the circuit's transfer function as the input's spectrum! This approach to finding inverse transforms breaking down a complicated expression into products and sums of simple components is the engineer's way of breaking down the problem into several subproblems that are much easier to solve and then gluing the results together. Along the way we may make the system serve as the input, but in the rule $Y(f) = X(f)\, H(f)$, which term is the input and which is the transfer function is merely a notational matter (we labeled one factor with an X and the other with an H).

4.9.1 Transfer Functions

The notion of a transfer function applies well beyond linear circuits. Although we don't have **all** we need to demonstrate the result as yet, all linear, time-invariant systems have a frequency-domain input-output relation given by the product of the input's Fourier transform and the system's transfer function. Thus, linear circuits are a special case of linear, time-invariant systems. As we tackle more sophisticated problems in transmitting, manipulating, and receiving information, we will assume linear systems having certain properties (transfer functions) **without** worrying about what circuit has the desired property. At this point, you may be concerned that this approach is glib, and rightly so. Later we'll show that by involving software that we really don't need to be concerned about constructing a transfer function from circuit elements and op-amps.

4.9.2 Commutative Transfer Functions

Another interesting notion arises from the commutative property of multiplication (exploited in an example above (Example 4.6)): We can rather arbitrarily choose an order in which to apply each product. Consider a cascade of two linear, time-invariant systems. Because the Fourier transform of the first system's output is $X(f)\, H1(f)$ and it serves as the second system's input, the cascade's output spectrum is $X(f)\, H1(f)\, H2(f)$. Because this product also equals $X(f)\, H2(f)\, H1(f)$, the **cascade having the linear systems in the opposite order yields the same result**. Furthermore, the cascade acts like a single linear system, having transfer function $H_1(f)\, H_2(f)$. This result applies to other configurations of linear, time-invariant systems as well; see this Frequency Domain Problem (Problem 4.13). Engineers exploit this property by determining what transfer function they want, then breaking it down into components arranged according to standard confgurations. Using the fact that op-amp circuits can be connected in cascade with the transfer function equaling the product of its component's transfer function (see this analog signal processing problem (Problem 3.44)), we find a ready way of realizing designs. We now understand why op-amp implementations of transfer functions are so important.