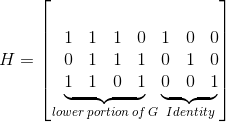
of the received block c. Using matrix notation, we make this calculation by multiplying the received block c by the matrix ***H*** known as the ***parity check matrix***. It is formed from the generator matrix ***G*** by taking the bottom, error-correction portion of ***G*** and attaching to it an identity matrix. For our (7,4) code,



(6.57)

The parity check matrix thus has size (*N - K*) × *N*, and the result of multiplying this matrix with a received word is a length- (*N - K*) binary vector. If no digital channel errors occur we receive a codeword so that



For example, the frst column of *G*, (1, 0, 0, 0, 1, 0, 1)T , is a codeword. Simple calculations show that multiplying this vector by *H* results in a length-(*N - K*) zero- valued vector.

###### Exercise 6.28.1

Show that *Hc* =0 for all the columns of *G*. In other words, show that *HG* =0 an (*N - K*) ×

*K* matrix of zeroes. Does this property guarantee that all codewords also satisfy *Hc*=0?

When the received bits do not form a codeword,

does not equal zero, indicating the presence of one or more errors induced by the digital channel. Because the presence of an error can be mathematically written as



with *e* a vector of binary values having a 1 in those positions where a bit error occurred.

###### Exercise 6.28.2

Show that adding the error vector (1, 0,..., 0)T to a codeword fps the codeword's leading bit and leaves the rest unafected.

Consequently,



Because the result of the product is a length-(*N − K*) vector of binary values, we can have 2*N − K* − 1 non-zero values that correspond to non-zero error patterns *e*. To perform our channel decoding,

* + 1. compute (conceptually at least)



* + 1. if this result is zero, no detectable or correctable error occurred;
    2. if non-zero, consult a table of length-(*N − K*) binary vectors to associate them with the **minimal** error pattern that could have resulted in the non-zero result; then