The frequency response of the simple IIR system (difference equation given in a previous example (Discrete-Time Systems in the Time-Domain (Page 228))) is given by

$$H(e^{j2\pi f}) = \frac{b}{1 - ac^{-(j2\pi f)}}$$

(5.50)

This Fourier transform occurred in a previous example; the exponential signal spectrum (Figure 5.10: Spectra of exponential signals) portrays the magnitude and phase of this transfer function. When the filter coefficient *a* is positive, we have a lowpass filter; negative *a* results in a highpass filter. The larger the coefficient in magnitude, the more pronounced the lowpass or highpass filtering.

Example 5.8

The length-*q* boxcar filter (difference equation found in a previous example (Discrete-Time Systems in the Time-Domain (Page 228))) has the frequency response

$$H(e^{j2\pi f}) = \frac{1}{q} \sum_{m=0}^{q-1} \left(e^{-(j2\pi f m)}\right)$$

(5.51)

This expression amounts to the Fourier transform of the boxcar signal (5.13). There we found that this frequency response has a magnitude equal to the absolute value of **dsinc(π*f*)**; see the length-10 filter's frequency response (Figure 5.11: Spectrum of length-ten pulse). We see that boxcar filters length-***q*** signal averages have a lowpass behavior, having a cutoff frequency of

$$\frac{1}{q}.$$

.

Exercise 5.13.1

Suppose we multiply the boxcar filter's coefficients by a sinusoid:

$$b_m = \frac{1}{q} \cos(2\pi f_0 m)$$

Use Fourier transform properties to determine the transfer function. How would you characterize this system: Does it act like a filter? If so, what kind of filter and how do you control its characteristics with the filter's coefficients?

These examples illustrate the point that systems described (and implemented) by difference equations serve as filters for discrete-time signals. The filter's **order** is given by the number *p* of denominator coefficients in the transfer function (if the system is IIR) or by the number q of numerator coefficients if the filter is FIR. When a system's transfer function has both terms, the system is usually IIR, and its order equals *p* regardless of *q*. By selecting the coefficients and filter type, filters having virtually any frequency response desired can be designed. This design flexibility can't be found in analog systems. In the next section, we detail how analog signals can be filtered by computers, offering a much greater range of filtering possibilities than is possible with circuits.