

# A Stepwise Simple Self-Calibration Method for Low-Cost MEMS-IMU

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January 27, 2023

## Abstract

The low-cost MEMS-IMU is often used for navigation applications. The quality of the IMU measurement, angular velocity from the gyros and specific force from the accelerometers, has crucial affect on the navigation accuracy. Thus, these sensors need to be calibrated to minimize sensor errors. These errors can be categorised to deterministic and stochastic error elements. For the low cost IMUs, the deterministic errors are a major part of the overall sensor error. In the article we discuss, the authors proposed a new calibration method that compensate the affect of some of the deterministic errors such as biases, scale factors and the misalignment errors of the gyros and the accelerometers without any use of external equipment, such as a turntable. In this work we will discuss about the proposed calibration method, implement it and the 6 turns method for the sensors in our smartphone and compare the calibrations parameters with recordings of pedestrian dead reckoning with known walking distance.

## Introduction

Dead reckoning systems that based on MEMS-IMU measurements are widely used in the military and civilian fields, because in some cases, like indoor navigation for example, GPS readings are unavailable. The accuracy of Dead reckoning relies deeply on the accuracy of the IMU readings and thus calibration procedures are required for minimizing the sensors errors.

### Accelerometers calibration

The common way for calibrating the accelerometer bias, scale factor and misalignment is by setting the IMU in a set of static positions and record the specific force values of each axis. Usually we consider at least 9 position sets so that the number of samples or interval averages is not too small for statistical reasoning to be applied. The procedure exploits the fact that, in static conditions, the modulus of the accelerometer output vector matches that of the gravity acceleration. Some calibration algorithms require a specific set of precise static positions in a known inclinations, so addition calibration equipment is needed, and more advance ones doesn't rely on the data of the precise inclinations.

### Gyro calibration

The gyro calibration process is usually separated into 2 steps: 1) the bias calibration and 2) the scale factor and misalignment calibration. This is because gyroscope biases change over time, while

the other model parameters remain relatively constant. Therefore, the gyro biases have to be modeled separately. For the short duration of the second calibration, the random gyroscope biases are effectively removed by calibrating it right before the second calibration. For gyro bias calibration it is common to record a long static recordings. For the scale factor and the misalignment calibration it's possible to calibrate by a set of dynamic recordings with known angular velocity using external equipment such as turntable, or by a set of dynamic recording that fed to optimisation process.

## Proposed approach

In the article we chose, the authors proposed a calibration method in which we use the data of static intervals in different orientations to calibrate the error parameters of accelerometer and the biases of gyroscope, and use the data of dynamic interval between the static intervals to calibrate the scale factors and misalignment errors of gyroscope. Following a schematic overview of the method:

1. First we calculate the bias of the accelerometer from the static recordings using optimization process.
2. We use the bias values as initial conditions to optimization process in order to find the scale factor and misalignment parameters for the accelerometer. In this stage we use the assumption that the x-axis of the accelerometer coincides with the x-axis of the IMU carrier, and the y-axis lies on the x-y axis plane of the carrier coordinate system. Based on this assumption, we first set  $\alpha_{xz}$ ,  $\alpha_{xy}$ ,  $\alpha_{yx}$  to zero and estimate the other nine parameters, that is, the parameter vector to be estimated is set to:

$$\theta_{acce} = [S_{ax}, S_{ay}, S_{az}, \alpha_{yz}, \alpha_{zy}, 0, \alpha_{zx}, 0, 0, b_{ax}, b_{ay}, b_{az}]$$

3. The bias of the gyro is calculated by the average value of the static measurements.
4. We use another optimization process with the estimated bias parameters in order to find the gyro's scale factor and misalignment parameters.

Besides that, in order to deal with the bias instability of the gyro, an online calibration is done by taking the average values of the three-axis of the gyroscope after the compensation of the self-calibration parameters with any static time greater than 20 seconds under the working state. these values are taken as an addition bias to the estimated bias values.

## Problem formulation

Let's define the sensor measurement models we used in the calibration process and the optimization's cost functions:

### Accelerometer

We want to estimate the biases, scale factors and misalignment error of the accelerometer, hence the error model we define is as follows:

$$\vec{a} = T_a(\vec{A} - \vec{b}_a)$$

Where  $\vec{a}$  represent the calibrated measurement,  $\vec{A}$  is the raw measurement,  $T_a = \begin{bmatrix} S_{ax} & \alpha_{yz} & \alpha_{zy} \\ \alpha_{xz} & S_{ay} & \alpha_{zx} \\ \alpha_{xy} & \alpha_{yx} & S_{az} \end{bmatrix}$

where  $[S_{ax} S_{ay} S_{az}]$  are the scale factors and  $[\alpha_{yz} \alpha_{zy} \alpha_{xz} \alpha_{zx} \alpha_{xy} \alpha_{yx}]$  are the misalignment errors between the accelerometer three axes and the IMU carrier coordinate system and  $\vec{b}_a$  is the bias.

