**The circuit's output to a sinusoidal input is also a sinusoid, having a gain equal to the magnitude of the circuit's transfer function evaluated at the source frequency and a phase equal to the phase of the transfer function at the source frequency.** It will turn out that this input-output relation description applies to any linear circuit having a sinusoidal source.

This input-output property is a special case of a more general result. Show that if the source can be written as the imaginary part of a complex exponential ***vin*** (***t***) = ***Im (Vej2πft)*** the output is given by***vout*** (***t***)

= ***Im(VH (f) e j2πft***). Show that a similar result also holds for the real part.

**Exercise 3.13.1**

The notion of impedance arises when we assume the sources are complex exponentials. This assumption may seem restrictive; what would we do if the source were a unit step? When we use impedances to fnd the transfer function between the source and the output variable, we can derive from it the diferential equation that relates input and output. The diferential equation applies no matter what the source may be. As we have argued, it is far simpler to use impedances to fnd the diferential equation (because we can use series and parallel combination rules) than any other method. In this sense, we have not lost anything by temporarily pretending the source is a complex exponential.

In fact we can also solve the diferential equation using impedances! Thus, despite the apparent restrictiveness of impedances, assuming complex exponential sources is actually quite general.

### Designing Transfer Functions

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If the source consists of two (or more) signals, we know from linear system theory that the output voltage equals the sum of the outputs produced by each signal alone. In short, linear circuits are a special case of linear systems, and therefore superposition applies. In particular, suppose these component signals are complex exponentials, each of which has a frequency diferent from the others. The transfer function portrays how the circuit afects the amplitude and phase of each component, allowing us to understand how the circuit works on a complicated signal. Those components having a frequency less than the cutof frequency pass through the circuit with little modifcation while those having higher frequencies are suppressed. The circuit is said to act as a **flter**, fltering the source signal based on the frequency of each component complex exponential. Because low frequencies pass through the flter, we call it a **lowpass flter** to express more precisely its function.

We have also found the ease of calculating the output for sinusoidal inputs through the use of the transfer function. Once we fnd the transfer function, we can write the output directly as indicated by the output of a circuit for a sinusoidal input (3.18).