#### Derivative Systems and Integrators

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Systems that perform calculus-like operations on their inputs can produce waveforms signifcantly diferent than present in the input. Derivative systems operate in a straightforward way: A frst-derivative system would have the input-output relationship



. Integral systems have the complication that the integral's limits must be defned. It is a signal theory convention that the elementary integral operation have a lower limit of

−∞, and that the value of all signals at t = −∞ equals zero. A simple integrator would

have input-output relation



#### Linear Systems

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Linear systems are a **class** of systems rather than having a specifc input-output relation. Linear systems form the foundation of system theory, and are the most important class of systems in communications. They have the property that when the input is expressed as a weighted sum of component signals, the output equals the same weighted sum of the outputs produced by each component. When S (·) is linear,



for all choices of signals and gains. This general input-output relation property can be manipulated to indicate specifc properties shared by all linear systems.

* + - * ***S (Gx (t)) = GS (x (t))*** The colloquialism summarizing this property is "Double the input, you double the output." Note that this property is consistent with alternate ways of expressing gain changes: Since *2x (t)* also equals *x (t)+ x (t)*, the linear system Defnition provides the same output no matter which of these is used to express a given signal.
      * ***S (0) =0*** If the input is **identically zero for all time**, the output of a linear system must be zero. This property follows from the simple derivation ***S (0) = S (x (t) − x (t)) = S (x (t)) − S (x (t)) = 0***.

Just why linear systems are so important is related not only to their properties, which are divulged throughout this course, but also because they lend themselves to relatively simple mathematical analysis. Said another way, "They're the only systems we thoroughly understand!"

We can fnd the output of any linear system to a complicated input by decomposing the input into simple signals. The equation above (2.34) says that when a system is