Example 4.2

Let's find the Fourier series representation for the half-wave rectified sinusoid.

$$s(t) = \begin{cases} \sin\left(\frac{2\pi t}{T}\right) & \text{if } 0 \leq t < \frac{T}{2} \\ 0 \text{ if } \frac{T}{2} \leq t < T \end{cases}$$

Begin with the sine terms in the series; to find **bk** we must calculate the integral

$$b_k = \frac{2}{T} \int_0^{\frac{T}{2}} sin\left(\frac{2\pi t}{T}\right) sin\left(\frac{2\pi k t}{T}\right) dt$$

Using our trigonometric identities turns our integral of a product of sinusoids into a sum of integrals of individual sinusoids, which are much easier to evaluate.

$$\begin{aligned} \int_0^{\frac{T}{2}} \sin\left(\frac{2\pi t}{T}\right) \sin\left(\frac{2\pi k t}{T}\right) dt &= \frac{1}{2} \int_0^{\frac{T}{2}} \cos\left(\frac{2\pi(k-1)t}{T}\right) - \cos\left(\frac{2\pi(k+1)t}{T}\right) dt \\ &= \begin{cases} \frac{1}{2} & \text{if } k = 1 \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

Thus,

$$b_1 = \frac{1}{2}$$
$$b_2 = b_3 = \ldots = 0$$

On to the cosine terms. The average value, which corresponds to a0, equals

$$\frac{1}{\pi}$$

. The remainder of the cosine coefficients are easy to fnd, but yield the complicated result

$$a_k = \begin{cases} -\left(\frac{2}{\pi} \frac{1}{k^2 - 1}\right) & \text{if } k \in \{2, 4, \ldots\} \\ 0 \text{ if } \text{ k odd} \end{cases}$$

Thus, the Fourier series for the half-wave rectified sinusoid has non-zero terms for the average, the fundamental, and the even harmonics.