Analogous to the analog pulse signal, let's find the spectrum of the length-*N* pulse sequence.

$$s(n) = \begin{cases} 1 \; if \; 0 \leq n \leq N-1 \\ 0 \; otherwise \end{cases}$$

(5.24)

The Fourier transform of this sequence has the form of a truncated geometric series.

$$S(e^{j2\pi f}) = \sum_{n=0}^{N-1} \left(e^{-(j2\pi f n)}\right)$$

(5.25)

For the so-called finite geometric series, we know that

$$\sum_{n=n_0}^{N+n_0-1} (\alpha^n) = \alpha^{n_0} \frac{1-\alpha^N}{1-\alpha}$$

(5.26)

for **all** values of α.

Exercise 5.6.2

Derive this formula for the finite geometric series sum. The "trick" is to consider the difference between the series' sum and the sum of the series multiplied by α. Applying this result yields (Figure 5.11 (Spectrum of length-ten pulse).)

$$\begin{aligned} S(e^{j2\pi f}) &= \frac{1-e^{-(j2\pi f N)}}{1-e^{-(j2\pi f)}} \\ &= e^{-(j\pi f(N-1))} \frac{\sin(\pi f N)}{\sin(\pi f)} \end{aligned}$$

The ratio of sine functions has the generic form of

$$\frac{\sin(Nx)}{\sin(x)},$$

which is known as the **discrete-time sinc function dsinc (x)**. Thus, our transform can be concisely expressed as

$$S\left(e^{j2\pi f}\right) = e^{-(j\pi f(N-1))} dsinc(\pi f),$$

The discrete-time pulse's spectrum contains many ripples, the number of which increase with *N*, the pulse's duration.