Final Project Retake - Evolutionary Games and Spatial Chaos

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Modelling Complex Systems

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1 Model description.

In the prisoner's dilemma model, we are dealing with an environment were individuals can be in two states: cooperation (C) or defection (D). We assume a certain pay-off for each individual. The cooperating individual will recive the reward (R) and the defecting individual will recive the Punishment (P) and if one individual defects on the cooperator, the defector will get the maximum amount of temptation (T), and the cooperator will recive the sucker's pay-off (S). We will assume that $T > R > P > S$, the pay-off function can be decribed in table format (Table 1). R/R means both cooperate, S/T means one cooperated and the other defected and so on. Also we assume that $2R > T + P$.

An individual A runs the simulation, or 'plays the game', in an iterated fashion against another individual B. So on a grid each individual plays with

	C	D
C	R/R	S/T
D	T/S	P/P

Table 1: Pay-off function for two individuals in the Prisoners Dilemma.

his neighbor each time step. The elements of a grid, a square matrix, will be represented by either 0 or 1, where 0 means defector and 1 means cooperator. The simulation evoles from an initial state of randomly distributed defectors and cooperators which interacts with each other on each time step. The pay-off the individual will recive is based on the pay-off matrix A = $\begin{pmatrix} R & S \\ T & P \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ b & \epsilon \end{pmatrix}$. Where $b > 1$ is the temptation parameter and ϵ is a small number, we set $\epsilon = 0$. We compare the pay-off for an individual with all of its neighbors and find the individual with the best pay-off score, if the individual with the best pay-off is a defector the element becomes a defector in the next round. Otherwise if it is a cooperator it will become a cooperator. This defines the evolutionary strategy of choosing the best strategy to maximize the pay-off. We also make the assumption that players have no memory of previous rounds and are solely defined as defector or cooperator. Each individual have 8 neighbors (Moore Neighborhood) and it interacts with itself so that's 9 interaction (unless it's a border cell).

2 Simulation results.

We simulate the model on a 200x200 grid with an even 50-50 split between cooperators and defectors randomly distributed on the grid and the simulation is for 200 generations. We look at how the final state of the map looks in the final state and how the poulation of defectors and cooperators change over time for different values of the temptation parameter $b = 1.1, b = 1.5, b = 1.7$ and $b = 1.9$. Since the map is a 200x200 square grid we have a total population of 40,000, we will notice in the graphs that the inital number of cooperators and defectors are 20,000 each.

In the graph of the final state the cooperators are painted yellow, the defectors ar painted blue, Defectors to Cooperators (DtoC) are painted green and Cooperators to Defectors (CtoD) are painted red. In the graphs we see a sudden drop in the number of cooperators within the first generations due to them being isolated and surronded by defectors, so the few cooperators that survive will create small clusters and then rise in numbers.

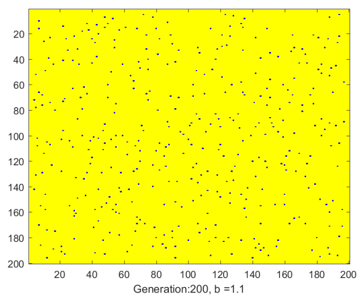

Figure A1: Final state for $b = 1.1$

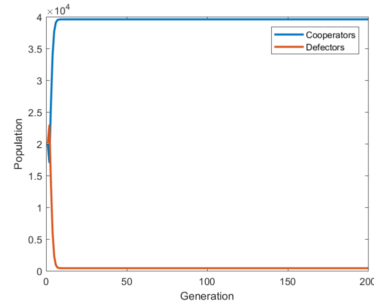

Figure A2: How the population changes over time for $b = 1.1$

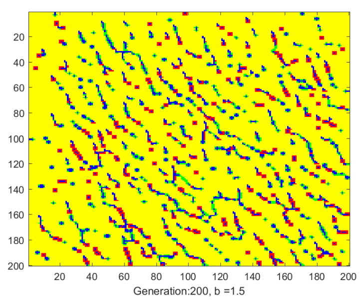

Figure A3: Final state for $b = 1.5$

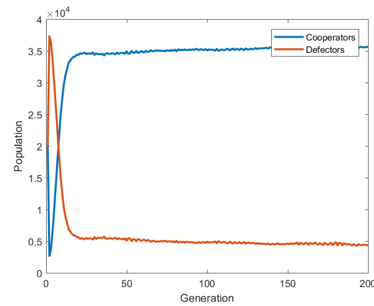

Figure A4: How the population changes over time for $b = 1.5$

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So, for the lower values of the temptation, b we see that the cooperators quickly become dominant over the defectors after a few generations. Up until $b = 1.7$ the cooperators are dominant but when the temptation becomes higher, that is for $b = 1.9$ the defectors are outnumbering the cooperators. This means that there should exist some transition point between $b = 1.7$ and $b = 1.9$ when the defectors overtake the cooperators in the model. Now we can investigate how the parameter b influences the model by looking at the density of cooperators each round and over time, see how the model changes behavior for different values of b, especially in the region $b = 1.7 < b < b = 1.9$. Because that's where we have seen a transition shift between the defectors and the cooperators.

