In this example, the transmitter encodes 0 as 000. The channel creates an error (changing a 0 into a 1) that with probability *p*e. The first column lists all possible received datawords and the second the probability of each dataword being received. The last column shows the results of the majority-vote decoder. When the decoder produces 0, it successfully corrected the errors introduced by the channel (if there were any; the top row corresponds to the case in which no errors

occurred). The error probability of the decoders is the sum of the probabilities when the

decoder produces 1.

Thus, if one bit of the three bits is received in error, the receiver can correct the error; if more than one error occurs, the channel decoder announces the bit is 1 instead of transmitted value of 0. Using this repetition code, the probability of



This probability of a decoding error is always less than pe, the uncoded value, so long as

###### Exercise 6.25.1

Demonstrate mathematically that this claim is indeed true. Is



### Block Channel Coding

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Because of the higher datarate imposed by the channel coder, the probability of bit error occurring in the digital channel **increases** relative to the value obtained when no channel coding is used. The bit interval duration must be reduced by



in comparison to the no-channel-coding situation, which means the energy per bit *E*b goes **down** by the same amount. The bit interval must decrease by a factor of three if the transmitter is to keep up with the data stream, as illustrated here ([Figure 6.21](#_bookmark458): Repetition Code).

POINT OF INTEREST: It is unlikely that the transmitter's power could be increased to compensate.

Such is the sometimes-unfriendly nature of the real world.

Because of this reduction, the error probability *pe* of the digital channel goes up. The question thus becomes does channel coding **really** help: Is the efective error probability lower with channel coding even though the error probability for each transmitted bit is larger? The answer is **no**: Using a repetition code for channel coding cannot ultimately reduce the probability that a data bit is received in error. The ultimate reason is the repetition code's inefciency: transmitting one data bit for every three transmitted is too inefcient for the amount of error correction provided.

###### Exercise 6.26.1