|  |  |  |
| --- | --- | --- |
| ***N*** | ***K*** | ***E (efciency)*** |
| 3 | 1 | 0.33 |
| 7 | 4 | 0.57 |
| 15 | 11 | 0.73 |
| 31 | 26 | 0.84 |
| 63 | 57 | 0.90 |
| 127 | 120 | 0.94 |

**Table 6.3 Hamming Codes**

Unfortunately, for such large blocks, the probability of multiple-bit errors can exceed the number of single-bit errors unless the channel single-bit error probability *Pe* is very small. Consequently, we need to enhance the code's error correcting capability by adding double as well as single-bit error correction.

Exercise 6.29.1

What must the relation between *N* and *K* be for a code to correct all single-and double- bit errors with a "perfect ft"?

### Noisy Channel Coding Theorem

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As the block length becomes larger, more error correction will be needed. Do codes exist that can correct **all** errors? Perhaps the crowning achievement of Claude Shannon's creation of information theory answers this question. His result comes in two complementary forms: the Noisy Channel Coding Theorem and its converse.

#### Noisy Channel Coding Theorem

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Let *E* denote the efciency of an error-correcting code: the ratio of the number of data bits to the total number of bits used to represent them. If the efciency is less than the **capacity** of the digital channel, an error-correcting code exists that has the property that as the length of the code increases, the probability of an error occurring in the decoded block approaches zero.