$$Z_L = j2\pi f L$$

The impedance is, in general, a complex-valued, frequency-dependent quantity. For example, the magnitude of the capacitor's impedance is inversely related to frequency, and has a phase of

$$-\left(\frac{\pi}{2}\right).$$

This observation means that if the current is a complex exponential and has constant amplitude, the amplitude of the voltage decreases with frequency.

Let's consider Kirchof's circuit laws. When voltages around a loop are all complex exponentials of the same frequency, we have

$$\sum_n (v_n) = \sum_n \left(V_n e^{j2\pi f t}\right)$$
$$=0$$

which means

$$\sum_n (V_n) = 0$$

the complex amplitudes of the voltages obey KVL. We can easily imagine that the complex amplitudes of the currents obey KCL.

What we have discovered is that source(s) equaling a complex exponential of the same frequency forces all circuit variables to be complex exponentials of the same frequency. Consequently, the ratio of voltage to current for each element equals the ratio of their complex amplitudes, which depends only on the source's frequency and element values.

This situation occurs because the circuit elements are linear and time-invariant. For example, suppose we had a circuit element where the voltage equaled the square of the current:

$$v(t) = Ki^2(t).\ If\ i(t) = Ie^{j2\pi f t}, v(t) = KI^2 e^{j2\pi 2 f t},$$

meaning that voltage and current no longer had the same frequency and that their ratio was time-dependent.

Because for linear circuit elements the complex amplitude of voltage is proportional to the complex amplitude of current *V* = *ZI* assuming complex exponential sources means circuit elements behave as if they were resistors, where instead of resistance, we use impedance. **Because complex amplitudes for voltage and current also obey Kirchoff's laws, we can solve circuits using voltage and current divider and the series and parallel combination rules by considering the elements to be impedances.**

3.10 Time and Frequency Domains

When we find the differential equation relating the source and the output, we are faced with solving the circuit in what is known as the **time domain**. What we