Sammy is faced with computing **lots** of discrete Fourier transforms. He will, of course, use the FFT algorithm, but he is behind schedule and needs to get his results as quickly as possible. He gets the idea of computing **two** transforms at one time by computing the transform of *s (n)= s*1 *(n)+ js*2 *(n)* **,** where *s*1 (*n*) and *s*2 (*n*) are two real- valued signals of which he needs to compute the spectra. The issue is whether he can retrieve the individual DFTs from the result or not.

* 1. What will be the DFT *S (k)* of this complex-valued signal in terms of *S1 (k)* and

*S2 (k)*, the DFTs of the original signals?

* 1. Sammy's friend, an Aggie who knows some signal processing, says that retrieving the wanted DFTs is easy: "Just fnd the real and imaginary parts of *S (k)*." Show that this approach is too simplistic.
  2. While his friend's idea is not correct, it does give him an idea. What approach will work? **Hint**: Use the symmetry properties of the DFT.
  3. How does the number of computations change with this approach? Will Sammy's idea ultimately lead to a faster computation of the required DFTs?

**Problem 5.18:** Discrete Cosine Transform (DCT)

The discrete cosine transform of a length-***N*** sequence is defned to be



Note that the number of frequency terms is 2*N* − 1: *k* = {0,..., 2*N* − 1}.

1. Find the inverse DCT.
2. Does a Parseval's Theorem hold for the DCT?
3. You choose to transmit information about the signal s (n) according to the DCT coefcients. You could only send one, which one would you send?

**Problem 5.19**: A Digital Filter

A digital flter is described by the following diference equation:



1. What is this flter's unit sample response?
2. What is this flter's transfer function?
3. What is this flter's output when the input is



**Problem 5.20**: Another Digital Filter

A digital flter is determined by the following diference equation.

*y (n)= y (n −* 1*) + x (n) − x (n −* 4*)*

1. Find this flter's unit sample response.
2. What is the flter's transfer function? How would you characterize this flter (lowpass, highpass, special purpose, ...)?
3. Find the flter's output when the input is the sinusoid