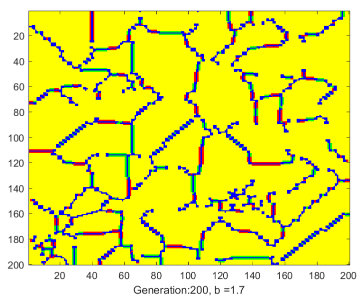

Figure A5: Final state for $b = 1.7$

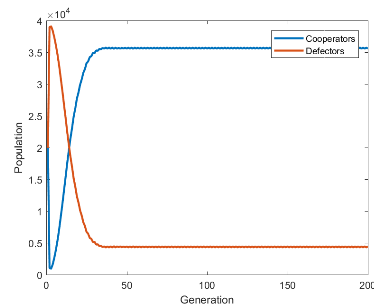

Figure A6: How the population changes over time for $b = 1.7$

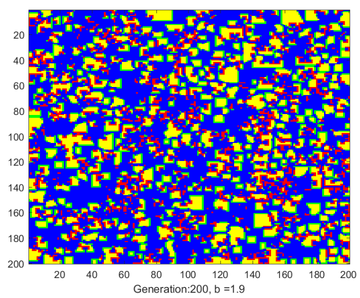

Figure A7: Final state for $b = 1.9$

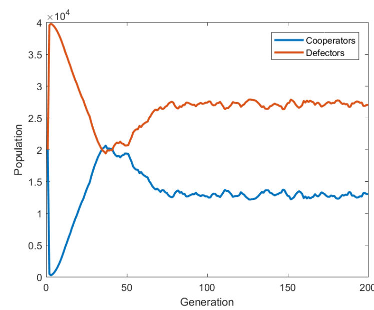

Figure A8: How the population changes over time for $b = 1.9$

3 Modifying a parameter.

The density of cooperators is determined by summing the number of cooperators each round and divide by the total grid. By varying the temptation parameter, b in small intervals and plot it against the density of cooperators for the final state we can obtain a phase transition plot.

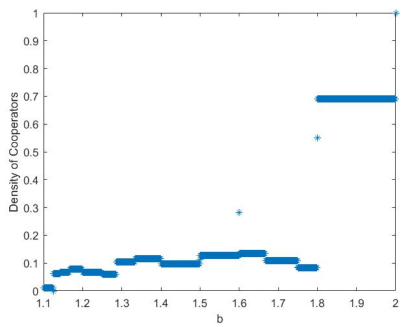

Figure A9: Phase transition plot for $b = 1.1$ to $b = 2.0$.

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As we can see from figure A9, there is a sharp phase transition for $b = 1.8$. Visulizing the simulation of $b = 1.8$ we see in figure A11 that there is a more chaotic and oscillating behavior in the model.

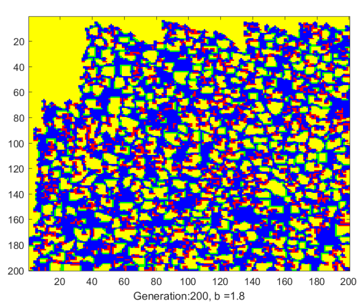

Figure A10: Final state for $b = 1.8$

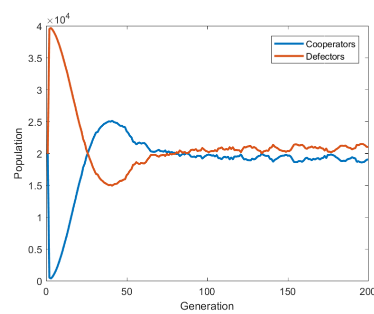

Figure A11: How the population changes over time for $b = 1.8$

4 Analytical results.

To understand mathmatically what happens when the temptaion parameter is at $b = 1.8$ and why there is sharp transition at $b = 1.8$, we can consider looking two matrices which represents a simplified scenario. One matrix, A which represets a block of 9 defectors (D) surrounded by cooperators (C) and another matrix, B which represents a block of 9 cooperators surrounded by defectors.

