$$\left[-\left(\frac{1}{2}\right),\frac{1}{2}\right]$$

or [0, 1], but the DTFT formula as it stands requires evaluating the spectra at all frequencies within a period. Let's compute the spectrum at a few frequencies; the most obvious ones are the equally spaced ones

$$f=\frac{k}{K},\ k\in\{0,\cdots,K-1\}$$

We thus define the **discrete Fourier transform** (DFT) to be

$$S(k)=\sum_{n=0}^{N-1}\left(s(n)e^{-\left(\frac{j2\pi nk}{K}\right)}\right)\ ,\ k\in\{0,...,K-1\}$$

Here, *S(k)* is shorthand for

$$S\left(e^{j2\pi\frac{k}{K}}\right).$$

We can compute the spectrum at as many equally spaced frequencies as we like. Note that you can think about this computationally motivated choice as **sampling** the spectrum; more about this interpretation later. The issue now is how many frequencies are enough to capture how the spectrum changes with frequency. One way of answering this question is determining an inverse discrete Fourier transform formula: given *S(k)*, *k = {0,...,K − 1}* how do we find *s(n)*, n = {0,...,N − 1}? Presumably, the formula will be of the form

$$s(n)=\sum_{k=0}^{K-1}\left(S(k)e^{\frac{j2\pi nk}{K}}\right).$$

Substituting the DFT formula in this prototype inverse transform yields

$$s(n)=\sum_{k=0}^{K-1}\left(\sum_{m=0}^{N-1}\left(s(m)e^{-\left(j\frac{2\pi mk}{K}\right)}e^{j\frac{2\pi nk}{K}}\right)\right)$$

Note that the orthogonality relation we use so often has a different character now.

$$\sum_{k=0}^{K-1}\left(e^{-\left(j\frac{2\pi km}{K}\right)}e^{j\frac{2\pi kn}{K}}\right)=\begin{cases}K\ if\ m=\{n,(n\pm K),(n\pm 2K),...\}\\ 0\ otherwise\end{cases}$$

We obtain nonzero value whenever the two indices difer by multiples of *K*. We can express this result as

$$K\sum_{l}\left(\delta(m-n-lK)\right).$$

Thus, our formula becomes

$$s(n)=\sum_{m=0}^{N-1}\left(s(m)K\sum_{l=-\infty}^{\infty}\left(\delta(m-n-lK)\right)\right)$$

The integers *n* and *m* both range over {0,...,*N* − 1}. To have an inverse transform, we need the sum to be a **single** unit sample for m, n in this range. If it did not, then *s (n)* would equal a sum of values, and we would not have a valid transform: Once going into the frequency domain, we could not get back unambiguously! Clearly, the term *l* =0 always provides a unit sample (we'll take care of the factor of *K* soon). If we