The cost function used in stage 2 of the calibration to optimize the error parameters of the accelerometer is:

$$L(\theta_{acc}) = \sum_{n=1}^N (g - \|a\|^2)$$

where N is the total number of static intervals recorded. g represent the local acceleration of gravity.

## Gyroscope

The error model of the gyroscope is similar to the accelerometer model, and it's as follows:

$$\vec{\omega} = T_g(\vec{W} - \vec{b}_g)$$

Where  $\vec{\omega}$  represent the calibrated measurement,  $\vec{W}$  is the raw measurement,  $T_g = \begin{bmatrix} S_{gx} & \beta_{yz} & \beta_{zy} \\ \beta_{xz} & S_{gy} & \beta_{zx} \\ \beta_{xy} & \beta_{yx} & S_{gz} \end{bmatrix}$

where  $[S_{gx} S_{gy} S_{gz}]$  are the scale factors and  $[\beta_{yz} \beta_{zy} \beta_{xz} \beta_{zx} \beta_{xy} \beta_{yx}]$  are the misalignment errors between the gyroscope three axes and the IMU carrier coordinate system and  $\vec{b}_g$  is the bias.

The optimization for the gyro try to minimize the following cost function:

$$L(\theta_{gyro}) = \sum_{k=1}^K (\|u_{a,k} - u_{g,k}\|^2)$$

Where  $u_{a,k}$  is the calibrated average static acceleration vector in interval k measured by the accelerometer and  $u_{g,k}$  is the estimated acceleration vector obtained using the calibrated average static acceleration vector in interval k-1 ( $u_{a,k-1}$ ) and the gyro measurements in the dynamic section ( $\vec{\omega}_{k-1 \rightarrow k}$ ). In order to calculate  $u_{g,k}$  we first convert  $u_{a,k-1}$  to unit quaternion that represent the direction of the gravity (q). then we use the following iterative equation:

$$\begin{bmatrix} q'_0 \\ q'_1 \\ q'_2 \\ q'_3 \end{bmatrix} = \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} -\omega_x q_1 - \omega_y q_2 - \omega_z q_3 \\ \omega_x q_0 + \omega_z q_2 - \omega_y q_3 \\ \omega_y q_0 - \omega_z q_1 + \omega_x q_3 \\ \omega_z q_0 + \omega_y q_1 - \omega_x q_2 \end{bmatrix} \cdot \Delta t$$

where q is the current orientation, q' is the next orientation and  $\omega$  is the measurement from the gyro. With the iterative equation we get the quaternion that describes the rotation in the dynamic section. From the result quaternion  $q_k = [a \ b \ c \ d]$  we can construct a rotation matrix by the relation:

$$R = \begin{bmatrix} a^2 + b^2 - c^2 - d^2 & 2(bc + ad) & 2(bd - ac) \\ 2(bc - ad) & a^2 - b^2 + c^2 - d^2 & 2(cd + ab) \\ 2(bd + ac) & 2(cd - ab) & a^2 - b^2 - c^2 + d^2 \end{bmatrix}$$

And finally,  $u_{g,k}$  is equal to:

$$u_{g,k} = R u_{a,k-1}$$

In order to evaluate the proposed calibration method we tested it versus the 6-turns calibration method we learned in class. We recorded a scenario of walking in known distance with a smartphone's IMU in texting position, and solve the equations of motion in order to estimate the distance with the calibrated measurements.

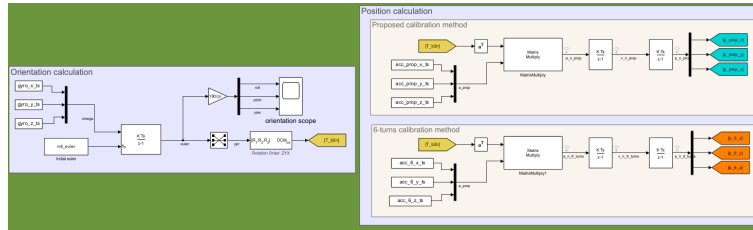
## Experiments

In order to evaluate the accuracy of the proposed calibration we preformed recordings of 2 types of scenarios using the IMU of the smartphone. Using the recording we solved the equations of motion using some reasonable assumptions. The Equations of motion we saw in class are:

