Exercise 5.14.2

Derive the minimal DFT length for a length-q unit-sample response using the Sampling Theorem. Because sampling in the frequency domain causes repetitions of the unit-sample response in the time domain, sketch the time-domain result for various choices of the DFT length N.

Exercise 5.14.3

Express the unit-sample response of a FIR filter in terms of difference equation coefficients. Note that the corresponding question for IIR filters is far more difficult to answer: Consider the example (Discrete-Time Systems in the Time-Domain (Page 228)).

For IIR systems, we cannot use the DFT to find the system's unit-sample response: aliasing of the unit-sample response will always occur. Consequently, we can only implement an IIR filter accurately in the time domain with the system's difference equation. **Frequency-domain implementations are restricted to FIR filters.**

Another issue arises in frequency-domain filtering that is related to time-domain aliasing, this time when we consider the output. Assume we have an input signal having duration N_x that we pass through a FIR filter having a length-q +1 unit-sample response. What is the duration of the output signal? The difference equation for this filter is

$$y(n) = b_0 x(n) + \ldots + b_q x(n-q)$$

(5.54)

This equation says that the output depends on current and past input values, with the input value q samples previous defining the extent of the filter's **memory** of past input values. For example, the output at index $\boldsymbol{N_x}$ depends on $\mathbf{x}\ (\boldsymbol{N_x})$ (which equals zero), $\boldsymbol{x\ (N_x - 1)}$, through $\boldsymbol{x\ (N_x - q)}$. Thus, the output returns to zero only after the last input value passes through the filter's memory. As the input signal's last value occurs at index $\boldsymbol{N_x - 1}$, the last nonzero output value occurs when $\boldsymbol{n - q = N_x - 1}$ or $\boldsymbol{n = q + N_x - 1}$. Thus, the output signal's duration equals $\boldsymbol{q + N_x}$.

Exercise 5.14.4

In words, we express this result as "The output's duration equals the input's duration plus the filter's duration minus one.". Demonstrate the accuracy of this statement.

The main theme of this result is that a filter's output extends longer than either its input or its unit-sample response. Thus, to avoid aliasing when we use DFTs, the dominant factor is not the duration of input or of the unit-sample response, but of the output. Thus, the number of values at which we must evaluate the frequency response's DFT must be at least $\boldsymbol{q + N_x}$ and we must compute the same length DFT of the input. To accommodate a shorter signal than DFT length, we simply zero-pad the input: Ensure that for indices extending beyond the signal's duration that the signal is zero. Frequency-domain filtering, diagrammed in Figure 5.20, is accomplished by storing the filter's frequency response as the DFT H (k), computing the input's DFT $X(k)$, multiplying them to create the output's DFT $Y(k) = H(k)\, X(k)$, and computing the inverse DFT of the result to yield $y(n)$.