



Figure 3.33 Node Voltage

The node method begins by finding all nodes places where circuit elements attach to each other in the circuit. We call one of the nodes the **reference node**; the choice of reference node is arbitrary, but it is usually chosen to be a point of symmetry or the "bottom" node. For the remaining nodes, we define node **voltages e_n** that represent the voltage between the **node** and the reference. These node voltages constitute the only unknowns; all we need is a sufficient number of equations to solve for them. In our example, we have two node voltages. **The very act of defining node voltages is equivalent to using all the KVL equations at your disposal.** The reason for this simple, but astounding, fact is that a node voltage is uniquely defined regardless of what path is traced between the node and the reference. Because two paths between a node and reference have the same voltage, the sum of voltages around the loop equals zero.

In some cases, a node voltage corresponds exactly to the voltage across a voltage source. In such cases, the node voltage is specified by the source and is not an unknown. For example, in our circuit, $e_1 = v_{in}$; thus, we need only to find one node voltage.

The equations governing the node voltages are obtained by writing KCL equations at each node having an unknown node voltage, using the v-i relations for each element. In our example, the only circuit equation is

$$\frac{e_2 - v_{in}}{R_1} + \frac{e_2}{R_2} + \frac{e_2}{R_3} = 0$$

A little reflection reveals that when writing the KCL equations for the sum of currents leaving a node, that node's voltage will **always** appear with a plus sign, and all other node voltages with a minus sign. Systematic application of this procedure makes it easy to write node equations and to check them before solving them. Also remember to check units at this point: Every term should have units of current. In our example, solving for the unknown node voltage is easy:

$$e_2 = \frac{R_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3} v_{in}$$

Have we really solved the circuit with the node method? Along the way, we have used KVL, KCL, and the v-i relations. Previously, we indicated that the set of equations resulting from applying these laws is necessary and sufficient. This result guarantees that the node method can be used to "solve" **any** circuit. One fallout of this result is that we must be able to find any circuit variable given the node voltages and sources. All circuit variables can be found using the v-i relations and voltage divider. For example, the current through **R3** equals