evaluate the spectrum at **fewer** frequencies than the signal's duration, the term corresponding to $m = n + K$ will also appear for some values of m, n = {0,...,N - 1}. This situation means that our prototype transform equals $s(n) + s(n + K)$ for some values of n. The only way to eliminate this problem is to require $K \geq N$: We **must** have at least as many frequency samples as the signal's duration. In this way, we can return from the frequency domain we entered via the DFT.

Exercise 5.7.1

When we have fewer frequency samples than the signal's duration, some discrete-time signal values equal the sum of the original signal values. Given the sampling interpretation of the spectrum, characterize this effect a different way.

Another way to understand this requirement is to use the theory of linear equations. If we write out the expression for the DFT as a set of linear equations,

$$s(0) + s(1) + ... + s(N-1) = S(0)$$
$$s(0) + s(1)e^{(-j)\frac{2\pi}{K}} + ... + s(N-1)e^{(-j)\frac{2\pi(N-1)}{K}} = S(1)$$

.....

$$s(0) + s(1)e^{(-j)\frac{2\pi(K-1)}{K}} + ... + s(N-1)e^{(-j)\frac{2\pi(N-1)(K-1)}{K}} = S(K-1)$$

we have K equations in N unknowns if we want to find the signal from its sampled spectrum. This require ment is impossible to fulfll if $K < N$; we must have $K \geq N$. Our orthogonality relation essentially says that if we have a sufcient number of equations (frequency samples), the resulting set of equations can indeed be solved.

By convention, the number of DFT frequency values K is chosen to equal the signal's duration N. The discrete Fourier transform pair consists of

Discrete Fourier Transform Pair

$$S(k) = \sum_{n=0}^{N-1} \left(s(n) e^{-\left(j \frac{2\pi nk}{N}\right)} \right)$$
$$s(n) = \sum_{k=0}^{N-1} \left(S(k) e^{j \frac{2\pi nk}{N}} \right)$$

Example 5.3

Use this demonstration to perform DFT analysis of a signal.

This media object is a LabVIEW VI. Please view or download it at

<DFTanalysis.llb>

Example 5.4

Use this demonstration to synthesize a signal from a DFT sequence.

This media object is a LabVIEW VI. Please view or download it at

<DFT_Component_Manipulation.llb>