Note that if carries are ignored, subtraction of two single-digit binary numbers yields the same bit as addition. Computers use high and low voltage values to express a bit, and an array of such voltages express numbers akin to positional notation. Logic circuits perform arithmetic operations.

Add twenty-five and seven in base 2. Note the carries that might occur. Why is the result "nice"? The variables of logic indicate truth or falsehood. **A** ∩ **B**, the AND of **A** and **B**, represents a statement that both **A** and **B** must be true for the statement to be true. You use this kind of statement to tell search engines that you want to restrict hits

to cases where both of the events **A** and **B** occur. **A** ∪ **B**, the OR of A

and B, yields a value of truth if either is true. Note that if we represent truth by a "1" and falsehood by a "0," **binary multiplication corresponds to AND and addition (ignoring carries)**

**to XOR**. XOR, the exclusive or operator, equals the union of **A** ∪ **B** and

**A** ∩ **B**. The Irish mathematician George Boole discovered this equivalence in the mid-nineteenth century. It laid the foundation for what we now call Boolean algebra, which expresses as equations logical statements. More importantly, any computer using base-2 representations and arithmetic can also easily evaluate logical statements. This fact makes an integer-based computational device much more powerful than might be apparent.

**Exercise 5.2.3**

### The Sampling Theorem

#### Analog-to-Digital Conversion

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Because of the way computers are organized, signal must be represented by a fnite number of bytes. This restriction means that **both** the time axis and the amplitude axis must be **quantized**: They must each be a multiple of the integers. l2 Quite surprisingly, the Sampling Theorem allows us to quantize the time axis without error for some signals. The signals that can be sampled without introducing error are interesting, and as described in the next section, we can make a signal "samplable" by fltering. In contrast, no one has found a way of performing the amplitude quantization step without introducing an unrecoverable error. Thus, a signal's value can no longer be any real number. Signals processed by digital computers must be **discrete-valued**: their values must be proportional to the integers. Consequently, **analog-to-digital conversion introduces error**.