## Problem 3.27: Linear, Time-Invariant Systems

For a system to be completely characterized by a transfer function, it needs not only be linear, but also to be time-invariant. A system is said to be time-invariant if delaying the input delays the output by the same amount. Mathematically, if *S* (x (t)) = *y* (*t*), meaning *y* (*t*) is the output of a system***S*** (**•**) when *x* (t) is the input, ***S***(**•**) is the time- invariant if *S* (*x* (*t* − τ)) = *y* (*t* − τ) for all delays τ and all inputs *x* (*t*).

Note that both linear and nonlinear systems have this property. For example, a system that squares its input is time-invariant.

* 1. Show that if a circuit has fixed circuit elements (their values don't change over time), its input-output relationship is time- invariant. Hint: Consider the differential equation that describes a circuit's input-output relationship. What is its general form? Examine the derivative(s) of delayed signals.
  2. Show that impedances cannot characterize time-varying circuit elements (R, L, and C). Consequently, show that linear, time- varying systems do not have a transfer function.
  3. Determine the linearity and time-invariance of the following. Find the transfer function of the linear, time-invariant (LTI) one(s).
     1. diode
     2. y (t)= x (t) sin (2πf0t) 3. y (t)= x (t − τ0)

4. y (t)= x (t)+ N (t)