Chapter 2 Signals and Systems

2.1 Complex Numbers

While the fundamental signal used in electrical engineering is the sinusoid, it can be expressed mathematically in terms of an even more fundamental signal: the **complex exponential**. Representing sinusoids in terms of complex exponentials is **not** a mathematical oddity. Fluency with complex numbers and rational functions of complex variables is a critical skill all engineers master. Understanding information and power system designs and developing new systems all hinge on using complex numbers. In short, they are critical to modern electrical engineering, a realization made over a century ago.

2.1.1 Definitions

The notion of the square root of −1 originated with the quadratic formula: the solution of certain quadratic equations mathematically exists only if the so-called imaginary quantity

$$\sqrt{-1}$$

could be defined. Euler first used i for the imaginary unit but that notation did not take hold until roughly Ampere's time. Ampere3 used the symbol i to denote current (intensite de current). It wasn't until the twentieth century that the importance of complex numbers to circuit theory became evident. By then, using i for current was entrenched and electrical engineers chose j for writing complex numbers.

An **imaginary number** has the form

$$jb = \sqrt{-b^2}$$

.A **complex number**, z, consists of the ordered pair (*a*,*b*), a is the real component and b is the imaginary component (the j is suppressed because the imaginary component of the pair is always in the second position). The imaginary number ***jb*** equals (**0**,***b***). Note that *a* and *b* are real-valued numbers.

Figure 2.1 shows that we can locate a complex number in what we call the **complex plane**. Here, a, the real part, is the x-coordinate and *b*, the imaginary part, is the ***y***-coordinate.

From analytic geometry, we know that locations in the plane can be expressed as the sum of vectors, with the vectors corresponding to the x and y directions. Consequently, a complex number *z* can be expressed as the (vector) sum *z* = *a* + *jb* where j indicates the y-coordinate. This representation is known as the **Cartesian form of *z***. An imaginary number can't be numerically added to a real number; rather,