Consider this mathematical question intuitively: Can a discontinuous function, like the square wave, be expressed as a sum, even an infinite one, of continuous signals? One should at least be suspicious, and in fact, it can't be thus expressed. This issue brought Fourier9 much criticism from the French Academy of Science (Laplace, Lagrange, Monge and LaCroix comprised the review committee) for several years after its presentation on 1807. It was not resolved for almost a century, and its resolution is interesting and important to understand from a practical viewpoint.

The extraneous peaks in the square wave's Fourier series **never** disappear; they are termed **Gibb'sphenomenon** after the American physicist Josiah Willard Gibbs. They occur whenever the signal is dis continuous, and will always be present whenever the signal has jumps.

Let's return to the question of equality; how can the equal sign in the definition of the Fourier series be justified? The partial answer is that **pointwise** each and every value of ***t*** equality is **not** guaranteed. However, mathematicians later in the nineteenth century showed that the rms error of the Fourier series was always zero.

$$\operatorname*{limit}_{K\to\infty} \mathrm{rms}\left(\epsilon_K\right) = 0$$

What this means is that the error between a signal and its Fourier series approximation may not be zero, but that its rms value will be zero! It is through the eyes of the rms value that we redefine equality: The usual Definition of equality is called **pointwise equality**: Two signals s1 (t), s2 (t) are said to be equal pointwise if s_1 (t)= s_2 (t) for all values of t. A new definition of equality is **mean-square equality**: Two signals are said to be equal in the mean square if **rms** ($\boldsymbol{s}_1 - \boldsymbol{s}_2$)=**0**. For Fourier series, Gibb's phenomenon peaks have finite height and zero width. The error differs from zero only at isolated points whenever the periodic signal contains discontinuities and equals about 9% of the size of the discontinuity. The value of a function at a finite set of points does not affect its integral. This effect underlies the reason why defining the value of a discontinuous function, like we refrained from doing in defining the step function (Section 2.2.4: Unit Step), at its discontinuity is meaningless. Whatever you pick for a value has no practical relevance for either the signal's spectrum or for how a system responds to the signal. The Fourier series value "at" the discontinuity is the average of the values on either side of the jump.

4.6 Encoding Information in the Frequency Domain

To emphasize the fact that every periodic signal has both a time and frequency domain representation, we can exploit both to encode information into a signal. Refer to the Fundamental Model of Communication Figure 1.4. We have an information source, and want to construct a transmitter that produces a signal x (t). For the source, let's assume we have information to encode every ***T*** seconds. For example, we want to represent typed letters produced by an extremely good typist (a key is struck every ***T*** seconds). Let's consider the complex Fourier series formula in the light of trying to encode information.