(Length-8 DFT decomposition)). Although most of the complex multiplies are quite simple (multiplying by

$$e-(j\pi)$$

means swapping real and imaginary parts and changing their signs), let's count those for purposes of evaluating the complexity as full complex multiplies. We have

$$\frac{N}{2}=4$$

complex multiplies and N =8 complex additions for each stage and $\log_2 N$ = 3 stages, making the number of basic computations

$$\frac{3N}{2}\log_2 N$$

as predicted.

Exercise 5.9.2

Note that the ordering of the input sequence in the two parts of Figure 5.12 (Length-8 DFT decomposition) aren't quite the same. Why not? How is the ordering determined?

Other "fast" algorithms were discovered, all of which make use of how many common factors the transform length ***N*** has. In number theory, the number of prime factors a given integer has measures how **composite** it is. The numbers 16 and 81 are highly composite (equaling 2^4 and 3^4 respectively), the number 18 is less so ($2^1 \cdot 3^2$), and 17 not at all (it's prime). In over thirty years of Fourier transform algorithm development, the original Cooley-Tukey algorithm is far and away the most frequently used. It is so computationally efficient that power-of-two transform lengths are frequently used regardless of what the actual length of the data.

Exercise 5.9.3

Suppose the length of the signal were 500? How would you compute the spectrum of this signal using the Cooley-Tukey algorithm? What would the length N of the transform be?

5.11 Spectrograms

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We know how to acquire analog signals for digital processing (pre-filtering), sampling (The Sampling Theorem (Page 201)), and A/D conversion (Analog-to-Digital Conversion (Page 201)) and to compute spectra of discrete-time signals (using the FFT algorithm (Fast Fourier Transform (FFT) (Page 221)), let's put these various components together to learn how the spectrogram shown in Figure 5.14 (Speech Spectrogram), which is used to analyze speech (Modeling the Speech Signal (Page 165)), is calculated. The speech was sampled at a rate of 11.025 kHz and passed through a 16-bit A/D converter.

POINT OF INTEREST: Music compact discs (CDs) encode their signals at a sampling rate of 44.1 kHz. We'll learn the rationale for this number later. The 11.025 kHz sampling rate for the speech is 1/4 of the CD sampling rate, and was the lowest available