At this point, we could learn how to solve differential equations. Note first that even finding the differential equation relating an output variable to a source is often very tedious. The parallel and series combination rules that apply to resistors don't directly apply when capacitors and inductors occur. We would have to slog our way through the circuit equations, simplifying them until we fnally found the equation that related the source(s) to the output. At the turn of the twentieth century, a method was discovered that not only made fnding the differential equation easy, but also simplifed the solution process in the most common situation. Although not original with him, Charles Steinmetzl9 presented the key paper describing the **impedance** approach in 1893. It allows circuits containing capacitors and inductors to be solved with the same methods we have learned to solved resistor circuits. To use impedances, we must master **complex numbers.** Though the arithmetic of complex numbers is mathematically more complicated than with real numbers, the increased insight into circuit behavior and the ease with which circuits are solved with impedances is well worth the diversion. But more importantly, the impedance concept is central to engineering and physics, having a reach far beyond just circuits.

3.9 The Impedance Concept

Rather than solving the differential equation that arises in circuits containing capacitors and inductors, let's pretend that all sources in the circuit are complex exponentials having the same frequency. Although this pretense can only be mathematically true, this fction will greatly ease solving the circuit no matter what the source really is.

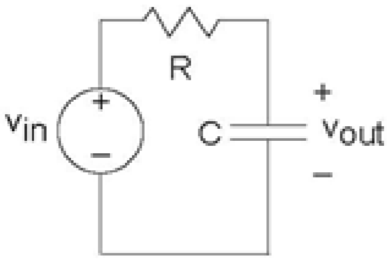

Figure 3.23 A simple *RC* circuit

For the above example ***RC*** circuit (Figure 3.23), let

$$v_{in} = V_{in} e^{j2\pi ft}.$$

The complex amplitude$\boldsymbol{V_{in}}$ determines the size of the source and its phase. The critical consequence of assuming that sources have this form is that all voltages and currents in the circuit are also complex exponentials, having amplitudes governed by KVL, KCL, and the ***v-i*** relations and the same frequency as the source. To appreciate why this should be true, let's investigate how each circuit element behaves when either the voltage or current is a complex exponential. For the resistor, ***v = Ri.*** When

$$v = Ve^{j2\pi ft}$$

; then

$$i = \frac{V}{R} e^{j2\pi ft}.$$