* The signal s (t) is **bandlimited** has power in a restricted frequency range to **W** Hz, and
* the sampling interval **Ts** is small enough so that the individual components in the sum do not overlap **Ts** < 1/2**W** ,

aliasing will not occur. In this delightful case, we can recover the original signal by lowpass fltering x (t) with a flter having a cutof frequency equal to **W** Hz. These two conditions ensure the ability to recover a bandlimited signal from its sampled version: We thus have the **Sampling Theorem**.

The Sampling Theorem (as stated) does not mention the pulse width Δ. What is the effect of this parameter on our ability to recover a signal from its samples (assuming the Sampling Theorem's two conditions are met)?

**Exercise 5.3.1**

The frequency



, known today as the **Nyquist frequency** and the **Shannon sampling frequency**, corresponds to the highest frequency at which a signal can contain energy and remain compatible with the Sampling Theorem. High-quality sampling systems ensure that no aliasing occurs by unceremoniously lowpass fltering the signal (cutof frequency being slightly lower than the Nyquist frequency) before sampling. Such systems therefore vary the **anti-aliasing** flter’s cutof frequency as the sampling rate varies. Because such quality features cost money, many sound cards do **not** have anti-aliasing flters or, for that matter, post-sampling flters. They sample at high frequencies, 44.1 kHz for example, and hope the signal contains no frequencies above the Nyquist frequency (22.05 kHz in our example). If, however, the signal contains frequencies beyond the sound card's Nyquist frequency, the resulting aliasing can be impossible to remove.



Let the sampling interval ***Ts*** be 1; consider two values for the square wave's period: 3.5 and 4. Note in particular where the spectral lines go as the period decreases; some will move to the left and some to the right. What property characterizes the ones going the same

direction?

To gain a better appreciation of aliasing, sketch the spectrum of a sampled square wave. For simplicity consider only the spectral

repetitions centered at

**Exercise 5.3.2**

If we satisfy the Sampling Theorem's conditions, the signal will change only slightly during each pulse. As we narrow the pulse, making Δ smaller and smaller, the nonzero values of the signal ***s(t) pTs (t)*** will simply be ***s*** (***nTs***), the signal's **samples**. If indeed the Nyquist frequency equals the signal's highest frequency, at least two samples will