energy. The BPSK signal set does perform much better than the FSK signal set once the signal-to-noise ratio exceeds about 5 dB.

Exercise 6.18.1

Derive the expression for the probability of error that would result if the FSK signal set were used.

The matched-filter receiver provides impressive performance once adequate signal-to-noise ratios occur. You might wonder whether another receiver might be better. The answer is that the matched-filter receiver is optimal: **No other receiver can provide a smaller probability of error than the matched filter regardless of the SNR**. Furthermore, no signal set can provide better performance than the BPSK signal set, where the signal representing a bit is the negative of the signal representing the other bit. The reason for this result rests in the dependence of probability of error p_e on the difference between the noise-free integrator outputs: For a given E_b, no other signal set provides a greater difference.

How small should the error probability be? Out of N transmitted bits, on the average Np_e bits will be received in error. Do note the phrase "on the average" here: Errors occur randomly because of the noise introduced by the channel, and we can only predict the probability of occurrence. Since bits are transmitted at a rate R, errors occur at an average frequency of Rp_e. Suppose the error probability is an impressively small number like 10^{-6} . Data on a computer network like Ethernet is transmitted at a rate R = 100Mbps, which means that errors would occur roughly 100 per second. This error rate is very high, requiring a much smaller p_e to achieve a more acceptable average occurrence rate for errors occurring. Because Ethernet is a wireline channel, which means the channel noise is small and the attenuation low, obtaining very small error probabilities is not difficult. We do have some tricks up our sleeves, however, that can essentially reduce the error rate to zero without resorting to expending a large amount of energy at the transmitter. We need to understand digital channels (Digital Channels (Page 292)) and Shannon's Noisy Channel Coding Theorem (Noisy Channel Coding Theorem (Page 313)).

6.19 Digital Channels

Let's review how digital communication systems work within the Fundamental Model of Communication (Figure 2.10: Fundamental model of communication). As shown in Figure 6.17 (DigMC), the message is a single bit. The entire analog transmission/reception system, which is discussed in Digital Communication (Digital Communication (Page 281)), Signal Sets25, BPSK Signal Set26, Transmission Bandwidth, Frequency Shift Keying (Frequency Shift Keying (Page 285)), Digital Communication Receivers (Digital Communication Receivers (Page 287)), Factors in Receiver Error (Digital Communication in the Presence of Noise (Page 289)), Digital Communication System Properties28, and Error Probability29, can be lumped into a single system known as the digital channel.