In this (7,4) code, 24 = 16 of the 27 = 128 possible blocks at the channel decoder correspond to error-free transmission and reception.

Error correction amounts to searching for the codeword ***c*** closest to the received block c in terms of the Hamming distance between the two. The error correction capability of a channel code is limited by how close together any two error-free blocks are. Bad codes would produce blocks close together, which would result in ambiguity when assigning a block of data bits to a received block. The quantity to examine, therefore, in designing code error correction codes is the minimum distance between codewords.



(6.56)

To have a channel code that can correct all single-bit errors, *d*min ≥ 3.

###### Exercise 6.27.2

Suppose we want a channel code to have an error-correction capability of n bits. What must the minimum Hamming distance between codewords ***d*min** be?

How do we calculate the minimum distance between codewords? Because we have *2K* codewords, the number of possible unique pairs equals *2K−1(2K–1)* , which can be a large number. Recall that our channel coding procedure is linear, with *c = Gb*.

Therefore *ci* ⊕ *cj* = *G* (*bi* ⊕ *bj*). Because *b*i ⊕ *b*j always yields another block of data bits,

we fnd that the diference between any two codewords is another codeword! Thus, to fnd *d*min we need only compute the number of ones that comprise all non-zero codewords. Finding these codewords is easy once we examine the coder's generator matrix. Note that the columns of *G* are codewords (why is this?), and that all codewords can be found by all possible pairwise sums of the columns. To fnd *d*min , we need only count the number of bits in each column and sums of columns. For our example (7, 4), *G*'s frst column has three ones, the next one four, and the last two three. Considering sums of column pairs next, note that because the upper portion of *G* is an identity matrix, the corresponding upper portion of all column sums must have exactly two bits. Because the bottom portion of each column difers from the other columns in at least one place, the bottom portion of a sum of columns must have at least one bit. Triple sums will have at least three bits because the upper portion of *G* is an identity matrix. Thus, no sum of columns has fewer than three bits, which means that *d*min =3, and we have a channel coder that can correct all occurrences of one error within a received 7-bit block.

### Error-Correcting Codes: Channel Decoding

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Because the idea of channel coding has merit (so long as the code is efcient), let's develop a systematic procedure for performing channel decoding. One way of checking for errors is to try recreating the error correction bits from the data portion