$$\frac{1}{e}$$

It equals the reciprocal of a(f), which depends on frequency, and is expressed by manufacturers in units of dB/m.

The presence of the imaginary part of γ, $b(f)$, also provides insight into how transmission lines work. Because the solution for $x > 0$ is proportional to $e^{-(jbx)}$, we know that the voltage's complex amplitude will **vary sinusoidally in space**. The complete solution for the voltage has the form

$$v(x,t) = Re\left(V_+ e^{-(ax)} e^{j(2\pi ft - br)}\right)$$

(6.12)

The complex exponential portion has the form of a **propagating wave**. If we could take a snapshot of the voltage (take its picture at t = t1), we would see a sinusoidally varying waveform along the transmission line. One period of this variation, known as the **wavelength**, equals

$$\lambda = \frac{2\pi}{b}$$

If we were to take a second picture at some later time $t = t_2$, we would also see a sinusoidal voltage. Because

$$2\pi f t_2 - bx = 2\pi f(t_1 + t_2 - t_1) - bx = 2\pi f t_1 - b\left(x - \frac{2\pi f}{b}(t_2 - t_1)\right)$$

the second waveform appears to be the first one, but delayed shifted to the right in space. Thus, the voltage appeared to move to the right with a speed equal to

$$\frac{2\pi f}{b}$$

(assuming $b > 0$). We denote this **propagation speed** by c, and it equals 2πf

$$c = \left|\frac{2\pi f}{Im\left(\sqrt{\left(\widetilde{G} + j2\pi f\ \widetilde{C}\right)\left(\widetilde{R} + 2\pi f\ \widetilde{L}\right)}\right)}\right|$$

(6.13)

In the high-frequency region where

$$\left(j2\pi f\ \widetilde{L} \gg \widetilde{R}\right)$$

and

$$\left(j2\pi f\ \widetilde{C} \gg \widetilde{G}\right).$$

the quantity under the radical simplifies to

$$-4\pi^2 f^2\ \widetilde{L}\widetilde{C},$$

and we find the propagation speed to be

$$\lim_{f\to\infty} c = \frac{1}{\sqrt{\widetilde{L}\widetilde{C}}}$$

(6.14)