

- For frequencies greater than  $f_c$ , the circuit strongly attenuates the amplitude. Thus, when the source frequency is in this range, the circuit's output has a much smaller amplitude than that of the source.

For these reasons, this frequency is known as the **cutoff frequency**. In this circuit the cutoff frequency depends only on the product of the resistance and the capacitance. Thus, a cutoff frequency of 1 kHz occurs when

$$\frac{1}{2\pi RC} = 10^3$$

or

$$RC = \frac{10^{-3}}{2\pi} = 1.59 \times 10^{-4}$$

. Thus resistance-capacitance combinations of 1.59 k $\Omega$  and 100nF or 10  $\Omega$  and 1.59  $\mu$ F result in the **same** cutoff frequency.

The phase shift caused by the circuit at the cutoff frequency precisely equals

$$-\left(\frac{\pi}{4}\right).$$

Thus, below the cutoff frequency, phase is little affected, but at higher frequencies, the phase shift caused by the circuit becomes

$$-\left(\frac{\pi}{2}\right).$$

This phase shift corresponds to the difference between a cosine and a sine.

We can use the transfer function to find the output when the input voltage is a sinusoid for two reasons. First of all, a sinusoid is the sum of two complex exponentials, each having a frequency equal to the negative of the other. Secondly, because the circuit is linear, superposition applies. If the source is a sine wave, we know that

$$\begin{aligned} v_{in}(t) &= A \sin(2\pi ft) \\ &= \frac{A}{2j} (e^{j2\pi ft} - e^{-j2\pi ft}) \end{aligned}$$

Since the input is the sum of two complex exponentials, we know that the output is also a sum of two similar complex exponentials, the only difference being that the complex amplitude of each is multiplied by the transfer function evaluated at each exponential's frequency.

$$v_{out}(t) = \frac{A}{2j} H(f) e^{j2\pi ft} - \frac{A}{2j} H(-f) e^{2(j\pi ft)}$$

As noted earlier, the transfer function is most conveniently expressed in polar form:  $H(f) = |H(f)| e^{j\angle(H(f))}$ . Furthermore,  $|H(-f)| = |H(f)|$  (even symmetry of the magnitude) and  $\angle(H(-f)) = -\angle(H(f))$  (odd symmetry of the phase). The output voltage expression simplifies to

$$\begin{aligned} v_{out}(t) &= \frac{a}{2j} |H(f)| e^{j2\pi ft + \angle(H(f))} - \frac{A}{2j} |H(f)| e^{-(j2\pi ft) - \angle(H(f))} \\ &= A |H(f)| \sin(2\pi ft + \angle(H(f))) \end{aligned}$$