat p_1 \hat p_4) = +\mu^2 \cdot S_{14}$

$\Rightarrow \langle q_1|3|4]^2\langle q_2|-\hat P|\hat 1] = -\mu^2 S_{14}\langle 1|3|4] \Rightarrow$

$$A_1 = (-)^2\, i\mu^2 S_{14}\,\frac{\langle 1|3|4]}{\langle 14\rangle^2\langle 41\rangle} = \left(\text{use } \frac{S_{14}}{\langle 14\rangle^2\langle 41\rangle} = -\frac{[41]}{\langle 14\rangle^2}\right)$$

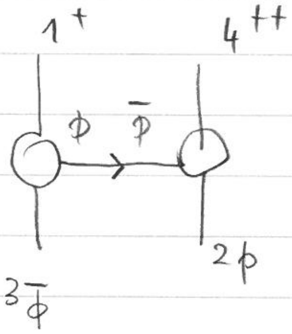
$$\boxed{\text{Diagram 1} = -i\mu^2\,\frac{[41]}{\langle 41\rangle^2}\,\langle 1|3|4]\,\frac{i}{(p_3+p_4)^2-\mu^2}}$$

NOW ADD NEW DIAGRAM

NEW DIAGRAM

still $\langle 41]$!

DIAGRAM 2



$$A_L = i\frac{\langle q_2|3|\hat{1}]}{\langle q_2 1\rangle}$$

$$A_R = -\frac{\langle q_1|-\hat{P}|4]^2}{\langle q_1\hat{4}\rangle^2}$$

$q_2 = \hat{4} \qquad q_1 = 1$

$$\langle q_1|-\hat{P}|4]^2\langle q_2|3|\hat{1}] = \langle 1|2|4]^2\langle\hat{4}|3|\hat{1}] =$$

$$= \langle 1|3|4]^2\langle\hat{4}|3|\hat{1}] = \langle 1|3|4]\,\mathrm{Tr}(\hat{1}3\hat{4}3) =$$

$$= \langle 1|3|4]\cdot 2\left[2\underbrace{(\hat{1}3)}_{\emptyset}(\hat{4}3) - \underbrace{3^2}_{\mu^2}(\hat{1}\hat{4})\right] =$$

$$= -\mu^2\langle 1|3|4]\cdot 2(p_1 p_4) = -\mu^2 S_{14}\langle 1|3|4]$$

$$\text{Diagram2} = \cancel{-}i\mu^2 S_{14}\,(\cancel{-})\frac{\langle 1|3|4]}{\langle 41\rangle\langle 14\rangle^2}\;\frac{i}{(p_2+p_4)^2-\mu^2}$$

$$\frac{S_{14}}{\langle 41\rangle\langle 14\rangle^2} = \frac{\cancel{\langle 14\rangle}[41]}{\langle 41\rangle\langle 14\rangle^{\cancel{2}}} = -\frac{[41]}{\langle 41\rangle^2} \Rightarrow$$

$$\text{Diagram2} = -i\mu^2\frac{[41]}{\langle 41\rangle^2}\langle 1|3|4]\;\frac{i}{(p_2+p_4)^2-\mu^2}$$

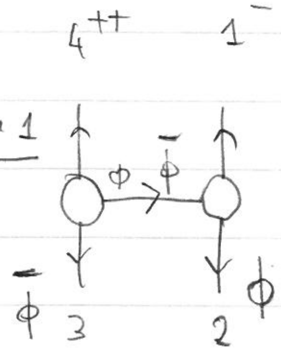
$$A_{EYM}(4^{++}1^+2_\phi 3_{\bar\phi}) = \text{Diag 1} + \text{Diag 2} \Rightarrow$$

$$\boxed{A_{EYM}(4^{++}1^+2_\phi 3_{\bar\phi}) = -i\mu^2\frac{[41]}{\langle 41\rangle^2}\langle 1|3|4]\left[\frac{i}{(p_3+p_4)^2-\mu^2}+\frac{i}{(p_2+p_4)^2-\mu^2}\right]}$$

$$\boxed{A_{EYM}(4^{++}1^{-}2_\phi 3_{\bar\phi})}$$

$$\boxed{4^{++}\,1^{-}\,2_\phi\,3_{\bar\phi}} \qquad \begin{aligned}\hat\lambda_4 &= \lambda_4 + z\lambda_1\\ \hat{\tilde\lambda}_1 &= \tilde\lambda_1 - z\tilde\lambda_4\end{aligned} \qquad \boxed{\langle 41]\text{ shift}}$$

