$$\sum_{k=-\infty}^{\infty}\left(c_k e^{j\frac{2\pi kt}{T}}\right)=\sum_{k=-\infty}^{\infty}\left((A_k+jB_k)e^{j\frac{2\pi kt}{T}}\right)$$

Simplifying each term in the sum using Euler’s formula.

$$\begin{aligned}(A_k+jB_k)e^{j\frac{2\pi kt}{T}} &= (A_k+jB_k)\left(cos\left(\frac{2\pi kt}{T}\right)+jsin\left(\frac{2\pi kt}{T}\right)\right)\\ &= A_k\ cos\left(\frac{2\pi kt}{T}\right)-B_k\ sin\left(\frac{2\pi kt}{T}\right)+j\left(A_k\ sin\left(\frac{2\pi kt}{T}\right)+B_k\ cos\left(\frac{2\pi kt}{T}\right)\right)\end{aligned}$$

We now combine terms that have the same frequency index **in magnitude**. Because the signal is real-valued, the coefficients of the complex Fourier series have conjugate symmetry: $\mathbf{c_{-k} = c_k^*}$or $\mathbf{A_{-k} = A_k}$ **and** $\mathbf{B_{-k} = -B_k}$. After we add the positive-indexed and negative-indexed terms, each term in the Fourier series becomes

$$2A_k\ cos\left(\frac{2\pi kt}{T}\right)-2B_k\ sin\left(\frac{2\pi kt}{T}\right)$$

. To obtain the classic Fourier series (4.11), we must have $2\mathbf{A_k}=\boldsymbol{a_k}$and $2\mathbf{B_k}=-\mathbf{b_k}$.

Solution to Exercise 4.3.2

The average of a set of numbers is the sum divided by the number of terms. Viewing signal integration as the limit of a Riemann sum, the integral corresponds to the average.

Solution to Exercise 4.3.3

We found that the complex Fourier series coefficients are given by

$$c_k=\frac{2}{j\pi k}.$$

The coefficients are pure imaginary, which means $\boldsymbol{a_k}=0$. The coefficients of the sine terms are given by $\boldsymbol{b_k}=-(\mathbf{2Im}(\boldsymbol{c_k}))$ so that

$$b_k=\begin{cases}\frac{4}{\pi k}\ if\ k\ odd\\ 0\ if\ k\ even\end{cases}$$

Thus, the Fourier series for the square wave is

$$sq(t)=\sum_{k\in\{1,3,\dots\}}\left(\frac{4}{\pi k}sin\left(\frac{2\pi kt}{T}\right)\right)$$

Solution to Exercise 4.4.1

The rms value of a sinusoid equals its amplitude divided by

$$\sqrt{2}.$$

As a half-wave rectified sine wave is zero during half of the period, its rms value is

$$\frac{A}{2\sqrt{2}}.$$

since the integral of the squared half-wave rectified sine wave equals half that of a squared sinusoid.

Solution to Exercise 4.4.2

Total harmonic distortion equals