A DEEPER DIVE: ON-SHELL AND OFF-SHELL

To better understand the implications of a recent paper by Perimeter Faculty member Freddy Cachazo et al., we delve deeper into the meaning of on- and off-shell particles.

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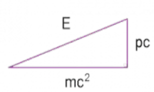
The new paper by Perimeter Faculty member Freddy Cachazo et al. outlines a new and much more user-friendly system for calculating scattering amplitudes. Scattering amplitudes – which predict what happens when two or more particles interact – are perhaps the most fundamental calculation in particle physics. Unfortunately, calculating them has, until now, been extremely difficult.

The standard way of doing these calculations was with Feynman diagrams, and modelling even simple collisions of a few particles can involve thousands of diagrams each introducing many terms into the calculation. As collisions become more complex, the Feynman diagram technique becomes too unwieldy to use.

The new calculation method outlined in "Scattering Amplitudes and the Positive Grassmannian" makes calculating scattering amplitudes far easier. It accomplishes this by bringing the entire calculation "on-shell."

In modern physics parlance, particles are said to be "on the mass shell," or simply "on-shell," if their behaviour satisfies the relationship between energy and momentum given by Einstein in his theory of special relativity. Virtual particles are those that don't satisfy this relationship, and are said to be "off-shell." This relationship is:



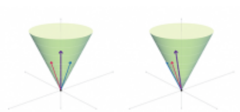
Algebraically, that's:

$$E^2 = (pc)^2 + (mc2)^2$$

Graph that out and you get a parabolic surface for massive particles, and a cone for massless particles, like photons. This is known as the mass shell. The momentum of a real particle can be represented by a vector lying along the shells' surface. The point is that real particles have momentum vectors that are on the shell – not inside it, but on it.

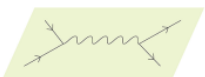
The mass shell for massless (m = 0) particles | The mass shell for massive particles

It's the stuff of high school physics that when two particles (or any other objects) collide, their total momentum has to be preserved. To calculate the momentum in the final state, the momentum vectors from the initial state should be added together. And here's the catch: pretend that two massless on-shell particles interact and produce a third particle. The momentum vector of the final particle won't lie along the surface of the cones.



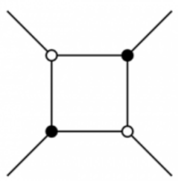
Here are the momentum vectors for two colliding particles, one red and one blue. The momentum of the final state is given by adding the red and blue vectors together to get the purple vector. Notice that the purple vector is not on the cone, but inside the cone.

That's where Feynman's virtual particles come in – they are bookkeeper's tools, used to keep track of the total momentum of the system. They aren't physical – they're just representations of an intermediate step that must occur in the calculation. In the diagram below, they're the wavy line. Think of the momentum shell being 'rolled flat' into a plane. The Feynman diagram below could be read as two physical particles coming in and producing an off-shell particle or virtual particle (wavy line), which in turn produces two physical particles. But nothing is 'moving' along the wavy line.



The new approach from Cachazo et al is different. In the Feynman-like sketch below, two real on-shell particles come in. Each one produces two more on-shell particles – but these particles have complex momentum, meaning that the number representing their momentum comprises both real and imaginary numbers. They are drawn here going out of the plane because we are thinking of the plane as being a real space, not a complex space. Below, we see two on-shell particles coming in and producing two complex on-shell particles each, which then merge and produce two more on-shell particles with real momentum.

This gives the same result as Feynman diagrams, but they make the calculation far simpler. The on-shell calculations are represented by on-shell diagrams, which look like this:



Lines that are not connected to the black and white vertices at each end represent particles with real momenta. The internal lines, with vertices at each end, have complex momenta. Just as in Feynman diagrams, there are rules that govern how the black and white vertices are paired and arranged, and using these rules both structures and guides the calculation.

The external particles must still be real; the internal particles – which in Feynman's approach were off-shell – are now on-shell, but with complex momenta.

In other words, what Cachazo et al. have done is represent mass shell more generally, in a complex space. When you add the complex vectors, the resultant vectors will stay on-shell.

"People have always said that quantum mechanics has to do with off-shell particles running in loops," says Cachazo. "You're allowed to be a little bit off-shell if it's for a small amount of time, because of the uncertainty principle. What we have shown, in several theories, is that all of quantum mechanics can be reformulated purely on-shell. There is no need to go off-shell anywhere."

Why is that better? For a start, it makes calculations far easier. But Cachazo stresses the broader implications: "The advantage of doing things on-shell is that you don't have to introduce the redundancies that were needed when you work with off-shell physics. The reason Feynman diagrams are complicated to use is that they were designed to make locality manifest, introducing redundancies as the price. Get rid of the off-shell particles and the calculations become strikingly simple. The price to pay is manifest locality. This price is, in fact, not too high as general principles imply that locality is not a property of nature when quantum gravity shows its head – but this is part of another fascinating story."