Chapter 7 Appendix

7.1 Decibels

The decibel scale expresses amplitudes and power values **logarithmically**. The definitions for these differ, but are consistent with each other.

$$power(s, in\ decibels) = 10\log_{10}\left(\frac{power(s)}{power(s_0)}\right)$$
$$amplitude(s, in\ decibels) = 20\log_{10}\left(\frac{amplitude(s)}{amplitude(s_0)}\right)$$

(7.1)

Here **power** (s_0) and **amplitude** (s_0) represent a **reference** power and amplitude, respectively. Quantifying power or amplitude in decibels essentially means that we are comparing quantities to a standard or that we want to express how they changed. You will hear statements like "The signal went down by 3 dB" and "The filter's gain in the stopband is -60" (Decibels is abbreviated dB.).

Exercise 7.1.1

The prefix "deci" implies a tenth; a decibel is a tenth of a Bel. Who is this measure named for?

The consistency of these two definitions arises because power is proportional to the square of amplitude:

$$\left(power(s) \propto amplitude^2(s)\right)$$

(7.2)

Plugging this expression into the Definition for decibels, we find that

$$\begin{aligned}10\log_{10}\left(\frac{power(s)}{power(s_0)}\right) &= 10\log_{10}\left(\frac{amplitude^2(s)}{amplitude^2(s_0)}\right)\\ &= 20\log_{10}\left(\frac{amplitude(s)}{amplitude(s_0)}\right)\end{aligned}$$

(7.3)

Because of this consistency, **stating relative change in terms of decibels is unambiguous**. A factor of 10 increase in amplitude corresponds to a 20 dB increase in both amplitude and power!