DIAGRAM 1



$$\hat P = 1+2$$

$$A_1 = -\frac{\langle q_1|3|4]^2}{\langle q_1\hat 4\rangle^2}\,\frac{i}{(p_3+p_4)^2-\mu^2}\; i\,\frac{\langle 1|-\hat P|q_2]}{[\hat 1\, q_2]}$$

Choose $\quad q_2 = \hat 4 \qquad q_1 = 1 \qquad [\, p_2^2 = p_3^2 = \mu^2 \,].$

$$\langle q_1|3|4]^2\langle 1|-\hat P|q_2] = -\langle 1|3|4]^2\langle 1|2|4] = \; - \langle 1|2|4]^3$$

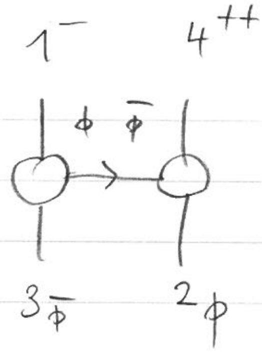
$$\langle q_1 4\rangle = \langle 14\rangle \qquad [\hat 1\, \hat q_2] = [14]$$

$$A_1 = (-)i^2\,\frac{\langle 1|2|4]^3}{\langle 14\rangle^2}\,\frac{1}{[14]}\,\frac{i}{(p_3+p_4)^2-\mu^2} \qquad \text{Use } \langle 14\rangle[14] = -s_{14}$$

$$\boxed{A_1 = -i\,\frac{\langle 1|2|4]^3}{s_{14}\langle 14\rangle}\,\frac{i}{(p_3+p_4)^2-\mu^2}}$$

<u>Next diagram 2</u> $\longrightarrow$

DIAGRAM 2 =



$$A_2 = -\frac{\langle q_1|-\hat{P}|4]^2}{\langle q_1 \hat{4}\rangle^2} \frac{i}{(p_2+p_4)^2-\mu^2} \cdot i\,\frac{\langle 1|3|q_2]}{[\hat{1}\, q_2]}$$

$q_2 = \hat{4} \quad q_1 = 1 \quad \Rightarrow$

$$\langle 1|-\hat{P}|4]^2 \langle 1|3|4] = \langle 1|3|4]^2 \langle 1|3|4] = -\langle 1|2|4]^3 \Rightarrow$$

$$A_2 = (-)^2\, i\, \frac{\langle 1|2|4]^3}{\underbrace{\langle 14\rangle^2 [14]}_{-\langle 14\rangle s_{14}}} \frac{i}{(p_2+p_4)^2-\mu^2} \Rightarrow$$

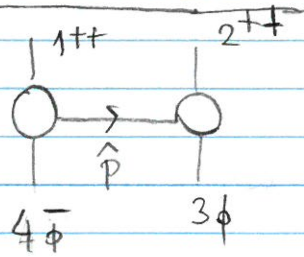
$$A_2 = -i\, \frac{\langle 1|2|4]^3}{\langle 14\rangle s_{14}} \frac{i}{(p_2+p_4)^2-\mu^2} \Rightarrow$$

$$A = A_1 + A_2 \Rightarrow$$

$$\boxed{A_{EYM}(4^{++}\, 1^-\, 2_\phi\, 3_{\bar\phi}) = -i\,\frac{\langle 1|2|4]^3}{s_{14}\langle 14\rangle}\left[\frac{i}{(p_3+p_4)^2-\mu^2} + \frac{i}{(p_2+p_4)^2-\mu^2}\right]}$$

$$\boxed{1^{++}\, 2^{++}\, 3_\phi\, 4_{\bar\phi} \quad \text{IN EYM}}$$

$$\hat\lambda_1 = \lambda_1 + z\lambda_2$$
$$\hat{\tilde\lambda}_2 = \tilde\lambda_2 - z\tilde\lambda_1$$



$+ \quad 1 \longleftrightarrow 2$

$$A_1 = -\frac{\langle q_1|4|1]^2}{\langle q_1 \hat 1\rangle^2} \; \frac{i}{(p_4+p_1)^2-\mu^2} \; (-)\frac{\langle q_2|-\hat P|\hat 2]^2}{\langle q_2 2\rangle^2}$$

Choose $\quad q_2 = \hat 1, \quad q_1 = 2 \quad \Rightarrow$

$$\langle q_1|4|1]\langle q_2|-\hat P|\hat 2] = \langle 2|4|1]\langle \hat 1|-\hat P|\hat 2] =$$

$$= -\mathrm{Tr}(\hat 2\, 4\, \hat 1\, \hat P) = -\mathrm{Tr}(\hat 2\, 4\, \hat 1\, 3) = \mathrm{Tr}(\hat 2\, 3\, \hat 1\, 3) =$$

$$= 2\left[\,2\underbrace{(\hat 2 3)}_{\emptyset}(\hat 1 3) - 3^2(\hat 1 2)\right] = -\mu^2 s_{12} \quad \Rightarrow$$

