Of practical importance is the conjugate symmetry property: When s (t) is real-valued, the spectrum at negative frequencies equals the complex conjugate of the spectrum at the corresponding positive frequencies. Consequently, we need only plot the positive frequency portion of the spectrum (we can easily determine the remainder of the spectrum).

Exercise 4.8.2

How many Fourier transform operations need to be applied to get the original signal back:

$$F(...(F(s))) = s(t)?$$

Note that the mathematical relationships between the time domain and frequency domain versions of the same signal are termed **transforms**. We are transforming (in the nontechnical meaning of the word) a signal from one representation to another. We express Fourier transform **pairs** as ($s\ (t) \leftrightarrow S\ (f)$). A signal's time and frequency domain representations are uniquely related to each other. A signal thus "exists" in both the time and frequency domains, with the Fourier transform bridging between the two. We can define an information carrying signal in either the time or frequency domains; it behooves the wise engineer to use the simpler of the two.

A common misunderstanding is that while a signal exists in both the time and frequency domains, a single formula expressing a signal must contain **only** time or frequency: Both cannot be present simultaneously. This situation mirrors what happens with complex amplitudes in circuits: As we reveal how communications systems work and are designed, we will define signals entirely in the frequency domain without explicitly finding their time domain variants. This idea is shown in another module (Section 4.6) where we define Fourier series coefficients according to letter to be transmitted. Thus, a signal, though most familiarly defined in the time-domain, really can be defined equally as well (and sometimes more easily) in the frequency domain. For example, impedances depend on frequency and the time variable cannot appear.

We will learn (Section 4.9) that finding a linear, time-invariant system's output in the time domain can be most easily calculated by determining the input signal's spectrum, performing a simple calculation in the frequency domain, and inverse transforming the result. Furthermore, understanding communications and information processing systems requires a thorough understanding of signal structure and of how systems work in **both** the time and frequency domains.

The only difficulty in calculating the Fourier transform of any signal occurs when we have periodic signals (in either domain). Realizing that the Fourier series is a special case of the Fourier transform, we simply calculate the Fourier series coefficients instead, and plot them along with the spectra of nonperiodic signals on the same frequency axis.