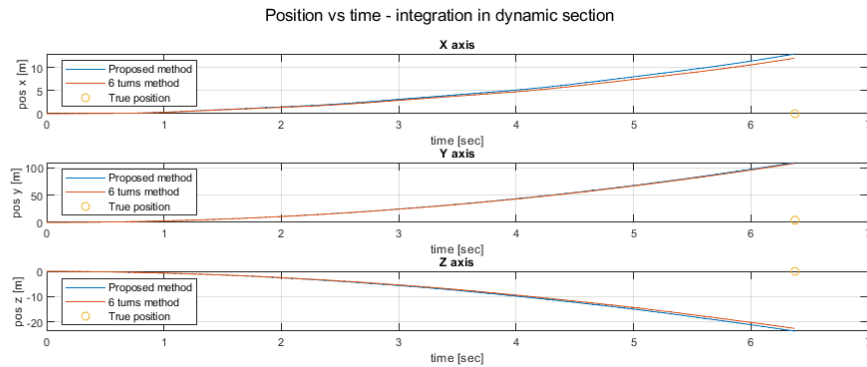
$$\begin{cases} \dot{v}^n = T_b^n f^b + g^n - (\Omega_{en}^n + 2\Omega_{ie}^n)V^n \\ \dot{p}^n = v^n \\ \dot{T}_b^n = T_b^n \Omega_{nb}^b \end{cases} \Rightarrow \begin{cases} \dot{v}^n = T_b^n \underbrace{f^b}_{Acc\ meas} + g^n \\ \dot{p}^n = v^n \\ \dot{T}_b^n = T_b^n \underbrace{\Omega_{nb}^b}_{Gyro\ meas} \end{cases}$$

*in our case  $\Omega_{en}^n = \Omega_{ie}^n = 0$*

The first type of scenario we tested is walking 4m in a straight line and holding the phone in texting position. The iterative calculations of the states from the measurements preformed using Matlab Simulink. The simulation model is:

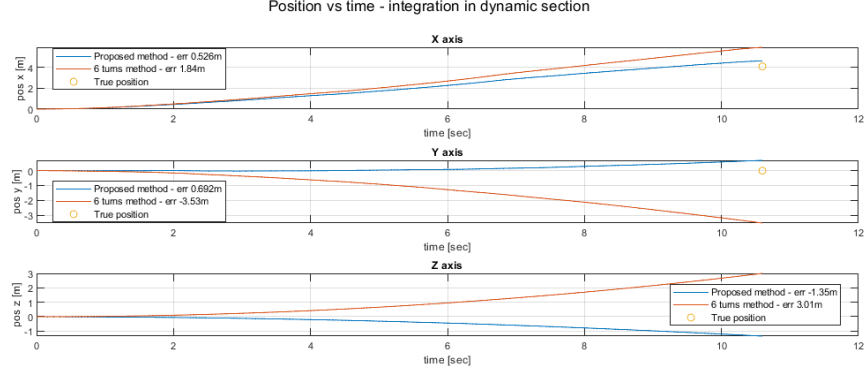


In order to compensate the bias instability of the gyro, a static interval of 30 seconds was made before the dynamic scenario and the integration of the measurements were made only in the dynamic part. Following the results of the position calculation using both calibration methods for this scenario:



As can be seen, the position diverge in all axes and it's not possible to conclude anything about the accuracy of the calibrations.

In order to evaluate the accuracy of the accelerometer calibration we preformed another type of scenario. This time we moved a cart in a straight line for 4m, and because the cart surface is flat, the orientation was leveled during the recording, so the position calculation was made only by integrating the accelerometers measurements. Following a plot that illustrates the results:



Following a table that summarize the position errors with the two calibration methods during 6 scenarios in which the direction of movement was aligned with the plus and minus of each axis.

| movement direction | Proposed calibration errors [m] |      |      | 6-turns errors [m] |      |      |
|--------------------|---------------------------------|------|------|--------------------|------|------|
|                    | X                               | Y    | Z    | X                  | Y    | Z    |
| +X                 | 0.53                            | 0.69 | 1.35 | 1.84               | 3.53 | 3.01 |
| -X                 | 1.02                            | 0.11 | 1.23 | 2.17               | 3.74 | 2.58 |
| +Y                 | 5.73                            | 2.69 | 0.88 | 6.49               | 5.19 | 1.68 |
| -Y                 | 7.17                            | 1.67 | 1.32 | 8.55               | 6    | 3.13 |
| +Z                 | 2.96                            | 0.79 | 5.05 | 0.32               | 2.1  | 7.87 |
| -Z                 | 3.7                             | 1.45 | 5.86 | 0.34               | 3.13 | 9.38 |
| average            | 3.52                            | 1.23 | 2.61 | 3.28               | 3.95 | 4.61 |

## Conclusions

As we can see from the table above, overall the proposed calibration method results a slight improvement in measurements accuracy. we can see a large error in x axis due to movement in y axis in both calibration which means that the misalignment calibration in this axis wasn't accurate. Moreover, we can see a large error in z axis due to movement in that axis meaning that there is an error in the scale factor of that axis or the bias value.

Regarding the article, in our opinion, the main idea of the article don't really innovate compare to previous works. The authors suggested a step-wise optimization process for the accelerometer parameters calibration and we didn't see the need for it. However, the explanation about the calibration process was very clear and it was noticeable that the authors tried to share their intuition about the chosen parameters they used such as static time needed to estimate the bias values of the gyro and the criterion for static and dynamic sections detection.

As for future research, we can suggest using recording from more than one IMU or other sensors such as GPS, vision, barometers and magnetometers in order to get more information about the reference signals used for the optimization process.

## References

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