Positive values of *a* are used in population models to describe how population size increases over time. Here, *n* might correspond to generation. The diference equation says that the number in the next generation is some multiple of the previous one. If this multiple is less than one, the population becomes extinct; if greater than one, the population fourishes. The same diference equation also describes the effect of compound interest on deposits. Here, n indexes the times at which compounding occurs (daily, monthly, etc.), *a* equals the compound interest rate plus one, and *b* =1 (the bank provides no gain). In signal processing applications, we typically require that the output remain bounded for any input. For our example, that means that we restrict $|a| < 1$ and choose values for it and the gain according to the application.

**Exercise 5.12.2**

Note that the difference equation (5.42),

$$y(n) = a_1 y(n-1) + \ldots + a_p y(n-p) + b_0 x(n) + b_1 x(n-1) + \ldots + b_q x(n-q)$$

does not involve terms like *y (n + 1)* or *x (n + 1)* on the equation's right side. Can such terms also be included? Why or why not?

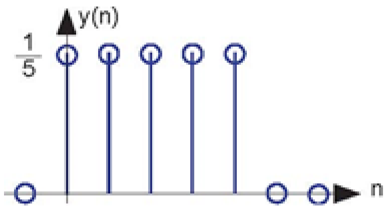


**Figure 5.19 The plot shows the unit-sample response of a length-5 boxcar filter.**

**Example 5.6**

A somewhat different system has no "*a*" coefficients. Consider the difference equation

$$y(n) = \frac{1}{q}(x(n) + \ldots + x(n-q+1))$$

(5.46)

Because this system's output depends only on current and previous input values, we need not be concerned with initial conditions. When the input is a unit-sample, the output equals

$$\frac{1}{q}$$

for $n = \{0,\ldots,q-1\}$, then equals zero thereafter. Such systems are said to be **FIR** (**F**inite **I**mpulse **R**esponse) because their unit sample responses have finite duration. Plotting this response (Figure 5.19) shows that the unit-sample response is a pulse of width q and height

$$\frac{1}{q}.$$

This waveform is also known as a boxcar, hence the name **boxcar filter** given to this system. We'll derive its frequency response and develop its filtering interpretation in the next section. For now, note that the difference equation says that each output value equals the **average** of the input's current and previous values. Thus, the output