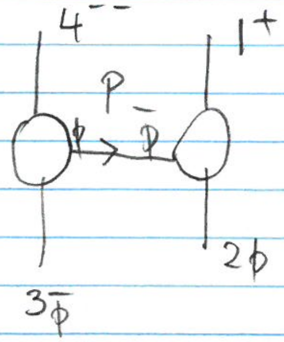
$$A = \mu^4 \frac{s_{12}^2}{\langle 12\rangle^2\langle 12\rangle^2}\left[\frac{i}{(p_1+p_4)^2-\mu^2} + 3 \longleftrightarrow 4\right]$$

$$\boxed{A(1^{++}2^{++}3_\phi 4_{\bar\phi}) = \mu^4 \frac{[12]^2}{\langle 12\rangle^2}\left[\frac{i}{(p_1+p_4)^2-\mu^2} + \frac{i}{(p_3+p_4)^2-\mu^2}\right]}$$

$\boxed{4^{--}\,1^{+}\,2_{\phi}\,3_{\bar\phi}}$ $\qquad \hat{\tilde\lambda}_4 = \tilde\lambda_4 + z\tilde\lambda_1$ $\qquad \boxed{[41\rangle}$

$\hat\lambda_1 = \lambda_1 - z\lambda_4$

(1)



$$A_L = -\frac{\langle 4|3|q_1]^2}{[\hat4 q_1]^2}$$

$$A_R = i\,\frac{\langle q_2|-\hat P|1]}{\langle q_2\hat1\rangle}$$

$$A_1 = -i\,\frac{\langle 4|3|q_1]^2}{[\hat4\, q_1]^2}\;\frac{i}{(p_3+p_4)^2-\mu^2}\;\frac{\langle q_2|-\hat P|1]}{\langle q_2\hat1\rangle}$$

Choose $q_2 = \hat4 \qquad q_1 = 1$ and

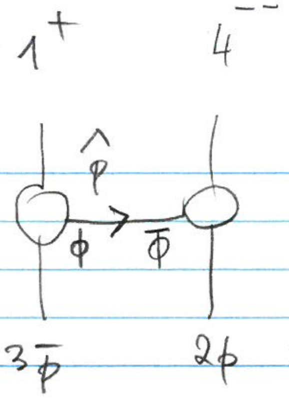
$$\langle 4|3|q_1]^2\langle q_2|-\hat P|1] = \langle 4|3|1]^2\langle 4|-\hat P|1] =$$

$$= \langle 4|3|1]^2\langle 4|-(-)(4+3)|1] = \langle 4|3|1]^3 \Rightarrow$$

$$A_1 = -i\,\frac{\langle 4|3|1]^3}{[14]^2\langle 41\rangle}\;\frac{i}{(p_3+p_4)^2-\mu^2} = +i\,\frac{\langle 4|3|1]^3}{[41]\,s_{14}}\;\frac{i}{(p_3+p_4)^2-\mu^2}$$

The second diagram:

(2)



$$A_L = i\,\frac{\langle q_2|3|1]}{\langle q_2 \hat{1}\rangle}$$

$$A_R = -\,\frac{\langle 4|-\hat{P}|q_1]^2}{[\hat{4}\, q_1]^2}$$

$$\langle q_2|3|1]\,\langle 4|-\hat{P}|q_1]^2 = \langle 4|3|1]\,\langle 4|2|1]^2 = \langle 4|3|1]$$

$$A_2 = -i\,\frac{\langle 4|3|1]^3}{\langle 41\rangle [41]^2}\,\frac{i}{(p_1+p_3)^2-\mu^2} =$$

$$= +\,\frac{i\,\langle 4|3|1]^3}{[41]\, s_{14}}\,\frac{1}{(p_1+p_3)^2-\mu^2}$$

$$\boxed{A(4^{--}1^+2\phi\,3\bar{\phi}) = +\,\frac{i\,\langle 4|3|1]^3}{s_{14}\,[41]}\left[\frac{i}{(p_3+p_4)^2-\mu^2} + \frac{i}{(p_3+p_1)^2-\mu^2}\right]}$$

Note: prefactor could be rewritten as

$$\frac{i\,\langle 4|\overset{\phi}{2}|1]^3}{s_{14}\,[14]}$$