$$A = \begin{pmatrix} C & C & C & C & C \\ C & D & D & D & C \\ C & D & D & D & C \\ C & D & D & D & C \\ C & C & C & C & C \end{pmatrix}, \; B = \begin{pmatrix} D & D & D & D & D \\ D & C & C & C & D \\ D & C & C & C & D \\ D & C & C & C & D \\ D & D & D & D & D \end{pmatrix}$$

Now first we look at when the temptation parameter is $b > 2.0$ for the matrix, A the defector in the middle of the block is the one with lowest score. In order for the middle defector to change strategy to cooperator one of its neighbors would need to switch to cooperator. Calculating the lowest score of a defector, S_D from the pay-off matrix for $T = b > 2.0$: $S_D > 3T + 5P = 6$.

And the maximum score of a neighboring cooperator S_C is calculated from the pay-off matrix as: $S_C = 6R + 2C = 6$. This way since we'll always have $S_D > SC$, the minimum score of a defector will always be higher than the maximum score of a cooperator, the defectors will always overtake the cooperators when the temptation factor is $b = 2.0$.

Looking at the case when the temptation is $b = 1.8$, for matrix A the lowest possible defector score is: $S_D > 3T + 5P = 5.4$. And the maximum cooperator score will be $S_C = 6R + 2S = 6$. Now the defector neighbor with the highest score will appear on the corner and will have a score of $S_D > 5T + 3P = 9$ according to the pay-off matrix.

This means we will always have growth at the corners since $S_D > S_C$ and it will grow as block and until it creates a cluster of 5x5 defectors. So the 5x5 defector cluster is unstable and brings upon a chaotic behavior in the model. Since the defector at the center of the edge still has a score of $S_D > 5.4$ but no longer a corner defector of $S_D > 9$ this means that it could switch strategy to cooperator, so the corner continue to grow while the middle of the edge switches from defectors to cooperators. This explains why the 5x5 defector cluster is unstable at $b = 1.8$, obviously our example is simplified, by looking only at a localized phenomenon, but it allows us to get a grasp on the princple of why the model behaves the way it does.

5 Conclusions.

In this report we have studied the effects of parameter on a simple spatial model based on the spatial prisoner's dilemma game described by Nowak and May (1992) [1]. The parameter in question is the so called temptation parameter here denoted by, b. It is this parameter, b that Nowak and May vary to get their results. Our results in this paper indicates that there is threshold at $b = 1.8$, so in the interval $b < 1.8$ the cooperators take over and reach a final state with a cooperator dominance. In the interval $b > 1.8$ we see a shift to the opposite that is the defectors dominate the cooperators in the final state. Running the simulation for 200 generations the final state appears to be stable in both interval (apart from some small noise).

This suggests to us that cooperation is a viable evolutionary strategy since score only takes materialistic properties which are vital for survival in regards and not any forms of altruistic motives. The algorithm is all about maximizing ones resources, this may be in contradiction with natural selection. But of course cooperation is widespread among humans and in the animal kingdom (Dugatkin, 1997) [2]. In the real world individuals (animals and humans) perform action which seems go against their own interest, that seems contradictionary to the principle of natural selecetion. And not just among individuals that are related in Nowak (2006) [3] it is stated that coefficient of relatedness, r, must exceed the cost-to-benefit ratio of the altruistic act as $r > c/b$. Even cooperation among different spicies was observed and studied in Trivers (1971) [4].

As our model shows in the short-term defection is a viable strategy to defect but in the long-term cooperation wins, at least when the temptation factor is less than $b = 1.8$. This potentially shows that animals have the capability of thinking and acting with a long-term strategy.

[1] Nowak, M.A. and May, R.M. (1992) Evolutionary games and spatial chaos.
[2] Dugatkin, L.A. (1997) Cooperation among Animals: An Evolutionary Perspective.
[3] Nowak, M. A. (2006) Five rules for the evolution of cooperation.
[4] Trivers, R.L. (1971) The Evolution of Reciprocal Altruism.

But what determines the temptation factor b in the real world? It could be individual to some extent and also environmental. how much is environmental and how much is individual is hard to say. In Nowak et al. (2009) [5] the subjet of phenotypic similarity is studied in terms of willingness to cooperate. Nowak et al. (2009) concludes that 'cooperation is more likely to evolve if the strategy mutation rate is small and if the phenotypic mutation rate is large'.

[5] Antal, T., Ohtsuki, H., Wakeley, J., Taylor, P. D. and Nowak, M. A. (2009) Evolution of cooperation by phenotypic similarity.