

Chapter 4 Frequency Domain

4.1 Introduction to the Frequency Domain



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In developing ways of analyzing linear circuits, we invented the impedance method because it made solving circuits easier. Along the way, we developed the notion of a circuit's frequency response or transfer function. This notion, which also applies to all linear, time-invariant systems, describes how the circuit responds to a sinusoidal input when we express it in terms of a complex exponential. We also learned the Superposition Principle for linear systems: The system's output to an input consisting of a sum of two signals is the sum of the system's outputs to each individual component.

The study of the frequency domain combines these two notions a system's sinusoidal response is easy to find and a linear system's output to a sum of inputs is the sum of the individual outputs to develop the crucial idea of a signal's **spectrum**. We begin by finding that those signals that can be represented as a sum of sinusoids is very large. In fact, **all signals can be expressed as a superposition of sinusoids**.

As this story unfolds, we'll see that information systems rely heavily on spectral ideas. For example, radio, television, and cellular telephones transmit over different portions of the spectrum. In fact, spectrum is so important that communications systems are regulated as to which portions of the spectrum they can use by the Federal Communications Commission in the United States and by International Treaty for the world (see Frequency Allocations (Section 7.3)). Calculating the spectrum is easy: The **Fourier transform** defines how we can find a signal's spectrum.

4.2 Complex Fourier Series



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In an earlier module (Exercise 2.3.1), we showed that a square wave could be expressed as a superposition of pulses. As useful as this decomposition was in this example, it does not generalize well to other periodic signals: How can a superposition of pulses equal a smooth signal like a sinusoid? Because of the importance of sinusoids to linear systems, you might wonder whether they could be added together to represent a large number of periodic signals. You would be right and in good company as well. Euler and Gauss in particular worried about this problem, and Jean Baptiste Fourier⁵ got the credit even though tough mathematical issues were not settled until later. They worked on what is now known as the **Fourier series**: representing **any** periodic signal as a superposition of sinusoids.

But the Fourier series goes well beyond being another signal decomposition method. Rather, the Fourier series begins our journey to appreciate how a signal can be